RLC Series Circuit

Example 3.11.1. Given that $u(t) = 120\,sin\,(500t + 45^0)\,V$ and $i(t) = 10\,sin(500t - 15^0)\,A$, find the instantaneous power, the active, reactive and apparent power.

Solution:

The instantaneous power is given by $p(t) = u(t) \cdot i(t) = 1200 \sin(500t + 45^0) \sin(500t - 15^0)$. *Applying the trigonometric identity* $\sin\alpha \cdot \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$, *gives:* $p(t) = 600[\cos(60^0) - \cos(1000t + 30^0)]$, *or* $p(t) = 300 - 600\cos(1000t + 30^0)\ [W]$.

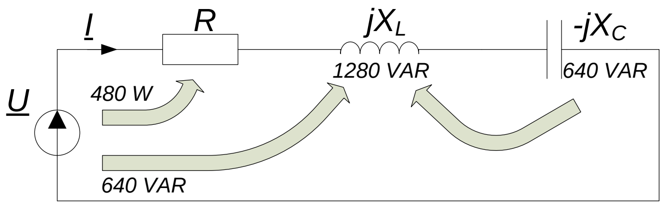
The active power is: $P = UI\cos(\varphi_U - \varphi_I) = \frac{120}{\sqrt{2}}\frac{10}{\sqrt{2}}\cos(45^0 - (-15^0)) = 600\cos(60^0) = 300\ (W)$.

The reactive power is: $Q = UI\sin(\varphi_U - \varphi_I) = \frac{120}{\sqrt{2}}\frac{10}{\sqrt{2}}\sin(45^0 - (-15^0)) = 600\sin(60^0) = 300\sqrt{3}(VAR)$.

The apparent power is: $S = UI = \frac{120}{\sqrt{2}}\frac{10}{\sqrt{2}} = 600\ (VA)$.

Using the apparent power operator: $\underline{S} = \underline{U}\underline{I}^* = \frac{120}{\sqrt{2}}\,e^{j45^0}\frac{10}{\sqrt{2}}e^{j15^0} = 600e^{j60^0} = 600\cos 60^0 + j600\sin 60^0 = 300 + j300\sqrt{3}$, *we identify* $P = Re(\underline{S}) = 300\ (W)$, $Q = Im(\underline{S}) = 300\sqrt{3}\ (VAR)$ *and* $S = |\underline{S}| = 600\ (VA)$.

Example 3.11.2. The parameters of the circuit below are $R = 30\Omega$, $X_L = \omega L = 80\Omega$, $X_C = \frac{1}{\omega C} = 40\Omega$ and the supplying voltage $u(t) = 200\sqrt{2}\,sin(100\pi t + 60^0)\,V$. Find the active, reactive and apparent power in the circuit.



Example 3.11.2.

Solution:

Using the KVL, $\underline{U} = \underline{I}R + jX_L\underline{I} - jX_C\underline{I}$, *the current is given by* $\underline{I} = \frac{\underline{U}}{R + jX_L - jX_C} = \frac{200\,e^{j60^0}}{30 + j80 - j40} = \frac{200\,e^{j60^0}}{30 + j40} = \frac{200\,e^{j60^0}}{50\,e^{j53^0}} = 4e^{j7^0}$. *Converting to the time domain* $i(t) = 4\sqrt{2}\sin(100\pi t + 60^0)\,A$.

The active power is: $P = UI\cos(\varphi_U - \varphi_I) = 200 \cdot 4\cos(60^0 - 7^0) = 800\cos(53^0) = 480\ (W)$. *The active power is generated by the source and dissipated in the resistance R,* $P = I^2R = 4^2 \cdot 30 = 480\ W$.

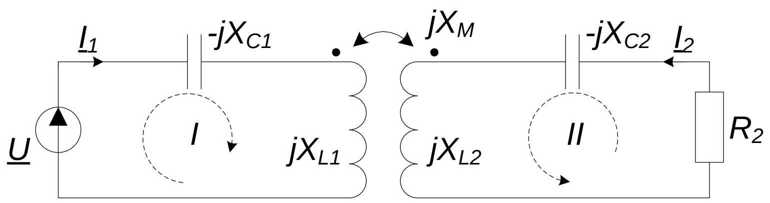
The reactive power is: $Q = UI\sin(\varphi_U - \varphi_I) = 200 \cdot 4\sin(60^0 - 7^0) = 800\sin(53^0) = 640(VAR)$. *The source generates 640 VAR reactive power. The same reactive power can be calculated by* $Q = I^2X = I^2(X_L - X_C) = 4^2(80 - 40) = 640\ (VAR)$. *The reactive power removed by the inductor is* $Q_L = I^2X_L = 4^2 \cdot 80 = 1280\ (VAR)$, *while the reactive power generated by the capacitor is* $Q_C =$

$-I^2 X_C = -4^2 \cdot 40 = -640\ (VAR)$. *The inductor removes the reactive power generated half from the source and half from the capacitor.*

The apparent power is: $S = UI = 200 \cdot 4 = 800\ (VA)$.

