MAC

1. Rez. sis. găsind factorizarea Cholesky a matricii coef. și fol. substituția înapoi ver. corect. fact. și a sol. obt. fol. înmulțirea matricilor

$$\begin{bmatrix} 4 & -2 & 0 \\ -2 & 2 & -1 \\ 0 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 7 \end{bmatrix}$$

$$\left.\begin{aligned} R_{11} &= \sqrt{a_{11}} = \sqrt{4} = 2 \\ R_{1,2:3} &= \frac{[-2, 0]}{R_{11}} = [-1, 0] = w^T \end{aligned}\right\} \Rightarrow R = \begin{bmatrix} 2 & -1 & 0 \\ & & \\ & & \end{bmatrix}$$

$$u \cdot w^T = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} = \cancel{1+1} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$

$$\left.\begin{aligned} R_{22} &= 1 \\ R_{23} &= \frac{-1}{1} = -1 \end{aligned}\right\} \Rightarrow R = \begin{bmatrix} 2 & -1 & 0 \\ & 1 & -1 \\ & & 2 \end{bmatrix}$$

$$5 - (-1)(-1) = 5 - 1 \Rightarrow R_{33} = \sqrt{4} = 2$$

$$\Rightarrow R = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$R^T \cdot R = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 2 & -1 \\ 0 & -1 & 5 \end{bmatrix} \checkmark$$

$$A \cdot x = b \iff R^T \cdot \underbrace{R \cdot x}_{c} = b$$

$$R^T \cdot c = b \iff \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -7 \end{bmatrix}$$

$$\begin{cases} 2c_1 = 0 & \Rightarrow c_1 = 0 \\ -1c_1 + c_2 = 3 & \Rightarrow c_2 = 3 \\ -c_2 + 2c_3 = -7 & \Rightarrow c_3 = \dfrac{-7 + c_2}{2} = -2 \end{cases}$$

$$R \cdot x = c \iff \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 2x_1 - x_2 = 0 & \Rightarrow x_1 = \dfrac{1}{2} = 0{,}5 \\ x_2 - x_3 = 3 & \Rightarrow x_2 = 1 \\ x_3 = -2 \end{cases}$$

$$\Rightarrow x = \begin{bmatrix} \frac{1}{2} \\ 1 \\ -2 \end{bmatrix}$$

2. Fol. interpolarea Lagrange pt. a găsi pol. care trece prin $(-1,0)$, $(2,1)$, $(3,1)$

$$P_2(x) = 0 \cdot \frac{(x-2)(x-3)}{(-1-2)(-1-3)} + 1 \cdot \frac{(x+1)(x-3)}{(2+1)(2-3)} +$$

$$+ 1 \cdot \frac{(x+1)(x-2)}{(3+1)(3-2)} =$$

$$= \frac{(x+1)(x-3)}{-3} + \frac{(x+1)(x-2)}{+4} =$$

$$= -\,{}^{4)}\frac{(x+1)(x-3)}{3} + {}^{3)}\frac{(x+1)(x-2)}{4} =$$

$$= \frac{-4(x+1)(x-3)+3(x+1)(x-2)}{12} =$$

$$= \frac{(x+1)[-4x+12+3x-6]}{12}$$

$$= \frac{(x+1)(-x+6)}{12} = \frac{-x^2+5x+6}{12} \; \left(\cancel{\frac{-5x^2-5x+12x+12}{12}}\right) =$$

$$= \boxed{-\frac{1}{12}x^2 + \frac{5}{12}x + \frac{1}{2}}$$

$$P_2(-1) = \cancel{-\frac{5}{6} - \frac{7}{6} + 2 = -2+2} = 0 \; ✓$$

$$P_2(2) = \cancel{-\frac{5}{6}\cdot 4 + \frac{7}{6}\cdot 2 + 2 = \frac{-20+14}{6}} + 2 = -1+2 =$$

$$= 1 \; ✓$$

$$P_2(3) = \cancel{-\frac{5}{6}\cdot 9 + \frac{7}{6}\cdot 3 + 2 = \frac{-45+21}{6}} + 2 = -4+2 =$$

$$= -2 \; ??$$

3. Găsiți cea mai bună parabolă care interpolează $(1,1), (1,2), (2,2), (2,3)$ și găsiți RCMP-ul ei.

• $y = c_1 + c_2 \cdot t \Rightarrow \begin{cases} c_1 + c_2 = 1 \\ c_1 + c_2 = 2 \\ c_1 + 2c_2 = 2 \\ c_1 + 2c_2 = 3 \end{cases}$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$

