Matricea Jacobi asociata functiei $f : D \subset \mathbb{R}^3 \to \mathbb{R}^2$, $f(x,y,z) = (\underbrace{x^2y^3 - 2z}_{f_1}, \underbrace{ze^{2x-y+3z}}_{f_2})$ in punctul $x_0 = (-1,1,1)$ este

a. $\begin{pmatrix} -2 & 3 & -2 \\ -2 & 1 & 4 \end{pmatrix}$

b. $\begin{pmatrix} -2 & 3 & -2 \\ 2e & e & 3e \end{pmatrix}$

c. $\begin{pmatrix} -2 & 3 & -2 \\ 2 & -1 & 4 \end{pmatrix}$

d. $\begin{pmatrix} -4 & 3 & -2 \\ 2 & 1 & -2 \end{pmatrix}$

e. $\begin{pmatrix} -2 & -3 & -2 \\ -2e^2 & e^2 & 3e^2 \end{pmatrix}$

$$\begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \end{pmatrix}$$

Fie $f : D \subset \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \frac{x^2y^3}{x^4+y^6}$, unde D este domeniul maxim de definiție al funcției f și ℓ_{12}, ℓ_{21} limitele iterate ale funcției f în punctul $a = (0,0)$, respectiv ℓ limita globală a funcției f în $a = (0,0)$. Se consideră următoarele propoziții:

- P1: Limitele iterate există și sunt egale, iar $\ell = \frac{1}{2}$
- P2: Limitele iterate există și sunt diferite, iar ℓ nu există.
- P3: Limitele iterate există și sunt diferite, iar $\ell = 0$.
- P4: Ambele limitele iterate există si sunt egale, iar ℓ nu exista.
- P5: Doar una din cele două limite iterate există, iar $\ell = 0$.

Care din propozițiile de mai sus este adevărată?

a. P3

b. P4

c. P1

d. P2

e. P5

$$\ell_{12} = \lim_{x\to 0} \lim_{y \to 0} \frac{x^2y^3}{x^4+y^6} = \lim_{x\to 0} \frac{0}{x^4} = 0$$

$$\ell_{21} = \lim_{y\to 0} \lim_{x\to 0} \frac{x^2y^3}{x^4+y^6} = \lim_{y\to 0} \frac{0}{y^6} = 0$$

$$\frac{a+b}{2} \geq \sqrt{ab} \Downarrow a+b \geq 2\sqrt{ab}$$

$$\left|\frac{x^2y^3}{x^4+y^6}\right| \leq \left|\frac{\cancel{x^2}\cancel{y^3}}{2\cancel{x^2}\cancel{y^3}}\right| = \frac{1}{2} \xrightarrow[(x,y)\to(0,0)]{} \frac{1}{2}$$

$$\Rightarrow \nexists \lim_{(x,y)\to(0,0)} f(x,y) \Rightarrow P_4 \quad b$$

Fie $f(x,y) = \frac{x^{15}y^3}{\sqrt{x^4y^8+x^3y^9}}$. Stiind ca $E(x,y) = x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$, sa se calculeze $E(1,1)$

a. $\frac{2\sqrt{2}}{3}$

b. $\frac{3}{2}$

c. $\frac{3\sqrt{2}}{2}$

d. $\frac{3}{4}$

e. $6\sqrt{2}$

$$f(tx, ty) = \frac{t^{18}x^{15}y^3}{\sqrt{t^{12}x^4y^8 + t^{12}x^3y^9}} = \frac{t^{18}}{\sqrt{t^{12}}} f(x,y) = \frac{t^{18}}{t^6} f(x,y) = t^{12} f(x,y)$$

$$\Rightarrow p = 12$$

$$\Rightarrow E(x,y) = 12 f(x,y)$$

$E(1,1) = 12 \quad \frac{1}{\ldots} \quad 12\sqrt{2}$

$\Rightarrow E(x,y) = 12 \cdot f(x,y)$

$E(1,1) = 12 \cdot \frac{1}{\sqrt{2}} = 12 \frac{\sqrt{2}}{2} = 6\sqrt{2}$ e

Diferentiala de ordinul intai a functiei $f(x,y,z) = x^2y^3z - z^2x + yz^3$ in punctul (-1,1,-1) este

- a. $d_{(-1,1,-1)}f = dx - 4dy + 2dz$
- b. $d_{(-1,1,-1)}f = dx - 4dy - 2dz$
- c. $d_{(-1,1,-1)}f = -dx + 4dy + 2dz$
- d. $d_{(-1,1,-1)}f = -dx - 4dy + 2dz$
- e. $d_{(-1,1,-1)}f = dx - 4dy - 2dz$

$$d_{(\bar{x})} f = \frac{\partial f}{\partial x}(x_0, y_0, z_0)\, dx + \frac{\partial f}{\partial y}(\bar{x})\, dy + \frac{\partial f}{\partial z}(\bar{x})\, dz$$

