RLC Series Circuit

Example 3.5.1. Transform these sinusoids in phasor form: $i(t) = 2\sqrt{2}\ sin(50t + 60^0)\,A$, and $u(t) = 80\,cos(30t - 50^0)$.

Solution:

The sinusoid $i(t) = 2\sqrt{2}\ sin(50t + 60^0)\,A$ *has the phasor* $\underline{I} = 2\,e^{j60^0}$, *while the sinusoid* $\mathrm{u(t)} = 80\,cos(30\mathrm{t} - 50^0) = 80\sin(30\mathrm{t} - 50^0 + 90^0)$ *has the phasor* $\underline{U} = \frac{80}{\sqrt{2}}\ e^{j40^0}$.

Example 3.5.2. Express these sinusoids as phasors: $i(t) = -4\sqrt{2}\ sin(10t + 20^0)\,A$, and $u(t) = 7\,cos(2t + 20^0)\ V$.

Solution:

The sinusoid $\mathrm{i(t)} = -4\sqrt{2}\ \sin(10\mathrm{t} + 20^0) = 4\sqrt{2}\ \sin(10\mathrm{t} + 20^0 + 180^0)\,\mathrm{A}$ *has the phasor* $\underline{I} = 4\,e^{j200^0} = 4\,e^{-j160^0}$, *while the sinusoid* $\mathrm{u(t)} = 7\,cos(2\mathrm{t} + 20^0) = 7\cos(2t + 20^0 + 90^0)\ \mathrm{V}$ *has the phasor* $\underline{U} = \frac{7}{\sqrt{2}}\ e^{j110^0}$.

Example 3.5.3. Find the sinusoids corresponding to the phasors:a) $\underline{I} = 4\,e^{j210^0}$; b) $\underline{I} = j\cdot(3 - j4)$; c) $\underline{U} = -97\,e^{j28^0}$.

Solution:

The phasor $\underline{I} = 4\,e^{j210^0}$ *has the corresponding sinusoid* $i(t) = 4\sqrt{2}\sin(\omega t + 210^2) = 4\sqrt{2}\sin(\omega t - 150)$. *The phasor* $\underline{\mathrm{I}} = \mathrm{j}\cdot(3 - \mathrm{j}4)$ *could be written in the form* $\underline{\mathrm{I}} = 4 + \mathrm{j}3 = 5\,\mathrm{e}^{\mathrm{j}37^0}$ *for which the corresponding sinusoid is* $\mathrm{i(t)} = 5\sqrt{2}\ \sin(\omega\mathrm{t} + 37^0)$. *The phasor* $\underline{\mathrm{U}} = -97\,\mathrm{e}^{\mathrm{j}28^0}$ *could be written in the form* $\underline{\mathrm{U}} = 97\,\mathrm{e}^{\mathrm{j}208^0} = 97\,\mathrm{e}^{-\mathrm{j}152^0}$ *and the corresponding sinusoid is* $\mathrm{i(t)} = 97\sqrt{2}\ \sin(\omega\mathrm{t} + 208^0)$ *or* $\mathrm{i(t)} = 97\sqrt{2}\ \sin(\omega\mathrm{t} - 152^0)$.

Example 3.5.4. Given $i_1(t) = 4\sqrt{2}\ sin(\omega t + 30^0)\,A$ and $i_2(t) = 6\sqrt{2}\,cos(\omega t - 30^0)\ A$, find their sum $i(t) = i_1(t) + i_2(t)$.

Solution:

To find the sum of the two currents is useful to perform it using the corresponding phasors. Here is an important use of phasors for summing the sinusoids of the same frequency. For the current $\mathrm{i_1(t)} = 4\sqrt{2}\ \sin(\omega\mathrm{t} + 30^0)\,\mathrm{A}$ *the phasor is* $\underline{\mathrm{I_1}} = 4\,\mathrm{e}^{\mathrm{j}30^0} = 4\,(\cos 30^0 + j\sin 30^0) = 2\sqrt{3} + j2$. *For the current* $\mathrm{i_2(t)} = 6\sqrt{2}\cos(\omega\mathrm{t} - 30^0) = 6\sqrt{2}\sin(\omega\mathrm{t} + 60^0)\,\mathrm{A}$, *the phasor is* $\underline{\mathrm{I_2}} = 6\,\mathrm{e}^{\mathrm{j}60^0} = 6\,(\cos 60^0 + j\sin 60^0) = 3 + j3\sqrt{3}$. *Summing the currents as phasors, we have:* $\underline{I} = \underline{I_1} + \underline{I_2} = (3 + 2\sqrt{3}) + j(2 + 3\sqrt{3}) = 9.66\,e^{j48^0}$ *and transforming to the time domain, we get* $\mathrm{i(t)} = \mathrm{i_1(t)} + \mathrm{i_2(t)} = 9.66\sqrt{2}\sin(\omega t + 48^0)\ (A)$.

Example 3.6.1. Using the phasor approach, determine the current i(t) in a circuit described by the integrodifferential equation:

$$4\sqrt{2}\,i(t)+8\sqrt{2}\int i(t)dt-3\sqrt{2}\,\frac{di(t)}{dt}=50\sqrt{2}\sin(2t-75^0).$$

Solution:

We transform each term from the time domain in the phasor form. The derivate multiplies the phasor by jω (in our case j2), while the integral multiplies the phasor bi $\frac{1}{j\omega}$. *In the phasor form, the integrodifferential equation becomes:* $10\underline{I}+8\cdot\frac{1}{j2}\,\underline{I}-3\cdot j2\,\underline{I}=50\,e^{-j75^0}$, *or* $(10-j4-j6)=50\,e^{-j75^0}$. *Solving for* $\underline{I}$, *we have:* $\underline{I}=\frac{50\,e^{-j75^0}}{10-j10}=\frac{50\,e^{-j75^0}}{10\sqrt{2}\,e^{-j45^0}}=\frac{5}{\sqrt{2}}\,e^{-j30^0}=2.5\sqrt{2}\,e^{-j30^0}$. *Converting this to the time domain, we get the solution of the initial equation:* $i(t)=5\sin(2t-30^0)$.

Example 3.6.2. The voltage $u(t)=15\sqrt{2}\,sin(50t+45^0)V$ is applied to a 0.1H inductance. Find the sinusoidal current through the inductance.

Solution:

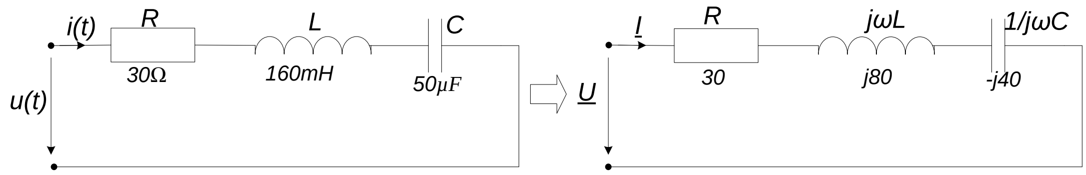
For the inductance, $\underline{U}=j\omega L\,\underline{I}$, *where* $\omega=50\frac{rad}{s}$ *and* $\underline{U}=15\,e^{j45^0}$. *Hence* $\underline{I}=\frac{\underline{U}}{j\omega L}=\frac{15\,e^{j45^0}}{5\,e^{j90^0}}=3\,e^{-j45^0}$. *Converting this to the time domain,* $\mathrm{i(t)}=3\sqrt{2}\,sin(50\mathrm{t}-45^0)\mathrm{A}$.

Example 3.6.3. the voltage $u(t)=10\sqrt{2}\,sin(100t+30^0)V$ is applied to a 50µF capacitance. Find the sinusoidal current through the capacitance.

Solution:

For the capacitance, $\underline{U}=\frac{1}{j\omega C}\,\underline{I}$, *where* $\omega=100\frac{rad}{s}$ *and* $\underline{U}=10\,e^{j30^0}$. *Hence* $\underline{I}=j\omega C\,\underline{U}=50\cdot10^{-3}e^{j120^0}$. *Converting this to the time domain,* $\mathrm{i(t)}=50\cdot10^{-3}\sqrt{2}\,sin(50\mathrm{t}-45^0)\mathrm{A}$, *or* $\mathrm{i(t)}=50\sqrt{2}\,sin(50\mathrm{t}-45^0)\mathrm{mA}$.