$A^T \cdot A \cdot c =$

• parabolă $\Leftrightarrow$ ec. de grad 2

$y = c_1 + c_2 t + c_3 t^2$

$$\Rightarrow \begin{cases} c_1 + c_2 + c_3 = 1 \\ c_1 + c_2 + c_3 = 2 \\ c_1 + 2c_2 + 4c_3 = 2 \\ c_1 + 2c_2 + 4c_3 = 3 \end{cases}$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 2 & 4 \end{bmatrix}}_{A} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$

sistemul inconsistent

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 2 & 4 \end{bmatrix} =$$

$$= \begin{bmatrix} 4 & 6 & 10 \\ 6 & 10 & 18 \\ 10 & 18 & 34 \end{bmatrix}$$

$$A^T \cdot b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 & 10 \\ 6 & 10 & 18 \\ 10 & 18 & 34 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 23 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} 4x_1 + 6x_2 + 10x_3 = 8 \\ 6x_1 + 10x_2 + 18x_3 = 13 \\ 10x_1 + 18x_2 + 34x_3 = 23 \end{cases}$$

$$\left[\begin{array}{ccc|c} 4 & 6 & 10 & 8 \\ 6 & 10 & 18 & 13 \\ 10 & 18 & 34 & 23 \end{array}\right] \xrightarrow[\cancel{L_2 \leftarrow L_2}]{L_1 \leftarrow L_1 \cdot \frac{1}{2}} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 4 \\ 6 & 10 & 18 & 13 \\ 10 & 18 & 34 & 23 \end{array}\right]$$

$$\xrightarrow[L_3 \leftarrow L_3 - 5L_1]{L_2 \leftarrow L_2 - 3L_1} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 4 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 9 & 3 \end{array}\right] \xrightarrow{L_3 \leftarrow L_3 - 3L_2}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 5 & 4 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \begin{cases} 2x_1 + 3x_2 + 5x_3 = 4 \\ x_2 + 3x_3 = 1 \end{cases} \Rightarrow \boxed{x_2 = 1 - 3x_3}$$

$$\Rightarrow 2x_1 + 3 - 9x_3 + 5x_3 = 4$$

$$2x_1 - 4x_3 = 1 \Rightarrow \boxed{x_1 = \frac{1 + 4x_3}{2}}$$

$$10 \cdot \frac{1 + 4x_3}{2} + 18 \cdot (1 - 3x_3) + 34 \cdot x_3 = 23 \quad | \cdot 2$$

$$10 + 40x_3 + 36 - 36 \cdot 3x_3 + 34 \cdot 2x_3 = 46$$

$$x_3(40 - 108 + 68) = 46 - 10 - 36$$

$$\cancel{x_3(} \Rightarrow 0 = 0 \Rightarrow x_3 = 0 \Rightarrow x_2 = 1 \Rightarrow x_1 = \frac{1}{2}$$

Verificare

$$4 \cdot \frac{1}{2} + 6 \cdot 1 = 2 + 6 = 8 \checkmark$$

$$\Rightarrow c = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}$$

<u>REMP</u>

$$y = \frac{1}{2} + t$$

$$y(1) = \frac{1}{2} + 1 = \frac{3}{2} = 1{,}5$$

$$y(1) = \frac{3}{2} = 1{,}5$$

$$y(2) = \frac{5}{2} = 2{,}5$$

$$y(2) = \frac{5}{2} = 2{,}5$$

$$\Rightarrow r = \begin{bmatrix} 0{,}5 \\ -0{,}5 \\ 0{,}5 \\ -0{,}5 \end{bmatrix}$$

$$\text{norm } r = \sqrt{4 \cdot \left(\frac{1}{2}\right)^2} = 1$$

$$\text{REMP} = \frac{1}{4} = 0{,}25$$

4. Care dintre următoarele ipf converge la $\sqrt{2}$? Ord. discuss, in fct. de rata de conv.

A $x \to \frac{1}{2}x + \frac{1}{x}$

B $x \to \frac{2}{3}x + \frac{2}{3x}$

C $x \to \frac{3}{4}x + \frac{1}{2x}$

A. $x \to \frac{1}{2}x + \frac{1}{x}$

$$f_1(x) = \frac{1}{2}x + \frac{1}{x}$$

$$f_1(\sqrt{2}) = \frac{1}{2}\sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ (conv)}$$

B. $x \to \frac{2}{3}x + \frac{2}{3x}$

$$f_2(x) = \frac{2}{3}x + \frac{2}{3x}$$

$$f_2(\sqrt{2}) = \frac{2}{3}\cdot\sqrt{2} + \frac{2}{3\sqrt{2}} = \frac{2\sqrt{2}}{3} + \frac{\sqrt{2}}{3} = \frac{3\sqrt{2}}{3} = \sqrt{2} \text{ (conv)}$$