Sa se aproximeze printr-un polinom de gadul intai urmatoarea functie $f(x,y) = \cos(x-y)$ in vecinatatea punctului $(\frac{\pi}{2}, \pi)$

- a. $x + y - \frac{\pi}{2}$
- b. $x - y - \frac{\pi}{2}$
- c. $x - y + \frac{\pi}{2}$
- d. $-x + y + \frac{\pi}{2}$
- e. $-x - y + \frac{\pi}{2}$

$$f(a,b) + \frac{1}{1!}\left[\frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b)\right]$$

Calculati $\frac{\partial^2 f}{\partial x \partial y}(-\frac{1}{3}, \frac{1}{2})$ pentru functia $f(x,y) = x^2 + 3y^2x - 2xy + x - 4y$

Răspuns:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$\frac{\partial f}{\partial y} = 6yx - 2x - 4$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = 6y - 2 \longrightarrow 3 - 2 = 1$$

10:47 4G

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8 întrebare

Timp rămas 0:02:06

Nu a primit răspuns încă

Marcat din 1,00

Întrebare cu flag

Derivatele partiale de ordinul intai pentru functia compusa $f : D \subset \mathbb{R}^2 \to \mathbb{R}, f(x,y) = g(u,v), g \in C^1(D)$, , unde $u(x,y) = \frac{y}{2x}, v(x,y) = x^2y^7$ sunt

- a. $\frac{\partial f}{\partial x} = \frac{y}{2x}\frac{\partial g}{\partial u} + 2x\frac{\partial g}{\partial v}$; $\frac{\partial f}{\partial y} = \frac{1}{2x}\frac{\partial g}{\partial u} + 7y^6\frac{\partial g}{\partial v}$
- b. $\frac{\partial f}{\partial x} = \frac{y}{4x^2}\frac{\partial g}{\partial u} + 12xy^6\frac{\partial g}{\partial v}$; $\frac{\partial f}{\partial y} = \frac{1}{2x}\frac{\partial g}{\partial u} + 7x^2y^6\frac{\partial g}{\partial v}$
- c. $\frac{\partial f}{\partial x} = \frac{y}{2}\frac{\partial g}{\partial u} + 2xy^7\frac{\partial g}{\partial v}$;

$$\boxed{\frac{\partial f}{\partial x} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial g}{\partial v} \cdot \frac{\partial v}{\partial x}}$$

$$= -\frac{y}{2x^2}\frac{dg}{du} + 2xy^7\frac{dg}{dv}$$

$$\boxed{\frac{\partial f}{\partial y} = \frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial g}{\partial v} \cdot \frac{\partial v}{\partial y}}$$

b. $\frac{\partial f}{\partial x} = \frac{y}{4x^2}\frac{\partial g}{\partial u} + 12xy^6\frac{\partial g}{\partial v}$;
$\frac{\partial f}{\partial y} = \frac{1}{2x}\frac{\partial g}{\partial u} + 7x^2y^6\frac{\partial g}{\partial v}$

c. $\frac{\partial f}{\partial x} = \frac{y}{2}\frac{\partial g}{\partial u} + 2xy^7\frac{\partial g}{\partial v}$;
$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial u} + 7x^2y^6\frac{\partial g}{\partial v}$

d. $\frac{\partial f}{\partial x} = -\frac{y}{2x^2}\frac{\partial g}{\partial u} + 2xy^7\frac{\partial g}{\partial v}$;
$\frac{\partial f}{\partial y} = \frac{1}{2x}\frac{\partial g}{\partial u} + 7x^2y^6\frac{\partial g}{\partial v}$

e. $\frac{\partial f}{\partial x} = -\frac{y}{4x^2}\frac{\partial g}{\partial u} + 2xy^7\frac{\partial g}{\partial v}$;
$\frac{\partial f}{\partial y} = \frac{1}{2x}\frac{\partial g}{\partial u} + 7x^2y^6\frac{\partial g}{\partial v}$

Trimite testul pentru evaluare

Navigare în test

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$= \frac{1}{2x}\frac{dg}{du} + 7x^2y^6\frac{dg}{dv}$$