Using the apparent power operator: $\underline{S} = \underline{U}\underline{I}^* = 200\ e^{j60^0} 4e^{j7^0} = 800e^{j53^0} = 800\cos 53^0 + j800\sin 53^0 = 480 + j640$, *we identify* $P = Re(\underline{S}) = 480\ (W)$, $Q = Im(\underline{S}) = 640\ (VAR)$ *and* $S = |\underline{S}| = 800\ (VA)$.

Example 3.11.3. The parameters of the coupled circuit below are $X_{L1} = X_{L2} = X_{C2} = 20\Omega$, $X_{C1} = 30$, $X_M = 10\Omega$, $R_2 = 10\Omega$ and the supplying voltage $\underline{U} = 100\ V$. Find the currents in the circuit, $\underline{I}_1$ and $\underline{I}_2$. Find the active, reactive and apparent power of the source. Analyze the active and reactive power distribution in the passive circuit elements (proof the active and reactive power conservation).



Example 3.11.3.

Solution:

We have two coupled circuits. Applying KVL for the two loops with reference directions as shown in the figure above, we have:

$$\begin{cases} -jX_{C1}\underline{I}_1 + jX_{L1}\underline{I}_1 + jX_M\underline{I}_2 = \underline{U} \\ -jX_{C2}\underline{I}_2 + jX_{L2}\underline{I}_2 + jX_M\underline{I}_1 + \underline{I}_2 R = 0. \end{cases}$$

Replacing with numerical values, the system becomes:

$$\begin{cases} (-j30 + j20)\underline{I}_1 + j10\underline{I}_2 = 100 \\ j10\underline{I}_1 + (-j20 + j20 + 10)\underline{I}_2 = 0. \end{cases}$$

Solving for $\underline{I}_1$ *and* $\underline{I}_2$*, we obtain:* $\underline{I}_1 = 5 + j5 = 5\sqrt{2}\ e^{j45^0}$ *and* $\underline{I}_2 = 5 - j5 = 5\sqrt{2}\ e^{-j45^0}$.

To analyze the active and reactive power distribution in the circuit elements, the apparent phasor operator will be calculated for each circuit element.

For the source: $\underline{S} = \underline{U}\underline{I}_1^* = 100(5 - j5) = 500 - j500$. *Hence, the source generates active power* $P = Re(\underline{S}) = 500\ W$ *and removes reactive power* $Q = Im(\underline{S}) = -500\ VAR$. *The source apparent power is:* $S = |\underline{S}| = 500\sqrt{2}\ VA$.

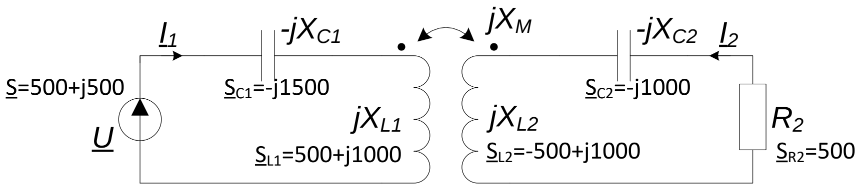
For the capacitor C_1*:* $\underline{S}_{C1} = \underline{U}_{C1}\underline{I}_1^* = -jX_{C1}\underline{I}_1\underline{I}_1^* = -jX_{C1}I_1^2 = -j1500$. *How we expect, the capacitor generates reactive power. Hence, reactive power is* $Q_{C1} = Im(\underline{S}_{C1}) = -1500\ VAR$.

Same, for the capacitor C_2*: :* $\underline{S}_{C2} = \underline{U}_{C2}\underline{I}_2^* = -jX_{C2}\underline{I}_2\underline{I}_2^* = -jX_{C2}I_2^2 = -j1000$. *Hence, reactive power generated by* C_2 *is* $Q_{C2} = Im(\underline{S}_{C2}) = -1000\ VAR$.

