Examen Analiza Matematica 1, anul 1, CTI - Varianta 3, partea a II-a

I. a) Să se dezvolte în serie Fourier de cosinusuri funcţia periodică de perioadă 2π, unde $f(x) = x+2$, $0 \leq x \leq \pi$.

b) Studiaţi continuitatea funcţiei $f(x,y) = \begin{cases} \dfrac{3x^2y^4}{x^6+2y^6}, & (x,y) \neq (0,0) \\ 2, & (x,y) = (0,0) \end{cases}$

II. a) Calculaţi $\dfrac{\partial^{37} f}{\partial x^{10} \partial y^{27}}$, unde $f(x,y) = (3x-2y)\cos(x+y)$.

b) Folosind funcţiile lui Euler, să se calculeze $\int_0^\infty \left(x^9 - 2x^4\right) \cdot e^{-x^2}\, dx$.

, CTI - Varianta 3, partea a II-a
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$\begin{cases} \dfrac{3x^2y^4}{x^6+2y^6}, & (x,y) \neq (0,0) \\ 2, & (x,y) = (0,0) \end{cases}$

$3x - 2y)\cos(x+y)$.

eze $\int_0^\infty \left(x^9 - 2x^4\right) \cdot e^{-x^2}\, dx$.

I a) $\sum_{n=1}^{\infty} 5$ ... nn

$\sum_{n=1}^{\infty} \frac{1}{n}$ - di

$\Rightarrow$ seria

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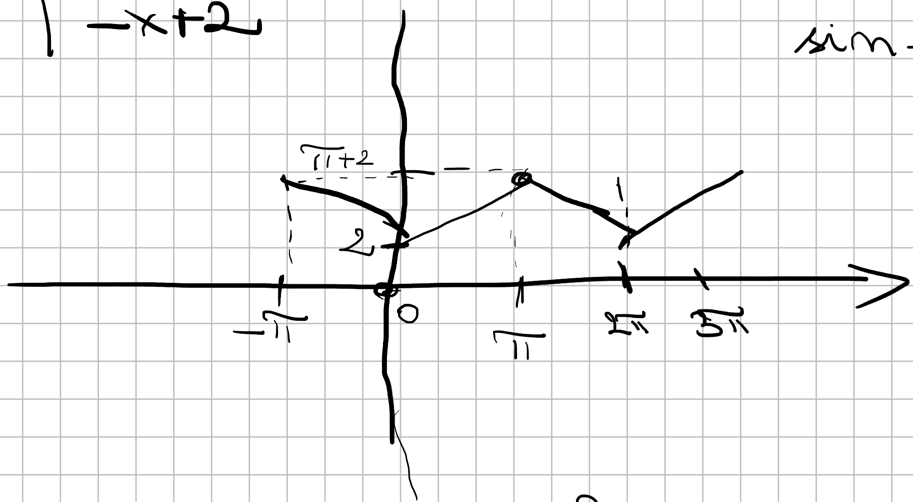
$f(x) = x+2 \quad , \quad x \in [0, \pi]$

$\begin{cases} f(x), & x \in [0,\pi] \\ f(-x), & x \in [-\pi, 0) \end{cases}$

$\begin{cases} x+2 \\ -x+2 \end{cases}$

cos $\to$ sim faţă de Oy

sin $\to$ sim faţă de O



$a_n \to \cos \qquad b_n \to \sin$

$b_n = 0 \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$a_0 =$

$a_n = \frac{4}{T} \int_0^{\frac{T}{2}} f(x) \cos(\omega n x)\, dx =$

$= \frac{4}{2\pi} \int_0^{\frac{2\pi}{2}} (x+2) \cos(nx)\, dx$

$= \frac{2}{\pi} \int_0^{\pi} (x+2) \left(\frac{\sin(nx)}{n}\right)' dx =$

$= \frac{2}{\pi} \left[(x+2) \frac{\sin(nx)}{n} \Big|_0^{\pi} - \int_0^{\pi} (x+2)' \frac{\sin(nx)}{n} dx = \right.$

$\sin \pi n = 0, \forall n \in \mathbb{N}$

$$= \frac{2}{\pi}\left[-\frac{1}{n}\int_0^{\pi} \sin(nx)\,dx\right] =$$

$$= \frac{2}{\pi}\left[-\frac{1}{n}\left(-\frac{\cos nx}{n}\right)\Big|_0^{\pi}\right] =$$

$$= \frac{2}{\pi}\,\frac{1}{n^2}\left[(-1)^n - 1\right] \qquad \cos \pi n = (-1)^n$$

$$n = 2k \Rightarrow a_{2k} = \frac{2}{\pi}\,\frac{1}{4k^2}\left[1-1\right] = 0$$

$$n = 2k+1 \Rightarrow a_{2k+1} = \frac{2}{\pi}\,\frac{1}{(2k+1)^2}\left[-1-1\right] = -\frac{4}{\pi(2k+1)^2}$$

$$\frac{\partial^{37} f}{\partial x^{10}\partial y^{27}} = \frac{\partial^{10}}{\partial x^{10}}\left(\frac{\partial^{27} f}{\partial y^{27}}\right) = A$$