Example 3.7.1. The circuit parameters for the RLC series circuit in the figure a) below are R=30Ω, L=160mH, C=50µF and the supplying voltage: $u(t)=200\sqrt{2}\;sin(500t+60^0)\,V$. Using the phasor method, calculate: a) the impedance phasor, $\underline{Z}$, the circuit impedance, Z, and the phase angle introduced by the circuit impedance, ϕ; b) the current phasor $\underline{I}$, the rms value I, and the instantaneous value of the current, i(t); c) the resistance voltage phasor $\underline{U}_R$, the rms value U_R, and the instantaneous value of the voltage on the resistance $u_R(t)$; d) the inductance voltage phasor $\underline{U}_L$, the rms value U_L, and the instantaneous value of the voltage on inductance $u_L(t)$; e) the capacitance-voltage phasor $\underline{U}_C$, the rms value U_C, and the instantaneous value of the voltage on the capacitance $u_C(t)$; f) draw the phasor diagram.

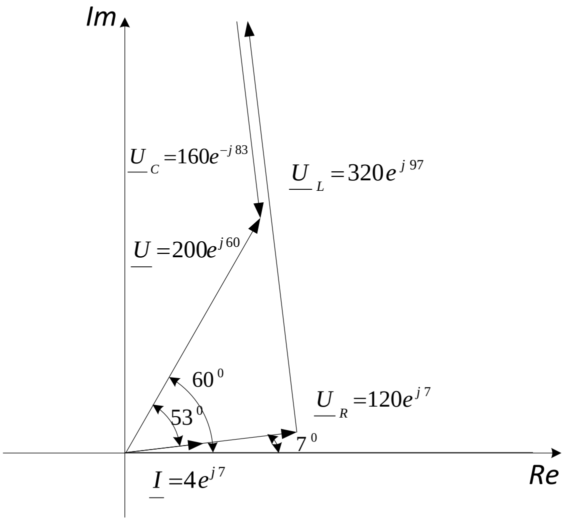


a) b)

Example 3.7.1.

Solution:

The inductive reactance is $X_L = \omega L = 80\ \Omega$, *and the capacitive reactance* $X_C = \frac{1}{\omega C} = 40\Omega$, *where ω=500rad/s. The phasor circuit is drawn in figure b) above. For this circuit*$\underline{U} = R\underline{I} + j\omega L\underline{I} - j\frac{1}{\omega C}\underline{I}$ *or* $200\ e^{j60^0} = 30\underline{I} + j80\underline{I} - j40\underline{I}$. *Solving for the* $\underline{I}$, *we have* $\underline{I} = \frac{200\ e^{j60^0}}{30+j40} = \frac{200\ e^{j60^0}}{50\ e^{j53^0}} = 4\ e^{j7^0}$. *Hence, I=4A and* $i(t) = 4\sqrt{2}\sin(500t + 7^0)\ A$. *The circuit impedance is Z=50Ω and the phase angle introduced by the impedance is φ=53*^{0}. *The resistance voltage phasor is* $\underline{U_R} = R\underline{I} = 120e^{j7^0}$ *and as a consequence,* U_R*=120V and* $u_R(t) = 120\sqrt{2}\sin(500t + 7^0)\ V$. *The inductance voltage phasor is* $\underline{U_L} = j\omega L\underline{I} = j320e^{j7^0} = e^{j90^0}320e^{j7^0} = 320e^{j97^0}$. *Hence,* U_L*=320V and* $u_L(t) = 320\sqrt{2}\sin(500t + 97^0)\ V$. *The capacitance-voltage phasor is* $\underline{U_C} = -j\frac{1}{\omega C}\underline{I} = -j160e^{j7^0} = e^{-j90^0}160e^{j7^0} = 320e^{-j83^0}$. *Hence,* U_C*=160V and* $u_C(t) = 160\sqrt{2}\sin(500t - 83^0)\ V$. *The phasor diagram is drawn in the figure below.*



The phasor diagram of the circuit