BITS Pilani K K Birla Goa Campus

Comprehensive Examination: PART- B (Closed Book)

Course Title : Mathematics III — **Max. Marks: 75**

Course No : MATH F211 — **Time: 2 hours**

Average: 28.02 / 75
Total av: 46.68/120

Instructions:

1. All questions are compulsory.
2. Start a new question in a new page and answer all its parts in the same place.
3. Write all the steps clearly and give explanations for complete credit.
4. Make an index on the front page of the main answer sheet, **otherwise penalty of 4 marks will be imposed.**

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1. Find two Frobenius series solutions of the equation $2xy'' + (x+1)y' + 3y = 0$ near $x = 0$. Also, write the region in which the solutions are valid. **[15]**

2. Find the Bessel series of $f(x) = (1 - x^2)x^p$, for $x \in [0, 1]$ and $p \geq 0$ in terms of positive zeros of $J_p(x)$. (Evaluate all the integrals involved). **[15]**

3. Solve the equation $(x-1)y'' + (3x-4)y' - (4x+5)y = 0,\ y(1) = 0$, by using Laplace transforms. **[15]**

4. Prove the recursion formula $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$. Use it to show that $(2n+1)P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$. **[15]**

5. Find the Fourier series for **[15]**

$$f(x) = \cos\left(\frac{x}{2}\right), \quad -\pi \leq x \leq \pi.$$

Use the Fourier series to find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1-4n^2}$.

The End

Useful Formulas:

(1) $nP_n(x) = xP'_n(x) - P'_{n-1}(x)$,

(2) $(n+1)P_n(x) = P'_{n+1}(x) - xP'_n(x)$,

(3) $\dfrac{1}{\sqrt{1-2xt+t^2}} = P_0(x) + P_1(x)t + P_2(x)t^2 + \cdots + P_n(x)t^n + \cdots$

(4) $J_p(x) = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(x/2)^{2n+p}}{n!(p+n)!}$,

(5) $\dfrac{d}{dx}[x^p J_p(x)] = x^p J_{p-1}(x)$,

(6) $\dfrac{d}{dx}[x^{-p} J_p(x)] = -x^{-p} J_{p+1}(x)$.