REGIMUL PERMANENT NESINUSOIDAL. ANALIZA FOURIER

Jean Baptiste Joseph Fourier (1768–1830), a French mathematician, first presented the series and transform that bear his name. Fourier’s results were not enthusiastically received by the scientific world. He could not even get his work published as a paper.

Born in Auxerre, France, Fourier was orphaned at age 8. He attended a local military college run by Benedictine monks, where he demonstrated great proficiency in mathematics. Like most of his contemporaries, Fourier was swept into the politics of the French Revolution. He played an important role in Napoleon’s expeditions to Egypt in the later 1790s. Due to his political involvement, he narrowly escaped death twice.

Serii Fourier

- Metoda complexului simplificat folosită în analiza circuitelor în regim sinusoidal, poate fi extinsă şi în cazul funcţiilor periodice nesinusoidale.
- În general, o funcţie periodică *f(t)=f(t+T)*, având perioada *T*, poate fi exprimată printr-o sumă de termeni în sinus şi cosinus, numită *serie Fourier*.
- În matematică, teorema asociată cu seriile Fourier arată că o funcţie *f(t)* poate fi scrisă în forma:

$$f(t)=a_0+a_{m1}\sin(\omega t)+a_{m2}\sin(2\omega t)+\ldots+b_{m1}\cos(\omega t)+b_{m2}\cos(2\omega t)+\ldots$$ sau

$$f(t)=a_0+\sum_{\nu=1}^{\infty}\left(a_{m\nu}\sin(\nu\omega t)+b_{m\nu}\cos(\nu\omega t)\right)$$

unde *ω=2π/T =2πf* este pulsaţia fundamentalei, *T* şi *f* reprezentând perioada şi respectiv frecvenţa funcţiei periodice iniţiale.

- Termenul a_0 reprezintă valoarea medie sau componenta continuă a funcţiei iniţiale.
- Termenul $a_{m1}\sin(\omega t)+b_{m1}\cos(\omega t)$ reprezintă componenta (armonica) fundamentală a funcţiei iniţiale având aceiaşi pulsaţie cu semnalul iniţial.
- Restul termenilor de forma $a_{m\nu}\sin(\nu\omega t)+b_{m\nu}\cos(\nu\omega t)$ dau armonica de ordinul *γ a funcţiei periodice iniţiale*.

- Valoarea coeficienţilor Fourier se poate determina după cum urmează:
- Coeficientul a_0 reprezintă valoarea medie a funcţiei *f(t)*: $a_0 = \frac{1}{T}\int_0^T f(t)dt$
- Pentru a determina coeficientul $a_{m\gamma}$, înmulţim fiecare termen al seriei cu *sin(γωt)* după care integrăm pe o perioadă (între *0* şi *T*). Astfel obţinem:

$$a_{m\nu} = \frac{2}{T}\int_0^T f(t)\sin(\nu\omega t)dt$$

- Pentru determinarea coeficientului $b_{m\gamma}$, înmulţim fiecare termen cu *cos(γωt)* după care integrăm pe o perioadă. Astfel se obţine:

$$b_{m\nu} = \frac{2}{T}\int_0^T f(t)\cos(\nu\omega t)dt$$

- Există câteva cazuri particulare care prezintă interes:

1. *f(t)=f(-t),* f este o funcţie pară, *f(t)=f(T-t):* $a_{m\nu} = 0,\quad b_{m\nu} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(\nu\omega t)dt$

2. *f(t)=-f(-t),* f este o funcţie impară, *f(t)=-f(T-t)* $a_{m\nu} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(\nu\omega t)dt,\quad b_{m\nu} = 0$

3. *f(t)=f(t+T/2),* f are doar coeficienţi pari: $a_{2n} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(2n\omega t)dt,\quad b_{2n} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(2n\omega t)dt$

4. *f(t)=-f(t+T/2),* f are doar coeficienţi impari:

$$a_{2n+1} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin((2n+1)\omega t)dt,$$
$$b_{2n+1} = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos((2n+1)\omega t)dt$$

- O reprezentare exactă a funcţiei periodice pe baza serie Fourier presupune un număr infinit de termeni Fourier. De menţionat că amplitudinea armonicilor în seriile Fourier scade progresiv cu creşterea ordinului armonicii. Aceasta permite realizarea unei bune aproximări a funcţiei periodice iniţiale cu un număr limitat de termeni Fourier.
- Armonica de ordinul γ dintr-o serie Fourier poate fi scrisă şi în forma:

$a_{m\nu}\sin(\nu\omega t)+b_{m\nu}\cos(\nu\omega t)=A_{m\nu}\sin(\nu\omega t+\varphi_\nu)$,unde $A_{m\nu}^2=a_{m\nu}^2+b_{m\nu}^2, \quad \varphi_\nu=arctg\left(\frac{b_{m\nu}}{a_{m\nu}}\right)$