For the resistor R_2*:* $\underline{S}_{R2} = \underline{U}_{R2}\underline{I}_2^* = R_2\underline{I}_2\underline{I}_2^* = R_2 I_2^2 = 500\ W$. *Hence, the resistor* R_2 *removes the active power* $P_{R2} = Re(\underline{S}_{R2}) = 500\ W$.

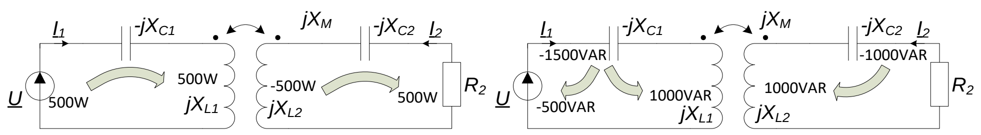
For the inductor L₁: $\underline{S}_{L1} = \underline{U}_{L1}\underline{I}_1^* = (jX_{L1}\underline{I}_1 + jX_M\underline{I}_2)\underline{I}_1^* = jX_{C1}I_1^2 + jX_M\underline{I}_2\underline{I}_1^* = 500 + j1000$. *As expected, inductor L₁ removes reactive power* $Q_{L1} = Im(\underline{S}_{L1}) = 1000\ VAR$. *Because of the coupling, we have to observe an active power component for coupling inductors,* $P_{L1} = Re(\underline{S}_{L1}) = 500\ W$. *It means the inductor L₁ removes active power.*

For the inductor L₂: $\underline{S}_{L2} = \underline{U}_{L2}\underline{I}_2^* = (jX_{L2}\underline{I}_2 + jX_M\underline{I}_1)\underline{I}_2^* = jX_{C2}I_2^2 + jX_M\underline{I}_1\underline{I}_2^* = -500 + j1000$. *How we expect, inductor L₂ removes reactive power* $Q_{L2} = Im(\underline{S}_{L1}) = 1000\ VAR$. *The active power component of the inductor L₂ is* $P_{L2} = Re(\underline{S}_{L2}) = -500\ W$. *The inductor L₂ generates active power.*



The apparent phasor operator for the circuit elements

The figure above indicates the apparent phasor operator for each circuit element. The real components refer to the active power while the imaginary parts refer to the reactive power. We should expect to satisfy the power conservation theorem for both active and reactive power. The figure below proved the power conservation. Active power conservation is illustrated in figure a). The source generates 500W active power, power which is absorbed by the coupled inductor L₁. Duo to the coupling, the active power is transferred to the inductor L₂. The inductor L₂ behaves like an active power source, generates 500W active power which is removed by the resistor R₂. The active power conservation is fulfilled. The reactive power conservation is illustrated in figure b) below. The two capacitors generate reactive power $Q_{C1} = -1500\ VAR$ *and* $Q_{C2} = -1000\ VAR$. *The reactive power is removed by the coupled inductors,* $Q_{L1} = 1000\ VAR$, $Q_{L2} = 1000\ VAR$ *and by the source,* $Q = 500\ VAR$.

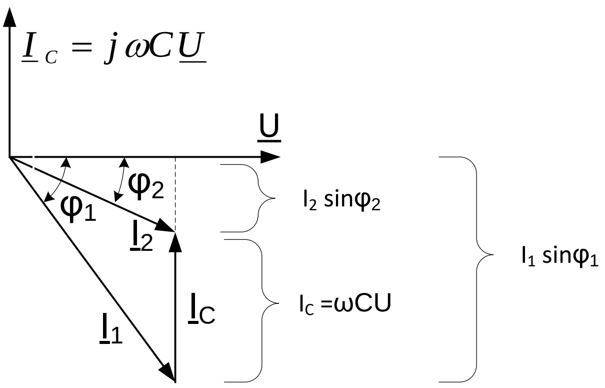


a) Active power conservation b) Reactive power conservation

Example 3.12.1. When connected to a 230V (rms), 50Hz power line, a load absorbs 10KW at a lagging power factor of $cos\ \varphi_1 = 0.8$. Find the value of capacitance necessary to raise the power factor to $cos\ \varphi_2 = 0.95$.