C. $x \to \frac{3}{4}x + \frac{1}{2x}$

$$f_3(x) = \frac{3}{4}x + \frac{1}{2x}$$

$$f_3(\sqrt{2}) = \frac{3}{2\sqrt{2}\cdot\sqrt{2}}\cdot\sqrt{2} + \frac{1}{2\sqrt{2}} = \frac{4^2}{2\sqrt{2}} = \sqrt{2} \text{ (conv)}$$

$\Rightarrow$ A, B, C – conv. la $\sqrt{2}$

$f_1'(x) = \frac{1}{2} - \frac{1}{x^2}$

$f_1'(\sqrt{2}) = \frac{1}{2} - \frac{1}{2} = 0$

$f_2'(x) = \frac{2}{3} - \frac{2}{3} \cdot \frac{1}{x^2} = \frac{2}{3}\left(1 - \frac{1}{x^2}\right)$

$f_2'(\sqrt{2}) = \frac{2}{3} - \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

$f_3'(x) = \frac{3}{4} - \frac{1}{2} \cdot \frac{1}{x^2}$

$f_3'(\sqrt{2}) = \frac{3}{4} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

$\Rightarrow f_3, f_2, f_1$

5. Găsiți curba spline cubică naturală care trece prin $(-1,1), (1,1), (2,4)$. Verificați apartenența punctelor la curba găsită și continuitatea și derivabilitatea ei

$3(3-0)'$

$x = [-1, 1, 2] \Rightarrow \delta_1 = 2, \delta_2 = 1$
$y = [1, 1, 4] = a \Rightarrow \Delta_1 = 0, \Delta_2 = 3$

ecuația matricială tridiagonală este

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 6 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} c_1 = 0 \\ 2c_1 + 6c_2 + c_3 = 9 \\ c_3 = 0 \end{cases} \Rightarrow c_2 = \frac{3}{2}$$

$\Rightarrow c = [0, \frac{3}{2}, 0]^T$

$$d_1 = \frac{c_2 - c_1}{3\delta_1} = \frac{\frac{\cancel{3}}{2}}{\cancel{3}\cdot 2} = \frac{1}{4}$$

$$d_2 = \frac{c_3 - c_2}{3\delta_2} = \frac{-\frac{\cancel{3}}{2}}{\cancel{3}\cdot 1} = -\frac{1}{2}$$

$$b_1 = \frac{\Delta_1}{\delta_1} - \frac{\delta_1}{3}(2c_1 + c_2) = \cancel{0 - \frac{2}{3} \cdot \frac{1}{2}} = -1$$

$$b_2 = \frac{\Delta_2}{\delta_2} - \frac{\delta_2}{3}(2c_2 + c_3) = \cancel{3 - \frac{1}{3} \cdot 1 = \frac{8}{9}} \; 2$$

$$\Rightarrow \cancel{S_1(x) = 1 - \frac{1}{3}(x+1) + 0 + \frac{1}{12}(x+1)^3}$$

$$\cancel{= 1 - \frac{1}{3}(x+1) + \frac{1}{12}(x+1)^3}$$

$$S_1(x) = 1 - (x+1) + \frac{1}{4}(x+1)^3 \text{ pe } [-1, 1]$$

$$\cancel{S_2(x) = 1 + \frac{8}{9}(x-1) + \frac{1}{2}(x-1)^2 +}$$

$$\cancel{- \frac{1}{6}(x-1)^3 \text{ pe } [1,2]}$$

$$S_2(x) = 1 + 2(x-1) + \frac{3}{2}(x-1)^2 - \frac{1}{2}(x-1)^3 \text{ pe } [1,2]$$

$$S_1(-1) = 1 - 0 + 0 + 0 = 1 \checkmark$$

$$S_1(1) = \cancel{1 - \frac{2}{3} + \frac{8}{12} = \frac{12-8+8}{12}} = 1 \checkmark$$

