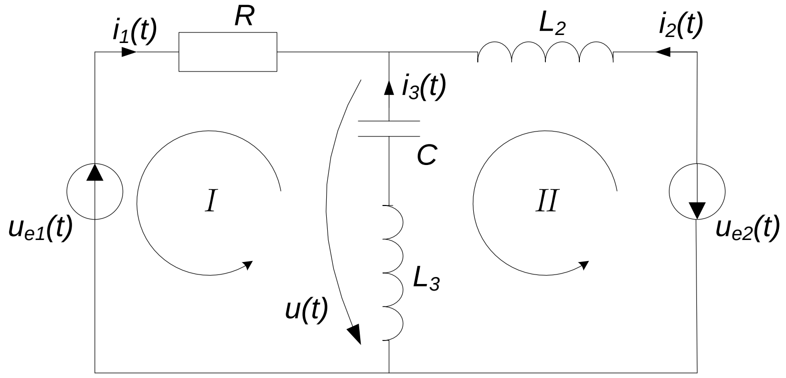
RLC Series Circuit

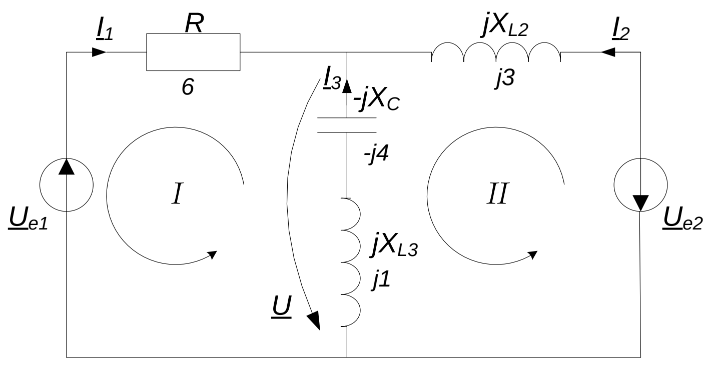
Example 3.8.1. For the circuit below we know: R=6Ω, $X_{L2}=\omega L_2=3\Omega$, $X_{L3}=\omega L_3=1\Omega$, $X_C=1/\omega C=4\Omega$, $u_{e1}(t) = 60\sqrt{2}\ sin(100\pi t + 30^0)$ and $u_{e2}(t) = 30\sqrt{2}\ sin(100\pi t - 45^0)\ V$. Find: a) the rms values and instantaneous values of the circuit currents, I_1, I_2, I_3, $i_1(t)$, $i_2(t)$, $i_3(t)$; b) the rms value U, and the instantaneous value u(t) of the voltage indicated in the figure.



Example 3.8.1.

Solution:

At first, we have to transform the time domain circuit in the phasor form circuit. The corresponding phasor circuit is drawn below. For the phasor circuit, we can apply KL in the same way as we proceed for DC circuits.



Circuit in the phasor domain

The phasor form of the supplying voltages are: $\underline{U}_{e1} = 60\ e^{j30^0}$ *and* $\underline{U}_{e2} = 30\ e^{-j45^0}$. *For the current reference directions and reference loop directions like chosen in figure above, the three equations system using the Kirchhoff laws, is:*

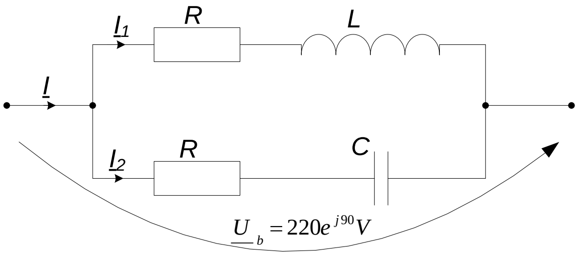
$$\begin{cases} \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0 \\ -\underline{I}_1 R + jX_{L3}\,\underline{I}_3 - jX_C\,\underline{I}_3 = -\underline{U}_{e1} \\ jX_{L2}\,\underline{I}_2 - \left(-jX_C\,\underline{I}_3\right) - jX_{L3}\,\underline{I}_3 = -\underline{U}_{e2} \end{cases},$$

which becomes:

