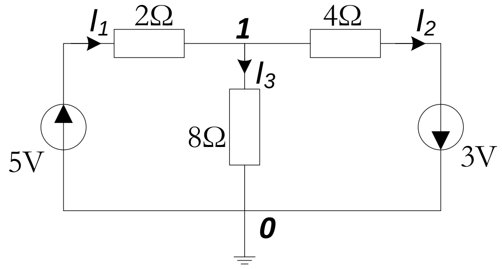
Nodal Analysis

Example 2.8.1. For the circuit below calculate the currents using the nodal analysis.



Example 2.8.1.

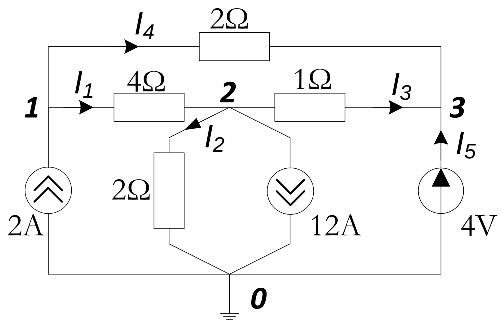
Solution:

As any of the nodes could be considered as a reference node, we will take node 0 as a reference node (of zero potential). The nodal equation for node 1 is:

$$V_1 \cdot \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{4}\right) = 5 \cdot \frac{1}{2} - 3 \cdot \frac{1}{4},$$

from where V_1=2V. Next, from $V_1 - V_0 = -2 \cdot I_1 + 5$, we get I_1=1.5A. In the same way, from $V_1 - V_0 = 4 \cdot I_2 - 3$, I_2=0.25A. The current I_3 could be calculated using the KCL or from $V_1 - V_0 = 8 \cdot I_3$, I_3=1.25A. As a verification, $I_1 = I_2 + I_3$.

Example 2.8.2. Find the node voltages for the circuit below. Calculate the currents through the circuit.



Example 2.8.2.

Solution:

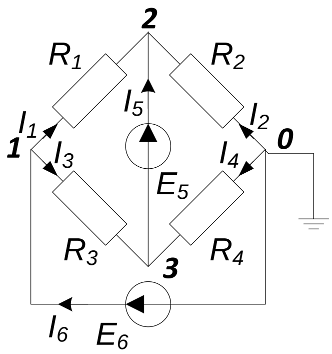
Because the branch between nodes 3 and 0 has no resistance (only voltage source), we have a supernode. We set one of these two nodes as a reference node. We will take node 0 as a reference node, V_0=0. Due to restriction between nodes 0 and 3, $V_3 - V_0 = 4V$, we have V_3=4V. The nodal equations for the nodes 1 and 2, are:

$$V_1 \cdot \left(\frac{1}{2} + \frac{1}{4}\right) - V_2 \cdot \frac{1}{4} - V_3 \cdot \frac{1}{2} = 2,$$

$$V_2 \cdot \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{1}\right) - V_1 \cdot \frac{1}{4} - V_3 \cdot \frac{1}{1} = -12.$$

Solving the system for V_1 and V_2, we get V_1=4V, V_2=-4V. using the value of the node potentials, the currents will be: $I_1 = \frac{V_1 - V_2}{4} = 2A$, $I_2 = \frac{V_2 - V_0}{2} = -2A$, $I_3 = \frac{V_2 - V_3}{1} = -8A$, $I_4 = \frac{V_1 - V_3}{2} = 0A$. For the current I_5 we have to use the KCL in node 3: $-I_3 - I_4 - I_5 = 0$, and I_5=8A.

Example 2.8.3. For the circuit below write the nodal equations for the calculation of the node voltages.



Example 2.8.3.

Solution:

We have two supernodes, one formed by nodes 1 and 0, and another one formed by nodes 2 and 3. Let's consider the reference node the node 0, V_0=0V. Having the restriction $V_1 - V_0 = E_6$, the potential of the node 1 is V_1=E_6. For the second supernode, we have the restriction $V_2 - V_3 = E_5$ and the corresponding nodal equation:

$$V_2 \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right) + V_3 \cdot \left(\frac{1}{R_3} + \frac{1}{R_4}\right) - V_1 \cdot \left(\frac{1}{R_1} + \frac{1}{R_3}\right) = 0.$$

The last two equations should be solved for the unknowns V_2 and V_3.