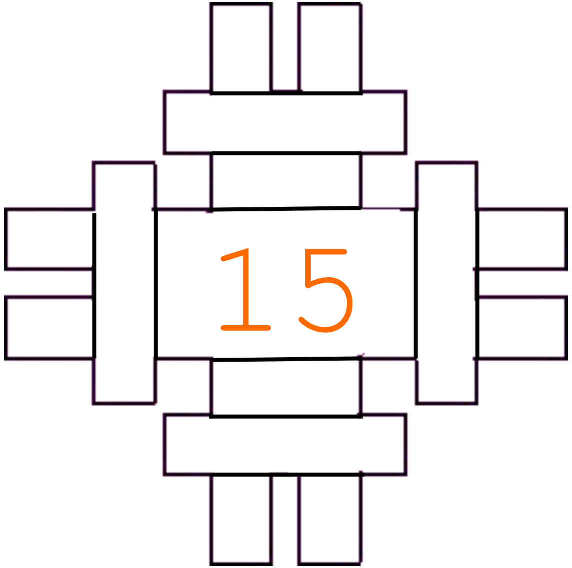
Minimum Convex Cover

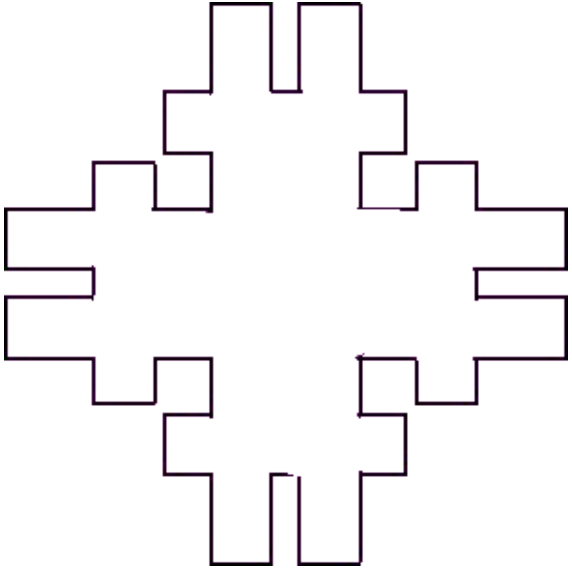
Decomposing the Poligon into its Convex Parts

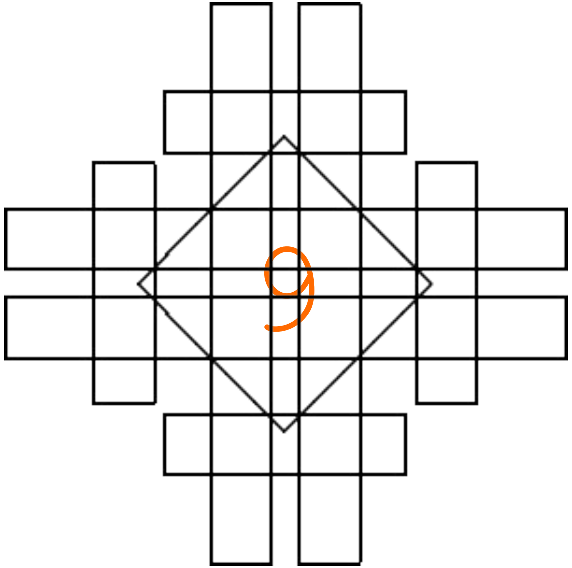
P
(Chazelle & Dobkin 1979)

Minimum Convex Cover

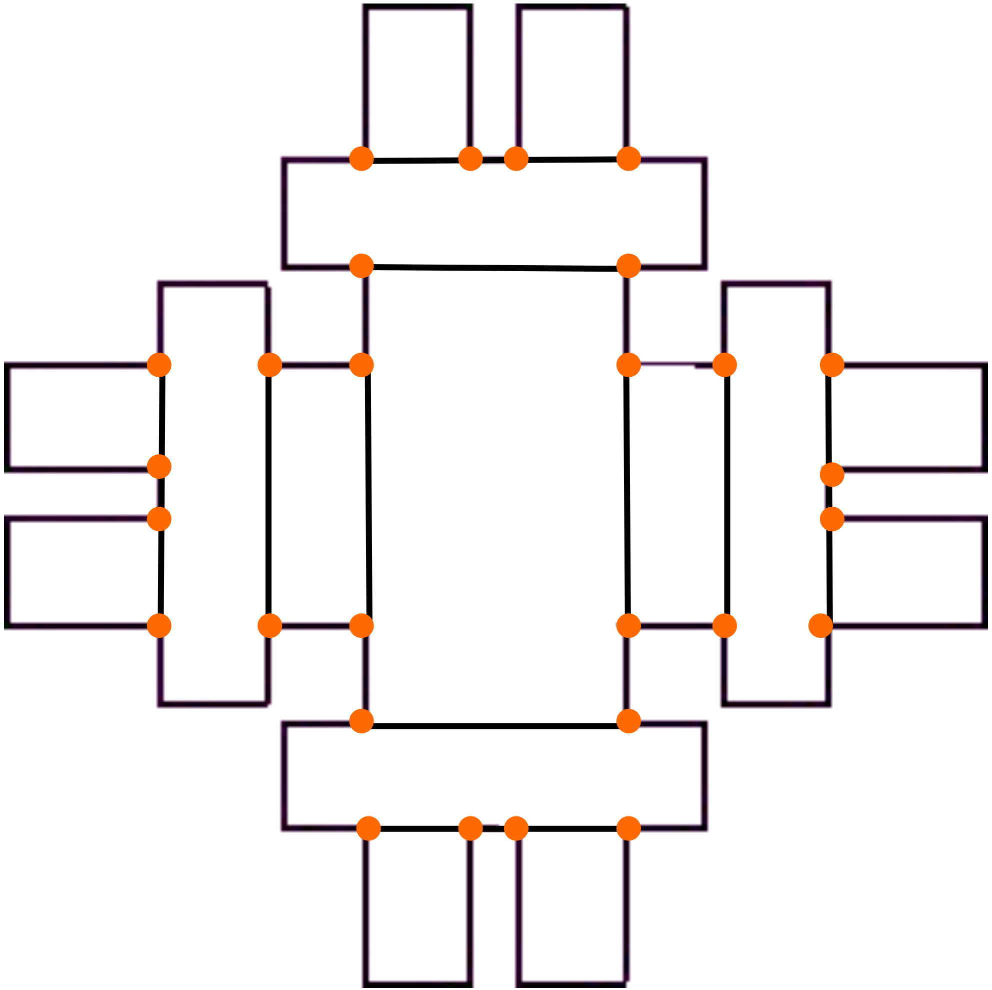
Decidable
(O'Rouke 1982)







Number of polygons in a convex decomposition is at least 1+N/2, where N is the number of notches.



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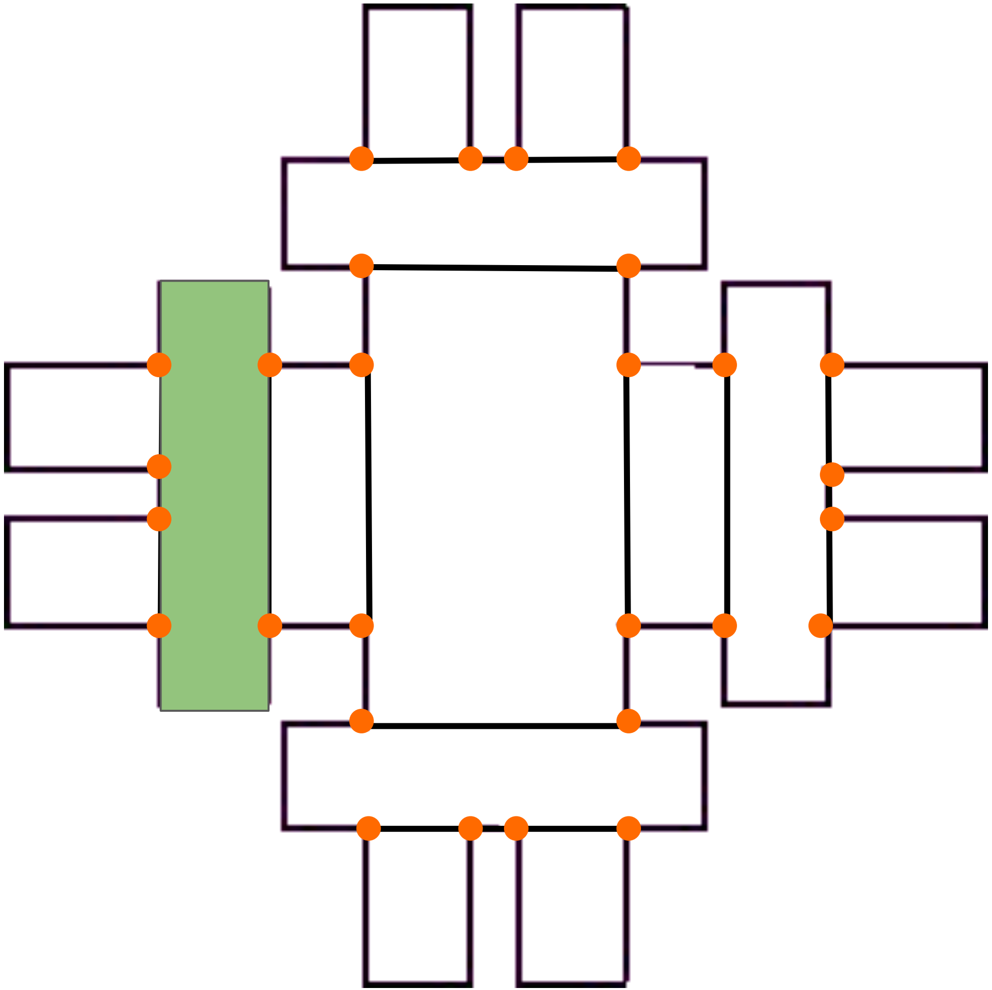
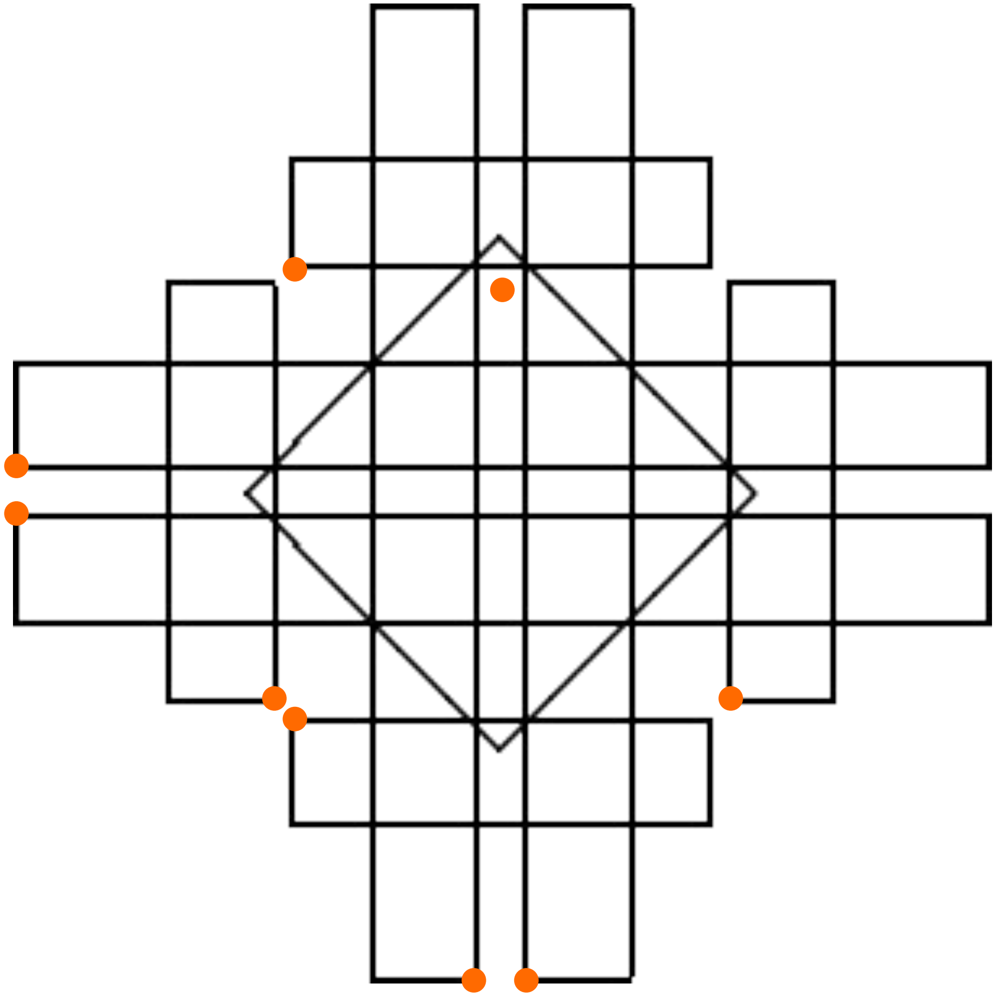


Number of polygons in a convex cover is at least the number of mutually invisible points.

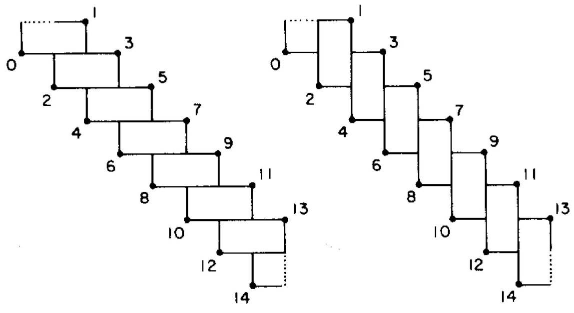


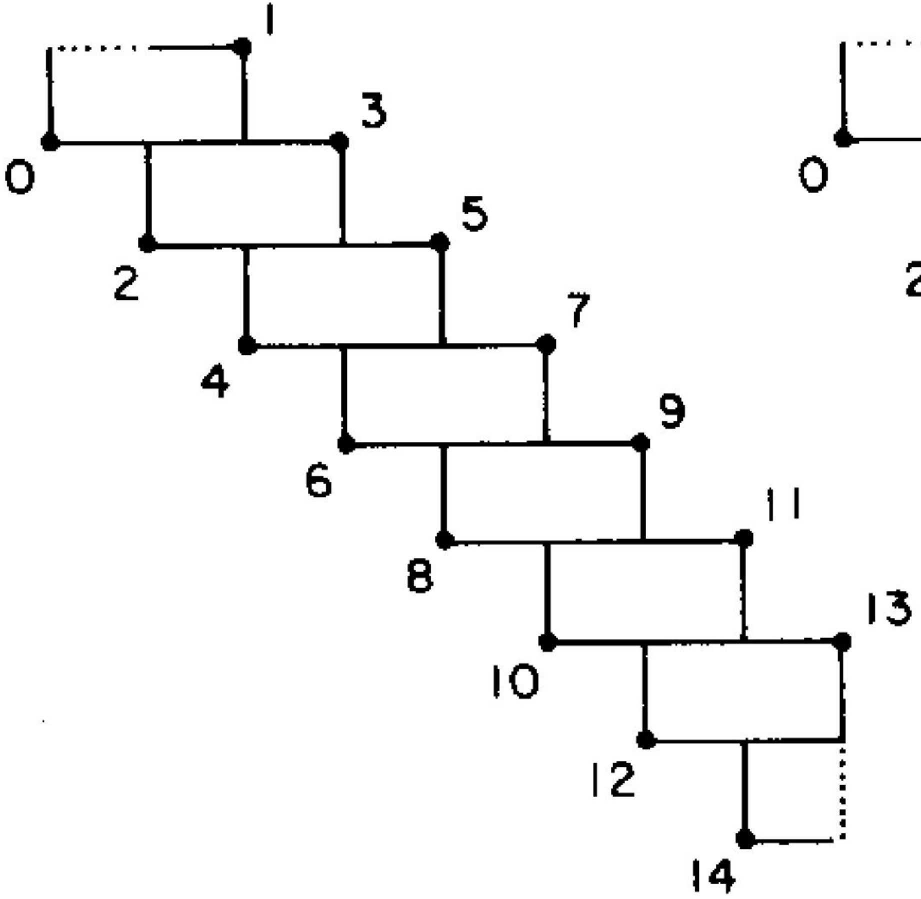
Some NP-Hard Polygon Decomposition Problems

(O'Rouke, Supowit 1983)

