BITS Pilani K K Birla Goa Campus

Comprehensive Examination: PART- A (Closed Book)

Course Title: Mathematics III | **Max. Marks: 45**

Course No : MATH F211 | **Time: 1 hour**

Date: 06/12/2023 | Average: 19.07/45 | **Day: Wednesday**

Name: Answer Key	**ID Number:**

Instructions:

(1) Circle ◯ your choice of the correct answer.

(2) Each question carries three marks, with a deduction of one mark for each incorrect answer.

(3) Incorrect, inappropriate marking, and overwriting will be treated as wrong answers.

(4) Rough work should be done on the back side of the main answer sheet.

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1. $P_{12}'(1)$ is equal to

   (A) 58 (B) 68 (C) 78 (D) None of these

2. The inverse Laplace transform of $\dfrac{1}{p^4-4}$, $p>\sqrt{2}$, is equal to

   (A) $\frac{1}{4\sqrt{2}}\left(\sinh\sqrt{2}x-\sin\sqrt{2}x\right)$ (B) $\frac{1}{4\sqrt{2}}\left(\sinh\sqrt{2}x+\sin\sqrt{2}x\right)$

   (C) $\frac{1}{2\sqrt{2}}\left(\sinh\sqrt{2}x-\sin\sqrt{2}x\right)$ (D) None of these

3. The coefficient of $\cos x$, in the Fourier series of $f(x)=x^2$, $\quad -\pi<x\leq\pi$ is

   (A) Zero (B) -2 (C) -4 (D) None of these

4. The Laplace transform of $f(x)=\displaystyle\int_0^x (x^2e^{5x}+1)dx$ is equal to

   (A) $\dfrac{1}{p^2}+\dfrac{2}{p(p-5)^2}$ (B) $\dfrac{1}{p^2}+\dfrac{2}{p^2(p-5)^2}$ (C) $\dfrac{1}{p^2}+\dfrac{2}{p(p-5)^3}$ (D) None of these

5. Consider the differential equations

   (i) $x^2y''+2yy'+\cos(x)y=0$, $\quad(ii)$ $\sin(x)y''+2xy'+\cos(x)y=0$. Which of the following equations are linear:

   (A) Only (i) (B) Only (ii) (C) Both (i) and (ii) (D) None of these

6. If $J_{-3/2}(x)=\sqrt{\dfrac{2}{\pi x}}\left(a\dfrac{\cos x}{x}+b\sin x\right)$, then

   (A) $a=-1,b=1$ (B) $a=-1,b=-1$ (C) $a=1,b=-1$ (D) None of these

7. Let Laplace transform of $f(x)=xe^x$ be $G(p)$, then the Laplace transform of $f'''(x)$ is

   (A) $p^3G(p)$ (B) $p^3G(p)-p-1$ (C) $p^3G(p)-p-2$ (D) None of these

8. Which of the following statements is correct?

   (i) The IVP $\dfrac{dy}{dx} = 5y^{4/5}, y(3) = 0$ has unique solution in the rectangle $|x| \leq 5$, $|y| \leq 4$.

   (ii) Let $P(x)$ and $Q(x)$ be continuous functions on a closed interval $[a, b]$. Then the equation $y'' + P(x)y' + Q(x)y = 0$ is uniquely determined by a pair of linearly independent solutions.

   (A) Only (i) (B) Only (ii) (C) Both (i) and (ii) (D) None of these

9. $\displaystyle\int_0^\infty e^{-3x} J_0(4x)dx =$

   (A) Zero (B) $\dfrac{2}{5}$ (C) $\dfrac{1}{5}$ (D) None of these

10. The solution of the integral equation $e^{-x} = y(x) + 2\displaystyle\int_0^x \cos(x-t)y(t)dt$ is

    (A) $e^x(x-1)^2$ (B) $e^{2x}(x-1)^2$ (C) $e^{-x}(x-1)^2$ (D) None of these

11. Let $f(x)$ is a polynomial of degree 10 such that $f(x)$ is orthogonal to every polynomial of degree less than 10 then $f(x)$ is equal to (here, C is some constant)

    (A) $CP_{10}(x)$ (B) $CP_{12}(x)$ (C) Zero (D) None of these

12. An integrating factor of the equation $(x - 2x^2y^3)dy + ydx = 0$ is

    (A) xy (B) $\dfrac{1}{xy}$ (C) $\dfrac{1}{x^2y^2}$ (D) None of these

13. The value of $h(3, e^{1/3})$, where $h(x, y)$ is a function in an integral curve $\dfrac{x^3}{3y^3} + h(x, y) = 1$ of the equation $x^2ydx - (x^3 - y^3)dy = 0$ passing through $(0, e)$, is

    (A) $\dfrac{e-9}{e}$ (B) 3 (C) $\dfrac{1}{3}$ (D) None of these

14. $(5/2)!$ is equal to

    (A) $\dfrac{15}{8}\sqrt{\pi}$ (B) $\dfrac{15}{4}\sqrt{\pi}$ (C) $\dfrac{5}{4}\sqrt{\pi}$ (D) None of these

15. The Fourier series of $\pi + \sin x$, for $-\pi < x < \pi$, is

    (A) $\pi + \displaystyle\sum_{n=0}^{\infty}(\sin nx + \cos nx)$ (B) $\pi + \sin x + \displaystyle\sum_{n=0}^{\infty}\cos nx$

    (C) $\pi + \displaystyle\sum_{n=0}^{\infty}\sin nx$ (D) None of these

The End