CLEȘTE

$$\left.\begin{array}{l} \lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = x \\ a_n \le x_n \le b_n \end{array}\right\} \lim_{n\to\infty} x_n = x$$

MAJORĂRI

1) Dacă $\exists\, x \in \mathbb{R}$ și $\exists\, (y_n)_{n\ge n_0}$ șir nenegativ, cu $\boxed{y_n \to 0}$ a.î. $\boxed{|x_n - x| \le y_n}$, $\forall n \ge n_0$ $\Rightarrow \boxed{x_n \to x}$.

2) Dacă $\exists\, (y_n)_{n\ge n_0}$, $\boxed{y_n \to -\infty}$ a.î. $\boxed{x_n \le y_n} \Rightarrow \boxed{x_n \to -\infty}$

3) Dacă $\exists\, (y_n)_{n\ge n_0}$ $\boxed{y_n \to \infty}$ a.î. $\boxed{x_n \ge y_n} \Rightarrow \boxed{x_n \to \infty}$

RAPORT

$$l = \lim_{n\to\infty} \frac{x_{n+1}}{x_n} \in [0, \infty]$$

a) $l < 1 \Rightarrow \lim_{n\to\infty} x_n = 0$

b) $l > 1 \Rightarrow \lim_{n\to\infty} x_n = \infty$

RAPORT GENERALIZAT

$$l = \lim_{n\to\infty} \left[\left(\frac{x_{n+1}}{x_n}\right)^n\right] \in [0, \infty]$$

a) $l < 1 \Rightarrow \lim_{n\to\infty} x_n = 0$

b) $l > 1 \Rightarrow \lim_{n\to\infty} x_n = \infty$

...-CESARO

$x_n = \frac{a_n}{b_n}, b_n \neq 0$

a) b_n - strict monoton

b) $b_n \to \infty$ SAU $\begin{cases} a_n \to 0 \\ b_n \to 0 \end{cases}$

c) $\exists \lim\limits_{n\to\infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = x \in \overline{\mathbb{R}}$

$\Rightarrow \lim\limits_{n\to\infty} x_n = x$

RĂDĂCINII

$\exists \lim\limits_{n\to\infty} \frac{x_{n+1}}{x_n} = l \in [0, \infty] \Rightarrow \lim\limits_{n\to\infty} \sqrt[n]{x_n} = l$

DIVERGENȚA

x_n – limita nenulă
x_n – nu are limită ($x_n \not\to 0$)

$\Rightarrow \sum x_n = \text{DIV}$

CONVERGENȚA LUI CAUCHY

$\sum x_n = \text{CONV}$ d.d. $\forall \varepsilon > 0, \exists N = N(\varepsilon) \geq n_0$ a.î. $|x_{n+p} - x_n| < \varepsilon, \forall n \geq N$.

$\sum\limits_{n\geq 1} \frac{1}{n^p} = \begin{cases} \text{CONV}, & p > 1 \\ \text{DIV}, & p \leq 1 \end{cases}$ seria armonică generalizată

$\sum\limits_{n\geq 0} r^n = \frac{1}{1-r} = \text{CONV}, r \in (-1, 1)$ seria geom.

...ARAȚIEI LA LIMITĂ

$\exists\, l = \lim_{n\to\infty} \frac{x_n}{y_n} \in [0,\infty]$

a) $l \in (0,\infty) \Rightarrow \sum x_n \sim \sum y_n$

b) $l = 0$, $\sum y_n = \text{CONV} \Rightarrow \sum x_n = \text{CONV}$

c) $l = \infty$, $\sum y_n = \text{DIV} \Rightarrow \sum x_n = \text{DIV}$

RAPORTULUI (D'ALAMBERT)

$\exists\, l = \lim_{n\to\infty} \frac{x_{n+1}}{x_n} \in [0,\infty]$

a) $l < 1 \Rightarrow \sum x_n = \text{CONV}$

b) $l > 1 \Rightarrow \sum x_n = \text{DIV}$

RAABE-DUHAMEL

$\exists\, l = \lim_{n\to\infty} n \cdot \left(\frac{x_n}{x_{n+1}} - 1\right) \in \overline{\mathbb{R}}$

a) $l < 1 \Rightarrow \sum x_n = \text{DIV}$

b) $l > 1 \Rightarrow \sum x_n = \text{CONV}$

RĂDĂCINII

$\exists\, l = \lim_{n\to\infty} \sqrt[n]{x_n} \in [0,\infty]$

a) $l < 1 \Rightarrow \sum x_n = \text{CONV}$

b) $l > 1 \Rightarrow \sum x_n = \text{DIV}$

CONDENSAREA LUI CAUCHY

x_n – $\downarrow$, neneg. $\Rightarrow \sum x_n \sim \sum 2^n \cdot x_{2^n}$

RAPORT (termeni oarecare)

$\exists\, l = \lim\limits_{n\to\infty} \left| \dfrac{x_{n+1}}{x_n} \right| \in [0,\infty]$

a) $l<1 \rightarrow \sum x_n$ = ABS CONV

b) $l>1 \Rightarrow \sum x_n$ = DIV

RĂDĂCINII (termeni oarecare)

$\exists\, l = \lim\limits_{n\to\infty} \sqrt[n]{|x_n|} \in [0,\infty]$

a) $l<1 \Rightarrow \sum x_n$ = ABS CONV

b) $l>1 \Rightarrow \sum x_n$ = DIV

LEIBNIZ

$\left.\begin{array}{l} a_n - \text{poz} \\ a_n \searrow_0^{n\to\infty} \end{array}\right\} \Rightarrow \sum\limits_n (-1)^n \cdot a_n$ = CONV