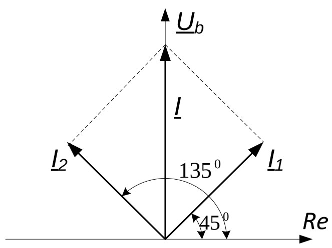
$$\begin{cases} \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0 \\ -6\underline{I}_1 + j\,\underline{I}_3 - j4\,\underline{I}_3 = -60\ e^{j30^0} \\ j3\,\underline{I}_2 + j4\,\underline{I}_3 - j\,\underline{I}_3 = -30\ e^{-j45^0}. \end{cases}$$

Solving for the currents $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$, *we get:* $\underline{I}_1 = -7.071 - j\,7.071 = 10\ e^{-j135^0}$, $\underline{I}_2 = -17.071 - j\,38.534 = 42.146\ e^{-j114^0}$ *and* $\underline{I}_3 = 24.142 - j\,31.463 = 39.658\ e^{-j52^0}$. *Hence, converting to the tome domain, the currents are:* I_1*=10A*, $\mathrm{i}_1(\mathrm{t}) = 10\sqrt{2}\ \sin(100\pi\mathrm{t} - 135^0)A$, I_2*=42.146A*, $\mathrm{i}_2(\mathrm{t}) = 42.146\sqrt{2}\ \sin(100\pi\mathrm{t} - 114^0)A$ *and* I_3*=39.658A*, $\mathrm{i}_3(\mathrm{t}) = 39.658\sqrt{2}\ \sin(100\pi\mathrm{t} - 52^0)A$. *The voltage phasor is* $\underline{U} = -\left(-jX_C\,\underline{I}_3\right) - jX_{L3}\,\underline{I}_3 = 94.388 + j\,72.426 = 118.973\ e^{j37.5^0}$. *Hence, U=118.973V and* $\mathrm{u}(\mathrm{t}) = 118.973\sqrt{2}\ \sin(100\pi\mathrm{t} + 37.5^0)V$.

Example 3.8.2. For the circuit below we know $R = X_L = X_C = 10\sqrt{2}\,\Omega$, and the supplying voltage $u_b(t) = 220\sqrt{2}\ sin(100\pi t + 90^0)\ V$. Calculate: a) the equivalent impedance, $\underline{Z}_e$; b) the rms and instantaneous values of the circuit currents, I, I_1, I_2, i(t), $i_1(t)$, $i_2(t)$. c) Draw the phasor diagram.



a)



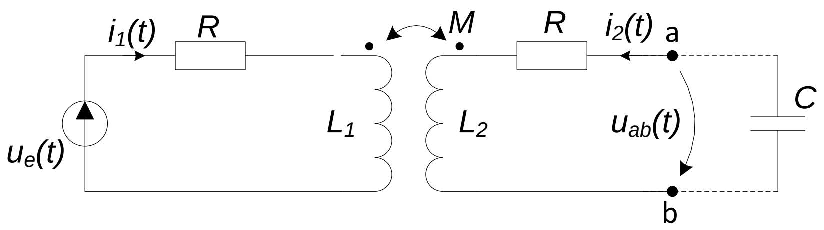
b)

Example 3.8.2.

Solution:

By denoting $\underline{Z}_1 = R + jX_L = 10\sqrt{2} + j10\sqrt{2}$ *and* $\underline{Z}_2 = R - jX_C = 10\sqrt{2} - j10\sqrt{2}$, *the equivalent impedance phasor is* $\underline{Z}_e = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{(10\sqrt{2}+j10\sqrt{2})\cdot(10\sqrt{2}-j10\sqrt{2})}{(10\sqrt{2}+j10\sqrt{2})+(10\sqrt{2}-j10\sqrt{2})} = 10\sqrt{2}\,\Omega$. *Hence,* $Z_e = |\underline{Z}_e| = 10\sqrt{2}\,\Omega$. *The phasor current is* $\underline{I} = \frac{\underline{U}_e}{\underline{Z}_e} = \frac{220\, e^{j90^0}}{10\sqrt{2}} = 11\sqrt{2}\, e^{j90^0} = j\,11\sqrt{2}$. *Converting to the time domain,* $I = 11\sqrt{2}\,A$ *and* $\mathrm{i(t)} = 22\,\sin(100\pi t + 90^0)$ A. *From* $\underline{U}_b = \underline{I}_1 R + jX_L\, \underline{I}_1$, *solving for* I_1, *we have* $\underline{I}_1 = \frac{\underline{U}_b}{R + jX_L} = \frac{220\, e^{j90^0}}{10\sqrt{2}+j10\sqrt{2}} = \frac{220\, e^{j90^0}}{20\, e^{j45^0}} = 11\, e^{j45^0}$. *Hence,* I_1*=11A and* $\mathrm{i}_1(\mathrm{t}) = 11\sqrt{2}\,\sin(100\pi t + 45^0)$ A. *From* $\underline{U}_b = \underline{I}_2 R - jX_C\, \underline{I}_2$, *solving for* I_2, *we have* $\underline{I}_2 = \frac{\underline{U}_b}{R - jX_C} = \frac{220\, e^{j90^0}}{10\sqrt{2}-j10\sqrt{2}} = \frac{220\, e^{j90^0}}{20\, e^{-j45^0}} = 11\, e^{j135^0}$. *Hence,* I_2*=11A and* $\mathrm{i}_2(\mathrm{t}) = 11\sqrt{2}\,\sin(100\pi t + 135^0)$ A. *The phasor diagram is drawn in the figure above.*

Example 3.8.3. For the circuit below we know R=10Ω, $\omega L_1=\omega L_2=\omega M=30\Omega$, $1/\omega C=20\Omega$ and the voltage $u_{e1}(t) = 100\sqrt{2}\, sin(100\pi t + 60^0)\, V$. Find the circuit currents $i_1(t)$, $i_2(t)$ and the voltage $u_{ab}(t)$, when: a) the terminals a and b are in open circuit; b) the terminals a and b are connected by the capacitance C.

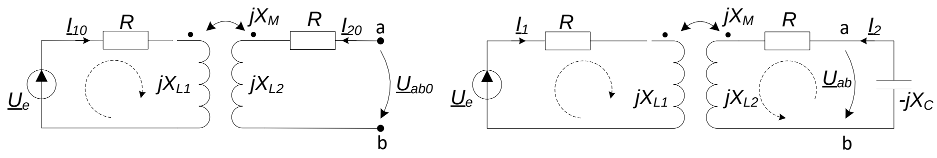


Example 3.8.3.

Solution:

The phasor form of the circuit with the open circuit terminals a and b is drawn in figure a) below. Due to the open circuit terminals a and b, the current $\underline{I}_{20}$*=0. Applying KVL for the primary circuit,* $\underline{U}_e = \underline{I}_{10} R + j\omega L_1\, \underline{I}_{10}$, *and solving for* $\underline{I}_{10}$, *we have:* $\underline{I}_{10} = \frac{\underline{U}_e}{R + j\omega L_1} = \frac{100\, e^{j60^0}}{10 + j30} = 3.098 - j0.634 = 3.162\, e^{-j11.56^0}$. *Hence, the rms primary current is* I_{10}*=3.162A, while the instantaneous value* $\mathrm{i}_{10}(\mathrm{t}) = 3.162\sqrt{2}\, sin(100\pi t - 11.56^0)\, A$. *The open circuit voltage is the voltage on the inductance* L_2, $\underline{U}_{ab0} = \underline{U}_{L2} = j\omega M\, \underline{I}_{10} =$

$19.019 + j92.942 = 94.868\, e^{j78.44^0}$. *Hence,* $U_{ab0}=94.868V$, *and* $u_{ab0}(t) = 94.868\sqrt{2}\, sin(100\pi t + 78.44^0)$ V. *Due to the coupling between the coils, a voltage on the terminals a and b exists,* $u_{ab0}\neq 0$, *even if the terminals a and b are in open circuit.*



a) b)

