**Problem 3 Report**

For project repository, please check: <https://github.com/DariMe20/Game-Theory-Applications>

**Introduction**

The objective of the game described in Problem 3 is to explore the concept of *subgame perfect Nash equilibrium (SPNE)* and determine if there are any Nash equilibrium that are not subgame perfect within the game described.

**Game Description**

1. Player 1 is tasked with dividing $10 with Player 2.
2. Player 1 proposes a division of the $10.
3. Player 2 decides whether to accept or reject the proposed division.
4. If Player 2 accepts, both players receive the amounts agreed upon.
5. If Player 2 rejects, both players receive nothing.

The game is a *one-shot* interaction, meaning there is no back-and-forth bargaining.

**What is the subgame perfect Nash equilibrium? Can you find a Nash equilibrium that is not subgame perfect in the normal form game associated to the game just described before?**

**Theoretical Framework**

A **Nash equilibrium** is reached when each player chooses their best strategy, given the choices of the other player.

A **subgame perfect Nash equilibrium (SPNE)** is a refinement of Nash equilibrium, ensuring that players make optimal decisions at every possible stage of the game, including every subgame. In other words, a strategy profile is SPNE if it constitutes a Nash equilibrium not only for the entire game but also for any subgame that might arise.

**The payoff matrix** for this game can be described as:

Player 2

**Accept Reject**

Player 1 **Offer** (x, 10-x) (0, 0)

**Game Analysis**

The **Subgame Perfect Nash Equilibrium (SPNE)** for this game is found using **backward induction**. We start from the last move (where Player 2 decides) and work backward to Player 1’s proposal. Player 2 has 2 options:

Option 1: accept offer => **payoff(Player2) = 10-x** where x is the sum proposed by Player1, and 0<=x<=10. This means that  **0 <= payoff(Player2)[‘accept’] <= 10.**

**Also, payoff for both players is = (x, 10-x).**

Option 2: reject the offer => Player 2 receives 0 dollars, which is the worst outcome.

=> **payoff(Player2)[‘accept’] = 0 and payoff for both players is = (0, 0).**

Knowing this, Player 1 must make a proposal that is acceptable to Player 2 while still maximizing their own payoff and not risking offering 0 dollars to win 10 dollars and player 2 rejecting to reduce the win of both to 0.

Since Player 2 will accept any offer greater than 0, Player 1 can propose 1 dollar for Player 2 and keep 9 dollars for themselves. This is the most efficient proposal for Player 1, as it maximizes their payoff without being rejected by Player 2.

1. **Subgame Perfect Nash Equilibrium**

In this game, the SPNE is as follows:

* **Player 1 proposes 9 dollars for themselves and 1 dollar for Player 2.**
* **Player 2 accepts the offer**.

This is the SPNE because it is rational for Player 2 to accept any positive amount, and Player 1 knows that Player 2 will accept a proposal of 1 dollar, thereby ensuring Player 1 gets the most they can (9 dollars).

1. **Nash Equilibrium that is Not Subgame Perfect**

In the normal form, Player 1 might propose a fairer split, for example, **$5 to Player 2** and **$5 for themselves**. If Player 2 accepts the offer, this would be a **Nash equilibrium**, as neither player would want to change their strategy. Player 1 wouldn't risk proposing less than $5 for Player 2 (Player 1 wants to avoid a rejection from Player 2), and Player 2 wouldn't want to reject the offer, since they would get nothing.

However, this is **not subgame perfect** because in the **actual sequential game**, backward induction shows that Player 1 would propose to keep a bigger amount of money ($9 for themselves and $1 for Player 2), since Player 2 will accept any amount.