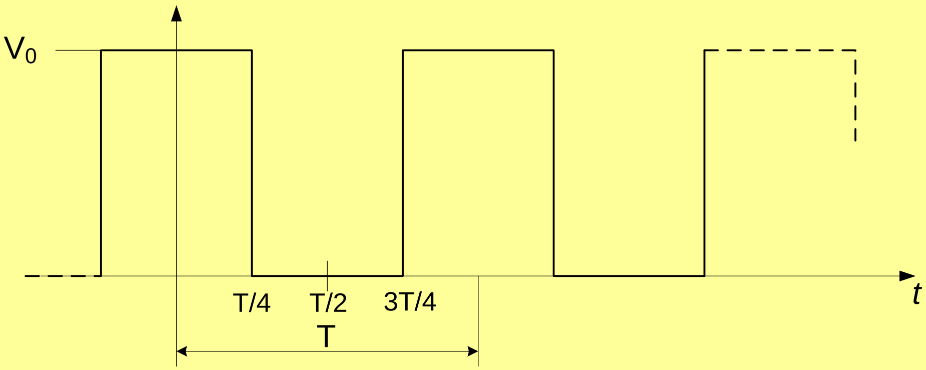
- O formă concentrată a seriei Fourier este:

$$f(t)=A_0+\sum_{\nu=1}^{\infty}A_{m\nu}\sin(\nu\omega t+\varphi_\nu)=A_0+\sum_{\nu=1}^{\infty}A_{m\nu}\sin(\omega_\nu t+\varphi_\nu)$$

unde $A_0=a_0$ şi: $\omega_\nu=\nu\omega=2\pi(\nu f)=\frac{2\pi}{T/\nu}$

- Pentru armonica de ordinul *γ*, frecvenţa f_γ devine de *γ* ori mai mare decât frecvenţa fundamentalei *f*, în timp ce perioada T_γ devine de *γ* ori mai mică decât perioada fundamentalei *T*.

Exemplu



f(t)=f(-t) funcţie pară

$$a_{m\gamma} = 0$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{T}\left(\int_0^{T/4} V_0\,dt + \int_{3T/4}^{T} V_0\,dt\right) = \frac{V_0}{2}$$

$$a_0 = \frac{V_0}{2}$$

$$b_{m\nu} = \frac{2}{T}\int_0^T f(t)cos(\nu\omega t)dt = \frac{2}{T}\left(\int_0^{T/4} V_0\,cos(\nu\omega t)dt + \int_{3T/4}^{T} V_0\,cos(\nu\omega t)dt\right)$$

$$b_{m\nu} = \frac{4}{T}\int_0^{T/2} f(t)\cos(\nu\omega t)dt = \frac{4}{T}\left(\int_0^{T/4} V_0\cos(\nu\omega t)dt\right) = \frac{4}{T}\frac{V_0}{\gamma\frac{2\pi}{T}}\sin\left(\nu\frac{2\pi}{T}t\right)\Big|_0^{T/4} = \frac{2V_0}{\gamma\pi}\left(\sin\left(\frac{\gamma\pi}{2}\right) - \sin(0)\right)$$

$$b_{m\gamma} = \begin{cases} -(-1)^n\dfrac{2V_0}{(2n-1)\pi}, & if\ \gamma = 2n-1 \\ 0 & ,if\ \gamma = 2n \end{cases}$$

$$b_{m1} = \frac{2V_0}{\pi},\ b_{m2} = 0,\ b_{m3} = -\frac{2V_0}{3\pi},\ b_{m4} = 0,\ b_{m5} = \frac{2V_0}{5\pi},\ b_{m6} = 0,\ b_{m7} = -\frac{2V_0}{7\pi},\ b_{m8} = 0,....$$

$$v(t) = \frac{V_0}{2} + \frac{2V_0}{\pi}\cos(\omega t) - \frac{2V_0}{3\pi}\cos(3\omega t) + \frac{2V_0}{5\pi}\cos(5\omega t) - \frac{2V_0}{7\pi}\cos(7\omega t) + \frac{2V_0}{9\pi}\cos(9\omega t) - \frac{2V_0}{11\pi}\cos(11\omega t) + \dots$$

Valoarea efectivă. Factorul de distorsiune (THD)

- Fie o funcţie periodică de perioadă *T*:

$$f(t)=A_0+\sum_{\nu=1}^{\infty}A_{m\nu}\sin(\nu\omega t+\varphi_\nu)$$

- Valoarea efectivă a funcţiei periodice *f*, este:

$$F=\sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$$

- Dorim să exprimăm valoarea efectivă în funcţie de armonici:

$$F^2=\frac{1}{T}\int_0^T\left(A_0+\sum_{\nu=1}^{\infty}A_{m\nu}\sin(\nu\omega t+\varphi_\nu)\right)^2 dt$$

$$F^2=A_0^2+\sum_{\nu=1}^{\infty}A_\nu^2$$

$$\frac{1}{T}\int_0^T A_{m\nu}^2\sin^2(\nu\omega t+\varphi_\nu)dt=\frac{A_{m\nu}^2}{2}=A_\nu^2$$

$$\frac{A_{m\nu}A_{m\lambda}}{T}\int_0^T\sin(\nu\omega t+\varphi_\nu)\sin(\lambda\omega t+\varphi_\lambda)dt=0$$

$$\frac{A_{m\nu}}{T}\int_0^T\sin(\nu\omega t+\varphi_\nu)dt=0$$

$$F=\sqrt{A_0^2+\sum_{\nu=1}^{\infty}A_\nu^2}$$

- Factorul de distorsiune *(total harmonic distortion (THD))* a unui semnal periodic (funcţie periodică) este o măsură a distorsiunii semnalului dată de prezenţa armonicilor superioare. Factorul de distorsiune este definit ca raport între suma puterilor active date de toate armonicile superioare şi puterea activă corespunzătoare fundamentalei.