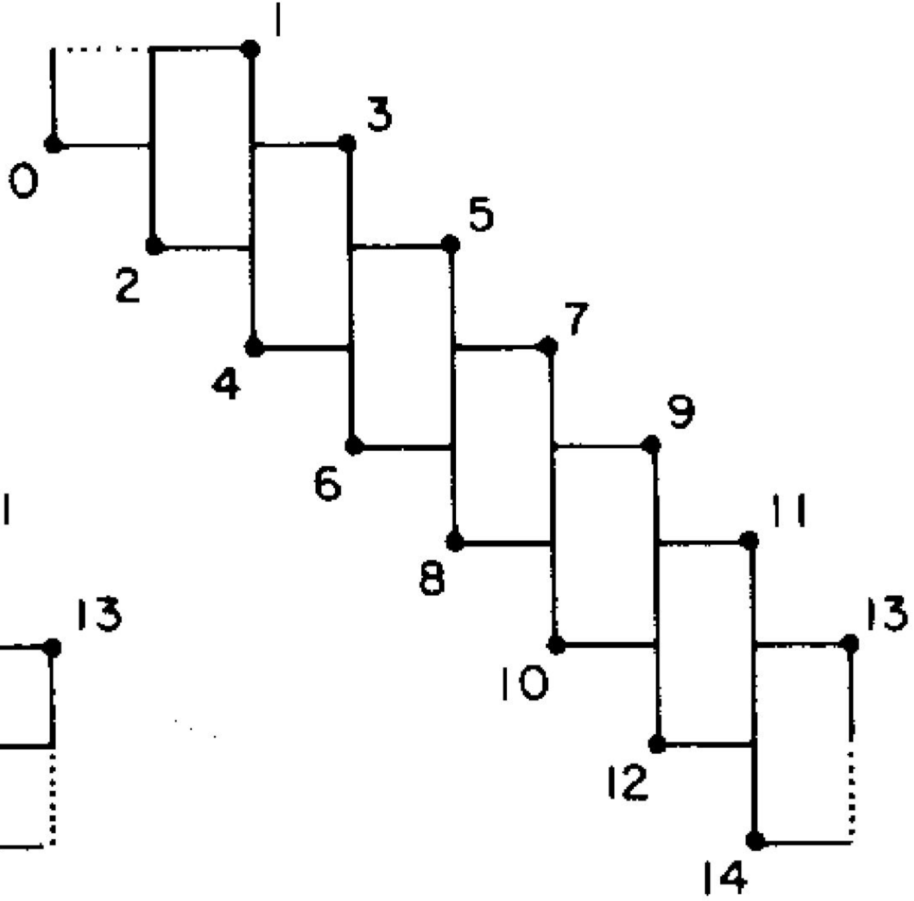
1 By reduction from 3SAT

2 Variable represented by a polygon that can be optimally covered in exactly 2 different ways

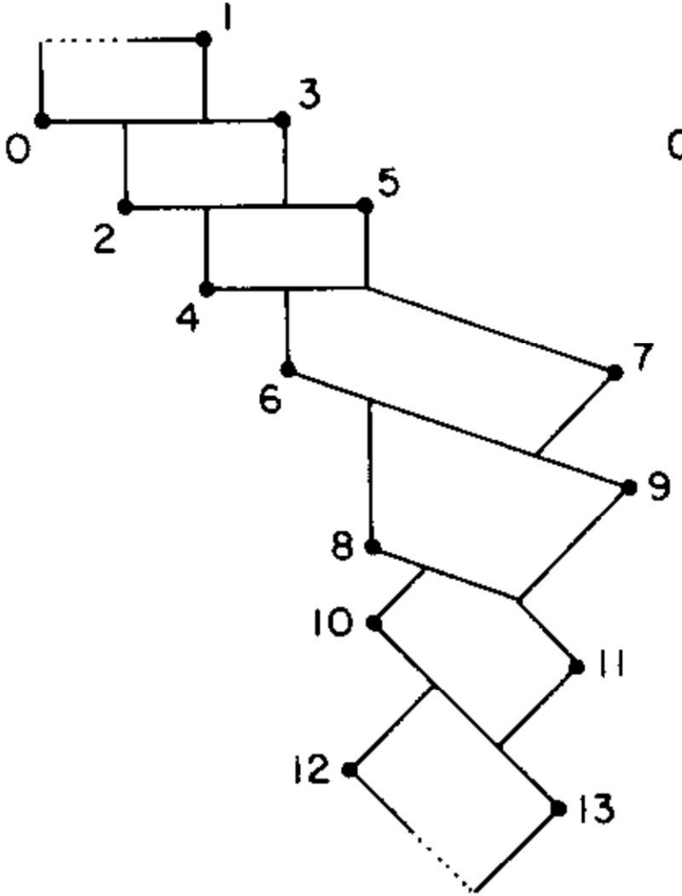




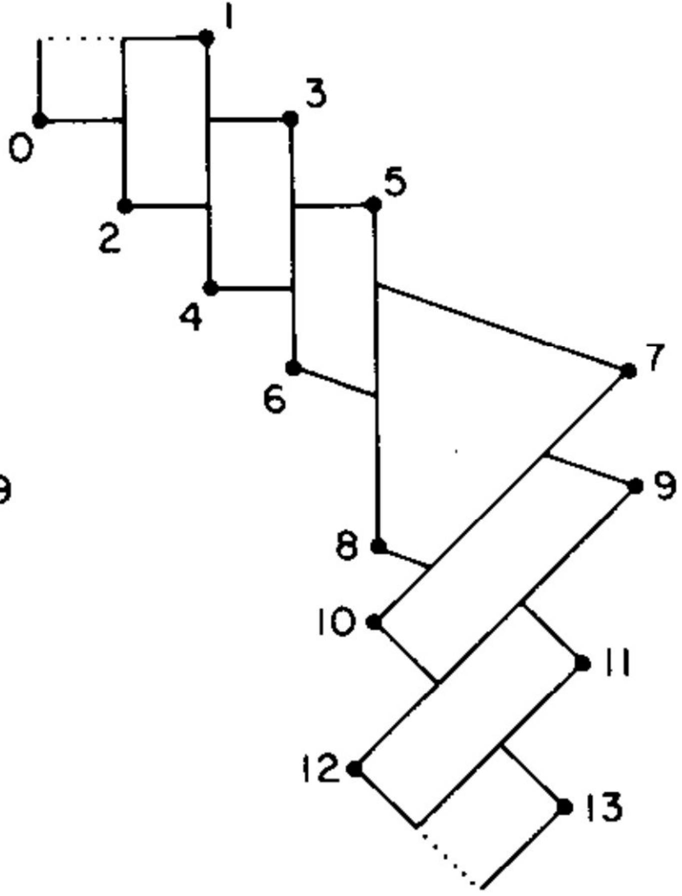
TRUE



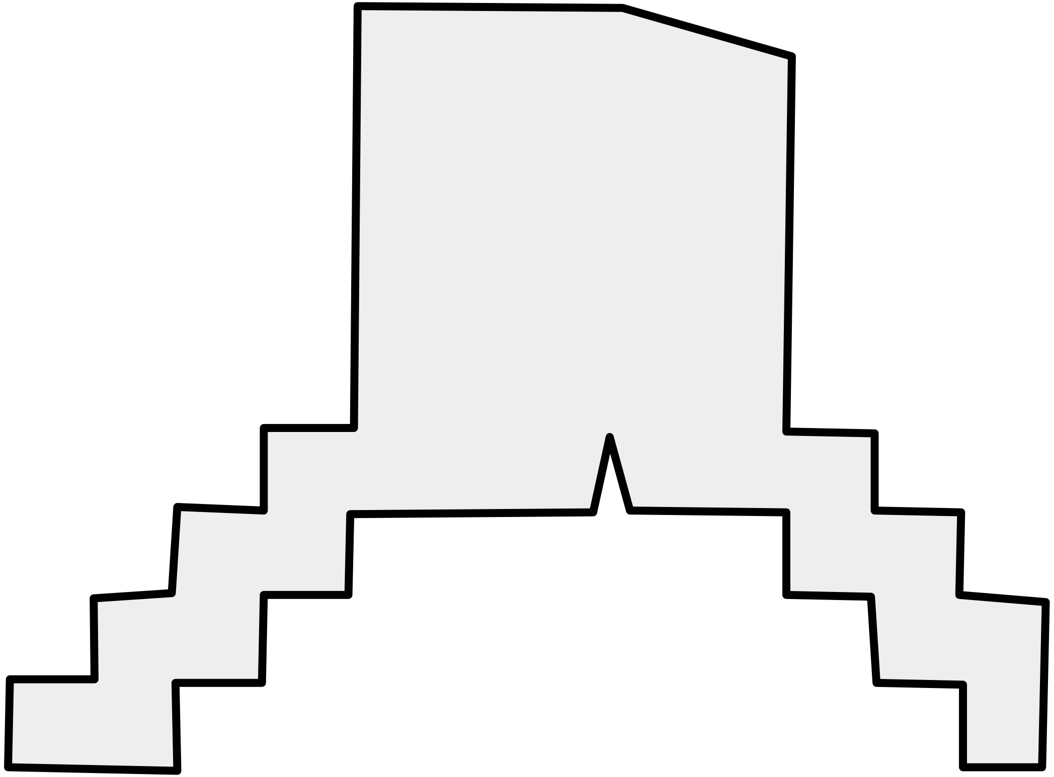
FALSE

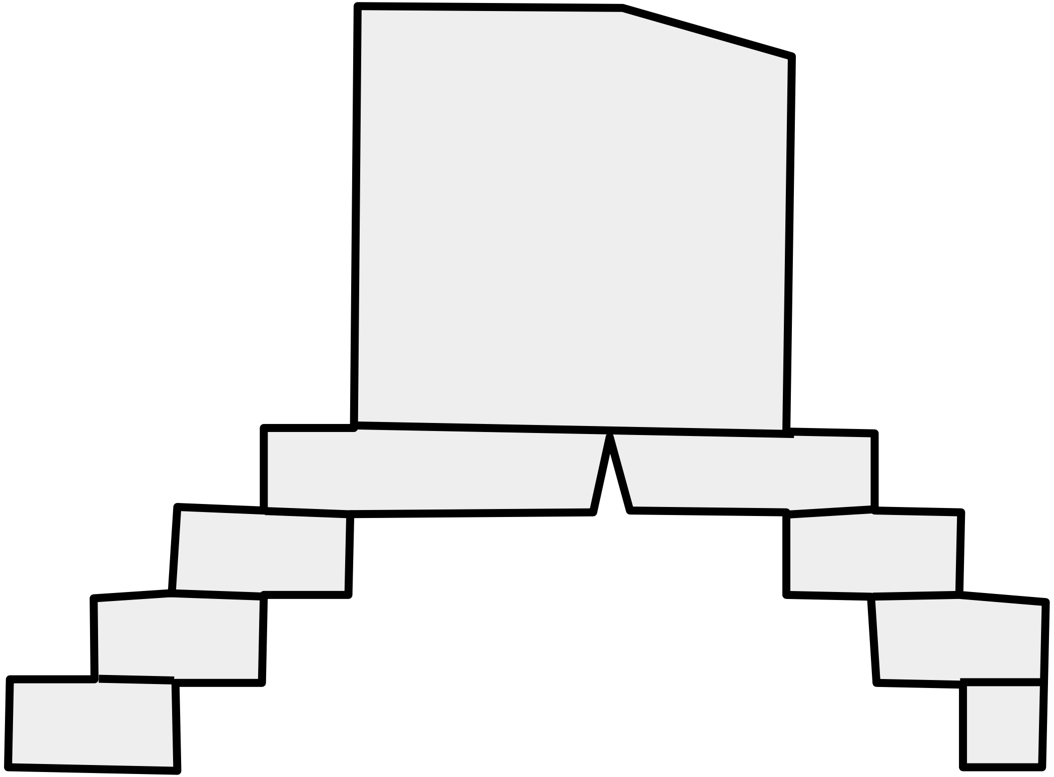


TRUE

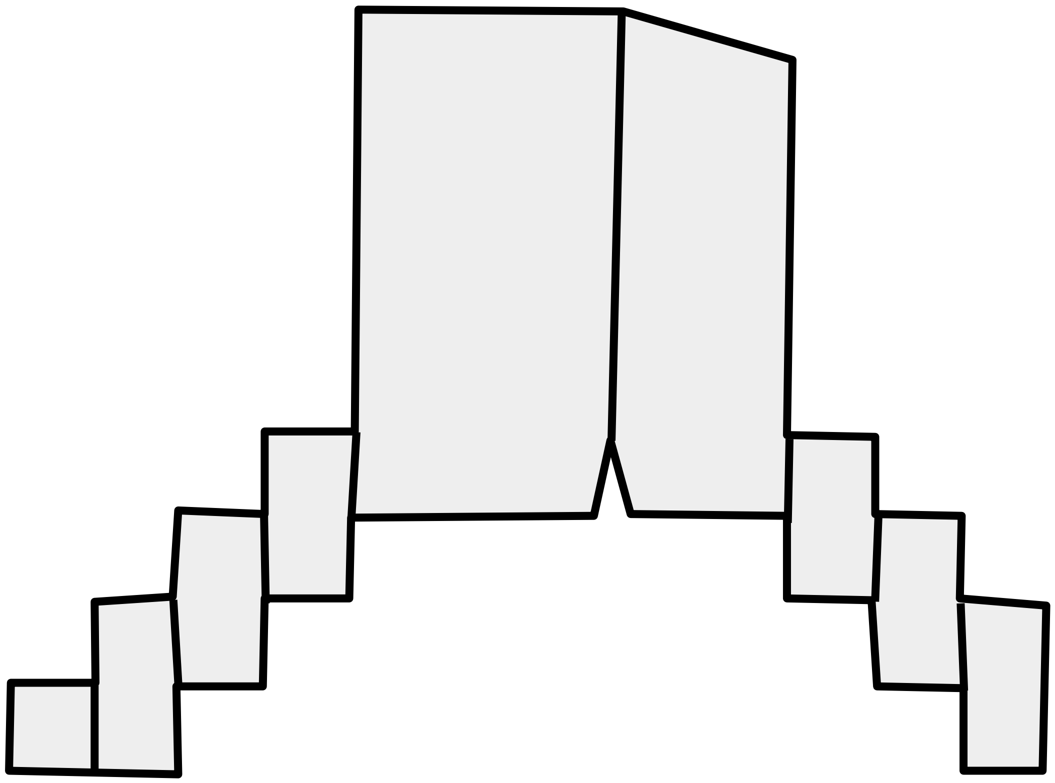


FALSE

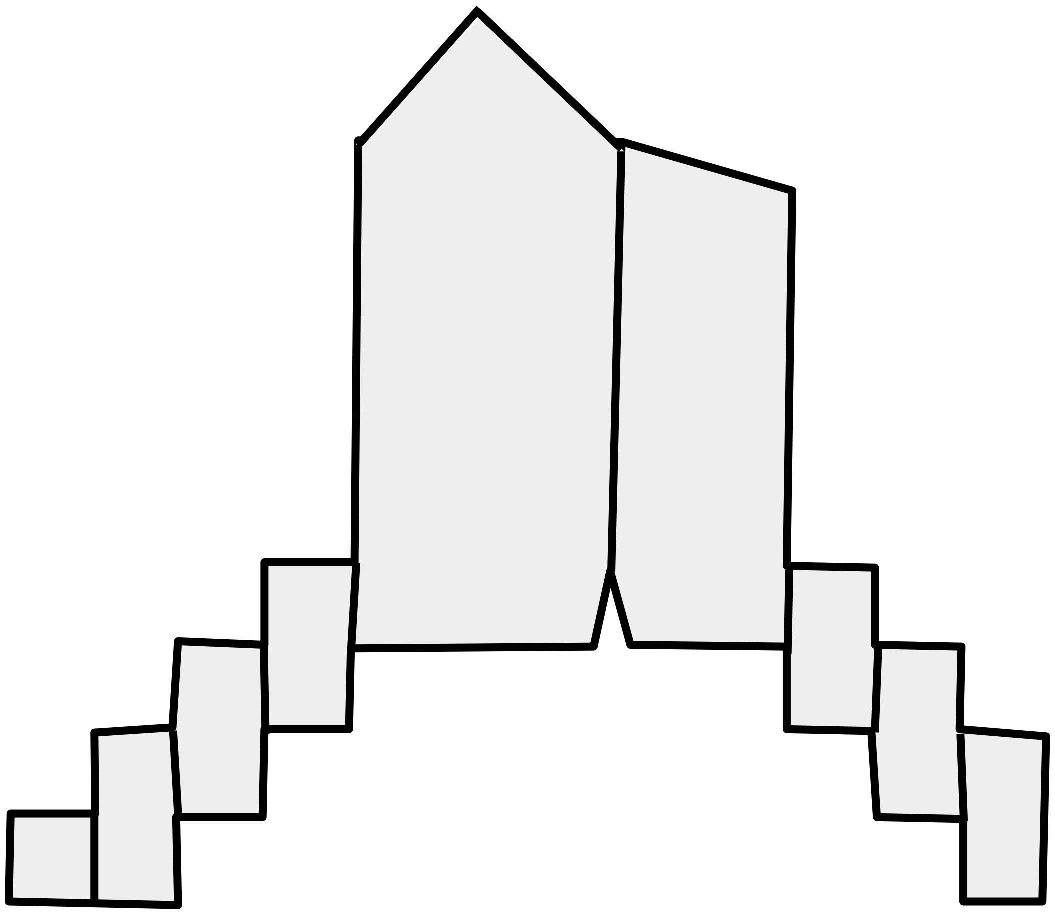




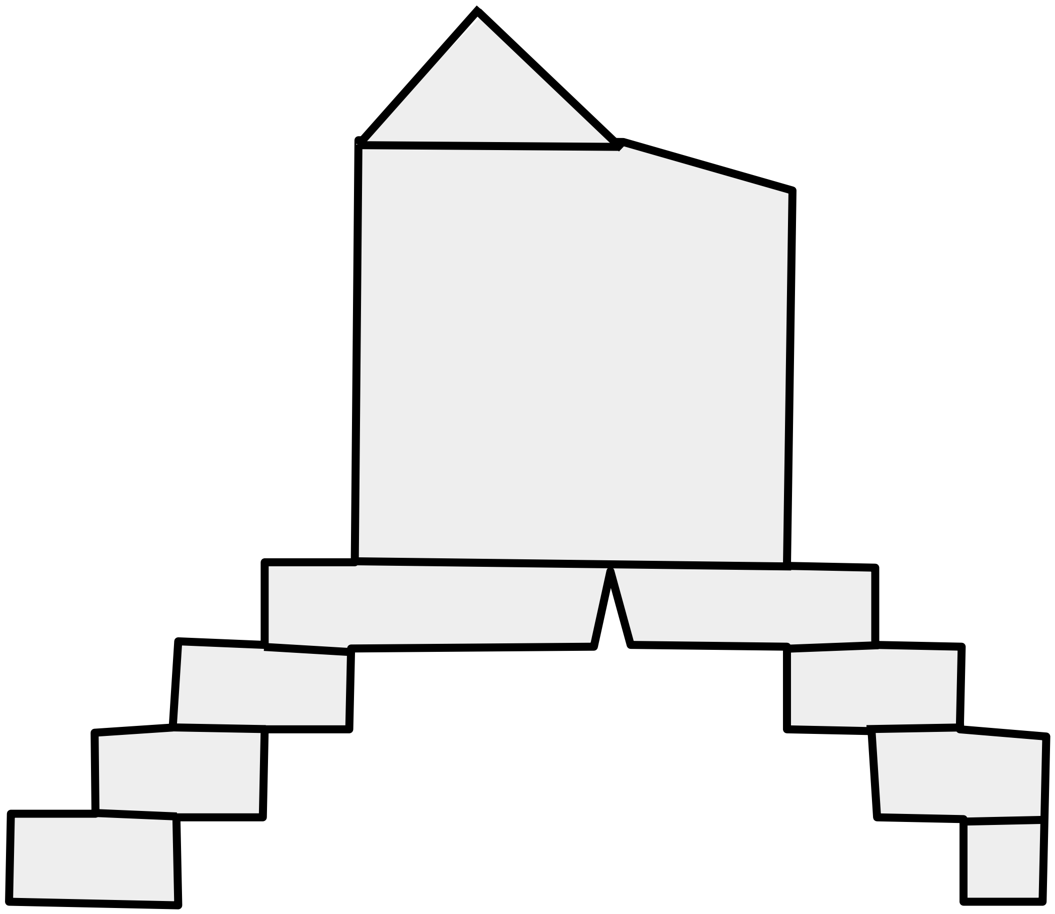
9
pieces
(True)



9
pieces
(False)

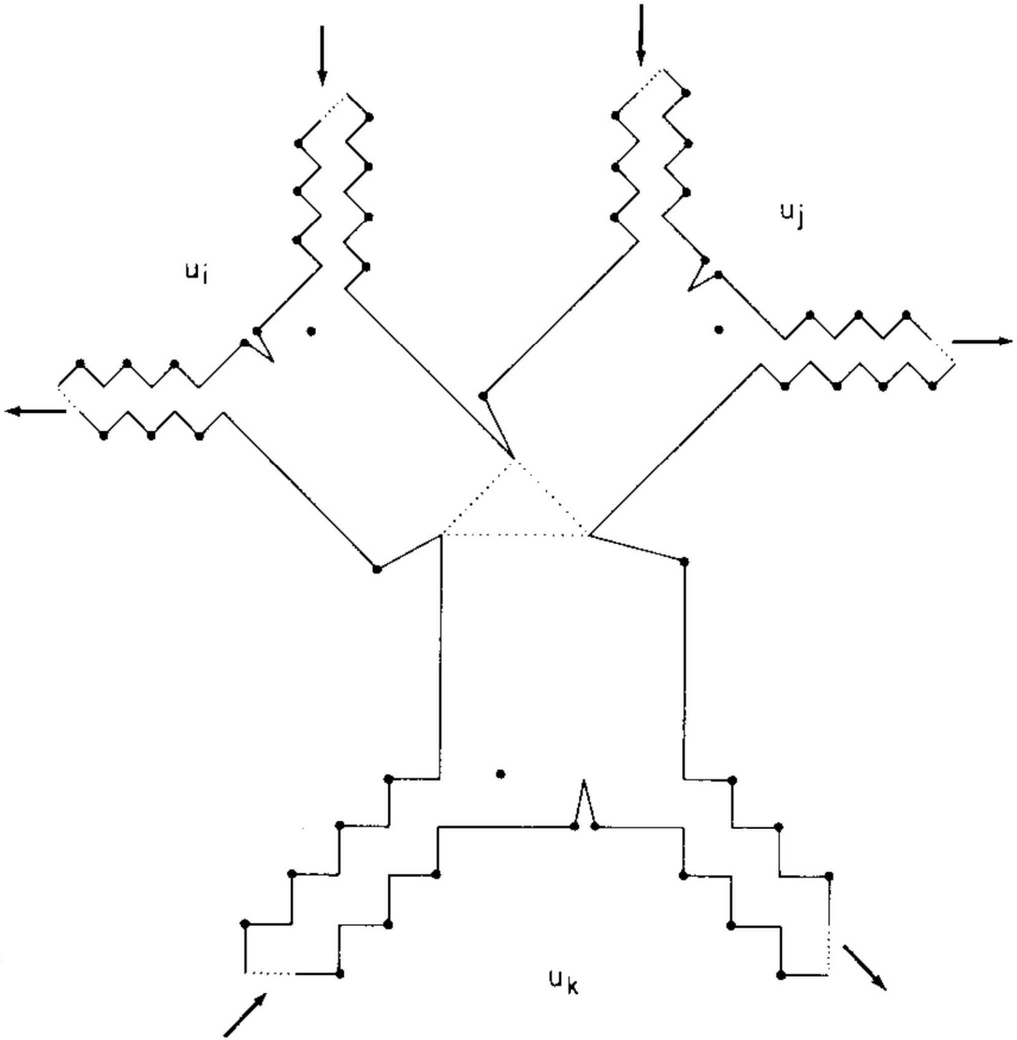


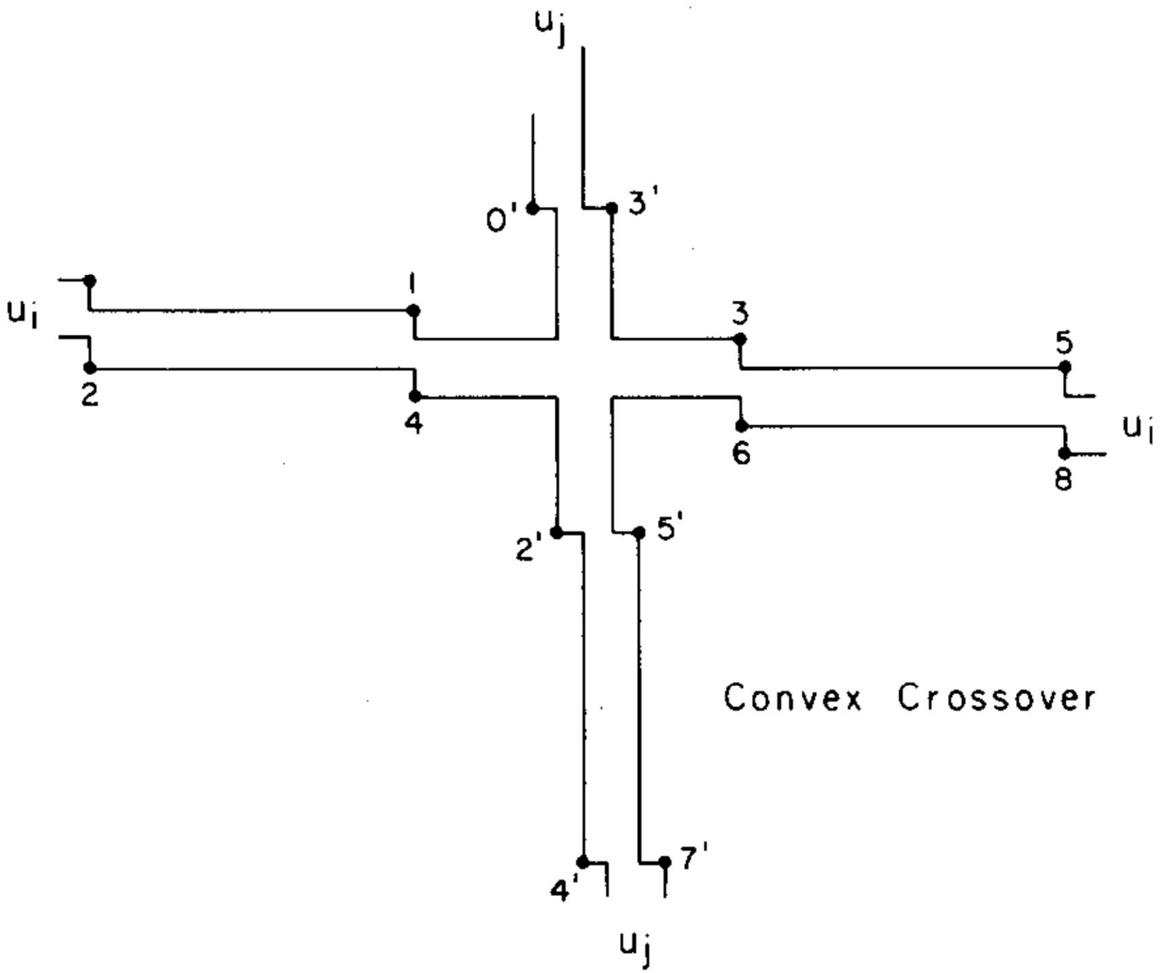
9
pieces
(False)