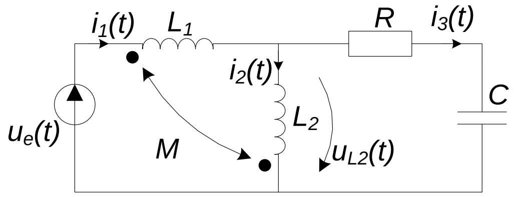
The circuit in the phasor form

The phasor form of the circuit with the connected capacitance between terminals a and b is drawn in figure b) below. Using KVL for the two fundamental loops, we have:

$$\begin{cases} \underline{I_1}R + j\omega L_1\, \underline{I_1} + j\omega M\, \underline{I_2} = \underline{U_e} \\ \underline{I_2}R + j\omega L_2\, \underline{I_2} + j\omega M\, \underline{I_1} + \frac{1}{j\omega C}\underline{I_2} = 0 \end{cases}, \quad or \quad \begin{cases} 10\, \underline{I_1} + j30\, \underline{I_1} + j30\, \underline{I_2} = 100\, e^{j60^0} \\ 10\, \underline{I_2} + j30\, \underline{I_2} + j30\, \underline{I_1} - j20\underline{I_2} = 0. \end{cases}$$

Solving for the $\underline{I_1}$ *and* $\underline{I_2}$, *we get* $\underline{I_1} = 0.446 + j1.696 = 1.754\, e^{j75.25^0}$, $\underline{I_2} = 1.875 - j3.214 = 3.721\, e^{-j59.75^0}$. *Converting to the time domain,* $I_1=1.754A$, $I_2=3.721A$ *and* $i_1(t) = 1.754\sqrt{2}\, sin(100\pi t - 75.25^0)\, A$, $i_2(t) = 3.721\sqrt{2}\, sin(100\pi t - 59.75^0)\, A$. *For the voltage* $\underline{U_{ab}}$, *the easiest way to calculate is* $\underline{U_{ab}} = -\frac{1}{j\omega C}\, \underline{I_2} = j20\, \underline{I_2} = 74.42\, e^{j30.25^0}$, *and converting to time-domain* $U_{ab}=74.42V$ *and* $u_{ab}(t) = 74.42\sqrt{2}\, sin(100\pi t + 30.25^0)$ V. *As a remark, the voltage* $\underline{U_{ab}}$ *could be calculated also using* $\underline{U_{ab}} = \underline{I_2}R + j\omega L_2\, \underline{I_2} + j\omega M\, \underline{I_1} = 74.42\, e^{j30.25^0}$.

Example 3.8.4. For the circuit below we know: R=10Ω, $X_{L1}=\omega L_1=3\Omega$, $X_{L2}=\omega L_2=4\Omega$, $X_M=\omega M=2\Omega$ $X_C=1/\omega C=2\Omega$ and $u_e(t) = 24\sqrt{2}\ sin(100\pi t)\, V$. Find: a) the rms values and instantaneous values of the circuit currents, I_1, I_2, I_3, $i_1(t)$, $i_2(t)$, $i_3(t)$; b) the rms value U_L, and the instantaneous value $u_L(t)$ of the voltage on the inductance L_2.

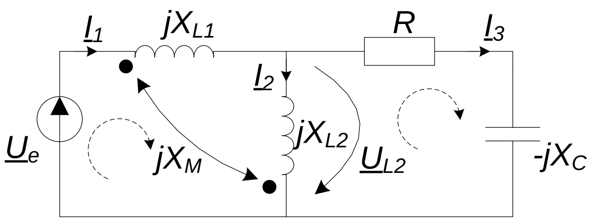


Example 3.8.4.