$$THD = \frac{\sum[harmonic\,powers]}{fundamntal\,frequency\,power} = \frac{P_2 + P_3 + P_4 + \ldots}{P_1}$$

- Unii autori definesc factorul de distorsiune ca raport între amplitudini (în loc de raport între puteri active).

$$THD = \frac{\sqrt{A_2^2 + A_3^2 + A_4^2 + \ldots}}{A_1} = \frac{\sqrt{\sum_{\nu=2}^{\infty} A_\nu^2}}{A_1}$$

- De exemplu, când un semnal sinusoidal trece printr-un element neliniar, apare un conţinut suplimentar de armonic la frecvenţa fundamentală. Spunem că semnalul a fost distorsionat. Factorul de distorsiune este o măsură a mărimii distorsiunii.

$$THD_U = \frac{\sqrt{U_2^2 + U_3^2 + U_4^2 + \ldots}}{U_1} = \frac{\sqrt{\sum_{\nu=2}^{\infty} U_\nu^2}}{U_1}$$

$$THD_I = \frac{\sqrt{I_2^2 + I_3^2 + I_4^2 + \ldots}}{I_1} = \frac{\sqrt{\sum_{\nu=2}^{\infty} I_\nu^2}}{I_1}$$

Răspunsul circuitelor la funcţii periodice

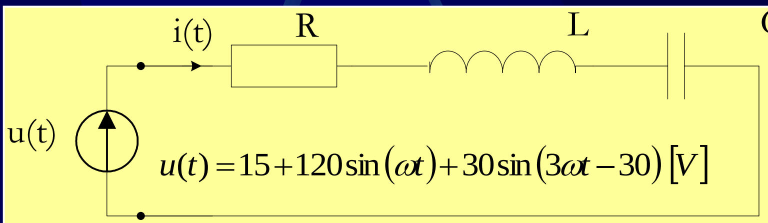
- În general circuitele electrice sunt alimentate cu tensiuni şi curenţi având forme periodice foarte apropiate de forma sinusoidală dar nu perfect sinusoidale.

- Faptul că un semnal periodic se poate scrie printr-o serie Fourier conduce la concluzia că toate cunoştinţele folosite la rezolvarea regimului sinusoidal pot fi folosite şi în acest caz. Fiecare termen din seria Fourier scrisă pentru fiecare sursă de tensiune sau curent este considerat ca o sursă separată (de tensiue sau curent).

- Se aplică principiul superpoziţiei conform căruia răspunsul total reprezintă suma răspunsurilor produse de fiecare armonică în parte.

- In general, numeroase funcţii periodice pot fi aproximate cu o bună precizie, printr-un număr redus de termeni în dezvoltarea Fourier, aceasta reducând substanţial calculul circuitului. Decizia de a include sau a neglija anumiţi termeni din seria Fourier, reprezintă o abordare inginerească care depinde în principal de acurateţea cu care dorim să obţinem rezultatele.
- Reprezentarea Fourier a excitaţiilor periodice (a surselor din circuit) şi principiul superpoziţiei permit utilizarea complexului simplificat pentru a obţine răspunsul sistemului. Fiecare componentă din răspunsul circuitului este produsă de armonica corespunzătoare din excitaţie; suma acestor răspunsuri reprezintă seria Fourier a semnalului căutat, semnal corespunzător regimului permanent nesinusoidal.

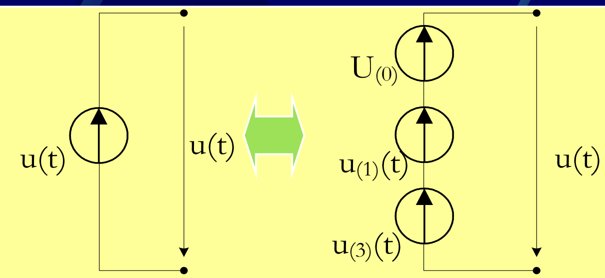
Example



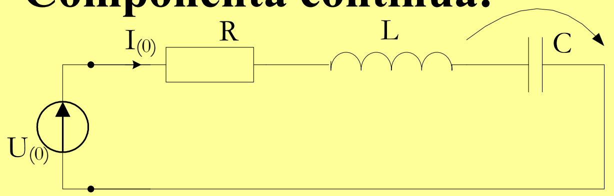
$$u(t)=15+120\sin(\omega t)+30\sin(3\omega t-30)\,[V]$$

R=6Ω
ωL=1Ω
1/ωC=9Ω

$I=?\qquad u_C(t)=?$



Componenta continuă:

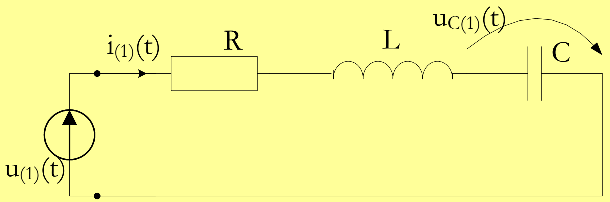


$$U_{(0)}=15V,\qquad I_{(0)}=0\,A,\qquad U_{C(0)}=U_{(0)}=15V$$

$$u_1(t)=120\sin(\omega t)\quad\rightarrow\quad \underline{U}_{(1)}=\frac{120}{\sqrt{2}}$$

Componenta fundamentală:

$$\underline{I}_{(1)}=\frac{\underline{U}_{(1)}}{\underline{Z}_{(1)}}=\frac{120}{\sqrt{2}\left[R+j\left(\omega L-\frac{1}{\omega C}\right)\right]}=\frac{120}{\sqrt{2}\,[6+j(1-9)]}=8.485\,e^{j53}$$

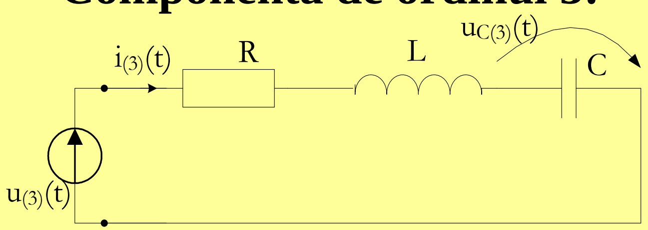


$$\underline{U}_{C(1)}=\frac{-j}{\omega C}\underline{I}_{(1)}=-9\,j\,8.485\,e^{j53}=76.365\,e^{-j37}$$

$$u_3(t)=30\sin\left(\omega t-30^0\right)\quad\rightarrow\quad \underline{U}_{(3)}=\frac{30}{\sqrt{2}}e^{-j30}$$

Componenta de ordinul 3:

$$\underline{I}_{(3)}=\frac{\underline{U}_{(3)}}{\underline{Z}_{(3)}}=\frac{30\,e^{-j30}}{\sqrt{2}\left[R+j\left(3\omega L-\frac{1}{3\omega C}\right)\right]}=\frac{30\,e^{-j30}}{\sqrt{2}\,[6+j(3-3)]}=3.536\,e^{-j30}$$



$$\underline{U}_{C(3)}=\frac{-j}{3\omega C}\underline{I}_{(3)}=-3\,j\,3.536\,e^{-j30}=10.608\,e^{-j120}$$

$$I=\sqrt{I_{(0)}^2+I_{(1)}^2+I_{(3)}^2}=\sqrt{0^2+8.485^2+3.536^2}=9.192\ (A)$$

$$u_C(t)=U_{(0)}+u_{C(1)}(t)+u_{C(3)}(t)=15+76.365\sqrt{2}\sin\left(\omega t-37^0\right)+10.608\sqrt{2}\sin\left(3\omega t-120^0\right)[V]$$

- Observaţie:
- Reactanţele inductive şi capacitive, şi ca urmare impedanţele sunt funcţii de pulsaţia ω a funcției sinusoidale.
- În analiza Fourier, reactanţele inductive şi capacitive, precum şi impedanţa, diferă de la o armonică la alta. Trebuie avut în vedere că reactanţa inductivă creşte cu ordinul armonicii în timp ce reactanţa capacitivă scade cu creştera armonicii.
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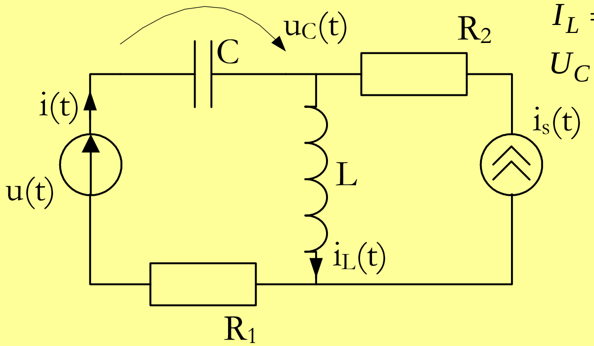
Example



$I_L = ?,\ i_L(t) = ?$

$U_C = ?,\ u_C(t) = ?$

R1=5Ω, R2=ωL=1/ωC=10Ω

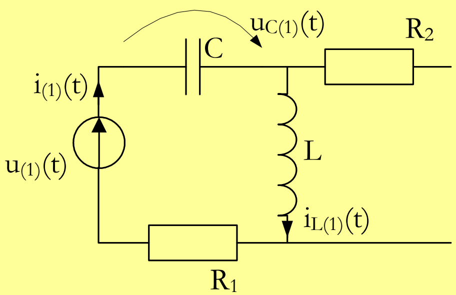
$$u(t) = 100 + 50\sin(\omega t - 30)\,[V] \qquad i_s(t) = 10 + 5\sin(2\omega t)\,[A]$$