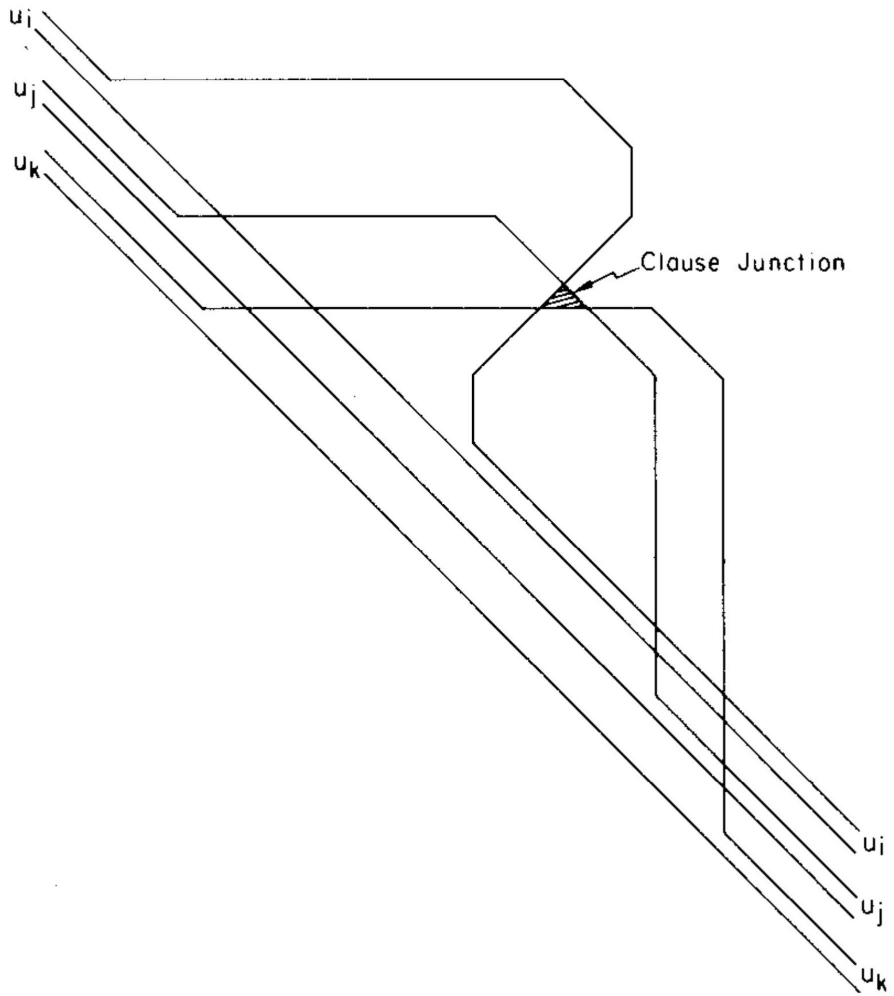
10
pieces
(True)

Formula is satisfiable if the triangle can be covered without increasing the number of pieces in the cover





Convex Crossover


u_i
u_j
u_k
Clause Junction
u_i
u_j
u_k

1 A number of pieces needed for a cover is never be more than 2n

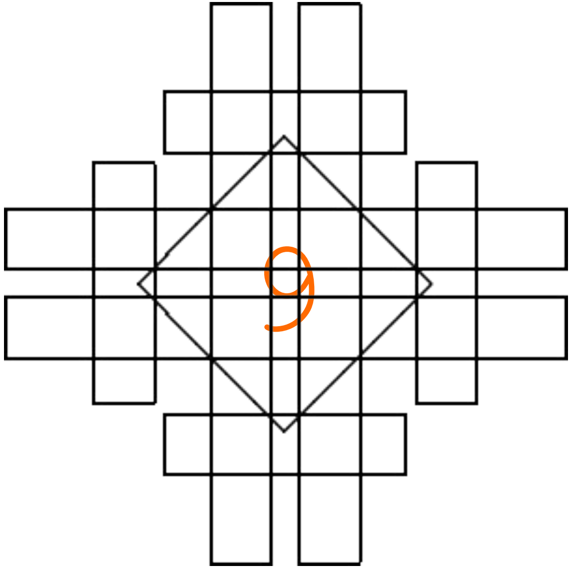
2 The number of vertices in each piece is bounded by a function of n

3 The problem can be stated as a set of algebraic equations

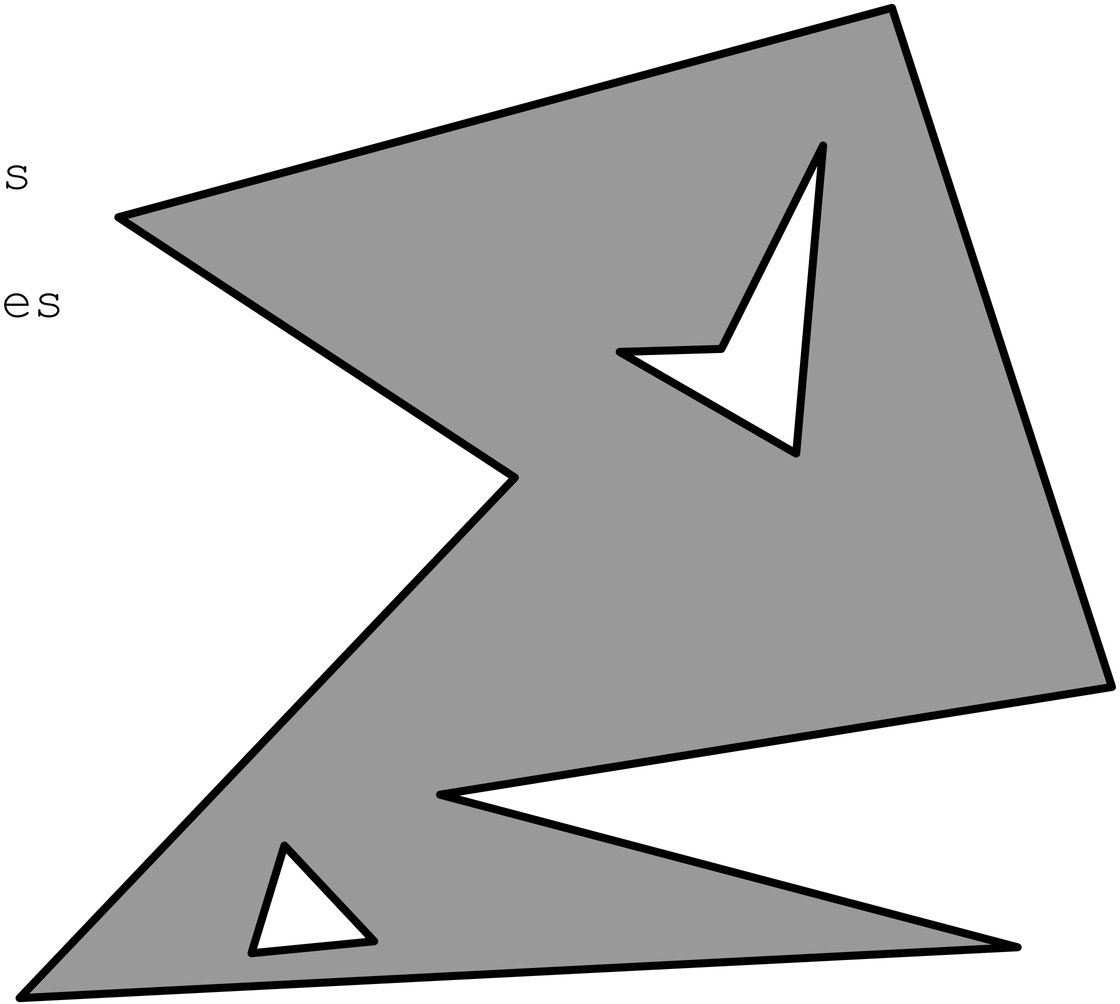
Minimum Convex Cover

Decidable

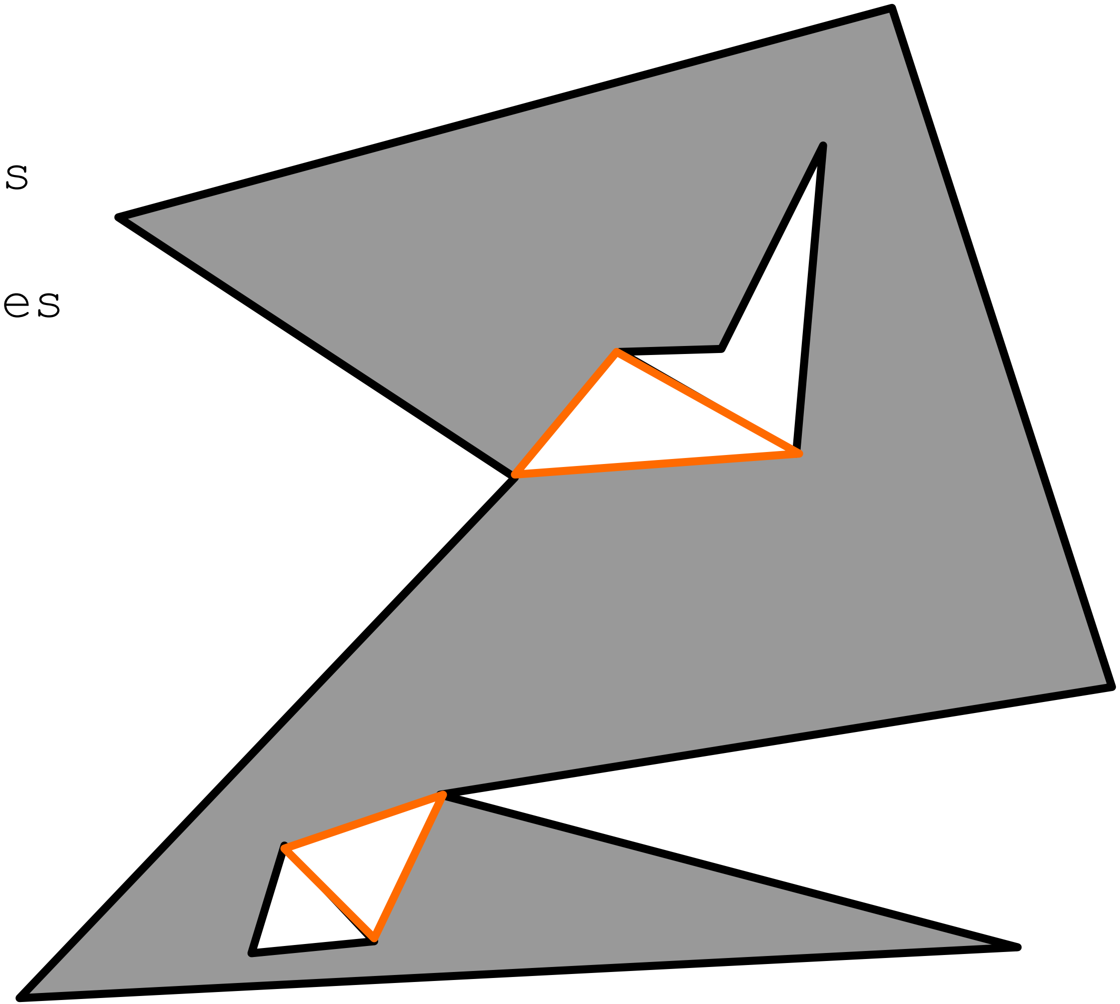
(O'Rouke 1982)



A polygon with n vertices and h holes can be triangulated into n-2+2h triangles

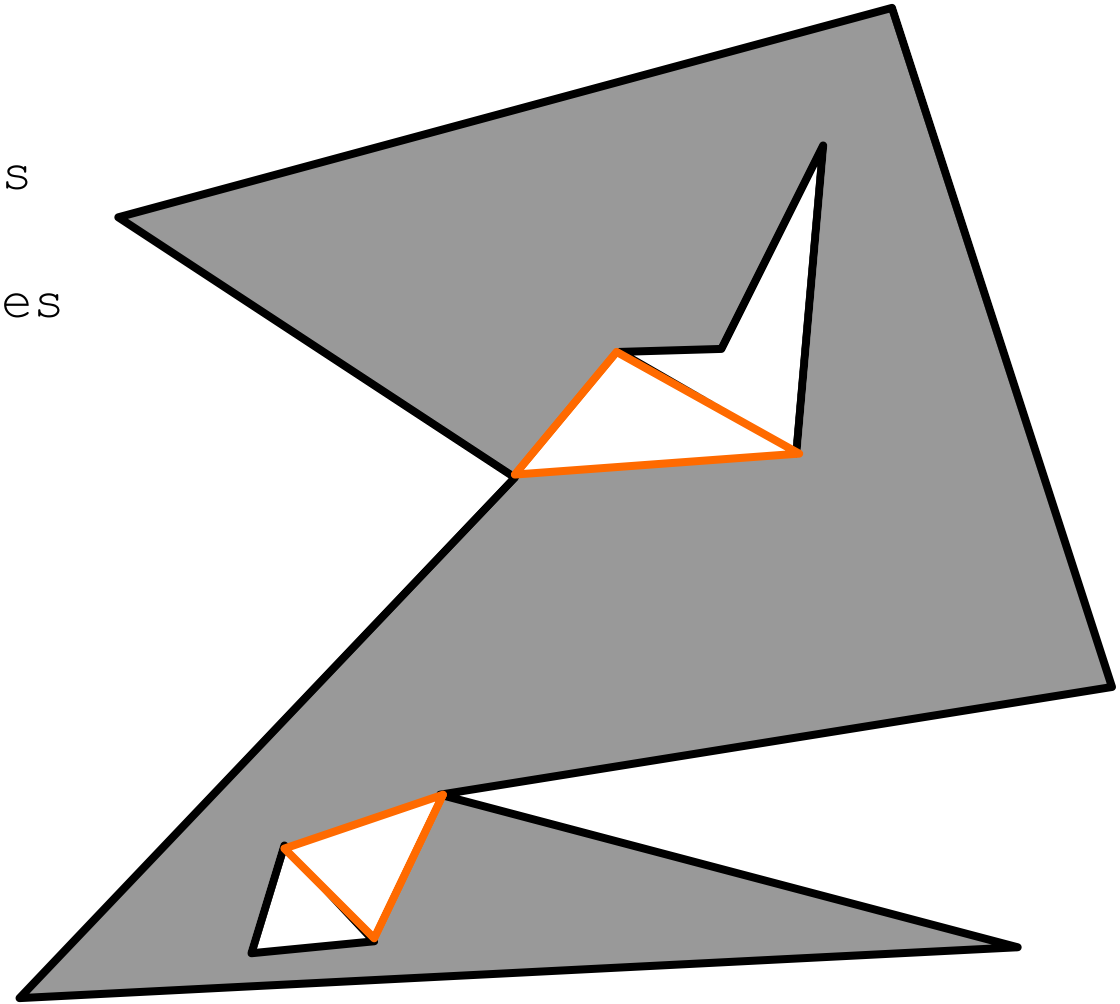


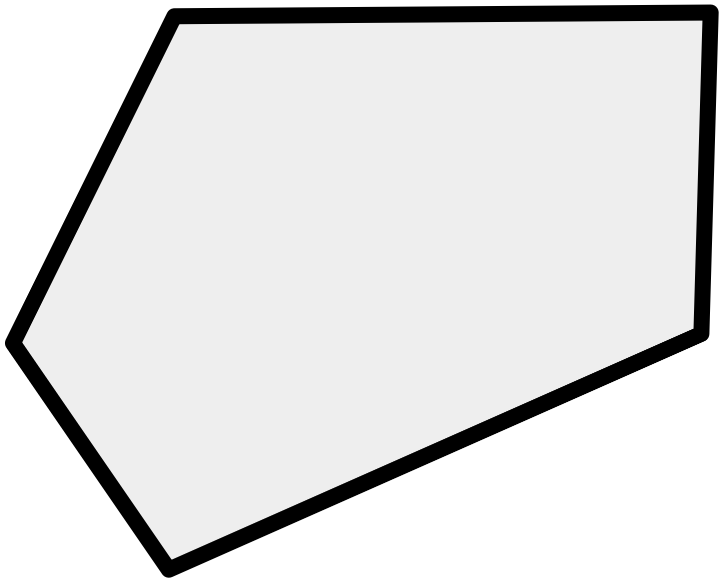
A polygon with n vertices and h holes can be triangulated into n-2+2h triangles



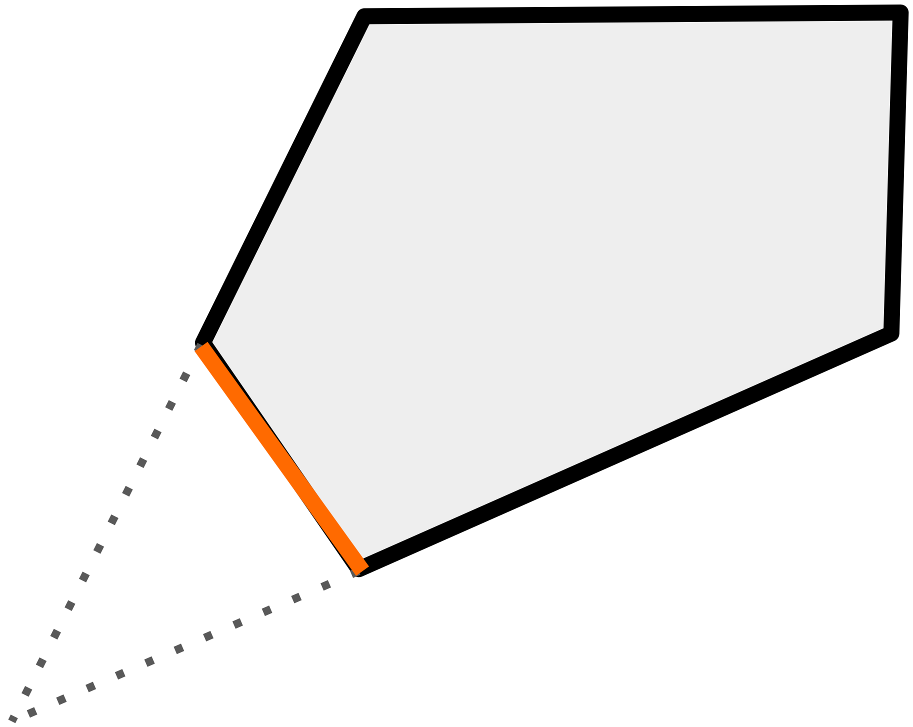
A polygon with n vertices and h holes can be triangulated into n-2+2h triangles

n >= 3 + 3h, so
n-2+2h <= 5n/3 - 4





The edge expansion procedure



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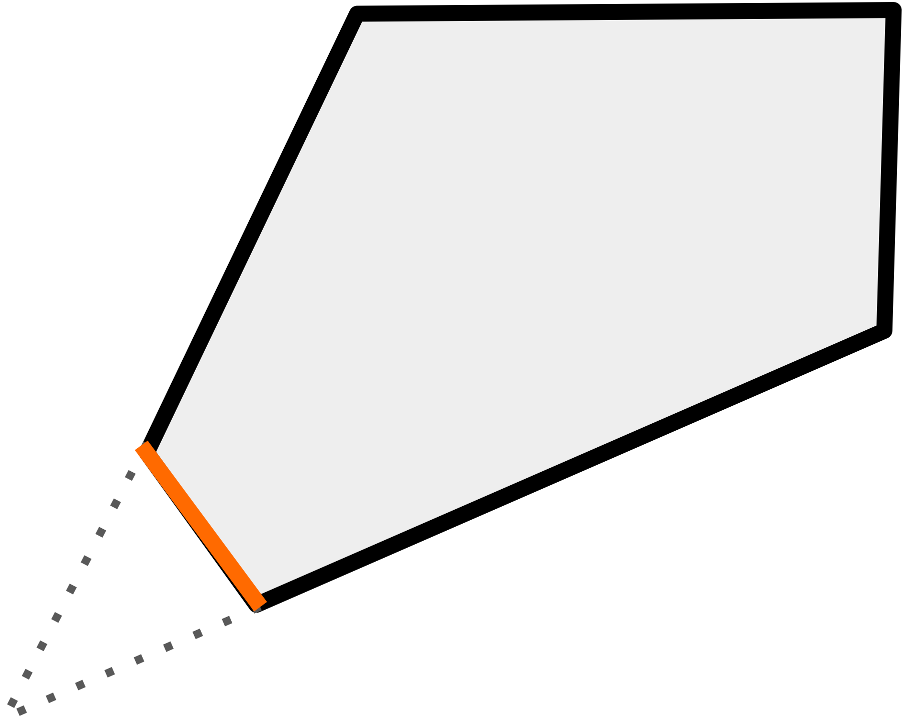

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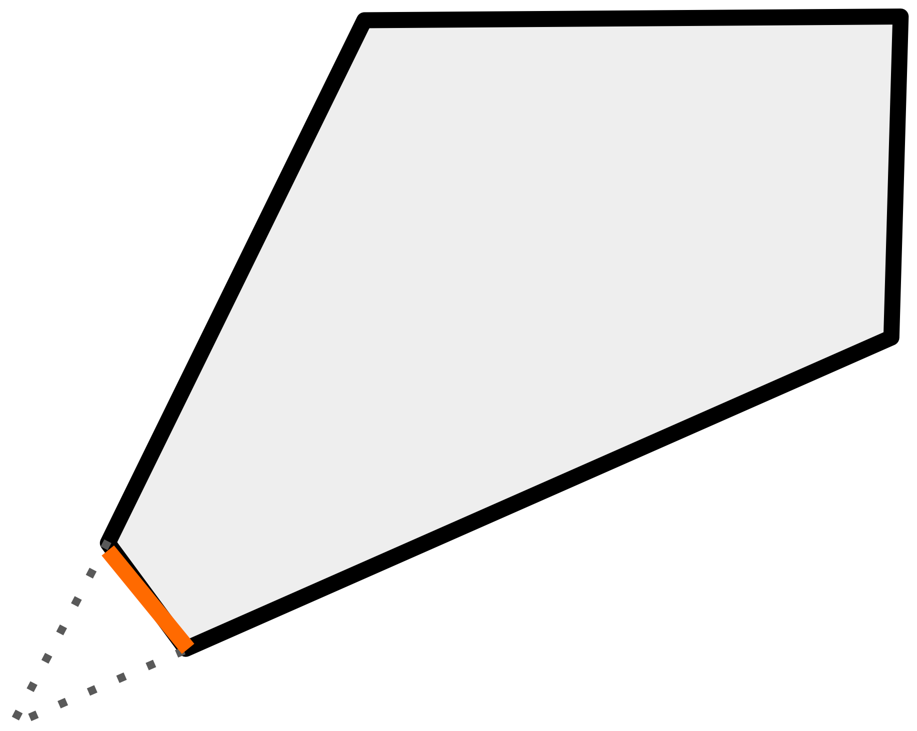