10:20 LTE

1 întrebare
Nu a primit răspuns încă
Marcat din 1,00
Ștergeți flag

Timp rămas 0:29:25

Fie campul scalar $f(x,y,z) = -x^2 + yz^3$ si campul vectorial $\vec{v} = x\vec{i} + zy\vec{j} + xyz\vec{k}$. Calculati ∇f si $\nabla \times \vec{v}$

a. $\nabla f = -2x\vec{i} + z^3\vec{j} + 3yz^2\vec{k}$;
$\nabla \times \vec{v} = (xz - y)\vec{i} - yz\vec{j}$

b. $\nabla f = -2x\vec{i} + (-x^2 + z^3)\vec{j} + (-x^2$
$\nabla \times \vec{v} = xz\vec{i} + yz\vec{j}$

c. $\nabla f = -2x\vec{i} + z^3\vec{j} + 3yz^2\vec{k}$;
$\nabla \times \vec{v} = z\vec{i} - y\vec{j} + x\vec{k}$

d. $\nabla f = -2x\vec{i} + z^3\vec{j} + 3yz^2\vec{k}$;
$\nabla \times \vec{v} = xz\vec{i} - (yz_x)\vec{j} + 2\vec{k}$

e. $\nabla f = -2x + z^3 + 3yz^2$;
$\nabla \times \vec{v} = xz\vec{i} - yz\vec{j}$

Pagina următoare

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$$\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$$

$$\boxed{\frac{\partial f}{\partial \vec{v}}(x_0,y_0,z_0) = \frac{\partial f}{\partial x}(\bar{x})a\vec{i} + \frac{\partial f}{\partial y}(\bar{x})b\vec{j} + \frac{\partial f}{\partial z}(\bar{x})c\vec{k}}$$

$$\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

$$= -2x\vec{i} + z^3\vec{j} + 3yz^2\vec{k}$$

$$\nabla \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2x & z^3 & 3yz^2 \\ x & zy & xyz \end{vmatrix} =$$

$$= xyz^4\vec{i} + 3xyz^2\vec{j} - 2xyz\vec{k} - xz^3\vec{k} - 3y^2z^3\vec{i} + 2x^2yz\vec{j} =$$

$$= yz^3(xz - 3y)\vec{i} +$$
$$+ xyz(2x + 3z)\vec{j} -$$
$$- xz(2y + z^2)\vec{k}$$

$$\boxed{\frac{\partial f}{\partial \vec{v}}(x_0,y_0,z_0) = \frac{\partial f}{\partial x}(\bar{x})a\vec{i} + \frac{\partial f}{\partial y}(\bar{x})b\vec{j} + \frac{\partial f}{\partial z}(\bar{x})c\vec{k}}$$

$$\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

$$\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

$$= -2x\vec{i} + z^3\vec{j} + 3yz^2\vec{k}$$

$$v = x\vec{i} + zy\vec{j} + xyz\vec{k}$$

$$= -2x \cdot x\vec{i} + z^3 \cdot zy\vec{j} + xyz\,3yz^2\vec{k} =$$

$$= -2x^2\vec{i} + z^4y\vec{j} + 3xy^2z^3\vec{k}$$

$$\begin{array}{c|ccc} & x & y & z \\ \hline x & \frac{\partial^2 f}{\partial x^2} & \frac{\partial}{\partial x} & \\ y & \frac{}{\partial x \partial y} & \frac{}{\partial y^2} & \\ z & & & \frac{}{\partial z^2} \end{array}$$

Studiati continuitatea functiei in origine

$$f(x,y) = \begin{cases} \dfrac{\ln(1+x^3+y^5)}{x^2+y^4}, (x,y) \neq (0,0) \\ \alpha, (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} \underbrace{\frac{\ln(1+x^3+y^5)}{x^3+y^5}}_{1} \cdot \frac{x^3+y^5}{x^2+y^4}$$

$|x|$

$$\left|\frac{x^3+y^5}{x^2+y^4}\right| \leq \left|\frac{x^3}{x^2+y^4} + \frac{y^5}{x^2+y^4}\right| \leq \left|\frac{x^3}{x^2+y^4}\right| + \left| \right.$$

$$\left| x^3 \right| \quad \left| y^5 \right|$$

$$\left| \frac{x^2}{x^2+y^4} \right| < |x|$$

$$\left| \frac{y^5}{x^2+y^4} \right|$$

$$\le \left|\frac{x^3}{x^2}\right| + \left|\frac{y^5}{y^4}\right| \le |x| + |y| \xrightarrow[(x,y)\to(0,0)]{} (0$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y) = 0$$

$\alpha = 0 \Rightarrow f$ - cont pe $\mathbb{R}$

$\alpha \neq 0 \Rightarrow f$ - cont pe $\mathbb{R}^*$

10:28 4G

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Analiza matematica - seria 1

Timp rămas 0:21:02

2 întrebare

Nu a primit răspuns încă

Marcat din 1,00

Întrebare cu flag

Sa se aproximeze printr-un polinom de gadul intai urmatoarea functie $f(x, y) = \sin^2(xy)$ in vecinatatea punctului $(\frac{\pi}{2}, \frac{1}{2})$.