Solution:

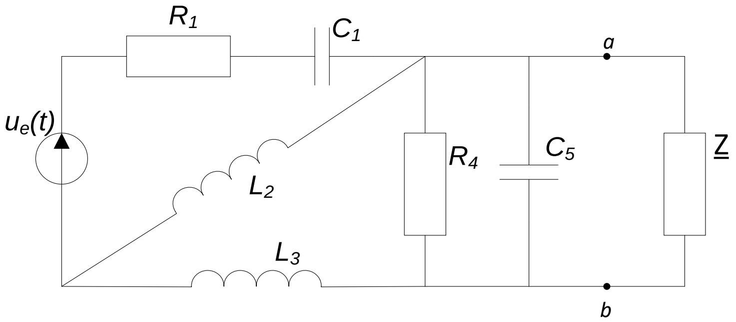
We have the power factor $\cos\varphi_1 = 0.8$, *then* $\varphi_1 = 36.87^0$, *where* φ_1 *is the phase between voltage and current. The power is absorbed under a current* $I_1 = \frac{P}{U\cos\varphi_1} = 54.35\ (A)$. *When the power factor rises to* $\cos\varphi_2 = 0.95$, *namely* $\varphi_2 = 18.19^0$, *the same power is absorbed under a current* $I_2 = \frac{P}{U\cos\varphi_2} = 45.75\ (A)$.



Example 3.12.1.

To determine the capacitance C, we have to refer to the figure above. The current through the capacitor of magnitude $I_C = \omega CU$*, leads the voltage by 90*^{0}*. We can write* $I_C = I_1 \sin\varphi_1 - I_2 \sin\varphi_2$*, or* $\omega CU = I_1 \sin\varphi_1 - I_2 \sin\varphi_2$*, from where* $C = \frac{I_1 \sin\varphi_1 - I_2 \sin\varphi_2}{\omega U} = \frac{P(\tan\varphi_1 - \tan\varphi_2)}{\omega U} = \frac{10\cdot 10^3(\tan 36.87 - \tan 18.19)}{2\cdot\pi\cdot 50\cdot 230} 252.7\ (\mu F)$.

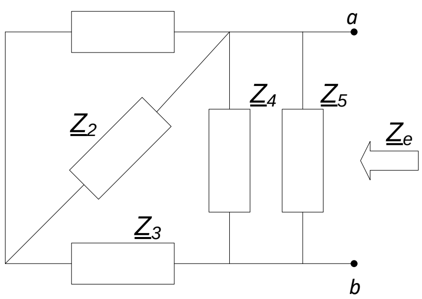
Example 3.13.1. For the circuit below determine the value of impedance $\underline{Z}$ which will absorb the greatest power from the circuit. The circuit parameters are $R_1 = X_{C1} = 2\Omega$, $X_{L2} = X_{L3} = 2\Omega$, $R_4 = 5\Omega$, $X_{C5} = 10\Omega$ and the voltage $u_e(t) = 10\sqrt{2}\, sin(314t - 30^0)\ V$.



Example 3.13.1.

Solution:

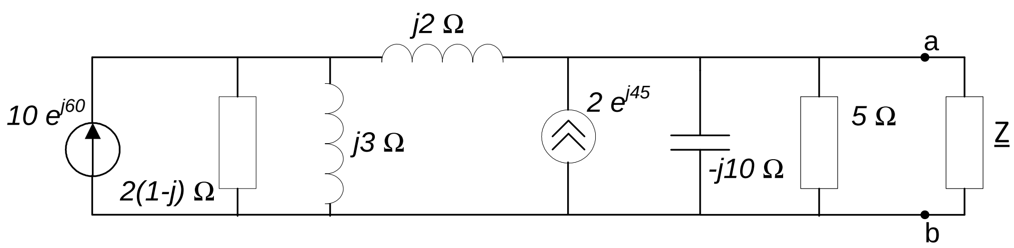
First, we have to transform the circuit in phasor form and obtain the equivalent impedance of circuit $\underline{Z}_e$*, suppressing all the sources. To get* $\underline{Z}_e$ *consider the circuit below where* $\underline{Z}_1 = R_1 - jX_{C1} = 2 - j2\ \Omega$, $\underline{Z}_2 = jX_{L2} = j2\ \Omega$, $\underline{Z}_3 = jX_{L3} = j2\ \Omega$, $\underline{Z}_4 = R_4 = 5\ \Omega$, $\underline{Z}_5 = -jX_{C6} = -j10\ \Omega$.



Example 3.13.1.

Successively we have $\underline{Z}_p = \frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} = 2 + j2\ (\Omega)$, $\underline{Z}_s = \underline{Z}_p + \underline{Z}_3 = 2 + j4\ (\Omega)$, $\frac{1}{\underline{Z}_e} = \frac{1}{\underline{Z}_s} + \frac{1}{\underline{Z}_4} + \frac{1}{\underline{Z}_5}$ *from* *where* $\underline{Z}_e = 3 + j\ (\Omega)$. *The load impedance draws the maximum power from the circuit when* $\underline{Z} = \underline{Z}_e^* = 3 - j\ (\Omega)$.