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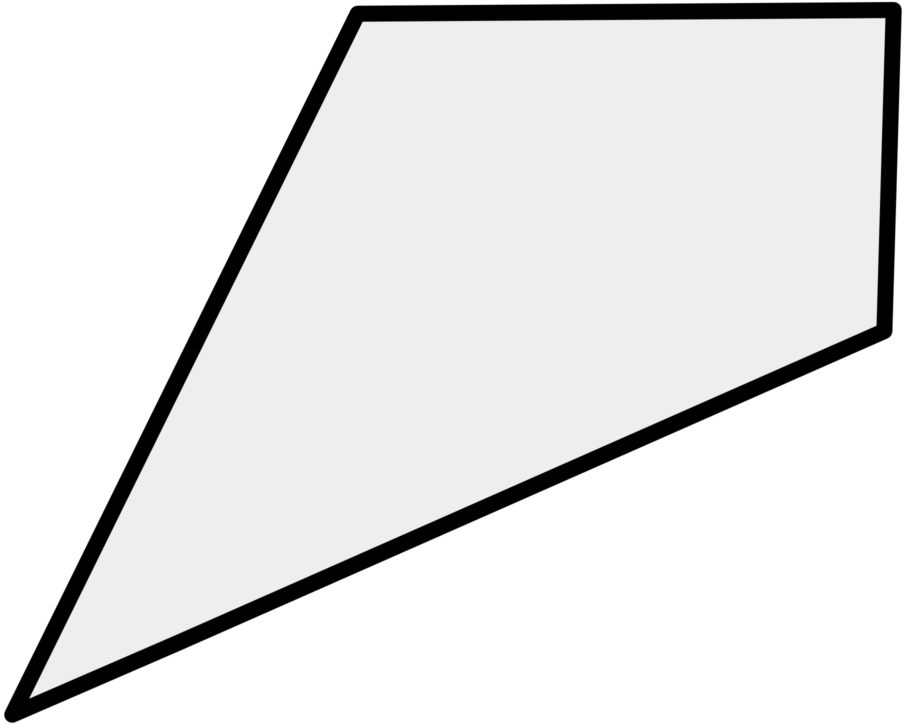

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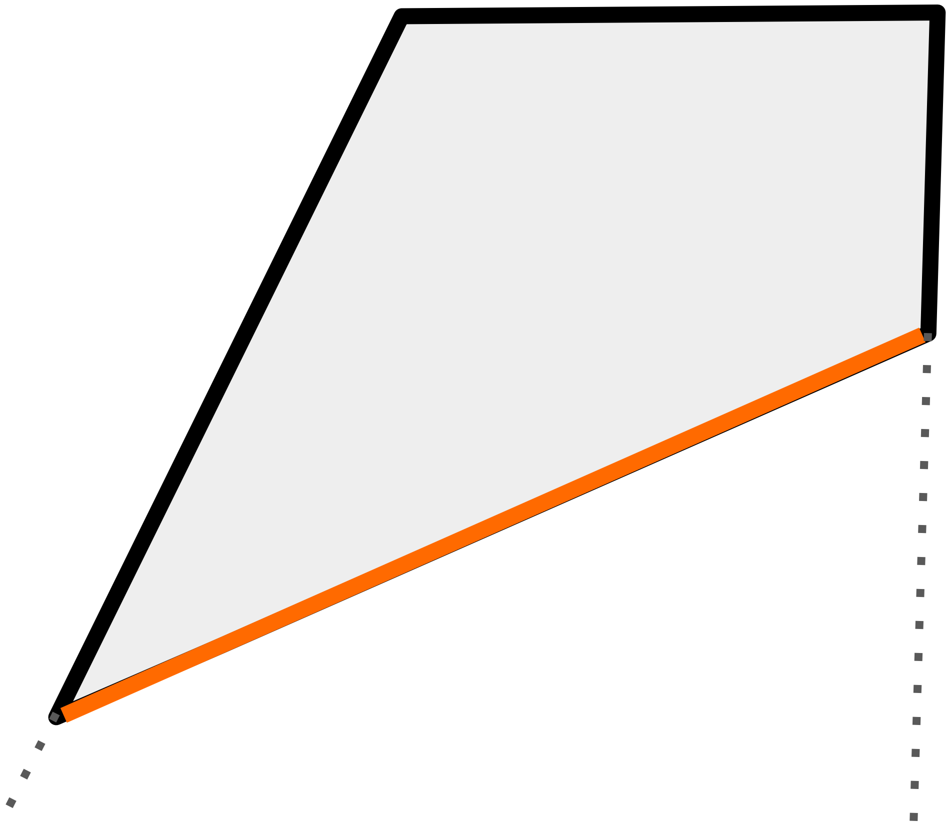

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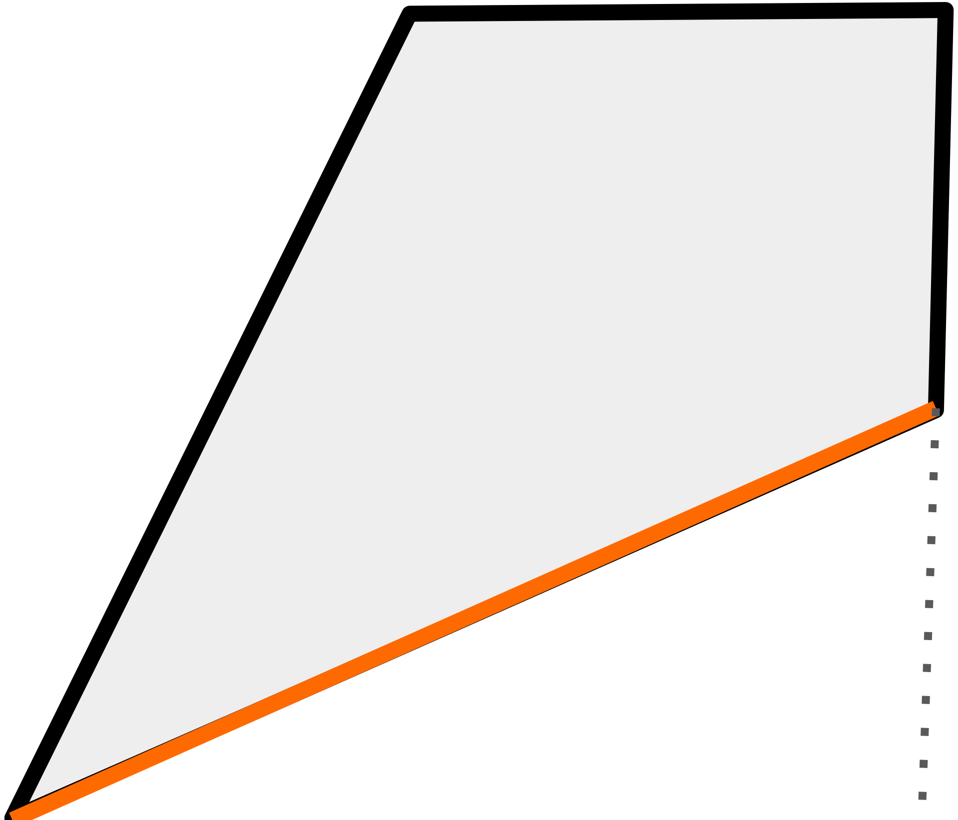
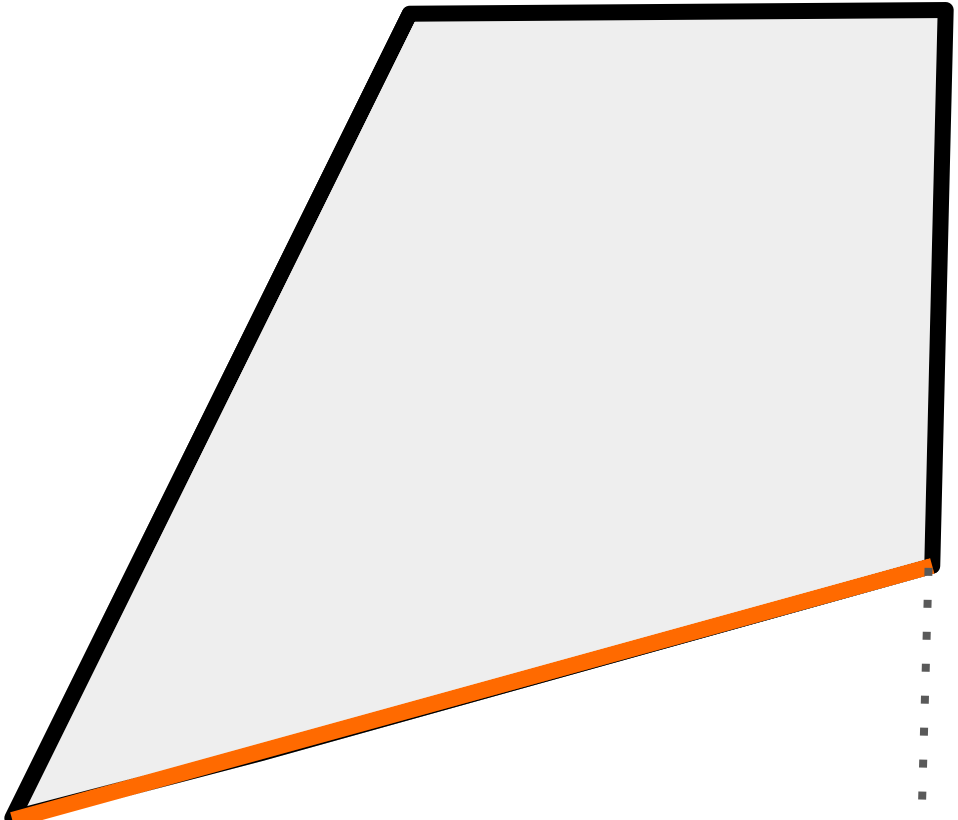


The edge expansion procedure



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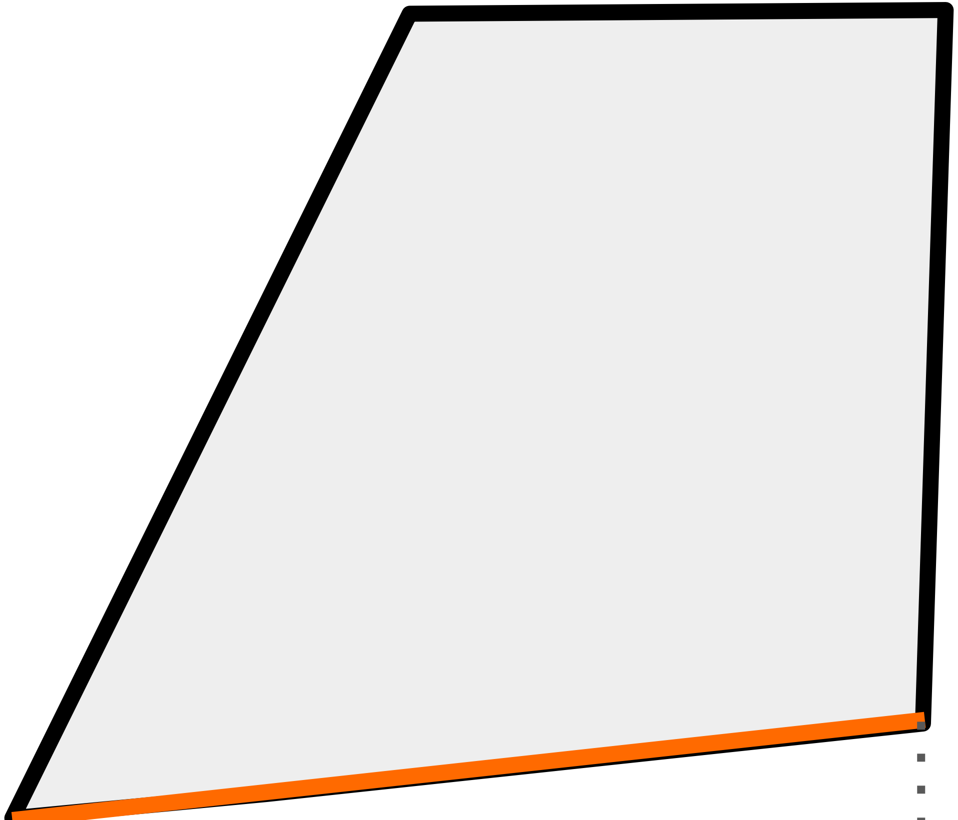

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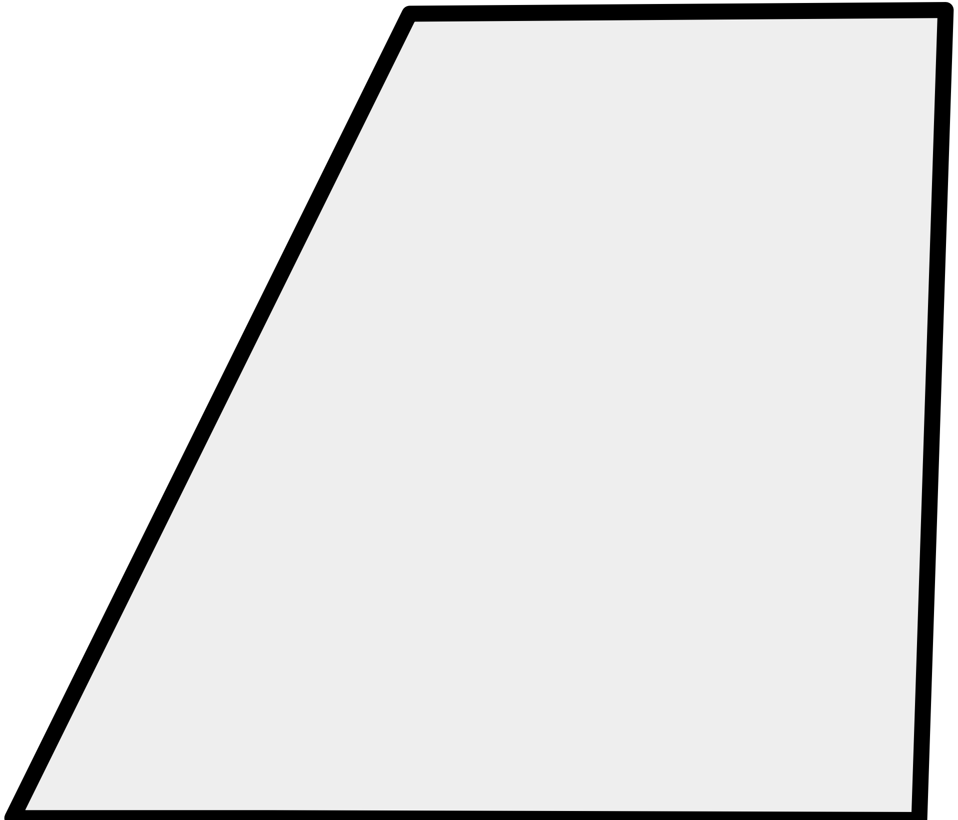

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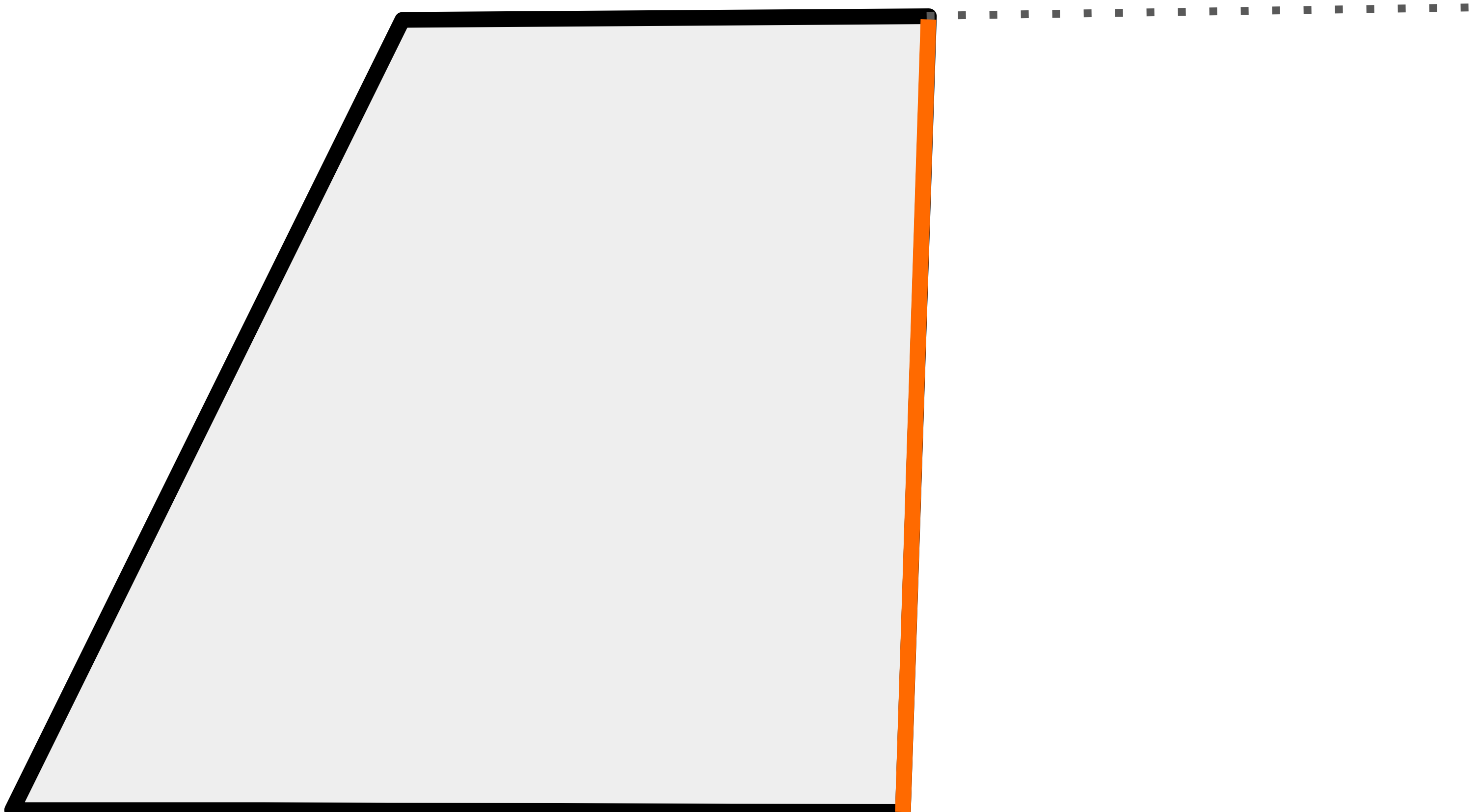

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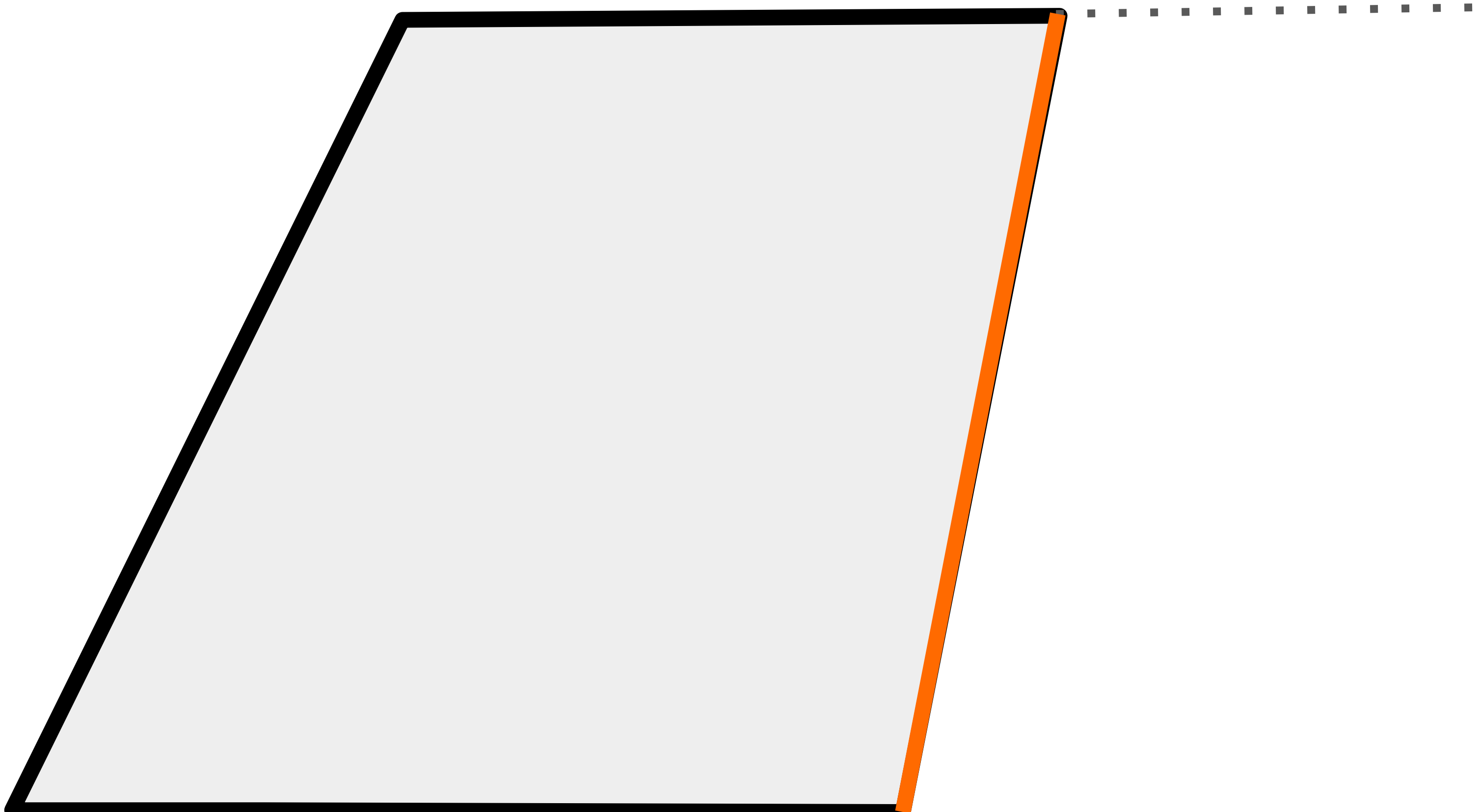