$$1 - 2 + 2 = 1$$

$$S_2(1) = 1 + 0 + 0 + 0 = 1 \checkmark$$

$$\cancel{S_2(2) = 1 + \frac{8}{9} + \frac{1}{2}} =$$

$$\cancel{= \frac{40}{18} + 4\; ?}$$

$$\cancel{\Rightarrow S_2 \notin \text{cu bei}\; ?}$$

$S_1(1) = S_2(1) = 1 \Rightarrow$

$\Rightarrow$ continuă

$$S_1'(x) = 1 + \frac{3}{4}(x+1)^2$$

$$S_2(2) = 1 + 2 + \frac{3}{2} - \frac{1}{2} = 3 + 1 = 4 \checkmark$$

$S_1(1) = S_2(1) = 1 \Rightarrow$

$\Rightarrow$ continuă

$S_1'(x) = 1 + \frac{3}{4}(x+1)^2$

$S_1'(1) = 2$

$S_2'(x) = 2 + \frac{6}{2}(x-1) - \frac{3}{2}(x-1)^2$

$S_2'(1) = 2$

$S_1'(1) = S_2'(1) \Rightarrow$

$\Rightarrow$ derivabilă

6. Calculați nodurile de interpolare Cebîșev $x_1, \dots, x_m$ pe $[-3, 7]$, unde $m = 3$. Găsiți limita superioară pt. $|(x - x_1) \cdot \dots \cdot (x - x_m)|$. Calculați pol. Cebîșev $T_m(x) = (x - x_1) \cdot \dots \cdot (x - x_n)$.

$$x_1 = \cos \frac{\pi}{10}$$

$$x_2 = \cos \frac{3\pi}{10}$$

$$x_3 = \cos \frac{5\pi}{10}$$

7. Deduceți ec care def o curbă Bezier în spațiu. Găsiți curba Bezier def de punctele $(1,0,0)$, $(2,0,0)$, $(0,2,1)$, $(0,1,0)$. Ver. apartenența capetelor la curba găsită.

$$bx = 3(x_2 - x_1) = 3(2-1) = 3$$
$$cx = 3(x_3 - x_2) - bx = 3(0-2) - 3 = -9$$
$$dx = x_4 - x_1 - bx - cx = 0 - 1 - 3 + 9 = 5$$

$$by = 3(y_2 - y_1) = 3(0-0) = 0$$
$$cy = 3(y_3 - y_2) - by = 3(2-0) - 0 = 6$$
$$dy = y_4 - y_1 - by - cy = 1 - 0 - 0 - 6 = -5$$

$$bz = 3(z_2 - z_1) = 0$$
$$cz = 3(z_3 - z_2) - bz = 3$$
$$dz = z_4 - z_1 - bz - cz = -3$$

Curba Bezier

$$\rightarrow \begin{cases} x(t) = x_1 + bx\,t + cx\,t^2 + dx\,t^3 \\ \quad\;\; = 1 + 3t - 9t^2 + 5t^3 \\ y(t) = 0 + 0t + 6t^2 - 5t^3 \\ z(t) = 0 + 0t + 3t^2 - 3t^3 \end{cases}$$

8 Găsiți factorizarea QR completă a matricii de mai jos fol. alg. Gramm-Schmidt complet. Ver. corectitudinea fact. fol. imm. mat

$$\begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix}$$

$$\cdot\ y_1 = A_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$r_{11} = \|y_1\|_2 = \sqrt{4+4+1} = 3 \Rightarrow q_1 = \frac{y_1}{\|y_1\|_2} = \begin{bmatrix} \frac{2}{3} \\ \frac{-2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\cdot\ y_2 = A_2 - q_1 \cdot q_1^T \cdot A_2 = \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ \frac{-2}{3} \\ \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{9} & \frac{-4}{9} & \frac{2}{9} \\ \frac{-4}{9} & \frac{4}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{-2}{9} & \frac{1}{9} \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{3} + \frac{8}{3} - 0 \\ -\frac{4}{3} - \frac{8}{3} + 0 \\ \frac{2}{3} + \frac{4}{3} + 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix} =$$

$$= \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix} \Rightarrow q_2 = \frac{y_2}{\|y_2\|_2} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$$

$$r_{12} = q_1^T \cdot A_2 = \begin{bmatrix} \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} =$$

$$= 2 + 4 = 6$$

$r_{22} = \|y_2\|_2 = 3$

$$\Rightarrow A = \cancel{\begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix}} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix}$$

$\neq$ Verificare

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} 3 & 6 \\ 0 & 3 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & \cancel{4}-1 \\ -2 & -4-2 \\ 1 & 2-2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix} = A \checkmark$$

9. Găsiți fact. Cholensky: $A = R^T \cdot R$ a.î.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\left.\begin{aligned} R_{11} &= \sqrt{a_{11}} = \sqrt{1} = 1 \\ R_{1,2:3} &= \frac{[1, 1]}{R_{11}} = [1, 1] = u^T \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow R = \left[\begin{array}{c|cc} 1 & 1 & 1 \\ \hline & & \\ & & \end{array}\right]$$

$$u \cdot u^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{c|cc} & & \\ \hline & 2 & 2 \\ & 2 & 3 \end{array}\right] - \left[\begin{array}{c|cc} & & \\ \hline & 1 & 1 \\ & 1 & 1 \end{array}\right] = \left[\begin{array}{c|cc} & & \\ \hline & 1 & 1 \\ & 1 & 2 \end{array}\right]$$

$$\left.\begin{aligned} R_{22} &= 1 \\ R_{23} &= \frac{1}{1} = 1 \end{aligned}\right\} \Rightarrow R = \left[\begin{array}{c|cc} 1 & 1 & 1 \\ \hline & 1 & 1 \\ & & \end{array}\right]$$

$$2 - 1 \cdot 1 = 1 \Rightarrow R_{33} = \sqrt{1} = 1$$

$$\Rightarrow R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^T \cdot R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = A \checkmark$$