Componenta continuă: $U_{(0)} = 100V, \quad I_{s(0)} = 10\,A$

$$U_{C(0)} = U_{(0)} = 100V, \quad I_{L(0)} = I_{s(0)} = 10\,A$$

Componenta fundamentală:

$$u_1(t) = 50\sin\left(\omega t - 30^0\right) \;\rightarrow\; \underline{U}_{(1)} = \frac{50\,e^{-j30}}{\sqrt{2}}$$

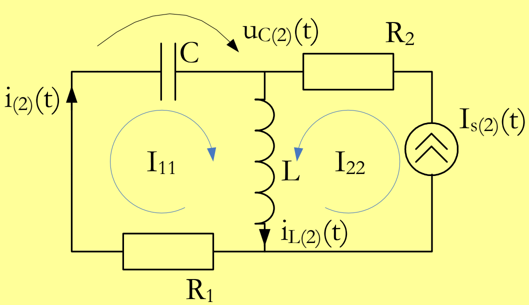


$$\underline{I}_{(1)} = \underline{I}_{L(1)} = \frac{\underline{U}_{(1)}}{\underline{Z}_{(1)}} = \frac{50\,e^{-j30}}{\sqrt{2}\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right]} = \frac{50\,e^{-j30}}{\sqrt{2}\left[5 + j(10-10)\right]} = \frac{10}{\sqrt{2}}e^{-j30}$$

$$\underline{U}_{C(1)} = \frac{-j}{\omega C}\underline{I}_{(1)} = -10j\frac{10}{\sqrt{2}}e^{-j30} = \frac{100}{\sqrt{2}}e^{-j120}$$

Componenta de ordinul 2:

$$i_{s(2)}(t) = 5\sin(2\omega t) \;\rightarrow\; \underline{I}_{s(2)} = \frac{5}{\sqrt{2}}$$



$$\begin{cases} \underline{I}_{22} = \underline{I}_{s(2)} = \frac{5}{\sqrt{2}} \\ \underline{I}_{11}\left[R_1 + j2\omega L - \frac{j}{2\omega C}\right] + j2\omega L\,\underline{I}_{22} = 0 \end{cases}$$

$$\underline{I}_{(2)} = \underline{I}_{11} = -4.243 - j1.414 = 4.472\,e^{-j162}$$

$$\underline{I}_{L(2)} = \underline{I}_{11} + \underline{I}_{22} = -0.707 - j1.414 = 1.581\,e^{-j117}$$

$$\underline{U}_{C(2)} = \frac{-j}{2\omega C}\underline{I}_{(2)} = -7.071 + j21.213 = 108.435\,e^{j108}$$

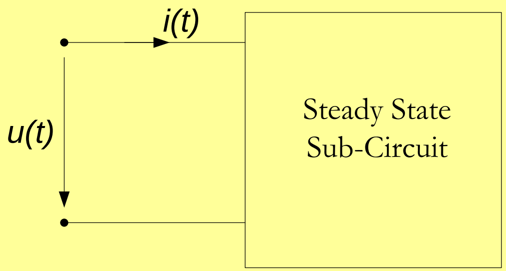
$$I=\sqrt{I_{(0)}^2+I_{(1)}^2+I_{(2)}^2}=\sqrt{10^2+\left(\frac{10}{\sqrt{2}}\right)^2+1.581^2}=12.349\ [A]$$

$$i_L(t)=I_{L(0)}+i_{L(1)}(t)+i_{L(2)}(t)=10+10\sin\left(\omega t-30^0\right)+1.581\sqrt{2}\sin\left(2\omega t-117^0\right)[A]$$

$$U_C=\sqrt{U_{C(0)}^2+U_{C(1)}^2+U_{C(2)}^2}=\sqrt{100^2+\left(\frac{100}{\sqrt{2}}\right)^2+108.435^2}=163.579\ (V)$$

$$u_C(t)=U_{C(0)}+u_{C(1)}(t)+u_{C(2)}(t)=100+100\sin\left(\omega t-120^0\right)+108.435\sqrt{2}\sin\left(2\omega t-108^0\right)[V]$$

Puteri în regim periodic nesinusoidsal



$$u(t)=U_0+\sum_{\gamma=1}^{\infty}u_{m\gamma}\sin(\gamma\omega t+\alpha_\gamma)$$

$$i(t)=I_0+\sum_{\gamma=1}^{\infty}i_{m\gamma}\sin(\gamma\omega t+\alpha_\gamma-\varphi_\gamma)$$

$$p(t)=u(t)i(t)$$

$$P=\frac{1}{T}\int_0^T p(t)dt$$

Puterea activă

$$P=\frac{1}{T}\int_0^T\left[U_0+\sum_{\gamma=1}^{\infty}u_{m\gamma}\sin(\gamma\omega t+\alpha_\gamma)\right]\left[I_0+\sum_{\gamma=1}^{\infty}i_{m\gamma}\sin(\gamma\omega t+\alpha_\gamma-\varphi_\gamma)\right]dt$$