DIRICHLET

$\sum a_n \cdot u_n$

a) $a_n \searrow_0^{n\to\infty}$

b) $|u_0+u_1+\dots+u_n| \le M$ $\Big\} \Rightarrow$

$\Rightarrow \sum a_n \cdot u_n = \text{CONV}$

ABEL

$\sum_n a_n \cdot u_n$

a) a_n - mărg., mon. $\Leftrightarrow a_n$ - CONV

b) $\sum_n u_n = \text{CONV}$ $\Big\} \Rightarrow$

$\Rightarrow \sum_n a_n \cdot u_n = \text{CONV}$

CONVERGENȚĂ NEUNIFORMĂ

$f_n \xrightarrow[D]{S} f$

$\lim_{n\to\infty} |f_n(x_n) - f(x_n)| \neq 0$ $\Big\} \Rightarrow f_n \not\xrightarrow[D]{u} f$

MAJORĂRI

$\exists f: D \to \mathbb{R}$

$a_n \to 0$; $|f_n(x) - f(x)| \le a_n$ $\Big\} \Rightarrow f_n \xrightarrow[D]{u} f$

$$\boxed{S(x) = \lim_{n\to\infty} S_n(x)}$$ suma seriei de funcții

CAUCHY

$$\boxed{\begin{array}{l}\sum f_n = \text{U.C. d.d. } \forall \varepsilon>0, \exists n(\varepsilon)\in\mathbb{N} \\ \text{a.î. } |f_{n+1}(x)+\ldots+f_{n+p}(x)| < \varepsilon, \forall n\geq n_\varepsilon\end{array}}$$

WEIERSTRASS

f_n – funcții, a_n – numeric

a) $|f_n(x)| \leq a_n$

b) $\sum a_n = \text{CONV}$

$\Rightarrow \sum f_n(x) = \text{U.C.}$

$$\boxed{\sum_{n\geq 0} x^n = \frac{1}{1-x}, \; x\in(-1,1) \Leftrightarrow |x|<1}$$

$$\boxed{\sum_{n\geq 0} (-1)^n \cdot x^n = \frac{1}{1+x}, \; |x|<1}$$

$$\boxed{\sum_{n\geq 0} x^{2n} = \frac{1}{1-x^2}, \; |x|<1}$$

$$\boxed{\sum_{n\geq 0} (-1)^n \cdot x^{2n} = \frac{1}{1+x^2}, \; |x|<1}$$

$$\frac{1}{\sqrt{1-x^2}} = 1 + \sum_{n \geq 1} \frac{(2n-1)!!}{(2n)!!} \cdot x^{2n}$$

$$\arcsin x = 1 + \sum_{n \geq 1} \frac{(2n-1)!!}{(2n)!!} \cdot \frac{x^{2n+1}}{2n+1}$$

$$R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| \quad R \in [0, \infty]$$

$$R^{\alpha} = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right|, \; R \in [0, \infty] \quad \text{RAZA}$$

POLINOMUL TAYLOR DE ORDIN N

$$T_n f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

RESTUL TAYLOR DE ORDIN N

$$R_n f(x) = f(x) - T_n f(x), \; x \in I$$

FORMULA LUI TAYLOR

$$f(x) = T_n f(x) + R_n f(x), \; x \in I$$

FORMULA LUI MAC-LAURIN

$$\boxed{x_0 = 0} \to f(x) = f(0) + \frac{f'(0)}{1!} \cdot x + \frac{f''(0)}{2!} \cdot x^2 + \dots + \frac{f^{(n)}(0)}{n!} \cdot x^n + \underbrace{\frac{f^{(n+1)}(0)}{(n+1)!} \cdot x^{n+1}}_{\to 0}$$

...IA TAYLOR CENTRATĂ ÎN XO

$$\sum_{n\geq 0} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \dots$$

SERIA MAC-LAURIN

$$\boxed{x_0=0} \Rightarrow \sum_{n\geq 0} \frac{f^{(n)}(0)}{n!}\cdot x^n = f(0) + \frac{f'(0)}{1!}\cdot x + \frac{f''(0)}{2!}\cdot x^2 + \dots + \frac{f^{(n)}(0)}{n!}\cdot x^n + \dots$$

SUMA SERIEI TAYLOR

$$S(x) = \sum_{n\geq 0} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$e^x = \sum_{n\geq 0} \frac{x^n}{n!}$$

$$\cos x = \sum_{n\geq 0} (-1)^n \cdot \frac{x^{2n}}{(2n)!}$$

$$\sin x = \sum_{n\geq 0} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$$

$$\ln(1+x) = \sum_{n\geq 1} (-1)^{n-1} \cdot \frac{x^n}{n}$$

$$\sin^2 x = \frac{1}{2} - \sum_{n\geq 0} (-1)^n \cdot \frac{2^{2n-1}}{(2n)!} \cdot x^{2n}$$

$$(1+x)^\alpha = 1 + \sum_{n\geq 1} \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!} x^n$$

$$(\sin x)^{(n)} = \sin\left(x + \frac{n\pi}{2}\right)$$

$$(\cos x)^{(n)} = \cos\left(x + \frac{n\pi}{2}\right)$$

$$\sum_{n\geq 1} \frac{\sin(nx)}{n} = \frac{\pi - x}{2}$$

$$\sum_{n\geq 1} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

$$\sum_{n\geq 1} \frac{(-1)^n}{n} = \frac{-\pi^2}{12}$$

$$\sum_{n\geq 1} \frac{1}{n^2} = \frac{\pi}{6}$$