$$\frac{\partial^{37} f}{\partial x^{10}\partial y^{27}} = \frac{\partial^{10}}{\partial x^{10}}\left(\frac{\partial^{27} f}{\partial y^{27}}\right)$$

$$f(x,y) = \underbrace{(3x-2y)}_{g}\underbrace{\cos(x+y)}_{h}$$

$g' = -2$

$g^{(n)} = 0, \ \forall\ n \geq 2$

$$\frac{\partial^{27} f}{\partial y^{27}} = C_{27}^{0}\, g\, h^{(27)} + C_{27}^{1}\, g'\, h^{(26)} =$$

$$= 1\cdot \underbrace{(3x-2y)}_{u}\,\underbrace{\cos\left(x+y+\frac{27\pi}{2}\right)}_{v} + 27(-2)\cos(x+y+13\pi)$$

$\frac{\partial}{\partial x^{10}}$

$u' = 3$

$u^{(n)} = 0, \ \forall\ n \geq 2$

$-54\cos\left(x+y+13\pi+5\pi\right)$

$(e^x)^{(n)} = e^x$

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 \end{cases}$$

$$\frac{x^2+y^2}{2} \geq \sqrt{x^2+y^2}$$

i) f - cont?

$$\left|\frac{xy^2}{x^2+y^2}\right| \leq \left|\frac{xy^2}{2xy}\right| = \frac{|y|}{2} \to 0$$

ii) f - dif în $(0,0)$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0}\frac{f(x,0)-f(0,0)}{x-0} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = 0$$

- $\exists\ \omega : \mathbb{R}^2 \to \mathbb{R}$ continuă și nulă în $(0,0)$

$f(x,y) = f(0,0) + \frac{\partial f}{\partial x}(0,0)(x-0) + \frac{\partial f}{\partial y}(0,0)(y-0) +$

• $\exists\ \omega : \mathbb{R}^2 \to \mathbb{R}$ continuă și nulă în $(0,0)$

$$f(x,y) = f(0,0) + \frac{\partial f}{\partial x}(0,0)(x-0) + \frac{\partial f}{\partial y}(0,0)(y-0) +$$
$$+\ \omega(x,y)\sqrt{(x-0)^2+(y-0)^2}$$

$$f(x,y) = 0 + 0 + 0 + \omega(x,y)\sqrt{x^2+y^2}$$

$$\omega(x,y) = \frac{f(x,y)}{\sqrt{x^2+y^2}} = \begin{cases} \dfrac{xy^2}{(x^2+y^2)\sqrt{x^2+y^2}}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

$$(x_n, y_n) = \left(\frac{1}{n}, \frac{1}{n}\right)$$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{1}{n^3}}{\frac{2}{n^2}\cdot\frac{\sqrt{2}}{n}} = \frac{1}{2\sqrt{2}}$$

$$(x_n', y_n') = \left(\frac{1}{n}, \frac{1}{\sqrt{n}}\right)$$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{1}{n^2}}{\left(\frac{1}{n^2}+\frac{1}{n}\right)\sqrt{\frac{1}{n^2}+\frac{1}{n}}} = \dots$$

$\Rightarrow \nexists \lim_{x,y\to(0,0)} \omega(x,y)$

$\Rightarrow \omega$ - nu e cont în origine

$\Rightarrow f$ - nu e dif

$$\frac{\partial f}{\partial s}(0,0) \quad \text{unde } \vec{s} = -\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$$

$$= \frac{\partial f}{\partial x}(0,0)\cdot\left(-\frac{1}{\sqrt{2}}\right) + \frac{\partial f}{\partial y}(0,0)\cdot\frac{1}{\sqrt{2}}$$

$$\vec{v} = -\vec{i} + 3\vec{j} - 2\vec{k}$$

$$d^2_{x_0} f(x,y) = \frac{\partial^2 f}{\partial x^2}(x,y)\,dx^2 + 2\cdot\frac{\partial f}{\partial x\partial y}(x,y)\,dx\,dy$$
$$+\ \frac{\partial^2 f}{\partial y^2}(x,y)\,dy^2$$