- a. $\frac{1}{2} + (x - \frac{\pi}{2})\frac{\pi}{2} + (y - \frac{1}{2})\frac{1}{2}$
- b. $\frac{1}{2} + (x - \frac{\pi}{2})\frac{1}{4} + (y - \frac{1}{2})\frac{\pi}{2}$
- c. $\frac{1}{2} + (x - \frac{\pi}{2})\frac{\pi}{4} + (y - \frac{1}{2})\frac{\pi}{4}$
- d. $\frac{1}{2} + (x - \frac{\pi}{2})\frac{1}{2} + (y - \frac{1}{2})\frac{\pi}{2}$
- e. $\frac{1}{2} + (x - \frac{\pi}{2})\frac{1}{2} + (y - \frac{1}{2})\frac{\pi}{4}$

Pagina următoare

Navigare în test

1 2 3 4 5 6 7 8

Trimite testul pentru evaluare

$$\frac{\partial f}{\partial x} = 2\sin xy\cos(xy)\cdot y$$

$$\frac{\partial f}{\partial y} = x\sin(2xy)$$

$$(T_1 f) = f\left(\frac{\pi}{2}, \frac{1}{2}\right) + \frac{1}{1!}\left[\frac{\partial f}{\partial x}\left(\frac{\pi}{2}, \frac{1}{2}\right)\right.$$

$$\left(y - \frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{1}{2}\left(x - \frac{\pi}{2}\right) + \frac{\pi}{2}$$

$,0)$

$= y \sin 2xy$

$)\left(x - \frac{\pi}{2}\right) + \frac{\partial f}{\partial y}\left(\frac{\pi}{2}, \frac{1}{2}\right)$

$] =$

$\left(y - \frac{1}{2}\right)$

7 întrebare

Nu a primit răspuns încă

Marcat din 2,00

Întrebare cu flag

Fie $f(x,y) = \frac{x^{15}y^9}{\sqrt[4]{x^4y^8+x^3y^9}}$. Stiind ca $E(x,y) = x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$, sa se calculeze $E(1,1)$

- a. $21\frac{\sqrt[4]{4}}{2}$
- b. $2\sqrt[4]{6}$
- c. $\frac{\sqrt[4]{2}}{2}$
- d. $21\frac{\sqrt[4]{8}}{2}$
- e. $17\frac{\sqrt[4]{8}}{2}$

$$f(tx, ty) = \frac{t^{24} x^{15} y^9}{\sqrt[4]{t^{12} x^4 y^8 + t^{12} x^3 y^9}} = \frac{t^{24}}{t^3} f$$

$$= t^{21} f(x,y) \Rightarrow n = 21$$

$$\Rightarrow E(x,y) = 21 f(x,y)$$

$$E(1,1) = 21 f(1,1) = 21 \frac{1}{\sqrt[4]{2}} = 21 \frac{\sqrt[4]{2^3}}{2}$$

10:39 4G

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Înapoi

Timp rămas 0:10:39

7 întrebare

Nu a primit răspuns încă

Marcat din 1,00

Întrebare cu flag

Fie functia $F : D \subset \mathbb{R}^2 \to \mathbb{R}, f(x,y) = \sqrt{x^2+3y^2}$. Stiind ca

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$\frac{3y}{} \Rightarrow 3y \left(- \right.$$

$(x, y) =$

$= \quad 2, \quad \frac{\sqrt[4]{8}}{2}$

$\frac{x}{\ }$

Fie

functia $F : D \subset \mathbb{R}^2 \to \mathbb{R}, f(x,y) = \sqrt{x^2 + 3y^2}$. Stiind ca $\frac{\partial^2 f}{\partial x \partial y}(-1,1) = \frac{a}{b}$, $\frac{a}{b}$ fractie ireductibila, sa se calculeze $a + b$.

Răspuns:

Pagina următoare

Navigare în test

1 2 3 4 5 6 7 8

Trimite testul pentru evaluare

$$\frac{3y}{\sqrt{x^2+3y^2}} \longrightarrow 3y\left(-\right.$$

$$-3\,\frac{(-1)}{\sqrt{64}} = \frac{3}{8} -$$

$$\frac{x}{\sqrt{(x^2+3y^2)^3}}$$

$\longrightarrow 11$