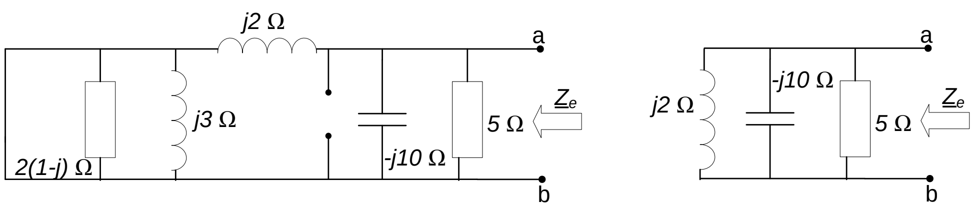
Example 3.13.2. For the circuit below determine the value of impedance $\underline{Z}$ which will absorb the greatest power from the circuit.



Example 3.13.2.

Solution:

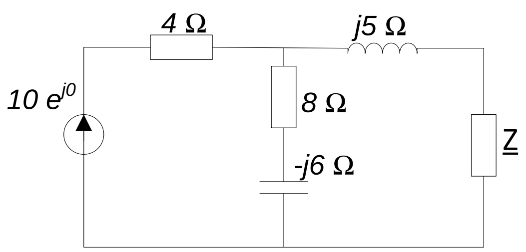
First, we obtain the equivalent impedance of circuit $\underline{Z}_e$*, suppressing all the sources. To get* $\underline{Z}_e$*, consider the passive circuit below:*



Example 3.13.2.

The wire corresponding to the voltage source short circuits the impedances $2(1-j)$ *and* j*., As a consequence we can neglect these impedances. We find* $\frac{1}{\underline{Z}_e} = \frac{1}{j2} + \frac{1}{-j10} + \frac{1}{5} = \frac{1}{5} - j\frac{2}{5}$ *from where* $\underline{Z}_e = 1 + j2\ (\Omega)$. *The load impedance draws the maximum power from the circuit when* $\underline{Z} = \underline{Z}_e^* = 1 - j2\ (\Omega)$.

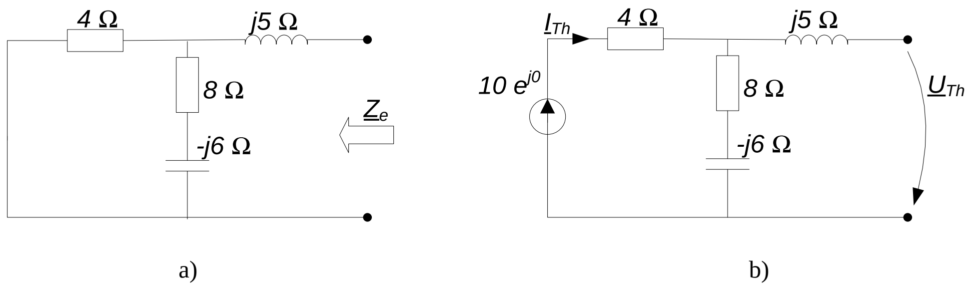
Example 3.13.3. Determine the load impedance $\underline{Z}$ that maximizes the active power drawn from the circuit below. What is the maximum active power?



Example 3.13.3.

Solution:

First, we obtain the equivalent impedance of circuit $\underline{Z}_e$*, suppressing all the sources. To get* $\underline{Z}_e$*, consider the passive circuit in figure a below, for which* $\underline{Z}_e = j5 + \frac{4(8-j6)}{4+8-j6} = 2.933 + j4.467\ (\Omega)$. *The load impedance draws the maximum power from the circuit when* $\underline{Z} = \underline{Z}_e^* = 2.933 - j4.467\ \Omega$.



Example 3.13.3.

To find $\underline{U}_{Th}$, consider the circuit in figure b) above. We find current $\underline{I}_{Th} = \frac{10}{4+6-j6} = \frac{2}{3} + j\frac{1}{3}$, *and* $\underline{U}_{Th} = (8 - j6)\,\underline{I}_{Th} = 7.33 - j1.33 = 7.454\; e^{-j10.3^0}$. *According to Eq.(3.75), the maximum active power is* $P_{max} = \frac{U_{Th}^2}{8\,R} = \frac{7.454^2}{8{\cdot}2.933} = 2.368\ (W)$.