$$f(x,y) = g(\overbrace{x\cos y}^{u};\ \overbrace{x^2y^3}^{v})$$

$$\frac{\partial^2 f}{\partial x\partial y} \qquad \frac{\partial f}{\partial x} = \frac{\partial g}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$\begin{cases} \dfrac{\partial f}{\partial x} = \cos y\,\dfrac{\partial g}{\partial u} + 2xy^3\,\dfrac{\partial g}{\partial v} \\ \dfrac{\partial f}{\partial y} = -x\sin y\,\dfrac{\partial g}{\partial u} + 3y^2x^2\,\dfrac{\partial g}{\partial v} \end{cases}$$

$$\frac{\partial f}{\partial x} = \cos y \frac{\partial g}{\partial u} + 2xy^3 \frac{\partial g}{\partial v}$$

$$\frac{\partial f}{\partial y} = -x \sin y \frac{\partial g}{\partial u} + 3y^2x^2 \frac{\partial g}{\partial v}$$

$$\partial_x \cdot = \cos y \frac{\partial \cdot}{\partial u} + 2xy^3 \frac{\partial \cdot}{\partial v}$$

$$\partial_y \cdot = -x \sin y \frac{\partial \cdot}{\partial u} + 3y^2x^2 \frac{\partial \cdot}{\partial v}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(-x \sin y \frac{\partial g}{\partial u} + 3y^2x^2 \frac{\partial g}{\partial v}\right) =$$

$$= -\sin y \frac{\partial g}{\partial u} - x \sin y\, \partial_x \cdot \frac{\partial g}{\partial u} + 6xy^2 \frac{\partial g}{\partial v} + 3x^2y^2\, \partial_x \cdot \frac{\partial g}{\partial v} =$$

$$= -\sin y \frac{\partial g}{\partial u} - x \sin y \left(\cos y \frac{\partial}{\partial u} \cdot \frac{\partial g}{\partial u} + 2xy^3 \frac{\partial}{\partial v}\frac{\partial g}{\partial u}\right)$$

$$+ 6xy^2 \frac{\partial g}{\partial v} + 3x^2y^2 \left(\cos y \frac{\partial}{\partial u}\frac{\partial g}{\partial v} + 2xy^3 \frac{\partial}{\partial v}\frac{\partial g}{\partial v}\right)$$

$$= -\sin y \frac{\partial g}{\partial u} - x \sin y \left(\cos y \frac{\partial^2 g}{\partial u^2} + 2xy^3 \frac{\partial^2 g}{\partial v \partial u}\right)$$

$$+ 6xy^2 \frac{\partial g}{\partial v} + 3x^2y^2 \left(\cos y \frac{\partial^2 g}{\partial u \partial v} + 2xy^3 \frac{\partial^2 g}{\partial v^2}\right)$$

4. i) *Să se demonstreze că funcţia* $f : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$,

$$f(x,y) = \frac{x^2 y}{x^4 + y^2}$$

are limită în origine după orice direcţie $h = (h_1, h_2) \in \mathbb{R}^2 \setminus \{(0,0)\}$, *dar limita lui* f *în origine nu există.*

$$\left|\frac{x^2 y}{x^4+y^2} - 0\right| \le \left|\frac{\cancel{x^2 y}}{2\cancel{x^2 y}}\right| = \frac{1}{2} \ne 0 \Rightarrow \nexists \lim f$$

$$l_{12} = 0$$

$$l_{21} = 0$$

iii) *Să se arate că pentru funcţia* $f : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$, $f(x,y) = x \sin\frac{1}{x} \cos\frac{1}{y}$, *o limită iterată există, cealaltă nu există şi totuşi* f *are limită în origine în raport cu ansamblul variabilelor.*

$$l_{12} = \lim_{x \to 0} \left(\lim_{y \to 0} x \sin\frac{1}{x} \cos\frac{1}{y} \right)$$

$$g(y) = \cos\frac{1}{y},\ y \neq 0$$

$$\left.\begin{array}{l} x_n = \frac{1}{2n\pi} \xrightarrow{n\to\infty} 0 \quad \lim\limits_{n\to\infty} g(x_n) = 1 \\ x_n' = \frac{1}{(2n+1)\pi} \xrightarrow{n\to\infty} 0 \quad \lim\limits_{n\to\infty} g(x_n') = -1 \end{array}\right\} \Rightarrow \nexists\, l_{12}$$

$$l_{21} = \lim_{y \to 0} \left(\lim_{x \to 0} x \sin\frac{1}{x} \cos\frac{1}{y} \right)$$

$$h(x) = x \sin\frac{1}{x}$$

$$\left| x \sin\frac{1}{x} - 0 \right| = |x| \underbrace{\left| \sin\frac{1}{x} \right|}_{\leq 1} \leq |x| \longrightarrow 0$$