10. Aplicați 2 pași ai met. lui Newton cu valoarea inițială $x_0 = 1$ pt. ec. $3x^2 + 2x = 1$. Det. rata de conv. Calc. eroarea de aprox.

f - compusă de ~~fct~~ fct. de grad 2 → f - cont.
~~f - dir~~.

$f(0) = -1 \Rightarrow f$ - neg. în 0
$\lim_{x\to\infty}(3x^2+2x-1) = \infty \Rightarrow f \to \infty$ ⟹

$f(x) = 3x^2 + 2x - 1$

$f'(x) = 6x + 2$

⟹ f - are răd.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{4}{8} = \frac{1}{2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{1}{2} - \frac{f(\frac{1}{2})}{f'(\frac{1}{2})} =$$

$$= \frac{1}{2} - \frac{3 \cdot \frac{1}{4} + 2 \cdot \frac{1}{2} - 1}{\cancel{6}^{3} \cdot \frac{1}{\cancel{2}} + 2} = \frac{1}{2} - \frac{\frac{3}{4}}{5} =$$

$$= \frac{\overset{10}{1}}{2} - \frac{3}{20} = \frac{7}{20}$$

Rădăcini

$3x^2 + 2x - 1 = 0$

$\Delta = 16$

$x_{1,2} = \frac{-2 \pm 4}{6}$ → $x_1 = -1$, $x_2 = \frac{1}{3}$

$\boxed{\text{Dacă } f'(r) \neq 0 \Rightarrow r - \text{răd.}}$

rata de convergență $M = \left| \frac{f''(x)}{2 \cdot f'(r)} \right|$

$f''(x) = 6$

$M = \left| \frac{6}{2 \cdot f'(r)} \right|$

$f'\left(\frac{1}{3}\right) = 6 \cdot \frac{1}{3} + 2 = 4$

$M = \left| \frac{6}{2 \cdot 4} \right| = \frac{6}{8}$ ⟵ rata de conv.

Eroarea după 2 pași

$e = \frac{1}{3} - x_2 = \frac{1}{3} - \frac{7}{20} = \frac{-1}{60}$

(${}^{20)}$ and ${}^{3)}$ written above the fractions)

$\longrightarrow$ eroare de $\frac{1}{60}$

11. Rezolvați ecuațiile normale și găsiți REMP-ul pt:

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}}_{b}$$

$$A' \cdot A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 7 & 3 & 6 \\ 3 & 2 & 2 \\ 6 & 2 & 7 \end{bmatrix}$$

$$A' \cdot b = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 3 & 6 \\ 3 & 2 & 2 \\ 6 & 2 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 11 \end{bmatrix}$$

$2 - \frac{6}{7}$ $\quad 2 - \frac{18}{7}$

$2 - \frac{18}{7}$ $\quad 7 - \frac{36}{7}$

$3 - \frac{30}{7}$ $\quad 11 - \frac{60}{7}$

$$\left[\begin{array}{ccc|c} 7 & 3 & 6 & 10 \\ 3 & 2 & 2 & 3 \\ 6 & 2 & 7 & 11 \end{array}\right] \xrightarrow[L_3 \leftarrow L_3 - \frac{6}{7} L_1]{L_2 \leftarrow L_2 - \frac{3}{7} L_1}$$

$\frac{50}{7} - \frac{9}{14}$

$$\left[\begin{array}{ccc|c} 7 & 3 & 6 & 10 \\ 0 & \frac{8}{7} & -\frac{4}{7} & -\frac{9}{7} \\ 0 & -\frac{4}{7} & \frac{13}{7} & \frac{50}{7} \end{array}\right] \xrightarrow{L_3 \leftarrow L_3 + \frac{1}{2} L_2}$$

$$\left[\begin{array}{ccc|c} 7 & 3 & 6 & 10 \\ 0 & \frac{8}{7} & -\frac{4}{7} & -\frac{9}{7} \\ 0 & 0 & \frac{11}{7} & \frac{91}{14} \end{array}\right]$$

$\rightarrow \frac{11}{7}x_3 = \frac{91}{14} \rightarrow x_3 = \frac{91}{\cancel{14}_2} \cdot \frac{\cancel{7}}{11} = \frac{91}{22}$