$$P=U_0I_0+\sum_{\gamma=1}^{\infty}U_\gamma I_\gamma\cos\varphi_\gamma \quad [W]$$

$$\int_0^T\sin(\gamma\omega t+\alpha_\gamma)dt=0,\quad \int_0^T\sin(\gamma\omega t+\alpha_\gamma-\varphi_\gamma)dt=0$$

$$\int_0^T u_{m\gamma}\sin(\gamma\omega t+\alpha_\gamma)i_{m\gamma}\sin(\gamma\omega t+\alpha_\gamma-\varphi_\gamma)dt=U_\gamma I_\gamma\cos\varphi_\gamma$$

$$\int_0^T\sin(\gamma\omega t+\alpha_\gamma)\sin(\xi\omega t+\alpha_\xi-\varphi_\xi)dt=0$$

$$Q=\sum_{\gamma=1}^{\infty}U_\gamma I_\gamma\sin\varphi_\gamma \quad [VAR]$$ **Puterea reactivă**

$$S=U\,I \quad [VA]$$ **Puterea aparentă**

$$U=\sqrt{U_0^2+\sum_{\nu=1}^{\infty}U_\nu^2}\qquad I=\sqrt{I_0^2+\sum_{\nu=1}^{\infty}I_\nu^2}$$

$$\cancel{S\neq U_0\,I_0+\sum_{\gamma=1}^{\infty}U_\gamma\,I_\gamma}$$

$S^2=P^2+Q^2$ pentru regim sinusoidal

$S^2=P^2+Q^2+D^2$ pentru regim periodic nesinusoidal

$$D=\sqrt{S^2-P^2-Q^2}\quad [VAD]$$ **Puterea deformantă**

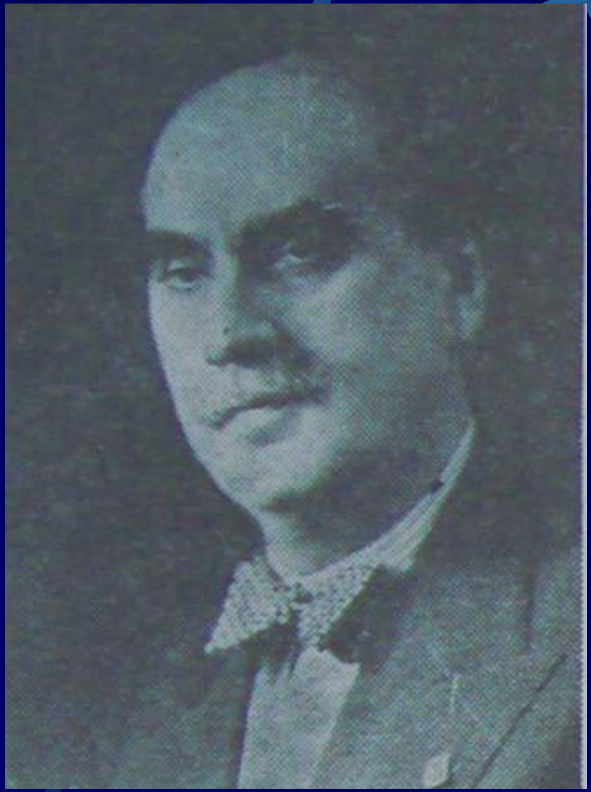
$$S^2 = P^2 + Q^2 + D^2$$

$$S = U\,I = \sqrt{U_0^2 + \sum_{\nu=1}^{\infty} U_\nu^2}\;\sqrt{I_0^2 + \sum_{\nu=1}^{\infty} I_\nu^2} \qquad P = U_0 I_0 + \sum_{\gamma=1}^{\infty} U_\gamma I_\gamma \cos\varphi_\gamma \qquad Q = \sum_{\gamma=1}^{\infty} U_\gamma I_\gamma \sin\varphi_\gamma$$

$$\left(U_0^2 + U_1^2 + \ldots + U_\gamma^2 + \ldots + U_\xi^2 + \ldots\right)\left(I_0^2 + I_1^2 + \ldots + I_\gamma^2 + \ldots + I_\xi^2 + \ldots\right) = \left(U_0 I_0 + \sum_{\gamma=1}^{\infty} U_\gamma I_\gamma \cos\varphi_\gamma\right)^2 + \left(\sum_{\gamma=1}^{\infty} U_\gamma I_\gamma \sin\varphi_\gamma\right)^2 + D^2$$