$$\overset{\text{crit.}}{\underset{\text{maj}}{\Rightarrow}} \lim_{x \to 0} x \sin\frac{1}{x} = 0$$

$$f(x, y, z) = \left(u(x,y,z),\ v(x,y,z) \right)$$

$$J = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{pmatrix}$$

$$\sum_{i=1}^{n} |x_i - y_i|$$

$$[0, \pi] \longrightarrow [-1, 1] \quad T = 2$$

$$\cos:\ f_p(x) = \begin{cases} f(x), & x \in [0, 1] \\ f(-x), & x \in [-1, 0) \end{cases} \quad T = 2$$

$\left\{ f(-x),\ x \in [-1,0) \right.$ $T=2$

$w = \frac{2\pi}{T} = \pi$

$b_n = 0$

$a_n = \frac{4}{T} \int_0^{\frac{T}{2}} f(x) \cos(w n x)\, dx$

$\cos n \pi$

$\Gamma(t) = \int_0^{\infty} e^{-x} x^{t-1} dx$ - conv, $\forall t > 0$

$B(p,q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx$ - conv, $\forall p,q > 0$

$= \frac{\Gamma(p)\cdot\Gamma(q)}{\Gamma(p+q)}$

$\Gamma\left(n+\frac{1}{2}\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$, $\forall n \in \mathbb{N}^*$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$\Gamma(n+1) = n!$, $n \in \mathbb{N}$

$\Gamma(t+1) = t\,\Gamma(t)$

$\Gamma(1) = 1$

$\Gamma(p)\,\Gamma(1-p) = \frac{\pi}{\sin(p\pi)}$ $p \in (0,1)$

$\int_0^{\infty} (x^9 - 2x^4) e^{-x^2} dx =$

$= \int_0^{\infty} x^9 e^{-x^2} dx - \int_0^{\infty} 2x^4 e^{-x^2} dx$

$x^2 = t$

$2x\,dx = dt$

$\int_0^{\infty} \ldots^{4} \ldots e^{-t} \ldots \qquad \int^{\infty} 2\,(x^2)^2 e^{-x^2} dx$

$$=\frac{1}{2}\int_0^\infty t^4 e^{-t}\,dt-\int_0^\infty 2(x^2)^2 e^{-x^2}\,dx$$

$p-1=4$ $\qquad$ $p'-1=2$

$p=5$ $\qquad$ $p'=3$

$$=\frac{1}{2}\Gamma(5)-2\Gamma(3)=$$

$$=\frac{1}{2}4!-2\,2!=$$

$$=12-4=8$$

$$\int_0^{\frac{\pi}{2}} \sin^{2p-1}x\cdot\cos^{2q-1}x\,dx=\frac{1}{2}\beta(p,q)$$

$$\int_0^\infty \frac{\sqrt{x}}{1+x^2}\,dx$$

$$\frac{1-t-(-t)}{(1-t)^2}=\frac{1}{(1-t)^2}$$

$$x^2=\frac{t}{1-t}$$

$$2x\,dx=\ldots\,dt$$

$$\int_1^\infty x^3\sqrt{x^2-1}\,e^{-x^2}\,dx \Rightarrow e^{-(x^2+1)}=e^{-x^2}\cdot e^{-1}$$

$x^2-1=t$ $\qquad$ $x=1\Rightarrow t=0$

$2x\,dx=dt$ $\qquad$ $x=\infty\Rightarrow t=\infty$

$$\frac{1}{2}\int_0^\infty \sqrt[3]{t}\,e^{-t}\cdot\frac{1}{e}\,dt=$$

$$=\frac{1}{2e}\int_0^\infty t^{\frac{1}{3}}e^{-t}\,dt$$

$$p-1=\frac{1}{3}\Rightarrow p=\frac{4}{3}$$

$$\Rightarrow I=\frac{1}{2e}\Gamma\left(\frac{4}{3}\right)-\text{conv}$$

$$\Rightarrow J = \frac{1}{2e}\Gamma\left(\frac{4}{3}\right) - \text{conv}$$

$$= \frac{1}{2e}\Gamma\left(1+\frac{1}{3}\right) = \frac{1}{2e}\cdot\frac{1}{3}\Gamma\left(\frac{1}{3}\right) = \frac{1}{6e}\Gamma\left(\frac{1}{3}\right)$$