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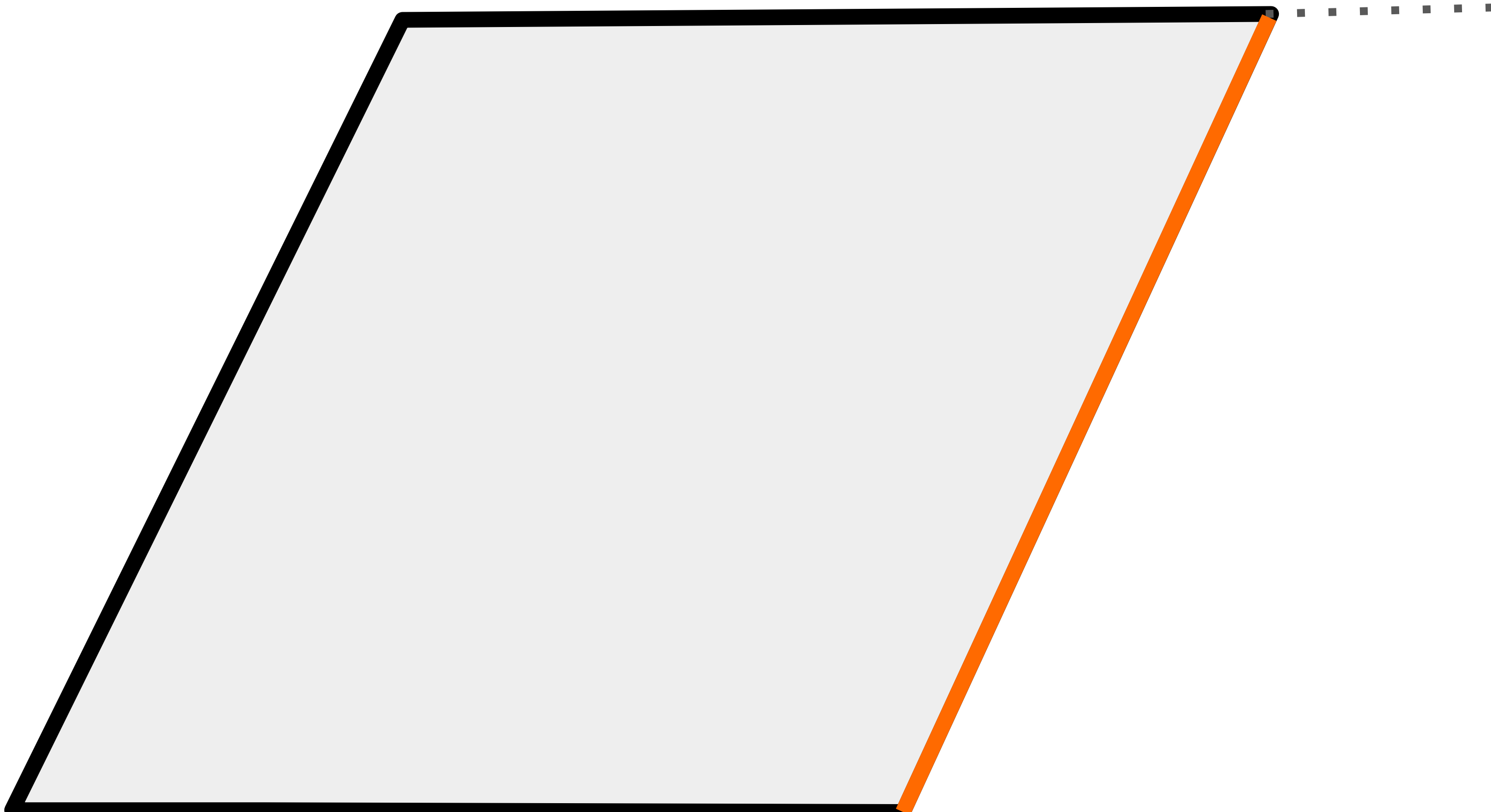

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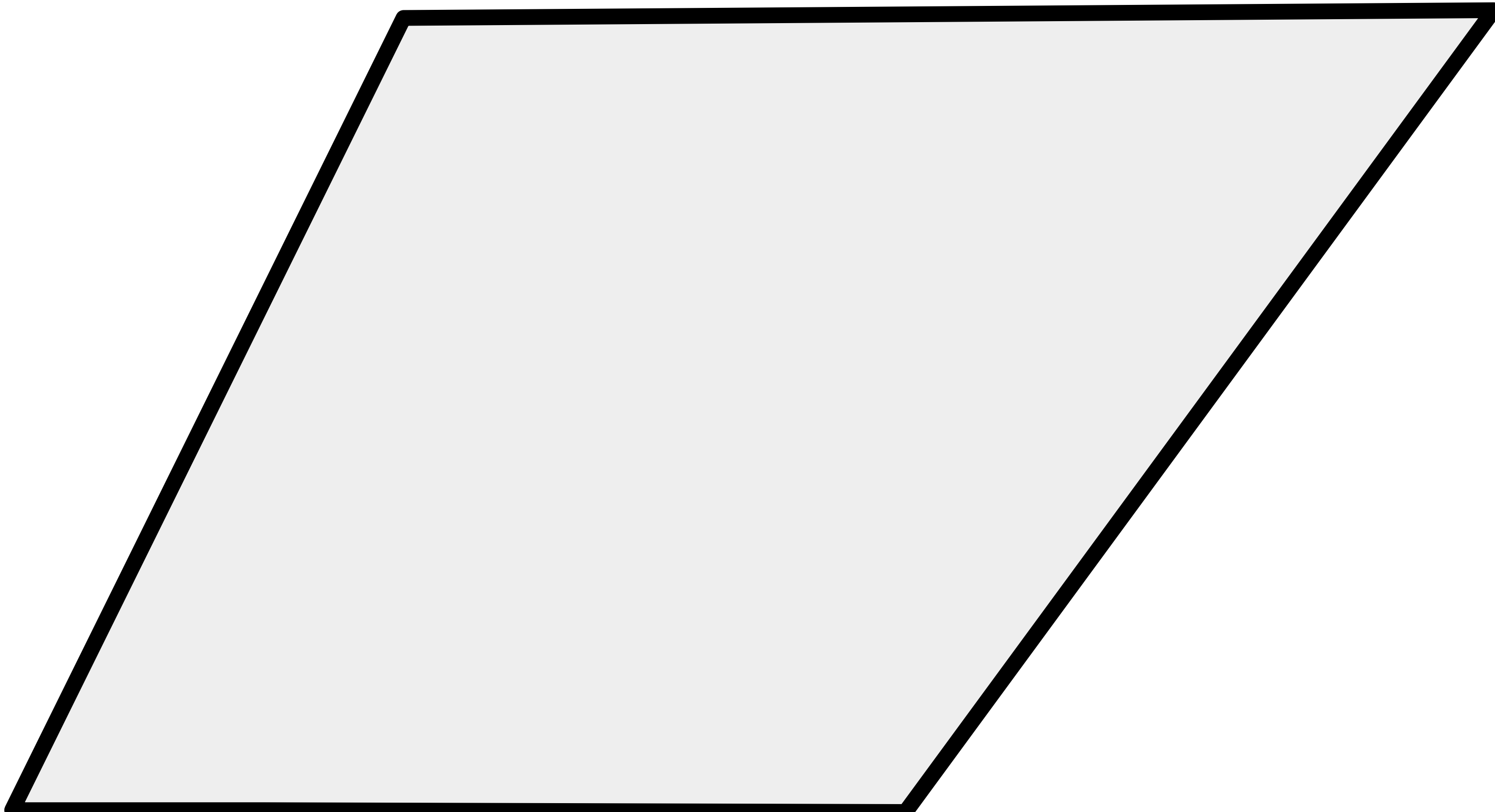

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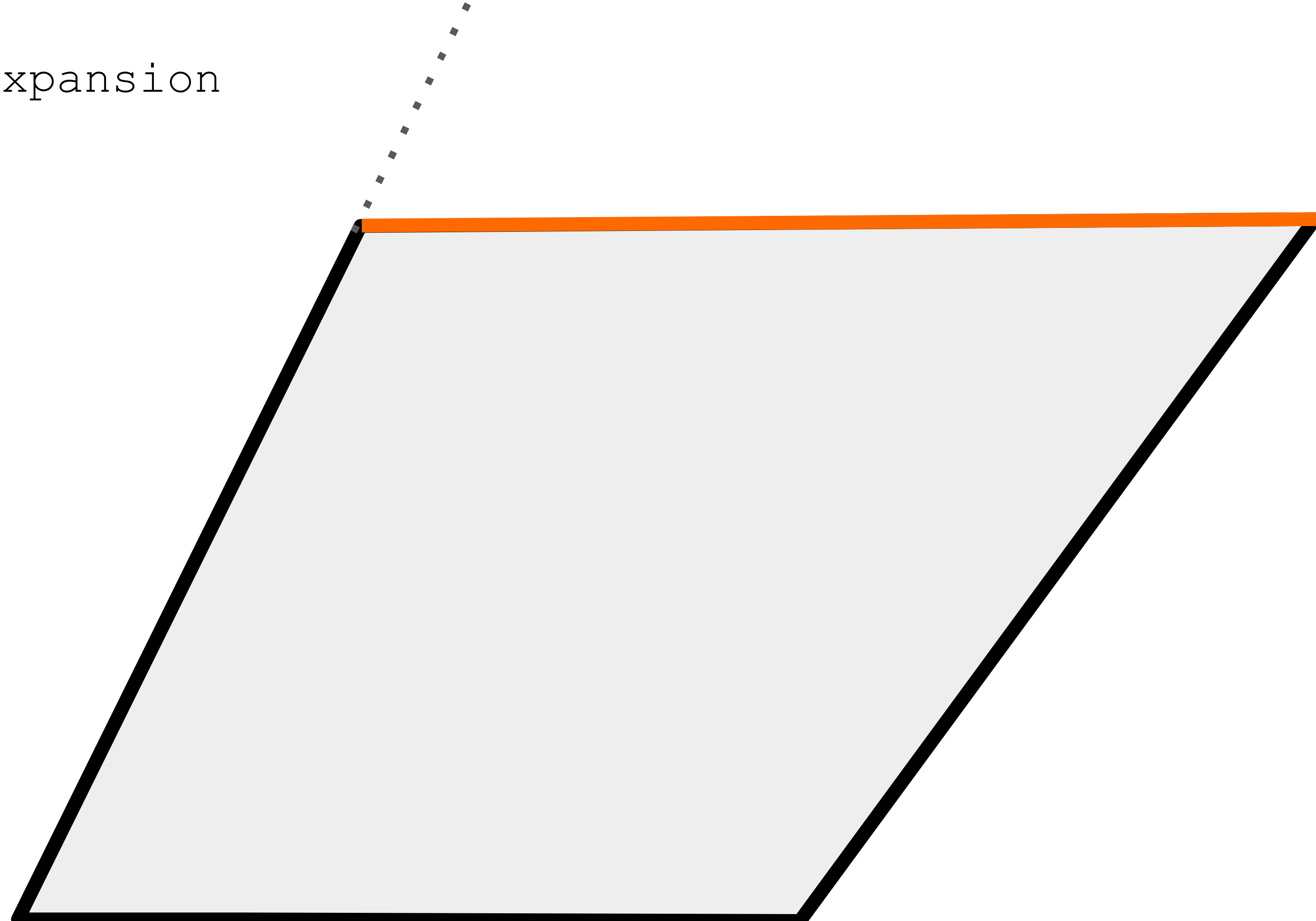

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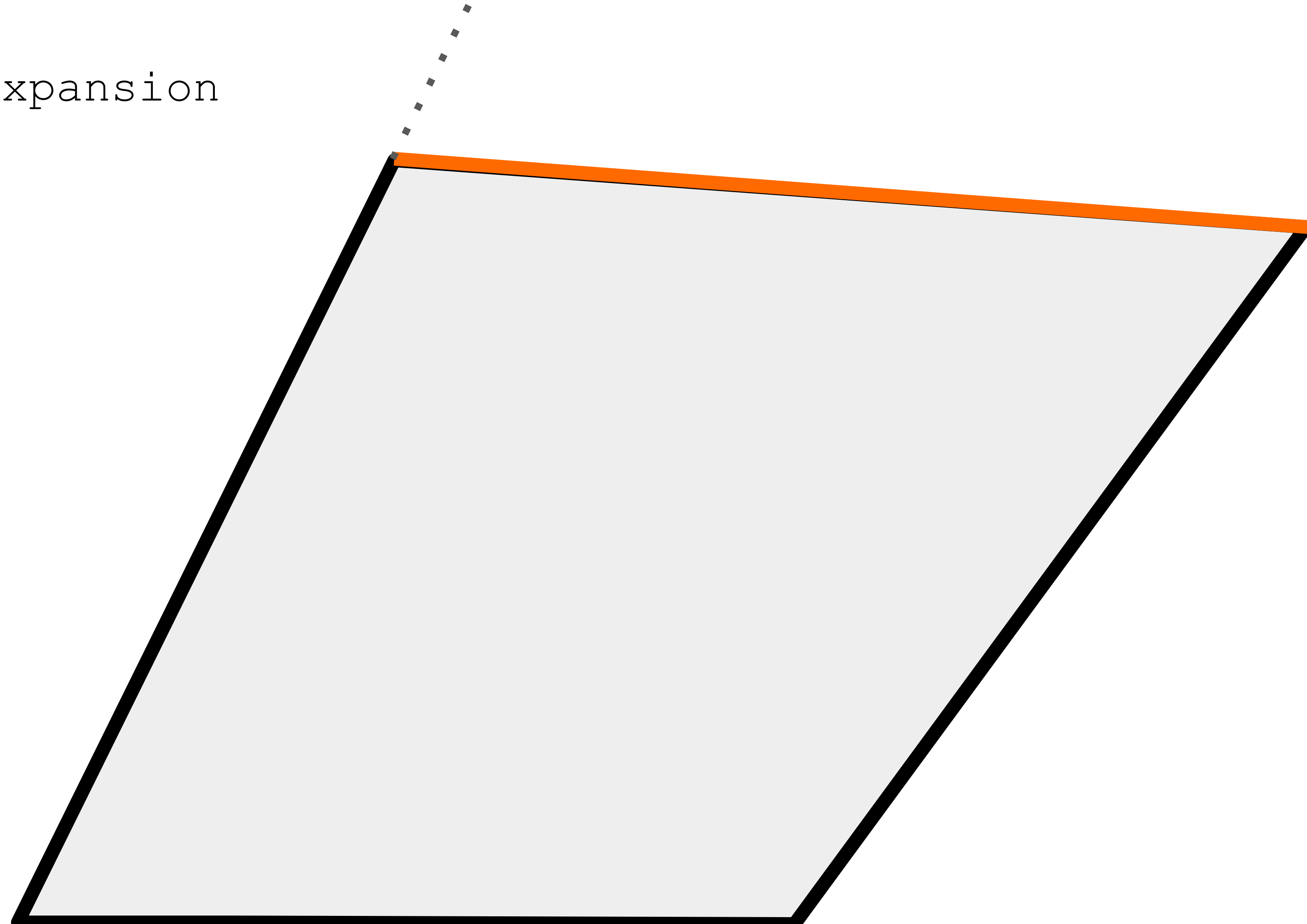

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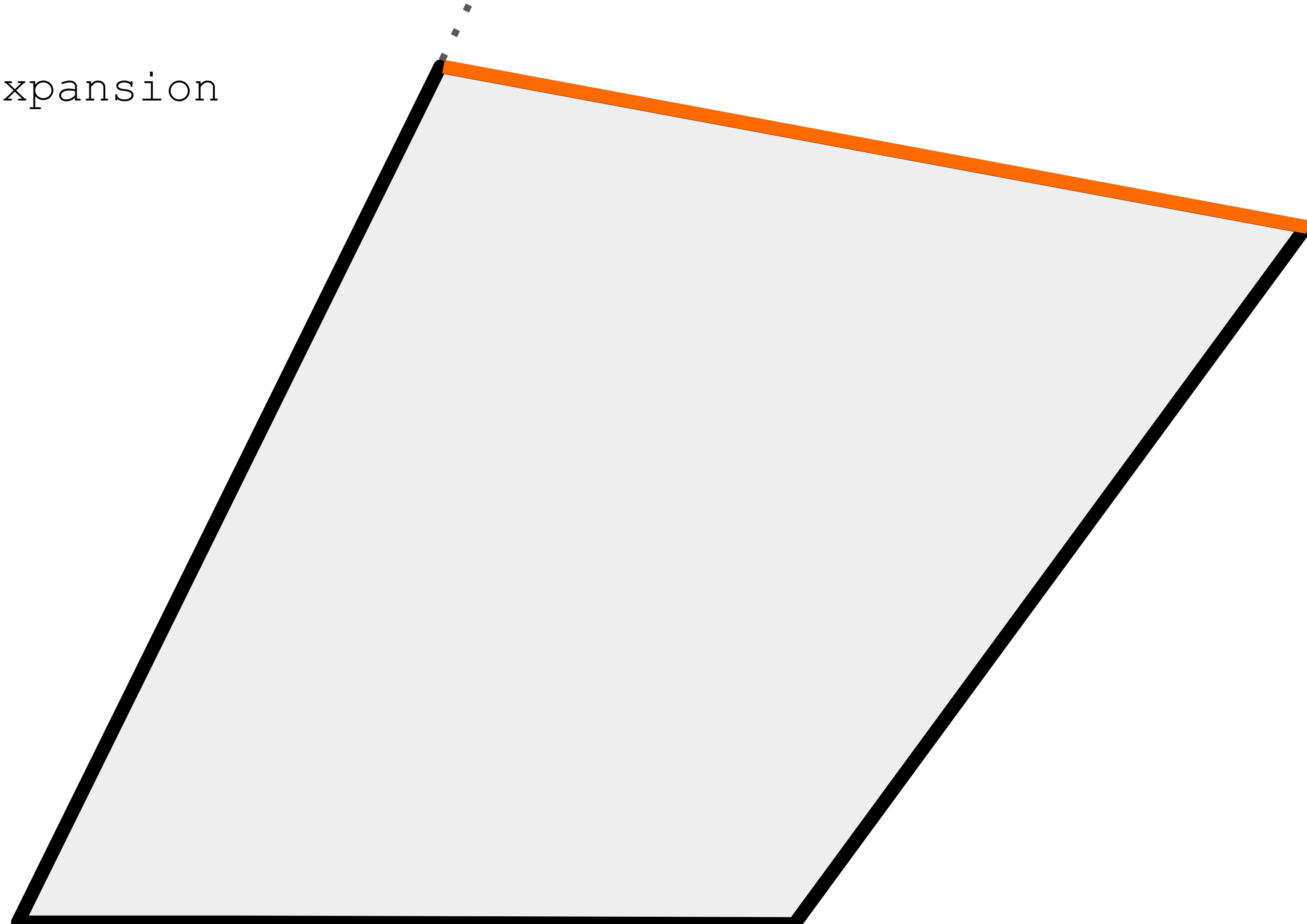

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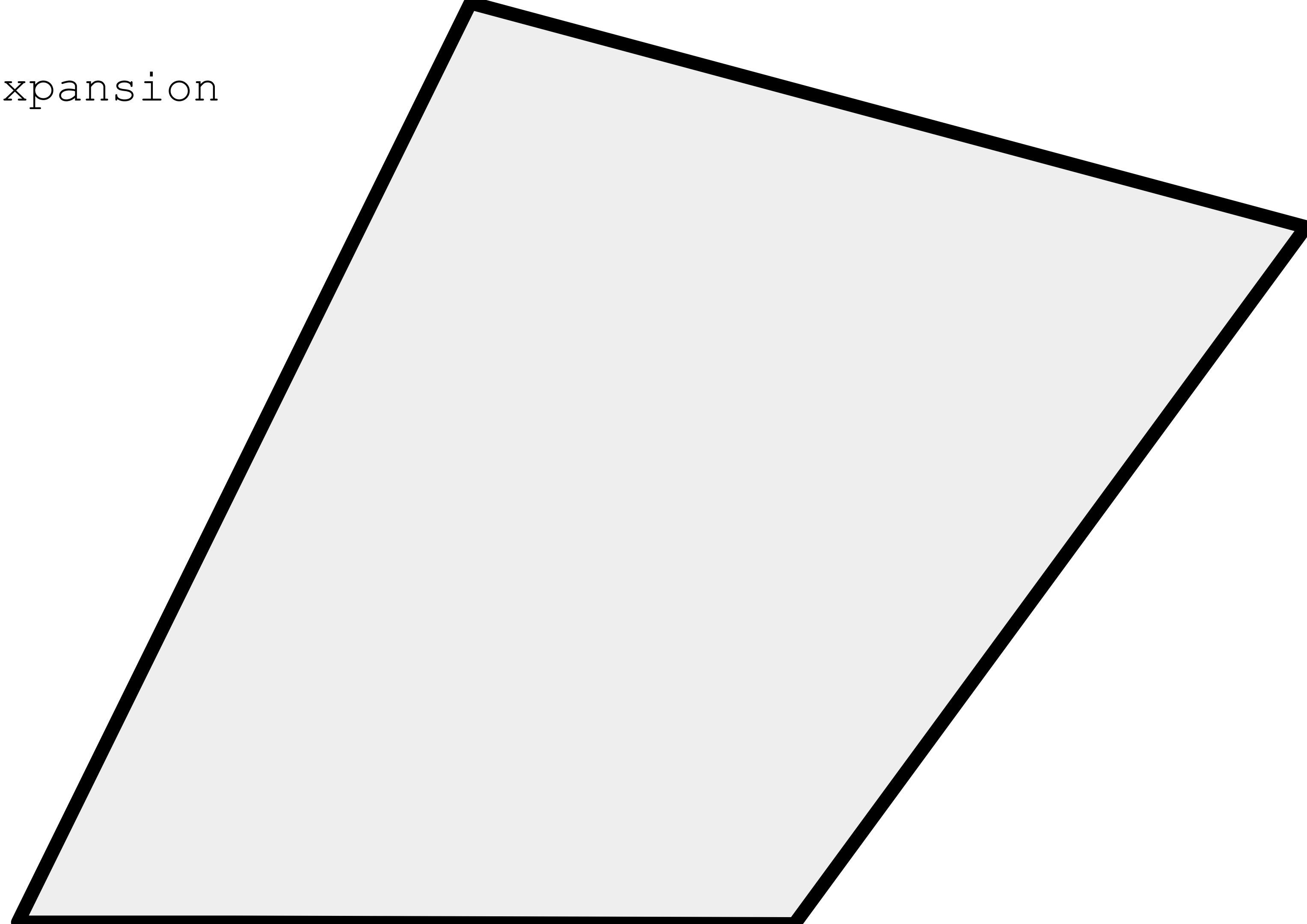