$\frac{8}{7}x_2 - \frac{4}{7}x_3 = -\frac{9}{7} \rightarrow \frac{8}{7}x_2 = -\frac{9}{7} + \frac{4}{7}x_3 =$

$= -\frac{9}{7} + \frac{4}{7} \cdot \frac{91}{22} * = \frac{83}{77} \rightarrow x_2 = \frac{83}{77} \cdot \frac{7}{8} = \frac{83}{88}$

$7x_1 + 3x_2 + 6x_3 = 10 \rightarrow$

$\rightarrow 7x_1 = 10 - 3x_2 - 6x_3 = 10 - 3 \cdot \frac{83}{88} - 6 \cdot \frac{91}{22}$

$= -\frac{1553}{88} \rightarrow x_1 = -\frac{1553}{88} \cdot \frac{1}{7} = -\frac{1553}{616}$

$\rightarrow x = \left[-\frac{1553}{616}, \frac{83}{88}, \frac{91}{22}\right]^T$

$$A \cdot x = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1553}{616} \\ \frac{83}{88} \\ \frac{91}{22} \end{bmatrix} = \dots = b_{final}$$

eroare $= b_{initial} - b_{final}$

apoi fac norma și calc. REMP

~~6. Calculați nodurile de interpolare Cebîșev $x_1, \dots, x_m$, unde $m=3$. Găsiți limita superioară pt $|(x-x_1)\cdot \ldots \cdot (x-x_m)|$. Calc. pol Cebîșev $T_m(x) = (x-x_1)\ldots(x-x_m)$~~

12. Interpolați datele din tabel folosind modelul periodic $y = F_3(t) = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$

t	y
0	10
1/4	3
1/2	2
3/4	0

• modelul periodic

$$y = F_3(t) = c_1 + c_2 \cdot \cos 2\pi t + c_3 \cdot \sin 2\pi t$$

$$F_3(0) = c_1 + c_2 \cdot \cos 0 + c_3 \cdot \sin 0$$

$$F_3\left(\tfrac{1}{4}\right) = c_1 + c_2 \cdot \cos \tfrac{\pi}{2} + c_3 \cdot \sin \tfrac{\pi}{2}$$

$$F_3\left(\tfrac{1}{2}\right) = c_1 + c_2 \cdot \cos \pi + c_3 \cdot \sin \pi$$

$$F_3\left(\tfrac{3}{4}\right) = c_1 + c_2 \cdot \cos \tfrac{3\pi}{2} + c_3 \cdot \sin \tfrac{3\pi}{2}$$

$$\Rightarrow \begin{cases} F_3(0) = c_1 + c_2 \\ F_3\left(\tfrac{1}{4}\right) = c_1 + c_3 \\ F_3\left(\tfrac{1}{2}\right) = c_1 - c_2 \\ F_3\left(\tfrac{3}{4}\right) = c_1 - c_3 \end{cases}$$

$$\Rightarrow \begin{cases} c_1 + c_2 = 1 \\ c_1 + c_3 = 3 \\ c_1 - c_2 = 2 \\ c_1 - c_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 3 \\ 1 & -1 & 0 & 2 \\ \cancel{1} & \cancel{0} & \cancel{-1} & \cancel{0} \end{array}\right] \xrightarrow[L_3 \leftarrow L_3 - L_1]{L_2 \leftarrow L_2 - L_1} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 2 \\ 0 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{L_3 \leftarrow L_3 - 2L_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & -2 & -3 \end{array}\right]$$

$$\Rightarrow c_3 = \frac{3}{2}$$

$$-c_2 + c_3 = 2 \Rightarrow c_2 = c_3 - 2 = \frac{3}{2} - \overset{4/2}{2} = -\frac{1}{2}$$

$$c_1 = 1 - c_2 = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\Rightarrow c = \left[\frac{3}{2}, -\frac{1}{2}, \frac{3}{2}\right]^T$$

$$F_3(t) = \frac{3}{2} - \frac{1}{2} \cdot \cos(2\pi t) + \frac{3}{2} \sin(2\pi t)$$

13. Rearanjați ecuațiile pt. a forma un sis. strict diagonal dominant. Aplicați un pas din metoda lui Gauss-Seidel cu vectorul inițial $[1, 2, -1]^T$.

$$\begin{cases} u + 4v = 5 \\ v + 2w = 2 \\ 4u + 3w = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 0 & 5 \\ 0 & 1 & 2 & 2 \\ 4 & 0 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 4 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 1 & 4 & 0 & 5 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 4 & 0 & 3 & 0 \\ 1 & 4 & 0 & 5 \\ 0 & 1 & 2 & 2 \end{array}\right]$$

$$\Rightarrow \begin{cases} 4u + 3w = 0 & \Rightarrow u = \frac{-3w}{4} \\ u + 4v = 5 & \Rightarrow v = \frac{5-u}{4} \\ \underbrace{v + 2w}_{D} = \underbrace{2}_{b} & \Rightarrow w = \frac{2-v}{2} \end{cases}$$

