Lecture 37 (Nov 22, 2024)

Subgroup :-

$G \leftarrow$ Group.

A subset $H \subseteq G$ is a subgroup if it satisfies Group properties.

i) Closure → Need to check $h_1 h_2 \in H$

ii) Identity → ✓

iii) Inverse → Need to check. $\forall h \to h^{-1}$

iv) Associativity → ✓

G

$H \subseteq G$

Subgroup :-

i) **Closure** - $\forall h_1, h_2 \in H$ $\quad h_1 h_2 \in H$

ii) **Inverse** $\forall h \in H \longrightarrow h^{-1} \in H$

G ⇐ finite group → Closure

$\exists\, r \geq 1$; $\boxed{a^r = e}$

$$a^{-1} a^r = a^{-1} \longrightarrow a^{r-1} = a^{-1}$$

Eg:- $G \Leftarrow$ group.

$Z \Longrightarrow$ set of all the elements of G which commute with all the elements of G.

$$Z = \{ z \in G \mid zu = uz ,\ \forall u \in G \}$$

subgroup of G. $\Longrightarrow$ center of G

i) Closure :- $z_1, z_2 \in Z \rightarrow$ Prove $z_1 z_2 \in Z$.

$$z_1 u = u z_1 , \quad z_2 u = u z_2$$

$$z_1 z_2 u = u z_1 z_2$$

$$z_1 u = u z_1$$

$$\Rightarrow z_2 z_1 u = \boxed{z_2 u} z_1$$

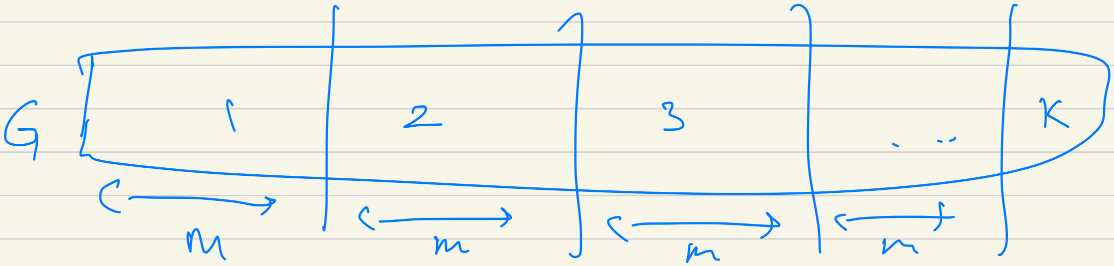
$$\boxed{z_2 z_1 u = u z_2 z_1}$$

Lagrange's Theorem

G ← finite group and H be a subgroup of G.

↓

$\overset{m}{\text{order}(H)}$ divides $\overset{n}{\text{order}(G)}$



$$\underline{n = k \cdot m}$$

$H = \{h_1, h_2, \ldots, h_m\}$; H is a subgroup of G.

$$\downarrow$$

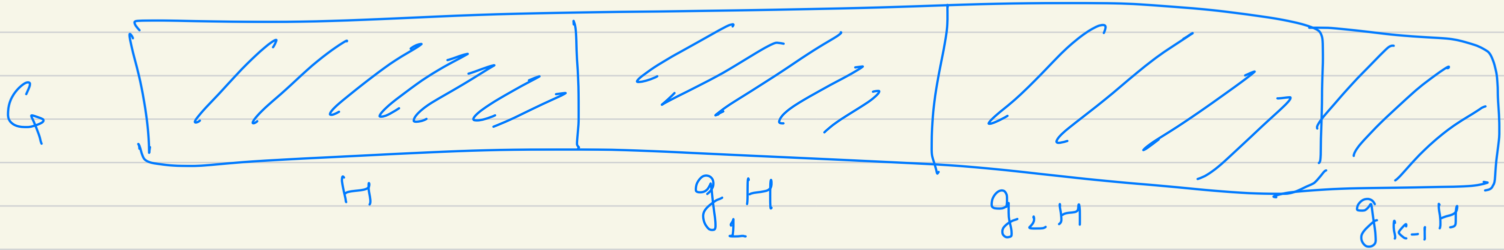
$\{h_1 h_2, h_2 h_3, \ldots, h_m h_1\} \subseteq H$

$\nexists H' \neq H$, $H' \longrightarrow$ subgroup of G.

$\forall g \in G$

$gH = \{gh_1, gh_2, \ldots, gh_m\}$

left coset



☆ $g_1, g_2 \in G$ s.t. $g_1 \neq g_2$

either $g_1H = g_2H$ or $g_1H \cap g_2H = \phi$

if $|g_1H \cap g_2H| \geq 1$ → $g_1H = g_2H$

$x = g_1h$; $x = g_2h'$

$g_1 H \subseteq g_2 H$ ✓

$y \in g_1 H$

$$y = g_1 \bar{h}$$

$$= (x h^{-1}) \bar{h}$$

$$= g_2 \underbrace{(h' h^{-1} \bar{h})}_{\in H}$$

$H, \; g_1 H, \; g_2 H, \; \cancel{g_3 H}, \; \ldots\ldots, \; g_n H$

$g_1 H = g_3 H$

$H, \; g_1' H, \; g_2' H, \; \ldots, \; g_t' H$

$|G| = (t+1)\,|H|$

$\cap = \phi$