1. Each piece must be convex

2. None of the vertices of the covered polygon are inside the interior of any of the convex pieces

3. Every edge of every piece is either in the outer boundary or is a member of a different piece, oriented in the opposite direction

1 Generating instances for the competition

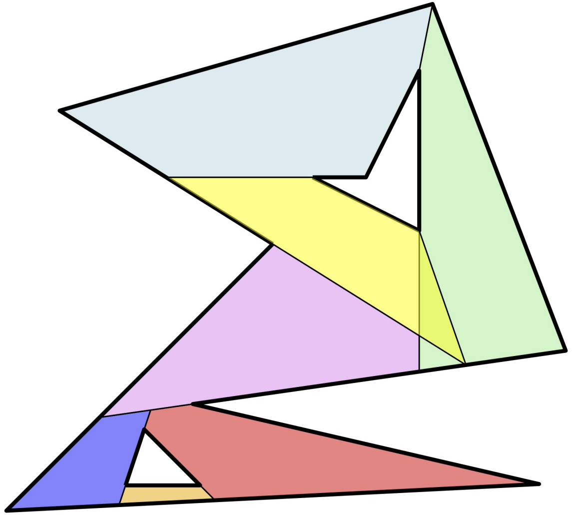
2 Baseline solution

3 Winning solutions

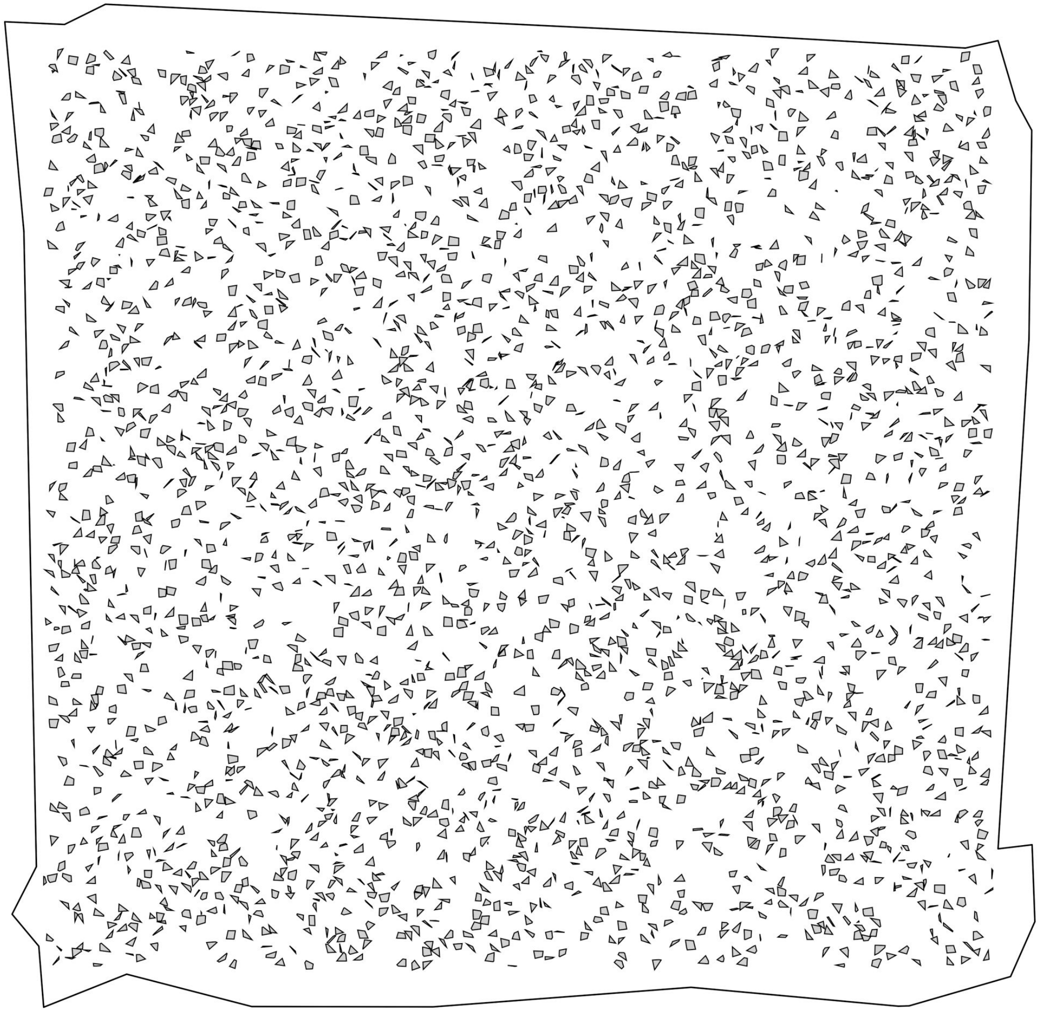
CG:SHOP Challenge 2023

Minimum Coverage by Convex Polygons

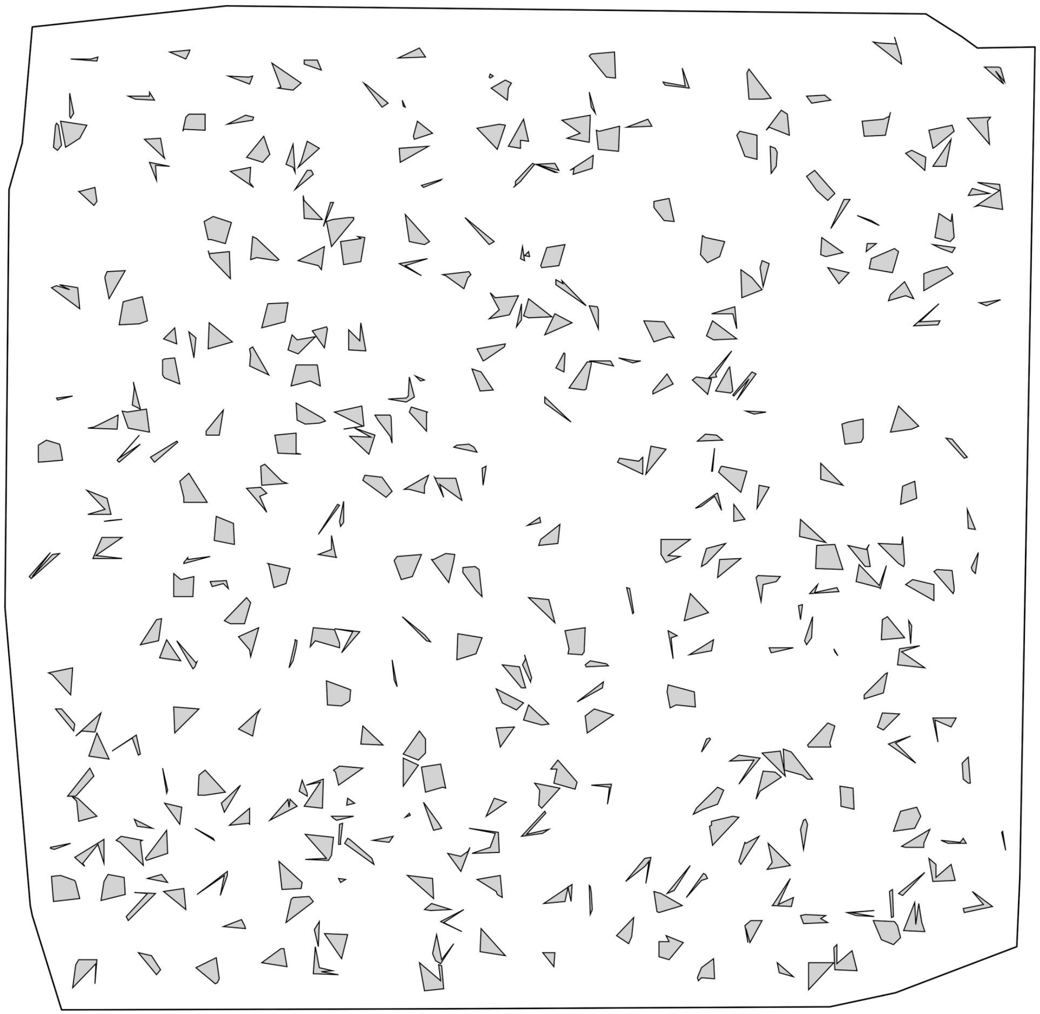
(Fekete, Keldenich, Krupke & Schirra 2023)