Solution:

At first, we have to transform the time domain circuit in the phasor form circuit. The corresponding phasor circuit in the figure below has two nodes and three branches. For the reference directions of the currents and fundamental loops directions chosen as indicated in the figure below, by applying KL, the three equations system is formed:

$$\begin{cases} \underline{I}_1 = \underline{I}_2 + \underline{I}_3 \\ jX_{L1}\,\underline{I}_1 - jX_M\,\underline{I}_2 + jX_{L2}\,\underline{I}_2 - jX_M\,\underline{I}_1 = \underline{U}_e \\ \underline{I}_3 R - jX_C\,\underline{I}_3 - \left(jX_{L2}\,\underline{I}_2 - jX_M\,\underline{I}_1\right) = 0 \end{cases}$$



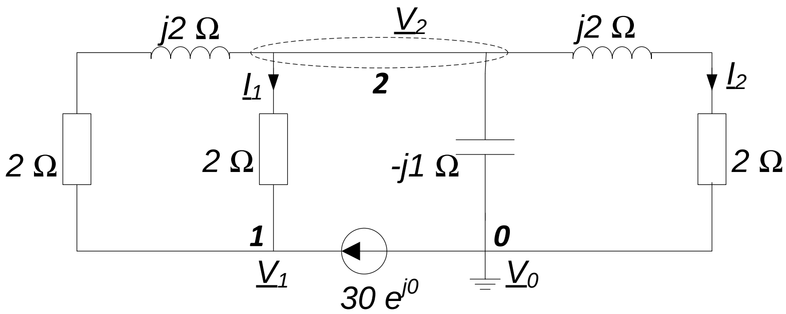
Circuit in the phasor domain

Replacing numerically, the system becomes:

$$\begin{cases} \underline{I}_1 = \underline{I}_2 + \underline{I}_3 \\ j\,\underline{I}_1 + 2j\,\underline{I}_2 = 24 \\ 2j\,\underline{I}_1 - 4j\,\underline{I}_2 + (10 - j4)\underline{I}_3 = 0, \end{cases}$$

and solving for the currents $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$, *we get:* $\underline{I}_1 = 0.878 - j7.902 = 7.951\, e^{-j83.66^0}$, $\underline{I}_2 = -0.439 - j8.049 = 8.061\, e^{-j93.12^0}$, $\underline{I}_3 = 1.317 - j0.146 = 1.325\, e^{j6.34^0}$. *Converting to the time domain, the rms values of the currents are* I_1*=7.951A,* I_2*=8.061A,* I_3*=1.325A, and the instantaneous values* $\mathrm{i}_1(\mathrm{t}) = 7.951\sqrt{2}\, sin(100\pi\mathrm{t} - 83.66^0)\, A$, $\mathrm{i}_2(\mathrm{t}) = 8.061\sqrt{2}\, sin(100\pi\mathrm{t} - 93.12^0)\, A$, $\mathrm{i}_3(\mathrm{t}) = 1.325\sqrt{2}\, sin(100\pi\mathrm{t} + 6.34^0)\, A$. *The voltage* $\underline{U}_{L2}$ *is:* $\underline{U}_{L2} = jX_{L2}\, \underline{I}_2 - jX_M\, \underline{I}_1 = 16.39 - j3.512 = 16.762\, e^{-j12.09^0}$, *and converting to time domain* U_{L2}*=16.762V and* $\mathrm{u}_{L2}(\mathrm{t}) = 16.762\sqrt{2}\, sin(100\pi\mathrm{t} - 12.09^0)\, \mathrm{V}$. *As a remark, the voltage* $\underline{U}_{L2}$ *could be calculated also using* $\underline{U}_{L2} = \underline{I}_3 R - jX_C\, \underline{I}_3 = 16.762\, e^{-j12.09^0}$, *or* $\underline{U}_{L2} = -jX_{L1}\, \underline{I}_1 + jX_M\, \underline{I}_2 + \underline{U}_e = 16.762\, e^{-j12.09^0}$.

Example 3.8.5. Using the nodal analysis calculate currents $\underline{I}_1$ and $\underline{I}_2$ for the circuit below.



Example 3.8.5.