$$f(x,y) = \overbrace{(3x-2y)}^{u}\,\overbrace{\cos(x+y)}^{v}$$

$$(\sin x)^{(n)} = \sin\left(x + \frac{n\pi}{2}\right)$$

$$(\cos x)^{(n)} = \cos\left(x + \frac{n\pi}{2}\right)$$

$$(e^x)^{(n)} = e^x \qquad (e^{2x})' = 2e^{2x} \quad (e^{2x})^{(n)} = 2^n e^{2x}$$

$$(\ln x)^{(n)} = \frac{(-1)^{n+1}(n-1)!}{x^n}$$

$$u' = -2$$

$$u'' = 0 \Rightarrow u^{(n)} = 0,\ \forall n \geq 2$$

$$\frac{\partial^{27} f}{\partial y^{27}} = \overbrace{C_{27}^{0} u^{(27)}\cdot v^{(0)} + C_{27}^{1} u^{(26)} v^{(1)} + \ldots}^{0} + C_{27}^{26} u^{(1)} v^{(26)} + C_{27}^{27} u^{(0)} v^{(27)}$$

$$= 27(-2)\cos\left(x+y+\frac{26\pi}{2}\right) + 1\underbrace{(3x-2y)}_{g}\underbrace{\cos\left(x+y+\frac{27\pi}{2}\right)}_{h}$$

$$\frac{\partial^{10}}{\partial x^{10}}$$

$g' = 3$

$g'' = 0 \Rightarrow g^{(n)} = 0, \forall n \geq 2$

$$\frac{\partial^{10}}{\partial x^{10}} = C_{10}^{0} g^{(10)} h^{(0)} + \dots + C_{10}^{9} g^{(1)} h^{(9)} + C_{10}^{10} g^{(0)} h^{(10)}$$

$$= 10 \cdot 3 \cos\left(x + y + \frac{27\pi}{2} + \frac{9\pi}{2}\right) + 1 \cdot (3x - 2y)\cos\left(x + y + \frac{27\pi}{2} + \frac{10\pi}{2}\right)$$

$$A = -54 \cos\left(x + y + 13\pi + \frac{10\pi}{2}\right) + 30 \cos(x + y + 18\pi)$$

$$+ (3x - 2y)\cos\left(x + y + \frac{37\pi}{2}\right)$$

I. b)

b) Studiaţi continuitatea funcţiei $f(x,y) = \begin{cases} \dfrac{3x^2y^4}{x^6+2y^6}, & (x,y) \neq (0,0) \\ 2, & (x,y) = (0,0) \end{cases}$

$A_m = \{(x,y) \in A_m / y = mx, \forall y\}$

$x \to 0 \Rightarrow y \to 0$

$$\lim_{x \to 0} \frac{3x^2 (mx)^4}{x^6 + 2(mx)^6} = \lim_{x \to 0} \frac{3\cancel{x^2} m^4 \cancel{x^4}}{\cancel{x^6} + 2m^6 \cancel{x^6}} = \frac{3m^4}{1 + 2m^6}$$

lim depinde de $m \Rightarrow f$ nu are lim în $(0,0)$

$\Rightarrow f(x,y)$ – nu e cont.

$$\Gamma(t)=\int_0^\infty e^{-x}x^{t-1}\,dx \text{ - conv, } \forall t>0$$

$$\beta(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}\,dx \text{ - conv; } \forall p,q>0$$

$$\int_0^{\frac{\pi}{2}} \sin^{2p-1}x\,\cos^{2q-1}x\,dx=\frac{1}{2}\beta(p,q)$$

$$\beta(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$\Gamma(1)=1$$

$$\Gamma\left(\tfrac{1}{2}\right)=\sqrt{\pi}$$

$$\Gamma(n+1)=n! \qquad \Gamma(6)=5!$$

$$\Gamma(t+1)=t\,\Gamma(t)$$

$$\Gamma\left(n+\tfrac{1}{2}\right)=\frac{(2n-1)!!}{2^n}\sqrt{\pi},\ \forall n\in\mathbb{N}^*$$

$$\Gamma(p)\Gamma(1-p)=\frac{\pi}{\sin(p\pi)},\quad p\in(0,1)$$

11. a) Calculaţi $\frac{\partial}{\partial x^{10}\partial y^{27}}$ ...

b) Folosind funcţiile lui Euler, să se calculeze $\int_0^\infty \left(x^9-2x^4\right)\cdot e^{-x^2}\,dx$.

$$\int_0^\infty x^9 e^{-x^2}dx-\int_0^\infty 2x^4e^{-x^2}dx=$$

$x^2=t \qquad x=0 \Rightarrow t=0$

$2x\,dx=dt \qquad x=\infty \Rightarrow t=\infty$

$$\frac{1}{2}\int_0^\infty t^4e^{-t}dt-\int_0^\infty 2(x^2)^2e^{-x^2}dx$$

$p-1=4 \quad p=5 \qquad q-1=2 \Rightarrow q=3$

$$\frac{1}{2}\Gamma(5)-2\Gamma(3)=\frac{1}{2}4!-2\cdot 2!=$$

$$=12-4=8$$