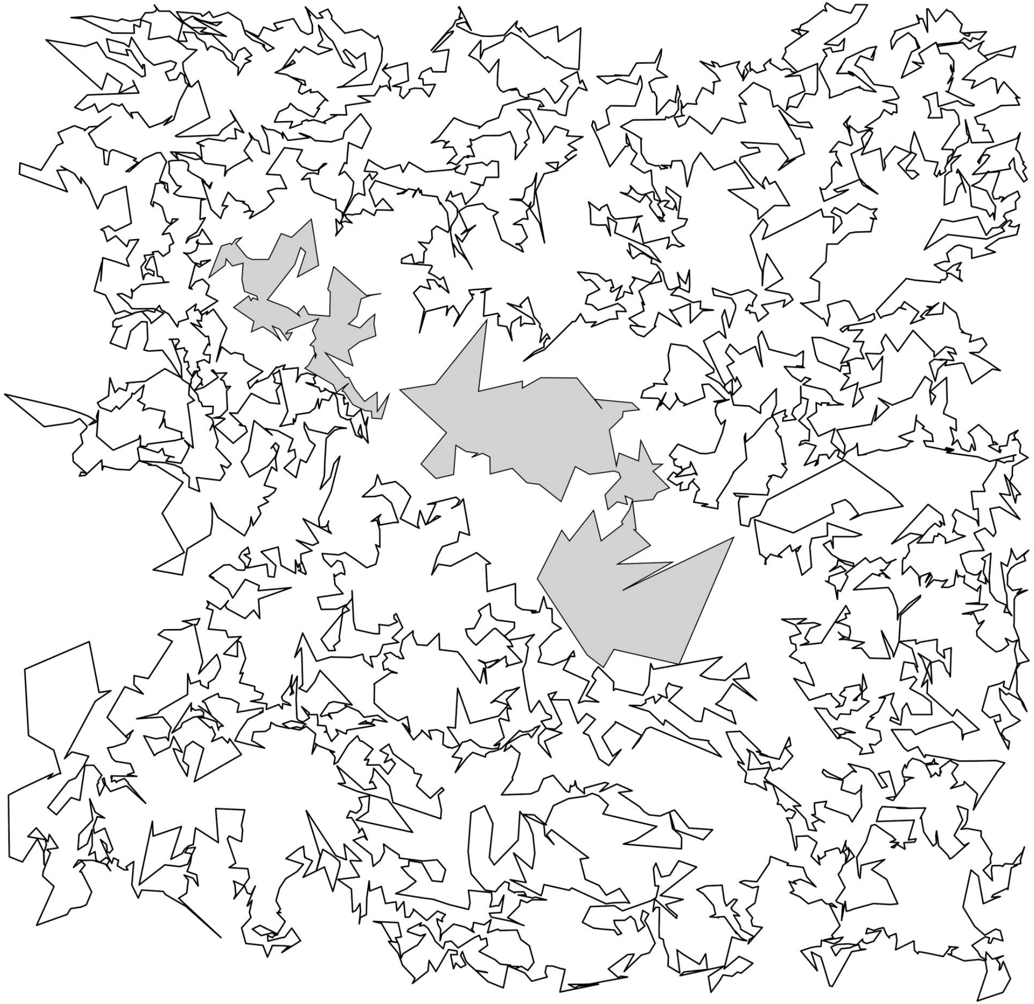
1 Large number of holes



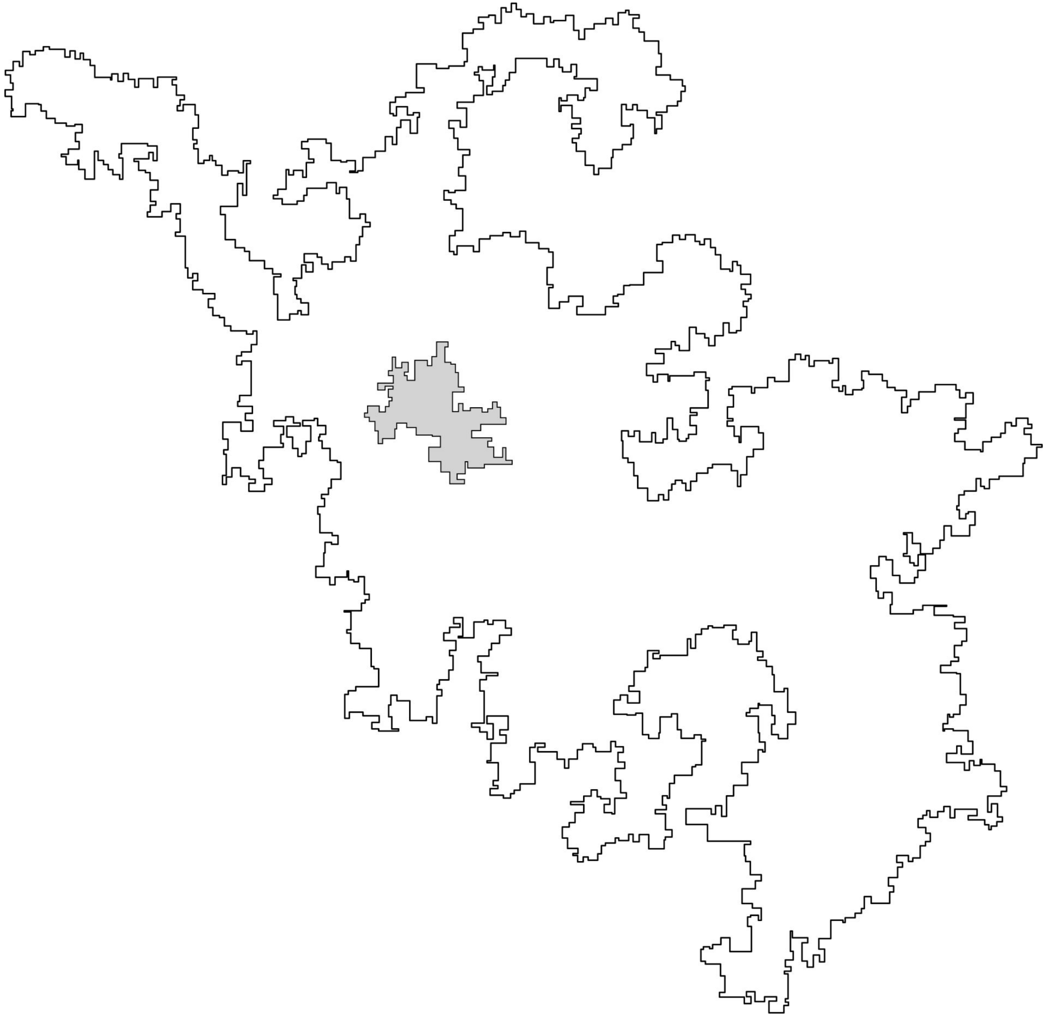
2 Large number of convex holes



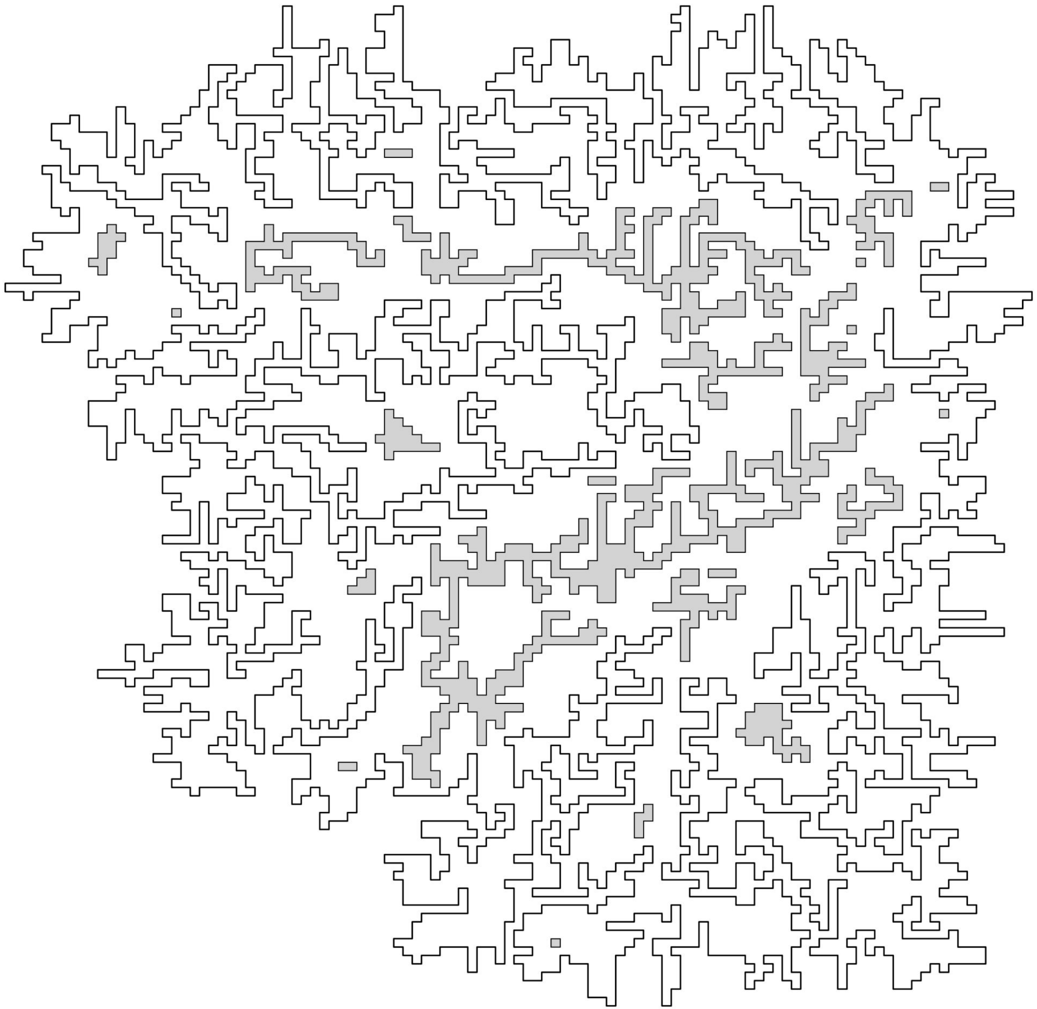
3 Triangulation perturbations



4 Orthogonal polygons

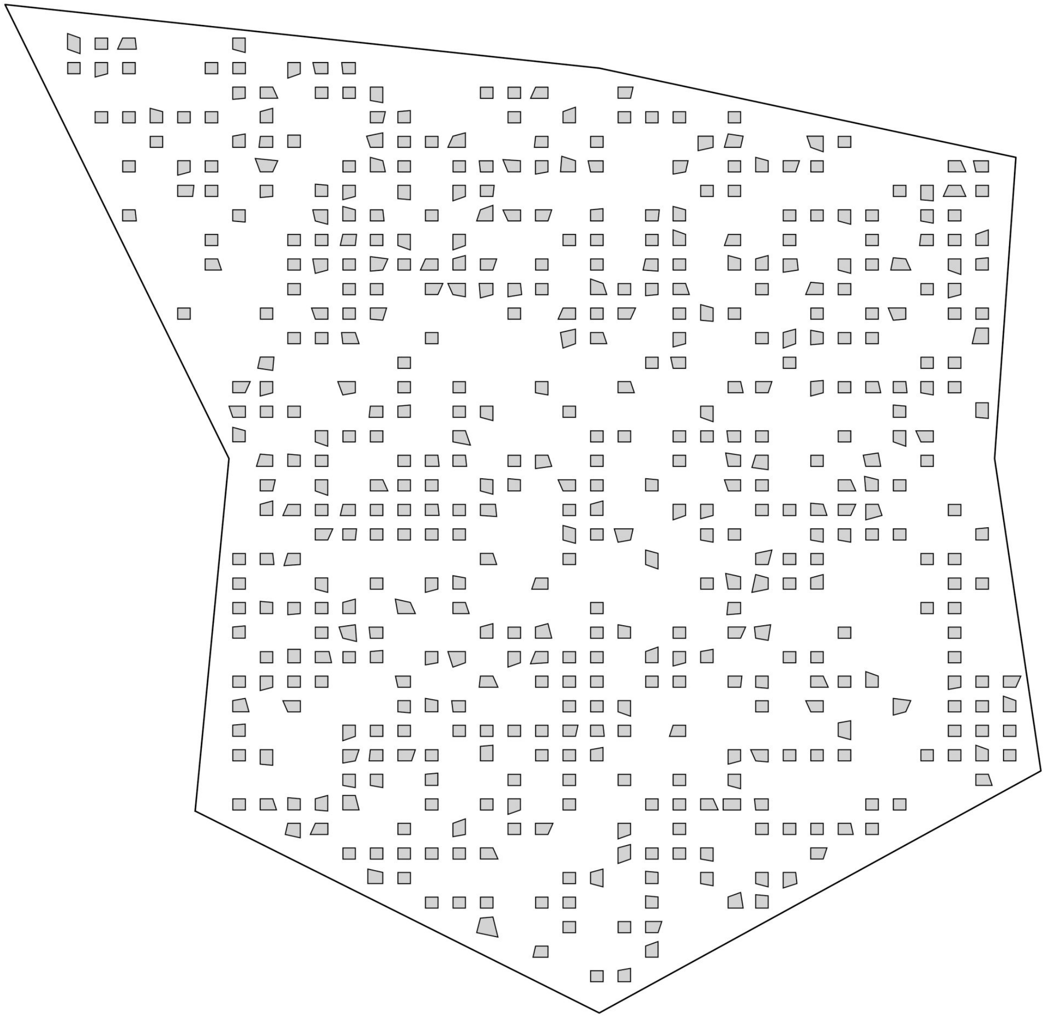


5 Orthogonal polygons with many points sharing a coordinate

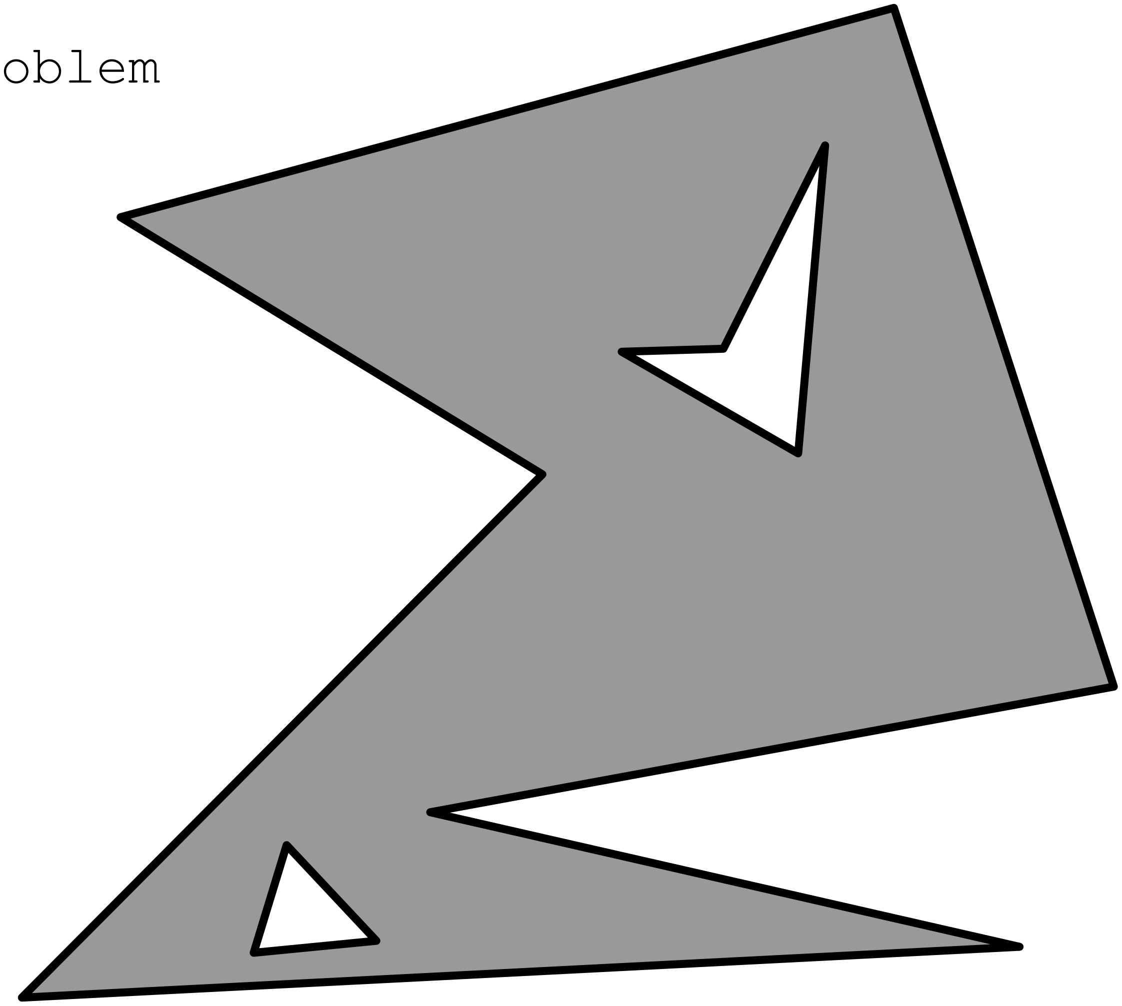


6

A pattern requiring highly overlapping polygons in solution

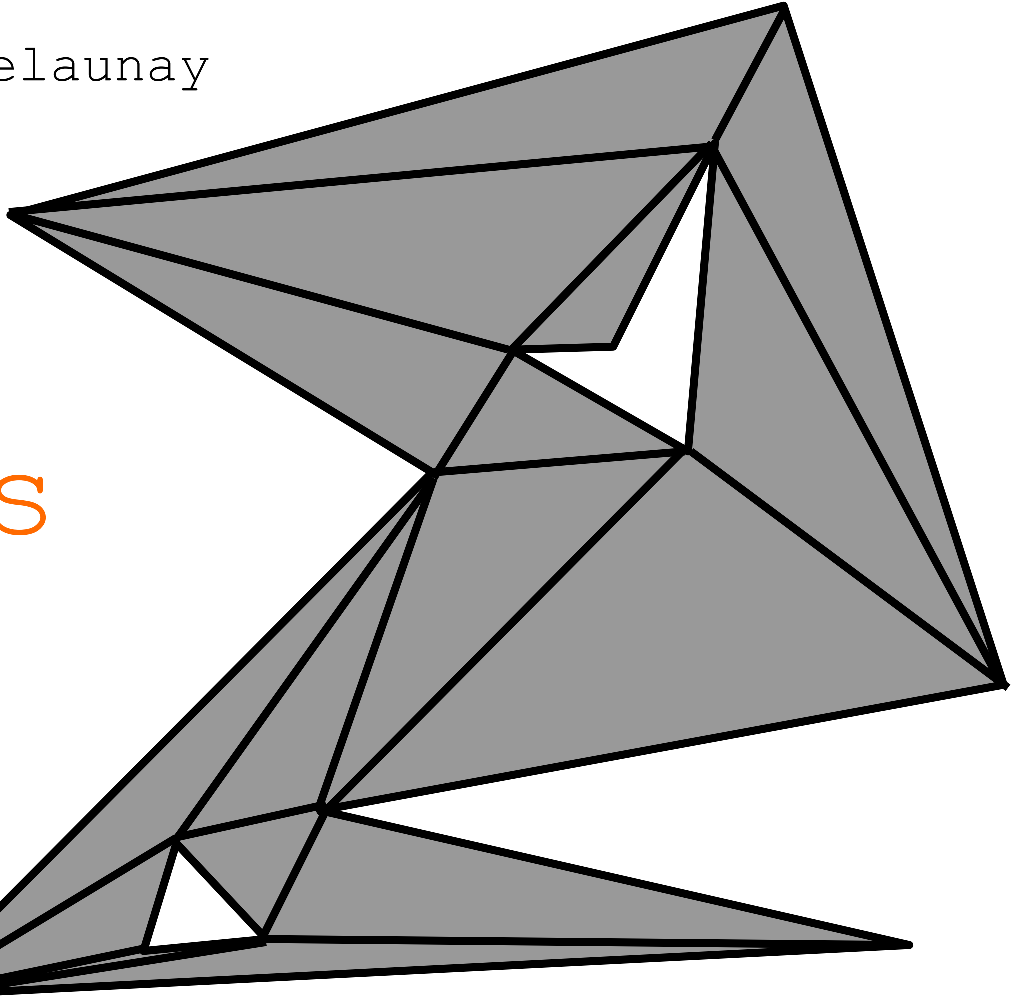


Example Problem
Instance



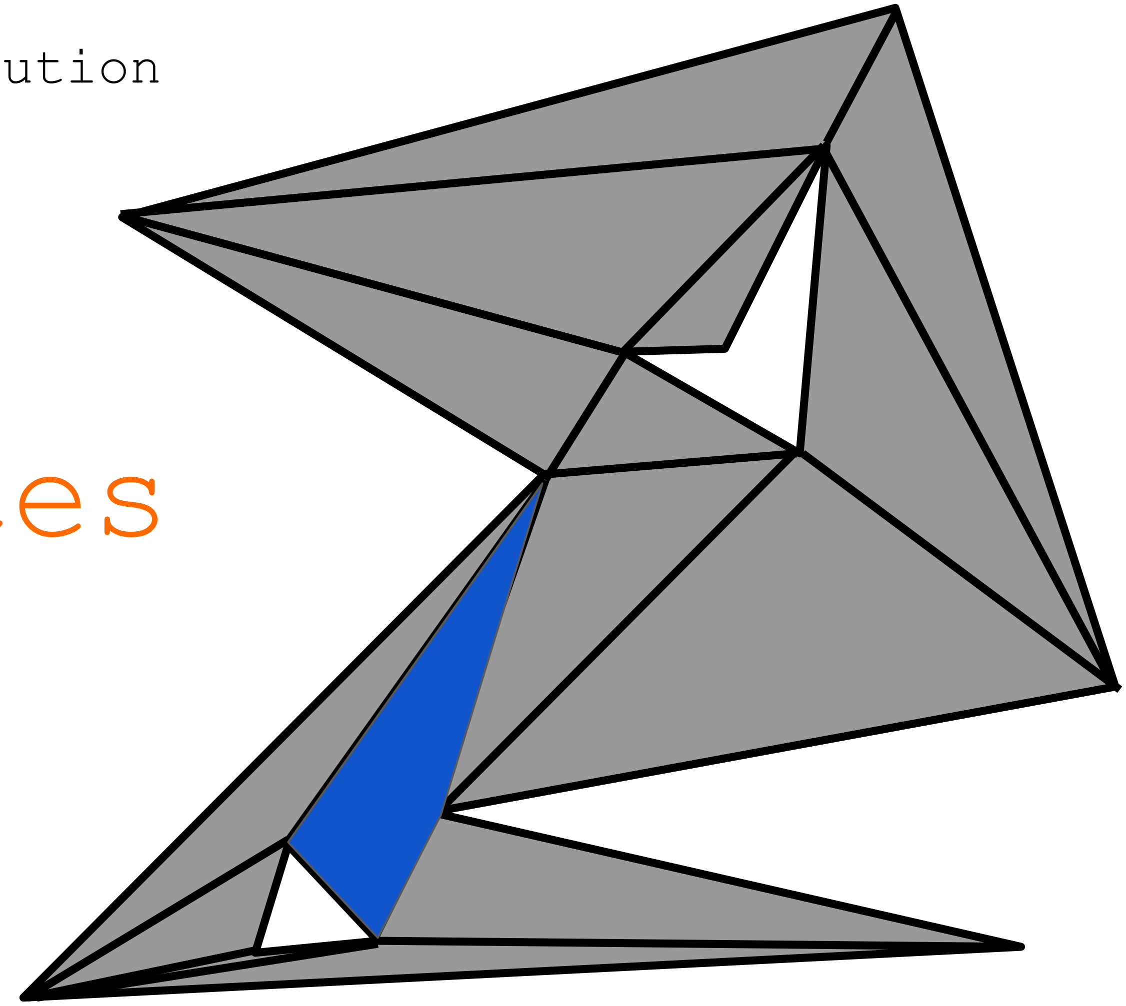
Constrained Delaunay Triangulation

16 pieces



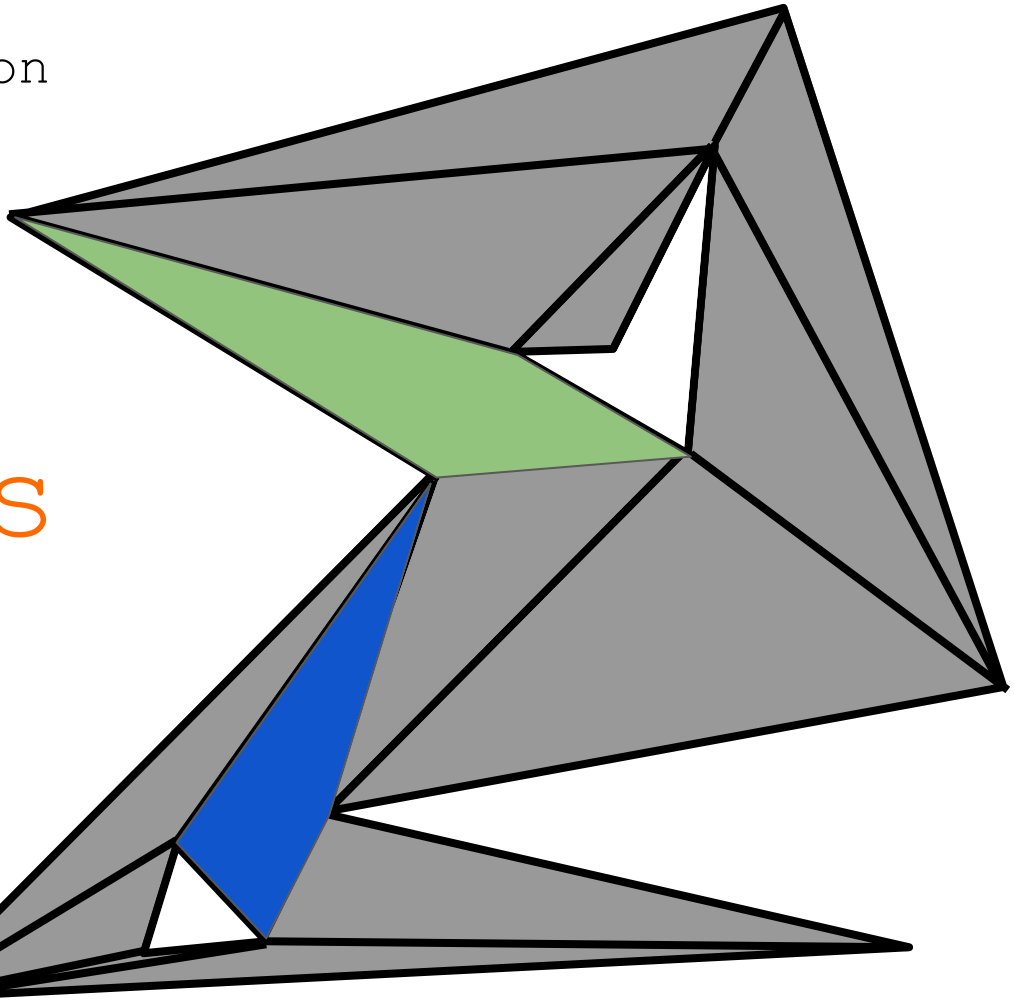
Greedy Solution

15 pieces



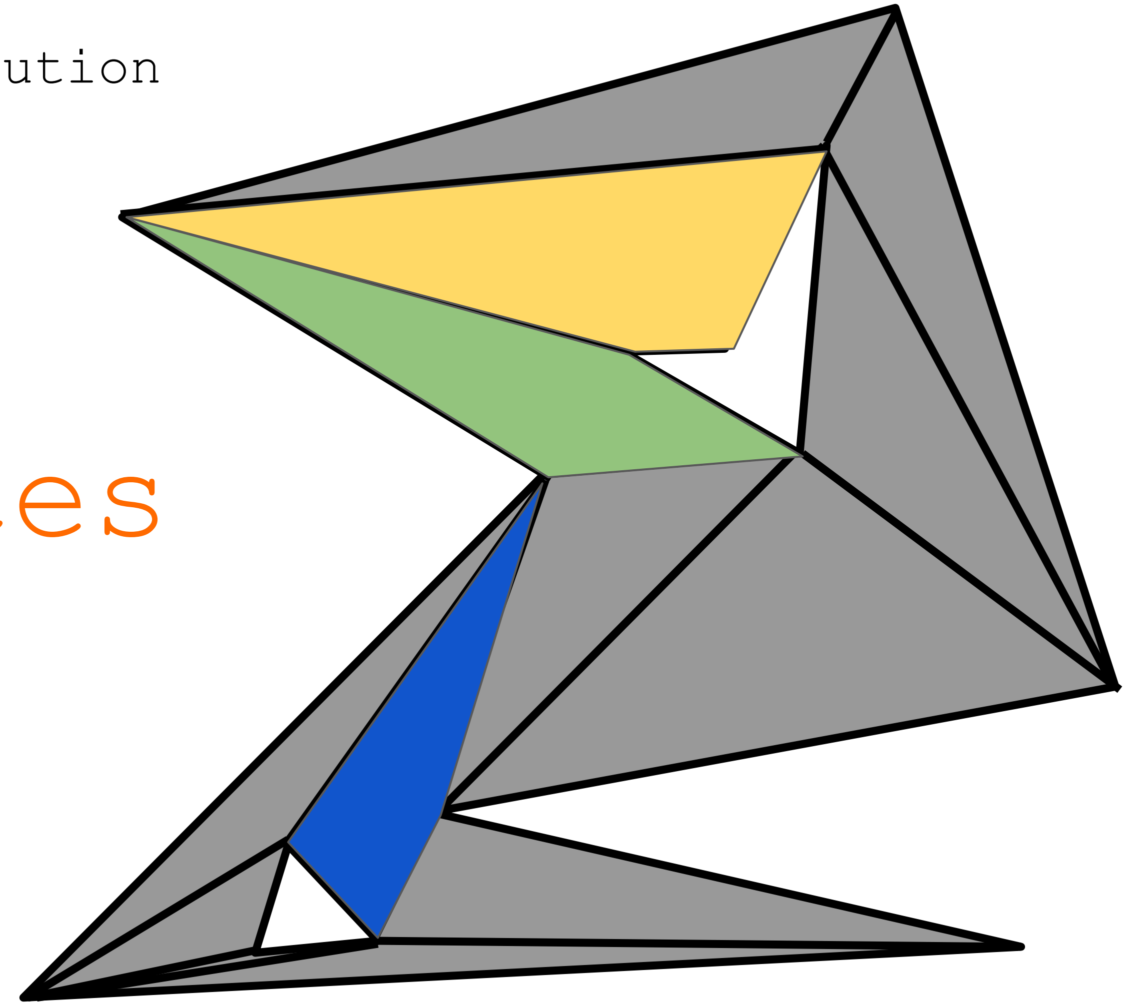
Greedy Solution

14 pieces



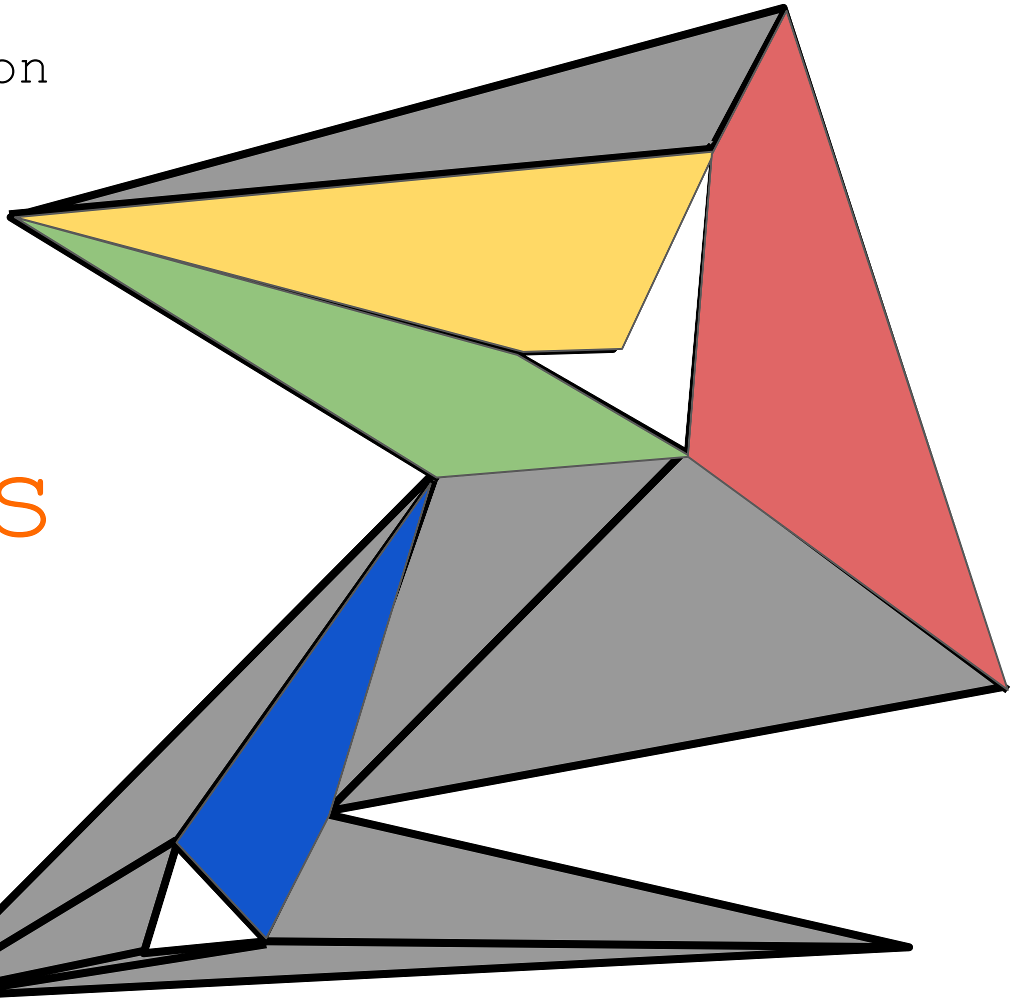
Greedy Solution

13 pieces



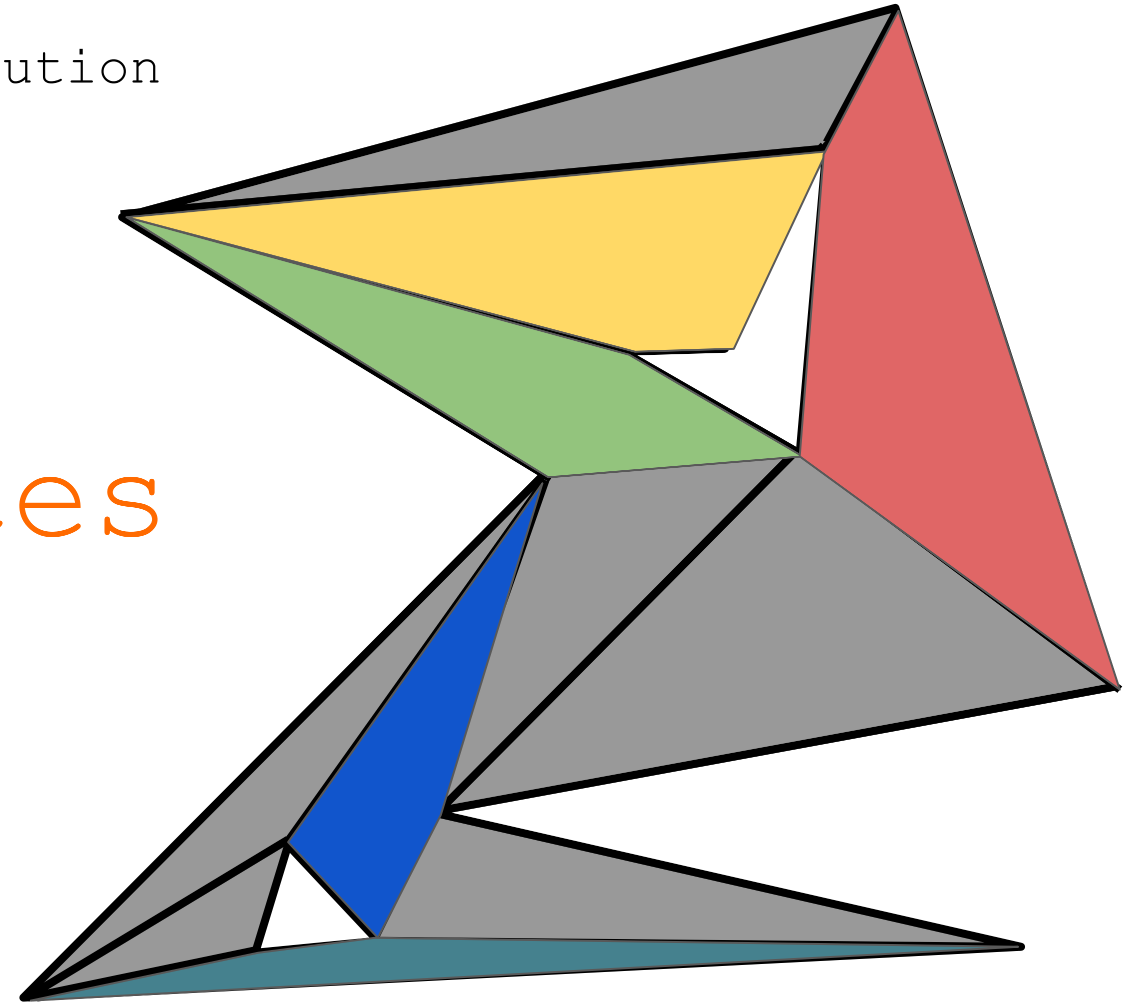
Greedy Solution

12 pieces



