Recapitulare

1) Det. pct. de extrem local

$$f(x,y,z) = xyz(4-x-y-z), \quad xyz \neq 0 \Rightarrow \begin{matrix} x\neq 0 \\ y \neq 0 \\ z \neq 0 \end{matrix}$$

$$\begin{cases} \dfrac{\partial f}{\partial x} = 0 \\ \dfrac{\partial f}{\partial y} = 0 \\ \dfrac{\partial f}{\partial z} = 0 \end{cases} \qquad \begin{cases} yz(4-x-y-z) + (xyz)\cdot(-1) = 0 \\ xz(4-x-y-z) + (xyz)\cdot(-1) = 0 \\ xy(4-x-y-z) + xyz\cdot(-1) = 0 \end{cases}$$

$$\begin{cases} yz(4-x-y-z) = xyz \;|\, : yz, \quad y\neq 0, z \neq 0 \\ xz(4-x-y-z) = xyz \;|\, : xz, \quad x \neq 0, z\neq 0 \\ xy(4-x-y-z) = xyz \;|\, : xy, \quad x\neq 0, y \neq 0 \end{cases}$$

$$\begin{cases} 4-x-y-z = x \\ 4-x-y-z = y \\ 4-x-y-z = z \end{cases} \qquad \begin{cases} -2x-y-z = -4 \\ -x-2y-z = -4 \\ -x-y-2z = -4 \end{cases}$$

$$\begin{cases} 2x+y+z = 4 \\ x+2y+z = 4 \\ x+y+2z = 4 \end{cases} \qquad \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 8+1+1-2-2-2 = 4 \neq 0$$

$$\boxed{x = \frac{\Delta x}{\Delta} = \frac{4}{4} = 1}$$

$$\Delta x = \begin{vmatrix} 4 & 1 & 1 \\ 4 & 2 & 1 \\ 4 & 1 & 2 \end{vmatrix} = 4 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4(4+1+1-2-2-1) = 4$$

$$\begin{cases} 1+2y+z = 4 \\ 1+y+2z = 4 \end{cases} \qquad \begin{cases} 2y+z = 3 \\ y+2z = 3 \end{cases} \Rightarrow \boxed{y = z = 1}$$

$M(1,1,1)$

$$Hf = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix}$$

$$f(x,y,z) = xyz(4-x-y-z)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(yz(4-x-y-z) - xyz\right) =$$

$$= -yz - yz = -2yz \quad \text{in } (1,1,1): -2$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(xz(4-x-y-z) - xyz\right) =$$

$$= -xz - xz = -2xz \quad \text{in } (1,1,1): -2$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial z}\left(xy(4-x-y-z) - xyz\right) =$$

$$= -xy - xy = -2xy \quad \text{in } (1,1,1): -2$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(xz(4-x-y-z) - xyz\right) =$$

$$= z(4-x-y-z) - xz - yz =$$

$$= 4z - 2xz - 2yz - z^2 \quad \text{in } (1,1,1): 4-2-2-1 = -1$$

$$\frac{\partial^2 f}{\partial x \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial x}\left(xy(4-x-y-z) - xyz\right) =$$

$$= y(4-x-y-z) - xy - yz =$$

$$= 4y - 2xy - 2yz - y^2 \quad \text{in } (1,1,1): -1$$

$$\frac{\partial^2 f}{\partial y \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right) = \frac{\partial}{\partial y}\left(xy(4-x-y-z) - xyz\right) =$$

$$= x(4-x-y-z) - xy - xz =$$

$$= 4x - 2xy - 2xz - x^2 \quad \text{in } (1,1,1): -1$$

$$Hf(1,1,1) = \begin{pmatrix} -2 & -1 & -1 \\ -1 & -2 & -1 \\ -1 & -1 & -2 \end{pmatrix}$$

$$\Delta_1 = -2 < 0$$

$$\Delta_2 = \begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix} = 4 - 1 = 3 > 0$$

$$\Delta_3 = \begin{vmatrix} -2 & -1 & -1 \\ -1 & -2 & -1 \\ -1 & -1 & -2 \end{vmatrix} = (-1)^3 \cdot \begin{vmatrix} 2 & 1 & 1 \\ +1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} =$$

$$= -\left[8 + \cancel{1} \cancel{+} 1 - \cancel{2} - 2 - 2 \right] = -4 < 0$$

$\Delta_1 < 0$, $\Delta_2 > 0$, $\Delta_3 < 0 \Rightarrow M(1,1,1)$ punct de maxim.