CS F222: Discrete Structures in Computer Science

COMPREHENSIVE EXAMINATION (40 points, 180 minutes)
December 08, 10 AM-1 PM

This is a CLOSED BOOK examination. Laptop, mobile phone and any other form of electronic gadget is NOT allowed. Any violation will be interpreted as unfair means and disciplinary action will be taken.

ANSWER ALL Questions. Detailed explanations are required for TRUE/FALSE type of answers in order to RECEIVE FULL CREDIT! Q.1-Q.10: 2 Marks each; Q.11-12: 10 Marks each

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Indicate if the following statements (**Q.1-10**) are TRUE or FALSE. Provide supporting arguments and proofs in the answer sheet.

1. A simple graph of order 4 and size 5 can't be a bi-partite graph. There is an important result on size and order (for simple, bi-partite graphs) that must be proved first.

2. For a graph with n vertices and m edges, if ϵ and δ are the minimum and maximum of degrees, then $\epsilon \leq \frac{2m}{n} < \delta$.

3. Two simple graphs are isomorphic iff their complements are isomorphic.

4. Let G be a disconnected graph of even order n with two components each of which is complete. G has a minimum of $\frac{n(n-1)}{4}$ edges.

5. If a tree has exactly two pendant vertices, the degree of every non-pendant vertex is 2.

6. Let T be a complete binary tree of order n, p leaves and q internal vertices. Then the following is not true.
   $n = 2p - 1$

7. If G_1 and G_2 are two connected subgraphs of G having at least one vertex in common, then $G_1 U G_2$ is connected.

8. Klein-4 group is cyclic.

9. $G = (Z_6, +), H = (Z_3, +), K = (Z_2, +)$. G and HXK are not isomorphic, X denotes cartesian product.

10. A closed cap on a sphere is a hemisphere including the circle on its boundary. For any 5 points on a sphere, there is some closed cap containing at least 4 of them.

11. Let e be an edge of K_n. Calculate the number of spanning trees of $K_n - e$. Explain how you got your answer.

12. Prove that at a cocktail party with ten or more people, there are either three mutual acquaintances or four mutual strangers.