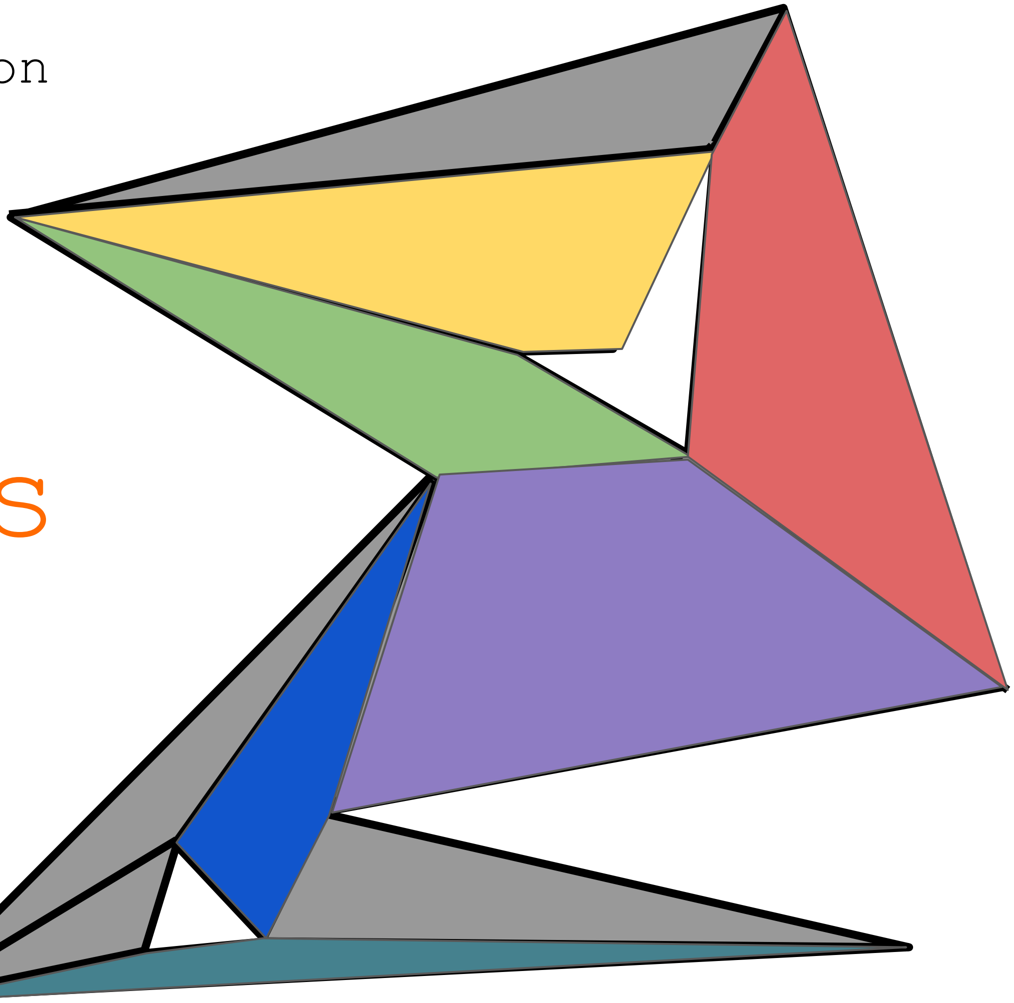
Greedy Solution

11 pieces



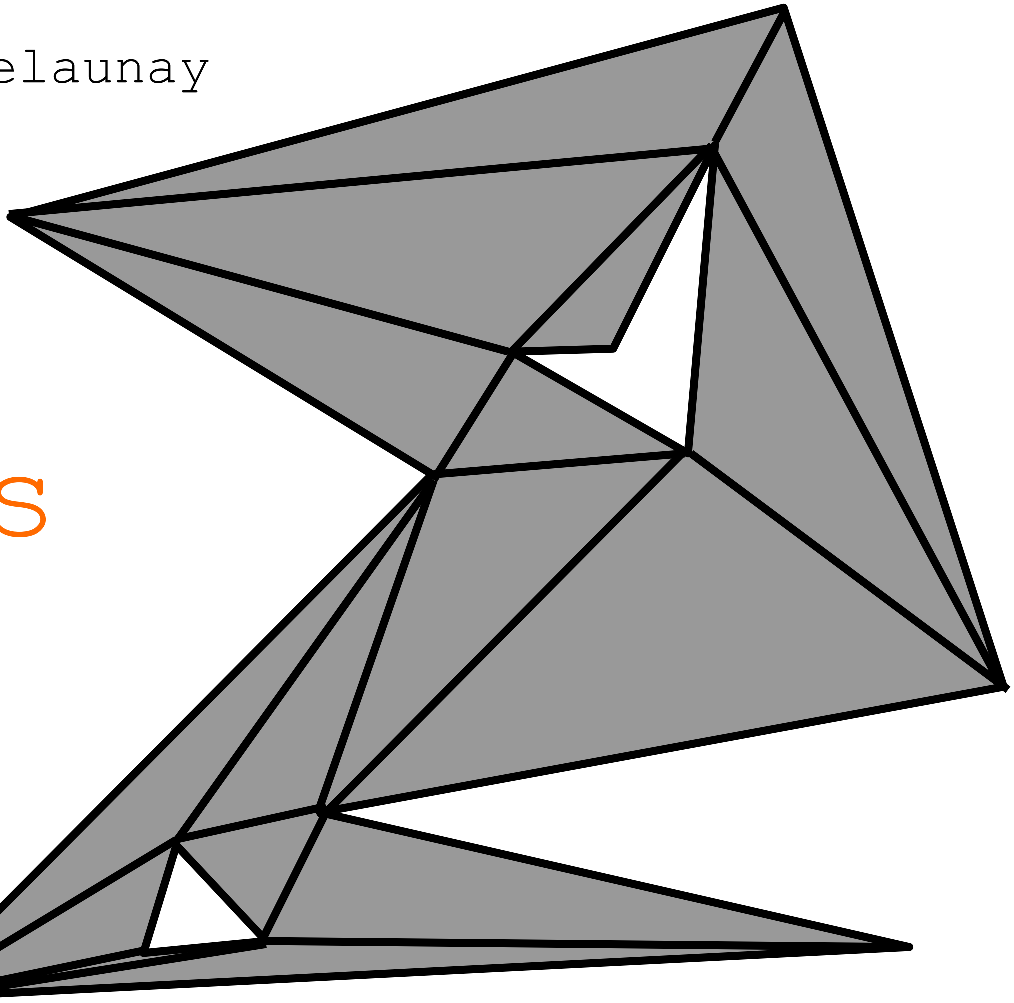
Greedy Solution

10 pieces

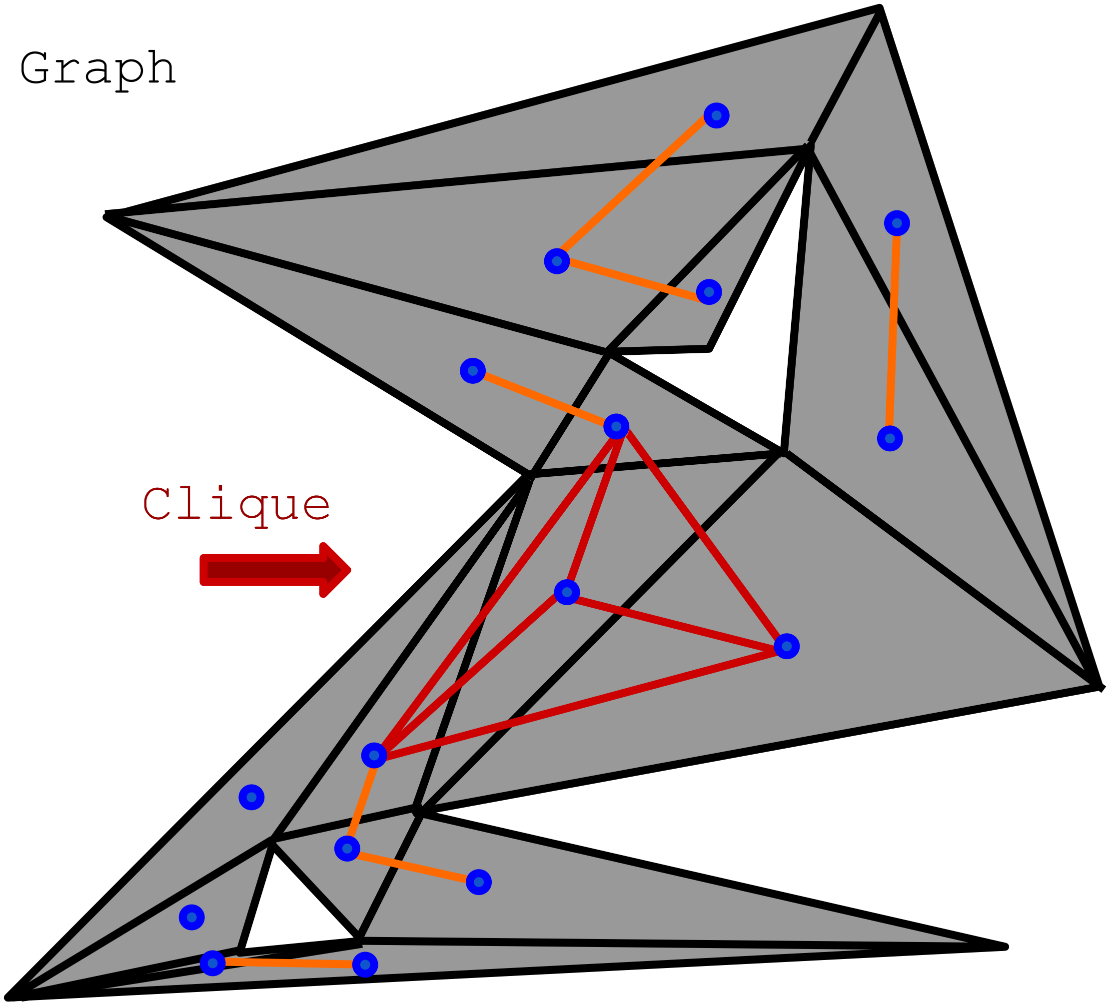


Constrained Delaunay Triangulation

16 pieces

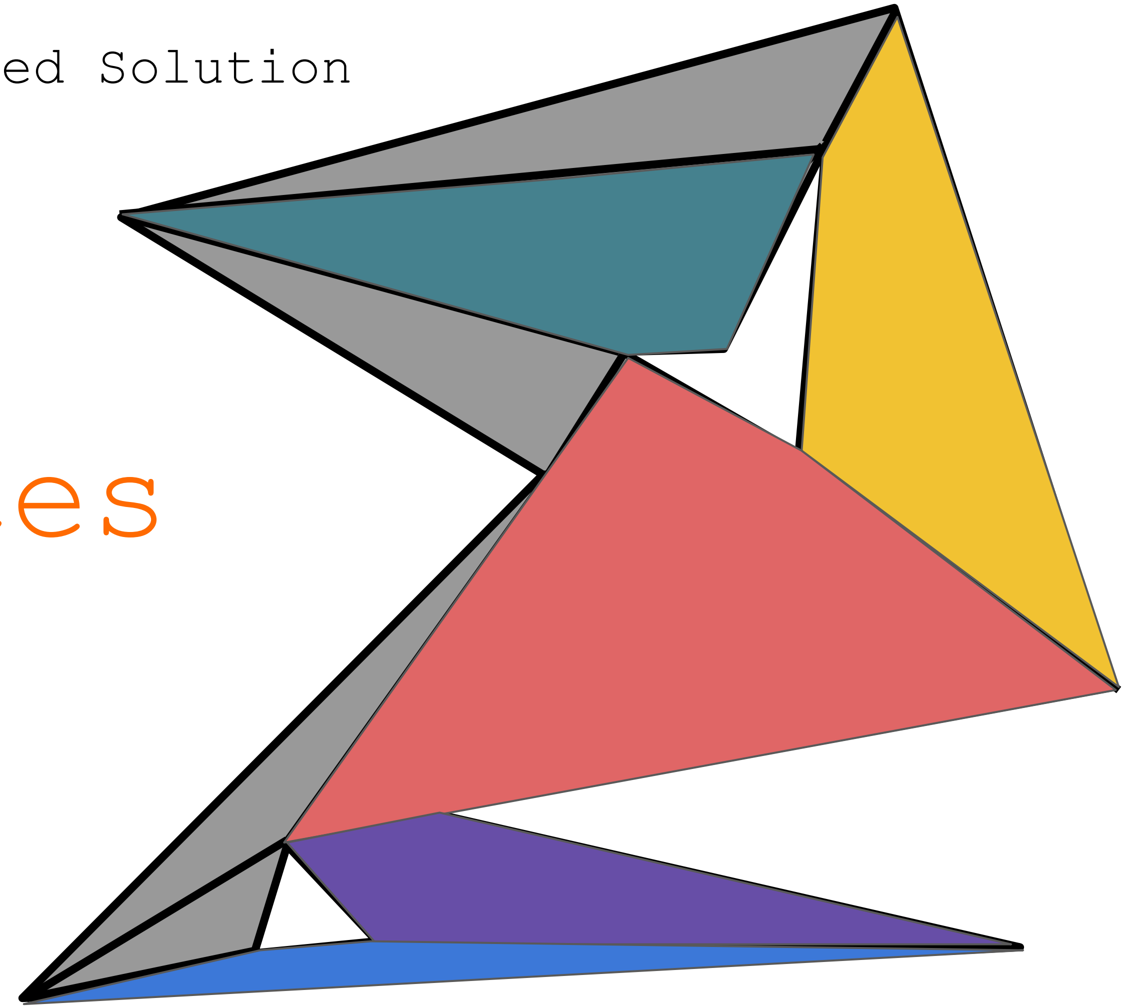


Visibility Graph



Clique Based Solution

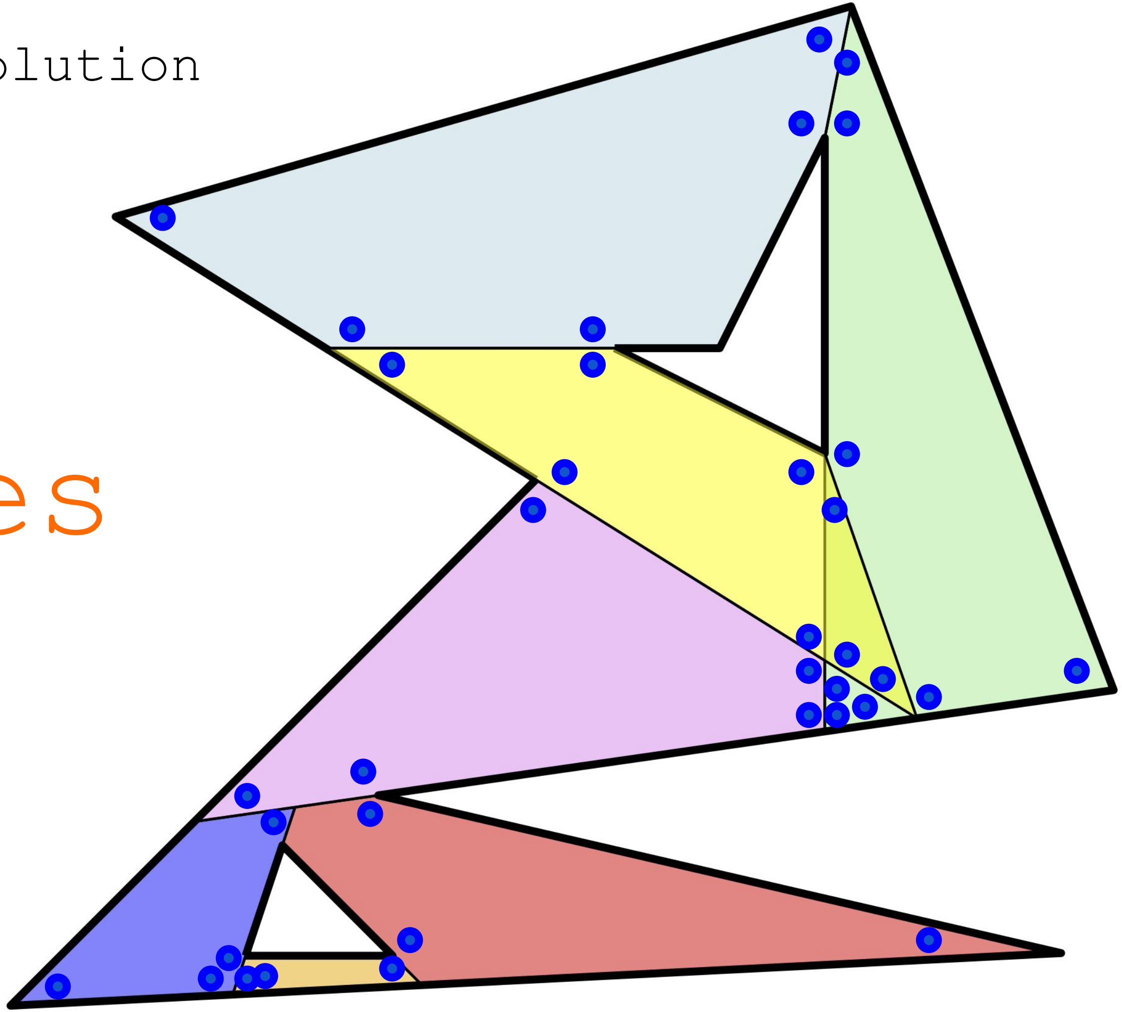
9
pieces



Set Cover Solution

7

pieces



Higher values mean better solutions
Mean score
1.00
0.95
0.90
0.85
0.80
0.75
10^2
10^3
10^4
10^5
Number of vertices in instance
Instance
ccheese
cheese
fpg
maze
srpg
srpg_iso
srpg_iso_aligned
srpg_octa
srpg_smo
srpg_smr
Team
DIKU (AMW)
Shadoks

1 Divide and Conquer algorithm

2 Add “infinite” vertices at at the corners of vertical strips

3 Any Delaunay edge of the graph must appear in a vertical strip containing the edge’s endpoints

Constrained Delaunay Triangulations
(Chew 1989)

O(nlogn)