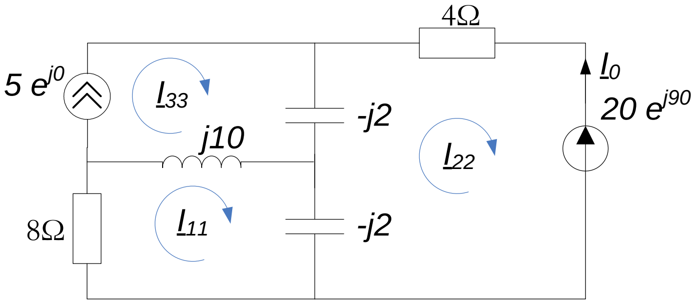
Solution:

Because we have a branch only with a voltage source, one of the nodes 0 or 1 will be considered as the reference node. Let's take node 0 as the reference node, $\underline{V}_0$*=0.* $\underline{V}_1 - \underline{V}_0 = 30$, *and as a consequence,* $\underline{V}_1$*=30V. A single nodal equation will be written for node 2:*

$$\underline{V}_2\left(\frac{1}{2+j2} + \frac{1}{2} + \frac{1}{2+j2} - \frac{1}{j}\right) - \underline{V}_1\left(\frac{1}{2+j2} + \frac{1}{2}\right) = 0.$$

Solving for the $\underline{V}_2$, *we get* $\underline{V}_2 = 15 - j15 = 15\sqrt{2}\, e^{-j45^0}$. *Hence, from* $\underline{V}_2 - \underline{V}_1 = 2\, \underline{I}_1$, *we have* $\underline{I}_1 = \frac{\underline{V}_2 - \underline{V}_1}{2} = -15 - j15 = 15\sqrt{2}\, e^{j225^0} = 15\sqrt{2}\, e^{-j135^0}$, *and from* $\underline{V}_2 - \underline{V}_0 = (2 + j2)\, \underline{I}_2$, $\underline{I}_2 = \frac{\underline{V}_2}{2+j2} = \frac{15\sqrt{2}\, e^{-j45^0}}{2\sqrt{2}\, e^{j45^0}} = 7.5\, e^{-j90^0}$.

Example 3.9.1. Using the loop analysis, find current $\underline{I}_0$ in the circuit below.



Example 3.9.1.

Solution:

As we proceeded for DC circuits, we identify the three fundamental loops. For the corresponding loop currents with reference directions as in the figure above, the loop equations are:

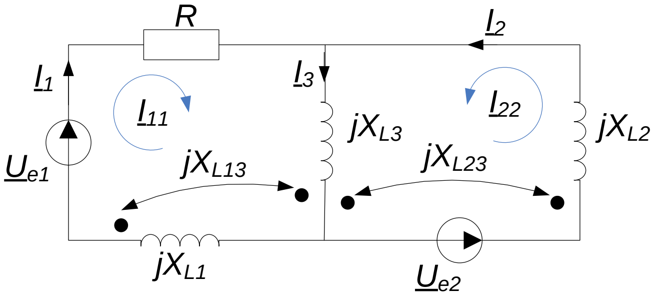
$$\begin{cases} \underline{I}_{11}(8 + j10 - j2) - \underline{I}_{22}(-j2) - \underline{I}_{33}(j10) = 0 \\ \underline{I}_{22}(4 - j2 - j2) - \underline{I}_{11}(-j2) - \underline{I}_{33}(-j2) = -20\, e^{j90^0} \\ \underline{I}_{33} = 5\, e^{j0} \end{cases}$$

Take into consideration that for the third loop, because of the current source, we do not write a loop equation. The loop current is equal to the current generated by the current source. The system becomes:

$$\begin{cases} 8(1 + j)\, \underline{I}_{11} + j2\, \underline{I}_{22} = j50 \\ j2\, \underline{I}_{11} + 4(1 - j)\, \underline{I}_{22} = 10(1 + j2). \end{cases}$$

Solving for the loop currents I_{11} and I_{22}, we get $\underline{I}_{22} = 6.12\, e^{-j35.22^0}$ *and the branch current* $\underline{I}_0 = -\underline{I}_{11} = -6.12\, e^{-j35.22^0} = 6.12\, e^{-j35.22^0} e^{j180^0} = 6.12\, e^{j144.78^0}$.