~~$x_0 = [1, 2, -1]^T$~~

~~$x_1 = D^{-1}(b - U \cdot x_0 - L \cdot x_1)$~~

$$\begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} \frac{-3w_0}{4} \\ \frac{5-u_1}{4} \\ \frac{2-v_1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ \frac{5-\frac{3}{4}}{4} \\ \frac{2-\frac{17}{16}}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ \frac{17}{16} \\ \frac{15}{32} \end{bmatrix}$$

14. Folosiți factorizarea QR pt. a rez. pb. de tip cele mai mici pătrate

$$\underbrace{\begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix}}_{b}$$

$\cancel{Rx = Q^T b}$

$$y_1 = A_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$r_{11} = \|y_1\|_2 = \sqrt{9} = 3 \Rightarrow q_1 = \frac{y_1}{\|y_1\|_2} = \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$y_2 = A_2 - q_1 \cdot q_1^T \cdot A_2 = \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{9} & -\frac{4}{9} & \frac{2}{9} \\ -\frac{4}{9} & \frac{4}{9} & -\frac{2}{9} \\ \frac{2}{9} & -\frac{2}{9} & \frac{1}{9} \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix} =$$

$$= \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{8+8+2}{27} \\ \frac{-8-8-2}{27} \\ \frac{4+4+1}{27} \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{18}{27} \\ \frac{-18}{27} \\ \frac{9}{27} \end{bmatrix} =$$

$$= \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{7}{3} \\ -\frac{16}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$49 + 256 + 1 = 256 +$
50
306
102

$$q_2 = \frac{y_2}{\|y_2\|_2} = \frac{1}{\sqrt{34}} \begin{bmatrix} \frac{7}{3} \\ -\frac{16}{3} \\ -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{7}{3\sqrt{34}} \\ \frac{-16}{3\sqrt{34}} \\ \frac{-1}{3\sqrt{34}} \end{bmatrix}$$

$$r_{12} = q_1^T \cdot A_2 = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -6 \\ 0 \end{bmatrix} =$$

$$= 2 + 6 = 8$$

$$r_{22} = \|y_2\|_2 = \sqrt{34}$$

$$\Rightarrow A = \underbrace{\begin{bmatrix} \frac{2}{3} & \frac{7}{3\sqrt{34}} \\ -\frac{2}{3} & \frac{-16}{3\sqrt{34}} \\ \frac{1}{3} & \frac{-1}{3\sqrt{34}} \end{bmatrix}}_{Q} \cdot \underbrace{\begin{bmatrix} 3 & 8 \\ 0 & \sqrt{34} \end{bmatrix}}_{R}$$

- $R \cdot x = Q^T \cdot b$

$$\begin{bmatrix} 3 & 8 \\ 0 & \sqrt{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{7}{3\sqrt{34}} & \frac{-16}{3\sqrt{34}} & \frac{-1}{3\sqrt{34}} \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} 3 & 8 \\ 0 & \sqrt{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2+2+2 \\ \frac{7+16-6}{\sqrt{34}} \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} 3 & 8 \\ 0 & \sqrt{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ \frac{3}{\sqrt{34}} \end{bmatrix}$$

$$\Rightarrow x_2 = \frac{3}{34}$$

$$3x_1 + 8x_2 = 6 \Rightarrow 3x_1 = 6 - 8x_2$$

$$\Rightarrow x_1 = \frac{6 - 8x_2}{3} = \frac{6 - 8^{4} \frac{3}{34_{17}}}{3} = \frac{17 \cdot 6 - 12}{17 \cdot 3}$$

$$\Rightarrow x = \left[\frac{17 \cdot 6 - 12}{17 \cdot 3} , \frac{3}{34} \right]^T$$

16. Nodurile Cebîșev pe $[-3,7]$, $n=3$

Calc. pol Cebîșev

$$x_1 = \cos \frac{(2-1)\pi}{2\cdot 3} = \cos \frac{\pi}{6}$$

$$x_2 = \cos \frac{(4-1)\pi}{2\cdot 3} = \cos \frac{\pi}{2}$$

$$x_3 = \cos \left(\frac{(6-1)\pi}{2\cdot 3}\right. = \cos \frac{5\pi}{6}$$

$x \in [-3,7]$

$a=-3$, $b=7$

$$\frac{b+a}{2} = \frac{7-3}{2} = \frac{4}{2} = 2$$

$$\frac{b-a}{2} = \frac{7+3}{2} = 5$$

$$\Rightarrow x_1 = 2+5\cdot \cos \frac{\pi}{6}$$

$$x_2 = 2+5\cdot \cos \frac{3\pi}{6}$$

$$x_3 = 2+5\cdot \cos \frac{5\pi}{6}$$

$$|(x-x_1)(x-x_2)(x-x_3)| \le \frac{5^3}{2^2}$$ prin sup.