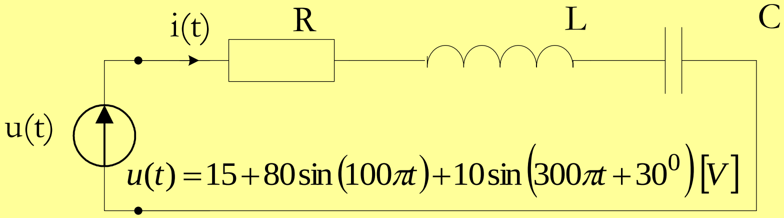
$$D^2 = \sum_{\gamma,\xi}\left(U_\gamma^2 I_\xi^2 + U_\xi^2 I_\gamma^2 - 2U_\gamma U_\xi I_\gamma I_\xi \cos\varphi_\gamma \cos\varphi_\xi - 2U_\gamma U_\xi I_\gamma I_\xi \sin\varphi_\gamma \sin\varphi_\xi\right)$$

$$D = \sqrt{\sum_{\gamma,\xi}\left[U_\gamma^2 I_\xi^2 + U_\xi^2 I_\gamma^2 - 2U_\gamma U_\xi I_\gamma I_\xi \cos\left(\varphi_\gamma - \varphi_\xi\right)\right]}$$



Constantin Budeanu (28 February 1886 - 1959) was a Romanian electrical engineer who contributed to the analysis of electric networks states and the SI system of units.

He studied electricity in Paris with a V. Adamachi scholarship gained after the completion of studies in Bucharest. He proposed the unit electric reactive power and he introduced the concept of *deformed power* in electric networks.



$$u(t)=15+80\sin(100\pi t)+10\sin\left(300\pi t+30^0\right)[V]$$

R=6Ω
L=100mH
C=200μF

$U=?$

$u_C(t)=?$

$P=?,\ Q=?,\ D=?$

$$U=\sqrt{U_{(0)}^2+U_{(1)}^2+U_{(3)}^2}=\sqrt{15^2+\left(\frac{80}{\sqrt{2}}\right)^2+\left(\frac{10}{\sqrt{2}}\right)^2}=59\ (V)$$

Componenta continuă: $U_{(0)}=15V,\quad I_{(0)}=0\,A,\quad U_{C(0)}=U_{(0)}=15V$

Componenta fundamentală: $u_1(t)=80\sin(100\pi t)\ \rightarrow\ \underline{U}_{(1)}=\frac{80}{\sqrt{2}}$

$$X_{L(1)}=\omega L=100\pi\,0.1=31.42(\Omega)$$

$$X_{C(1)}=\frac{1}{\omega C}=\frac{1}{100\pi\,200\cdot10^{-6}}=15.92(\Omega)$$

$$\underline{I}_{(1)}=\frac{\underline{U}_{(1)}}{\underline{Z}_{(1)}}=\frac{\underline{U}_{(1)}}{R+j\left(X_{L(1)}-X_{C(1)}\right)}=3.07\,e^{-j57}$$

$$\underline{U}_{C(1)}=-jX_{C(1)}\underline{I}_{(1)}=48.81e^{-j147}$$

Componenta de ordinul 3: $u_3(t)=10\sin\left(\omega t+30^0\right)\ \rightarrow\ \underline{U}_{(3)}=\frac{10}{\sqrt{2}}e^{j30}$

$$X_{L(3)}=3\omega L=3\cdot100\pi\,0.1=94.25(\Omega)$$

$$X_{C(3)}=\frac{1}{3\omega C}=\frac{1}{3\cdot100\pi\,200\cdot10^{-6}}=5.31(\Omega)$$

$$\underline{I}_{(3)}=\frac{\underline{U}_{(3)}}{\underline{Z}_{(3)}}=\frac{\underline{U}_{(3)}}{R+j\left(X_{L(3)}-X_{C(3)}\right)}=0.08\,e^{-j53}$$

$$\underline{U}_{C(3)}=-jX_{C(3)}\underline{I}_{(3)}=0.42\,e^{-j143}$$

$$u_C(t)=U_{(0)}+u_{C(1)}(t)+u_{C(3)}(t)=15+48.81\sqrt{2}\sin\left(100\pi t-147^0\right)+0.42\sqrt{2}\sin\left(300\pi t-143^0\right)[V]$$

$$I=\sqrt{I_{(0)}^2+I_{(1)}^2+I_{(3)}^2}=\sqrt{0^2+3.07^2+0.08^2}\cong3.07\ (A)$$

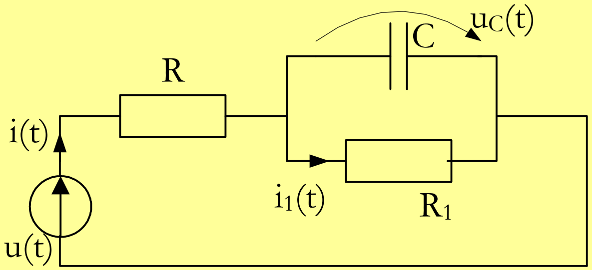
$$S=UI=59\cdot3.07=180.83[VA]$$

$$P=U_{(0)}I_{(0)}+U_{(1)}I_{(1)}\cos\varphi_1+U_{(3)}I_{(3)}\cos\varphi_3=\frac{80}{\sqrt{2}}3.07\cos(0-(-57))+\frac{10}{\sqrt{2}}0.08\cos(30-(-53))=94.11[W]$$

$$P=I^2R=94.11[W]$$

$$Q=U_{(1)}I_{(1)}\sin\varphi_1+U_{(3)}I_{(3)}\sin\varphi_3=\frac{80}{\sqrt{2}}3.07\sin(0-(-57))+\frac{10}{\sqrt{2}}0.08\sin(30-(-53))=146.33[VAR]$$

$$D=\sqrt{S^2-P^2-Q^2}=49.34\ [VAD]$$



$$u(t)=50+100\sin\left(400t+\frac{\pi}{6}\right)+50\sin\left(1200t-\frac{\pi}{4}\right)\,[V]$$