Example 3.9.2. For the circuit below we know: R=20Ω, X_{L1}= X_{L2}= X_{L3}=10Ω, X_{L12}= X_{L23}=5Ω, $u_{e1}(t) = 100\sqrt{2}\ sin(100\pi t)$, $u_{e2}(t) = 100\sqrt{2}\ sin(100\pi t + 90^0)\,V$. Using the loop analysis, calculate the branch currents in the circuit, I_1, I_2, I_3, $i_1(t)$, $i_2(t)$, $i_3(t)$.



Example 3.9.2

Solution:

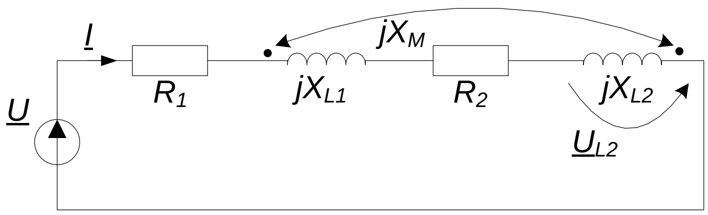
We chose the two-loop currents with the reference direction as shown in the figure above. The two-loop equations system in concentrated form is:

$$\begin{cases} \underline{Z}_{11}\underline{I}_{11} + \underline{Z}_{12}\underline{I}_{22} = \underline{U}_{e1} \\ \underline{Z}_{22}\underline{I}_{22} + \underline{Z}_{21}\underline{I}_{11} = \underline{U}_{e2}, \end{cases}$$

where: $$\begin{cases} \underline{Z}_{11} = R + jX_{L3} + jX_{L1} + 2jX_{L13} = 20 + j10 + j10 + j10 = 20 + j30 \\ \underline{Z}_{22} = jX_{L2} + jX_{L3} - 2jX_{L23} = j10 + j10 - j10 = j10 \\ \underline{Z}_{12} = \underline{Z}_{21} = jX_{L3} + jX_{L13} - jX_{L23} = j10 + j5 - j5 = j10. \end{cases}$$

The system becomes $\begin{cases} 10(2 + j3)\underline{I}_{11} + j10\,\underline{I}_{22} = 200 \\ j10\,\underline{I}_{11} + j10\,\underline{I}_{22} = j\,200 \end{cases}$*, and solving for the currents* $\underline{I}_{11}$ *and* $\underline{I}_{22}$*, we get:* $\underline{I}_{11} = -j10 = 10\,e^{-j90^0}$ *and* $\underline{I}_{22} = 20 + j10 = 22.36\,e^{j26^0}$*. Hence, the branch currents are* $\underline{I}_1 = \underline{I}_{11} = -j10 = 10\,e^{-j90^0}$, $\underline{I}_2 = \underline{I}_{22} = 10(2 + j) = 22.36\,e^{j26^0}$, $\underline{I}_3 = \underline{I}_{11} + \underline{I}_{22} = 20.$

Example 3.10.1. Determine the rms value of current, I, and the instantaneous value of the voltage drop on the inductance L_2, $u_{L2}(t)$. the circuit parameters are: $\underline{Z}_1 = 2 + j4\,\Omega$, $\underline{Z}_2 = 1 + j8\,\Omega$, $X_M = \omega M = 4\Omega$, $U = 180V$, $f = 50Hz$.



Example 3.10.1

Solution:

Using KVL we have: $\underline{U} = \underline{I}R_1 + jX_{L1}\underline{I} - jX_M\underline{I} + \underline{I}R_2 + jX_{L2}\underline{I} - jX_M\underline{I}$ *from where* $\underline{I} = \frac{\underline{U}}{(R_1+R_2)+j(X_{L1}+X_{L2}-2X_M)} = \frac{180}{3+j4} = \frac{180}{5e^{j53^0}} = 36e^{-j53^0}$. *Hence, the rms value of the current is:* $I = 36A$. *The voltage on the second inductance is:* $\underline{U}_{L2} = jX_{L2}\underline{I} - jX_M\underline{I} = j(8-4)\cdot 36e^{-j53^0} = 144e^{-j53^0}e^{j90^0} = 144e^{j37^0}$. *Hence, the instantaneous value is* $u_{L2}(t) = 144\sqrt{2}\sin(314t + 37^0)\,V$.