① $f_{X,Y}(x,y) = \begin{cases} \frac{1}{2}, & 0 \le x \le y \le 2 \\ 0, & \text{în rest} \end{cases}$

a) $P(Y \le \frac{1}{2} \mid X = 1)$

$f_X = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_{x}^{2} \frac{1}{2}\,dy = \frac{1}{2}(2-x), \quad x \in [0,2]$

$h(y/1) = \begin{cases} \dfrac{f_{X,Y}(1,y)}{f_X(1)} = \dfrac{\frac{1}{2}}{\frac{1}{2}} = 1, & y \in [1,2] \\ 0, \text{ în rest}, & y \in (-\infty,1) \cup (2,\infty) \end{cases}$

$P(Y \le \frac{1}{2} \mid X = 1) = \int_{-\infty}^{y} h(y|1)\,dy = \int_{-\infty}^{\frac{1}{2}} h(y|1)\,dy + \cancel{\int_{1}^{y=\frac{1}{2}} h(y|1)\,dy} =$ $\to 0$

$= 0$

b) $\operatorname{cov}(X,Y) = M(XY) - M(X)\,M(Y)$

$f_Y(y) = \int_0^y \frac{1}{2}\,dx = \frac{1}{2}y$

$M(X) = \int_{-\infty}^{\infty} x \cdot \frac{1}{2}(2-x)\,dx = \frac{1}{2}\int_0^2 2x - x^2\,dx =$

$= \frac{1}{2}\left(x^2 - \frac{x^3}{3}\right)\Big|_0^2 = \frac{1}{2}\left(4 - \frac{8}{3}\right) = \frac{1}{2}\cdot\frac{4}{3} = \frac{4}{6} = \frac{2}{3}$

$M(Y) = \int_0^2 \frac{1}{2}y^2\,dy = \frac{1}{2}\cdot\frac{y^3}{3}\Big|_0^2 = \frac{8}{6} = \frac{4}{3}$

$M(XY) = \int_0^2 \int_x^2 xy\,\frac{1}{2}\,dy\,dx = \frac{1}{2}\int_0^2 x\,\frac{y^2}{2}\Big|_x^2 dx =$

$= \frac{1}{2}\int_0^2 x\left(2 - \frac{x^2}{2}\right)dx = \frac{1}{2}\int_0^2 2x - \frac{x^3}{2}\,dx = \frac{1}{2}\left(x^2 - \frac{x^4}{8}\right)\Big|_0^2 =$

$= \frac{1}{2}\left(4 - \frac{16}{8}\right) = \frac{1}{2}(4-2) = \frac{1}{2}\,2 = 1$

$\operatorname{cov}(X,Y) = 1 - \frac{4}{3}\cdot\frac{2}{3} = 1 - \frac{8}{9} = \frac{1}{9}$

```
do{
    x = 2 * urand();
    y = 2 * urand();
}while( x > y)

return(x, y);
```

2) $\begin{pmatrix} 49 & -14 \\ -14 & 64 \end{pmatrix}$ $\quad cov(x,y) = -14$

$\sigma^2(x) = 49 \qquad \sigma(x) = 7$

$\sigma^2(y) = 64 \qquad \sigma(y) = 8$

$$\sigma^2(-x+2y) = \sigma^2(-x) + \sigma^2(2y) + 2\,cov(-x, 2y) =$$

$$= (-1)^2 \cdot 49 + 4 \cdot 64 - 4\,cov(x,y) =$$

$$= 49 + 256 + 4 \cdot 14 = \ldots$$

3) a) $P(X_5 = 3 \mid X_3 = 1) = Q^2(1,3) = 0{,}45$

$$Q^2 = \begin{pmatrix} 0{,}25^2 + 0{,}5 \cdot 0{,}2 & 0{,}25^2 + 0{,}5 \cdot 0{,}25 + 0{,}4 \cdot 0{,}5 & \underbrace{0{,}25 \cdot 0{,}5 + 0{,}25 \cdot 0{,}5 + 0{,}5\, 0{,}4} \\ & & \\ & & \end{pmatrix}$$

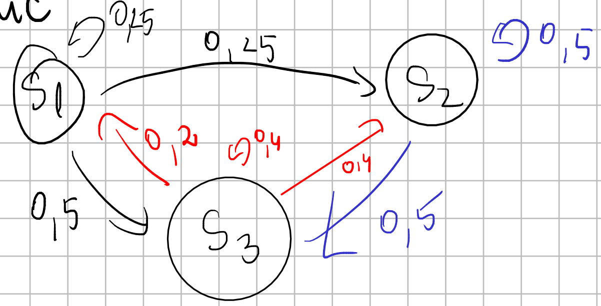
b) $P(X_4 = 1 \mid X_1 = 1, X_2 = 3, X_3 = 1) =$

$= P(X_4 = 1 \mid X_3 = 1) = Q(1,1) = 0{,}25$

c) – ired.

$\exists\, n \in \mathbb{N}^*$ a.î. $Q^n(i,j) > 0$

– aperiodic



$S_1 \to S_3 \to S_1 \quad 2$

$S_1 \to S_2 \to S_3 \to S_1 \quad 3$ $\Big\} \Rightarrow 1$ ired $\Big\} \Rightarrow$ toate nodurile sunt aperiodice

P2

$\sigma^2(x) = 25$

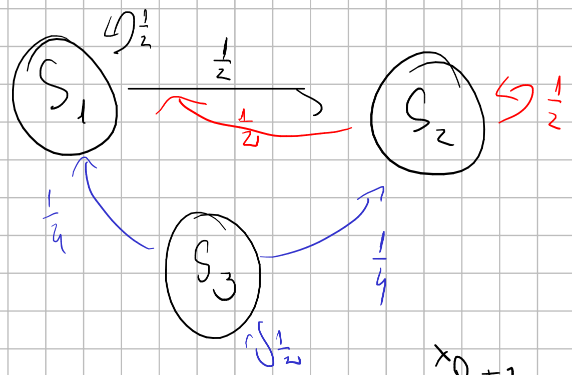
$y = -3x - 2 \Rightarrow \rho = -1$

$\sigma^2(y) = \sigma^2(-3x-2) = 9\,\sigma^2(x) = 9 \cdot 25$

$cov(x,y) = -\sigma(x) \cdot \sigma(y) = -5 \cdot 3 \cdot 5 = -75$

P3

a)



$x_0 + 2$

plec din 3 fără 1 ajung în 3

$S_3 \to S_3 \to S_3$

$$\frac{\pi_0(3) \cdot Q(3,3) \cdot Q(3,3)}{\pi_0(3)} = \frac{1}{4}$$

b) $\pi_0(3) \cdot Q(3,1) \cdot Q(1,2) \cdot Q(2,2) \cdot Q(2,1) =$

$= \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{96}$

c) $S_2 \to S_1 \to S_2 \quad 2$

$S_2 \to S_2 \quad 1$ $\Big\} \Rightarrow 1$