$$\frac{\left(\frac{b-a}{2}\right)^n}{2^{n-1}} \longrightarrow \frac{5^3}{2^2}$$

Polinomul

$$T_n(x) = (x-x_1)(x-x_2)(x-x_3)\cdot\frac{1}{2^{n-1}}$$

$$T_n(x) = \left(x-2-5\cos\frac{\pi}{6}\right)\left(x-2-5\cos\frac{3\pi}{6}\right)$$
$$\left(x-2-5\cos\frac{5\pi}{6}\right)\cdot\frac{1}{2^2}$$

17. Găsiți factorizarea LU a matricii de mai jos, fol. elim. gaussiană clasică. Verif. corectitudinea factorizării fol. înmulțirea matricilor

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow{L_3 \leftarrow L_3 - L_1} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow[L_4 \leftarrow L_4 - L_2]{L_3 \leftarrow L_3 - 2L_2} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$

15. Găsiți cea mai bună dreaptă care interpolează punctele $(-3,3), (-1,2), (0,1), (+1,-1)$ și găsiți REMP-ul

- $y = c_1 + c_2 \cdot t$

$$\begin{cases} c_1 - 3c_2 = 3 \\ c_1 - c_2 = 2 \\ c_1 = 1 \\ c_1 + c_2 = -1 \end{cases} \quad A = \begin{bmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 4 & -3 \\ -3 & 11 \end{bmatrix}$$

$$A^T \cdot b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \end{bmatrix} =$$

$$= \begin{bmatrix} 5 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ -3 & 11 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -12 \end{bmatrix}$$

$\frac{3}{4}$

$-11 - \frac{9}{4} = \frac{-44-9}{4} = -53$

$-12 + \frac{15}{4} = -48+15 =$

$$\begin{cases} 4x_1 - 3x_2 = 5 \Rightarrow 4x_1 = 5 + 3x_2 \\ -3x_1 - 11x_2 = -12 \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & -3 & 5 \\ -3 & -11 & -12 \end{array}\right] \xrightarrow{L_2 \leftarrow L_2 + \frac{3}{4} L_1} \left[\begin{array}{cc|c} 4 & -3 & 5 \\ 0 & \frac{-53}{4} & \frac{-33}{4} \end{array}\right]$$

$$\frac{-53}{4} x_2 = \frac{-33}{4} \Rightarrow x_2 = \frac{33}{4} \cdot \frac{4}{53} = \frac{33}{53}$$

$$4x_1 = 5 + 3x_2 \Rightarrow x_1 = \frac{5+3x_2}{4} =$$

$$= \frac{5 + 3 \cdot \frac{33}{53}}{4} = \frac{53 \cdot 5 + 3 \cdot 33}{53 \cdot 4}$$

$$x = \left[\frac{53 \cdot 5 + 3 \cdot 33}{53 \cdot 4} , \frac{33}{53} \right]^T$$

$$\Rightarrow y = \frac{53 \cdot 5 + 3 \cdot 33}{53 \cdot 4} - \frac{33}{53} t$$

SAIDA

$$y_1 = \underbrace{\frac{19}{35}}_{c_1} - \underbrace{\frac{33}{35}}_{c_2}(-3) = \frac{19-99}{35} = -\frac{80}{35}$$

$$y_2 = \frac{19}{35} - \frac{33}{35}(-1) = \frac{52}{35}$$

$$y_3 = \frac{19}{35} - 0 = \frac{19}{35}$$

$$y_4 = \frac{19}{35} - \frac{33}{35} = \frac{-14}{35}$$

$$r = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \end{bmatrix} = \frac{1}{35} \cdot \begin{bmatrix} -80-105 \\ 52-70 \\ 19-35 \\ -14+35 \end{bmatrix}$$

$$= \frac{1}{35} \cdot \begin{bmatrix} -185 \\ -18 \\ -16 \\ 21 \end{bmatrix}$$

$$\text{norma} = \sqrt{\left(\frac{1}{35}\right)^2 \cdot \left(185^2 + 18^2 + 16^2 + 21^2\right)}$$

$$m = 4$$

$$\text{REMP} = \frac{\text{norma}}{\sqrt{4}}$$