R=30Ω, R₁=50Ω, C=50μF

$U=?$
$I=?,\ I_1=?$
$u_C(t)=?$
$P=?,\ Q=?,\ D=?$

Componenta continuă:

$$U_{(0)}=50V,\qquad I_{(0)}=I_{1(0)}=\frac{U_{(0)}}{R+R_1}=0.625\,A\qquad U_{C(0)}=I_{1(0)}R_1=31.25V$$

Componenta fundamentală:

$$u_1(t)=100\sin\left(400t+\frac{\pi}{6}\right)\ \rightarrow\ \underline{U}_{(1)}=\frac{100}{\sqrt{2}}e^{j\frac{\pi}{6}}$$

$$X_{C(1)}=\frac{1}{\omega C}=\frac{1}{400\cdot 50\cdot 10^{-6}}=50(\Omega)$$

$$\underline{Z}_{p(1)}=\frac{R_1\left(-jX_{C(1)}\right)}{R_1-jX_{C(1)}}=25-25j\ (\Omega),\quad \underline{Z}_{e(1)}=R+\underline{Z}_{p(1)}=55-25j\ (\Omega)$$

$$\underline{I}_{(1)}=\frac{\underline{U}_{(1)}}{\underline{Z}_{e(1)}}=0.681+j0.952=1.17e^{j54}\qquad \underline{U}_{C(1)}=\underline{I}_{(1)}\underline{Z}_{p(1)}=40.819+j6.79=41.38e^{j9}\qquad \underline{I}_{1(1)}=\frac{\underline{U}_{C(1)}}{R_1}=0.816+j0.136=0.828e^{j9}$$

Componenta de ordinul 3:

$$u_3(t)=50\sin\left(1200t-\frac{\pi}{4}\right)\ \rightarrow\ \underline{U}_{(3)}=\frac{50}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

$$X_{C(3)}=\frac{1}{3\omega C}=\frac{1}{1200\cdot 50\cdot 10^{-6}}=16.667(\Omega)$$

$$\underline{Z}_{p(3)}=\frac{R_1\left(-jX_{C(3)}\right)}{R_1-jX_{C(3)}}=5-15j\ (\Omega),\quad \underline{Z}_{e(1)}=R+\underline{Z}_{p(1)}=35-15j\ (\Omega)$$

$$\underline{I}_{(3)}=\frac{\underline{U}_{(3)}}{\underline{Z}_{e(3)}}=0.862-j0.345=0.928e^{-j22}\qquad \underline{U}_{C(3)}=\underline{I}_{(3)}\underline{Z}_{p(3)}=-0.862-j14.655=14.68e^{-j93}\qquad \underline{I}_{1(3)}=\frac{\underline{U}_{C(3)}}{R_1}=-0.017-j0.293=0.294e^{-j93}$$

$$I=\sqrt{I_{(0)}^2+I_{(1)}^2+I_{(3)}^2}=\sqrt{0.625^2+1.17^2+0.928^2}\cong 1.619\ (A)\qquad I_1=\sqrt{I_{1(0)}^2+I_{1(1)}^2+I_{1(3)}^2}=\sqrt{0.625^2+0.828^2+0.294^2}\cong 1.078\ (A)$$

$$u_C(t)=U_{(0)}+u_{C(1)}(t)+u_{C(3)}(t)=31.25+41.38\sqrt{2}\sin\left(400t+9^0\right)+14.68\sqrt{2}\sin\left(300\pi t-93^0\right)[V]$$

$$P=U_{(0)}I_{(0)}+U_{(1)}I_{(1)}\cos\varphi_1+U_{(3)}I_{(3)}\cos\varphi_3=50\cdot 0.625+\frac{100}{\sqrt{2}}1.17\cos\left(-24^0\right)+\frac{50}{\sqrt{2}}0.928\cos\left(-23^0\right)=136.765[W]$$

$$P=I^2R+I_1^2R_1=136.765W$$

$$Q=U_{(1)}I_{(1)}\sin\varphi_1+U_{(3)}I_{(3)}\sin\varphi_3=\frac{100}{\sqrt{2}}1.17\sin\left(-24^0\right)+\frac{50}{\sqrt{2}}0.928\sin\left(-23^0\right)=-47.178[VAR]$$

$$U=\sqrt{U_{(0)}^2+U_{(1)}^2+U_{(3)}^2}=\sqrt{50^2+\left(\frac{100}{\sqrt{2}}\right)^2+\left(\frac{50}{\sqrt{2}}\right)^2}=93.541\ (V)$$

$$S=UI=93.541\cdot 1.619=151.484[VA]\qquad D=\sqrt{S^2-P^2-Q^2}=45\ [VAD]$$