1.6 Example SRT radix 2

$$\boxed{-\frac{b}{2} < r_i < \frac{b}{2}}$$

$$\boxed{r_{i+1} = 2r_i - q_i b}$$

COUNT	P (2n+1 bits)	A (n bits)	B (n bit)
000	000 00 0000	1110 1101	0000 0110
	000 01 1101	1010 0000	1100 0000
	$q_0 = 0$		$k = 5$
	000 11 1011	0100 0000	
001	$q_1 = 0$		
	001 11 0110	1000 0000	
010	$q_2 = 1$		
	011 10 1101	0000 0001	
	− 11 00 0000		
	000 10 1101		
011	$q_3 = 0$		
	001 01 1010	0000 0010	
100	$q_4 = 1$		
	010 11 0100	0000 0101	
	− 11 00 0000		
	111 11 0100		
101	$q_5 = 0$		
	111 10 1000	0000 1010	
110	$q_6 = 0$		
	111 01 0000	0001 0100	
111	$q_7 = 0$		
	110 10 0000	0010 1000	
cor	+ 11 00 0000	− 1	
	001 10 0000	0010 0111	
shift	000 00 0011	Quotient = 39_{ten}	

Remainder = 3_{ten}

$$\frac{b}{2} < r_i < b$$

255 −
18
237 | 6
18 | 39
= 57 Quotient
54
= 3
Remainder

$m/m = m \cdot 2^k / m \cdot 2^k$

Sweeney, Robertson, Tocher (SRT)

$n+1$	n	n
P	A	B
Partial remainder ⇓ Remainder	Dividend	Divisor

① If B has k leading $\emptyset$s when expressed on n bits, shift all registers (P.A.B) left k bits.

② For $i = 0$ to $n-1$

(a) If top 3 bits of P are equal, set $q_i = 0$, shift (P.A) one bit left

(b) If top 3 bits of P are not equal and $P \geq 0$, set $q_i = 1$ and shift (P.A) one bit left, and subtract B

(c) If top 3 bits of P are not equal and $P < 0$, set $q_i = \bar{1}$ and shift (P.A) one bit left, and add B.

End loop.

③ If the final reminder is negative ($P < 0$), correct the reminder by adding B, and correct the quotient by subtracting 1 from the quotient

④ The reminder must be shifted k bits right, if k is the initial shift.

213 | 5
20 | 42
= 13
10
= 3

213 | 17
17 | 12
= 43
34
= 9

213 -
128
= 85 -
64
21 -
16
= 5

1 1 0 1 0 1 0 1

COUNT	P	A	B
000	00000 0000 00000 0110 $q_0=0$ 00001101	1101 0101 1010 1000 0101 0000	00010001 10001000
001	$q_1=0$ 00011010	 10100000	
010	$q_2=0$ 00110101	 01000000	
011	$q_3=0$ 01101010	 10000000	
100	$q_4=1$ 01101 0101 − 1000 1000 0010 01101	 0000 0001	
101	$q_5=1$ 01001 1010 − 1000 1000 000010010	 0000 0011	
110	$q_6=0$ 000100100	 0000 0110	
111	$q_7=0$ 001001000	 0000 1100	
shift	000001001 Reminder = 9_{ten}	Quotient = 12_{ten}	

$k=3$

$q_i \in \{\bar{1}, 0, 1\}$

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	0	0	$\bar{1}$	0	1

$2^5 - 2^2 + 2^0 = 32 - 4 + 1 =$
$= 29_{ten}$

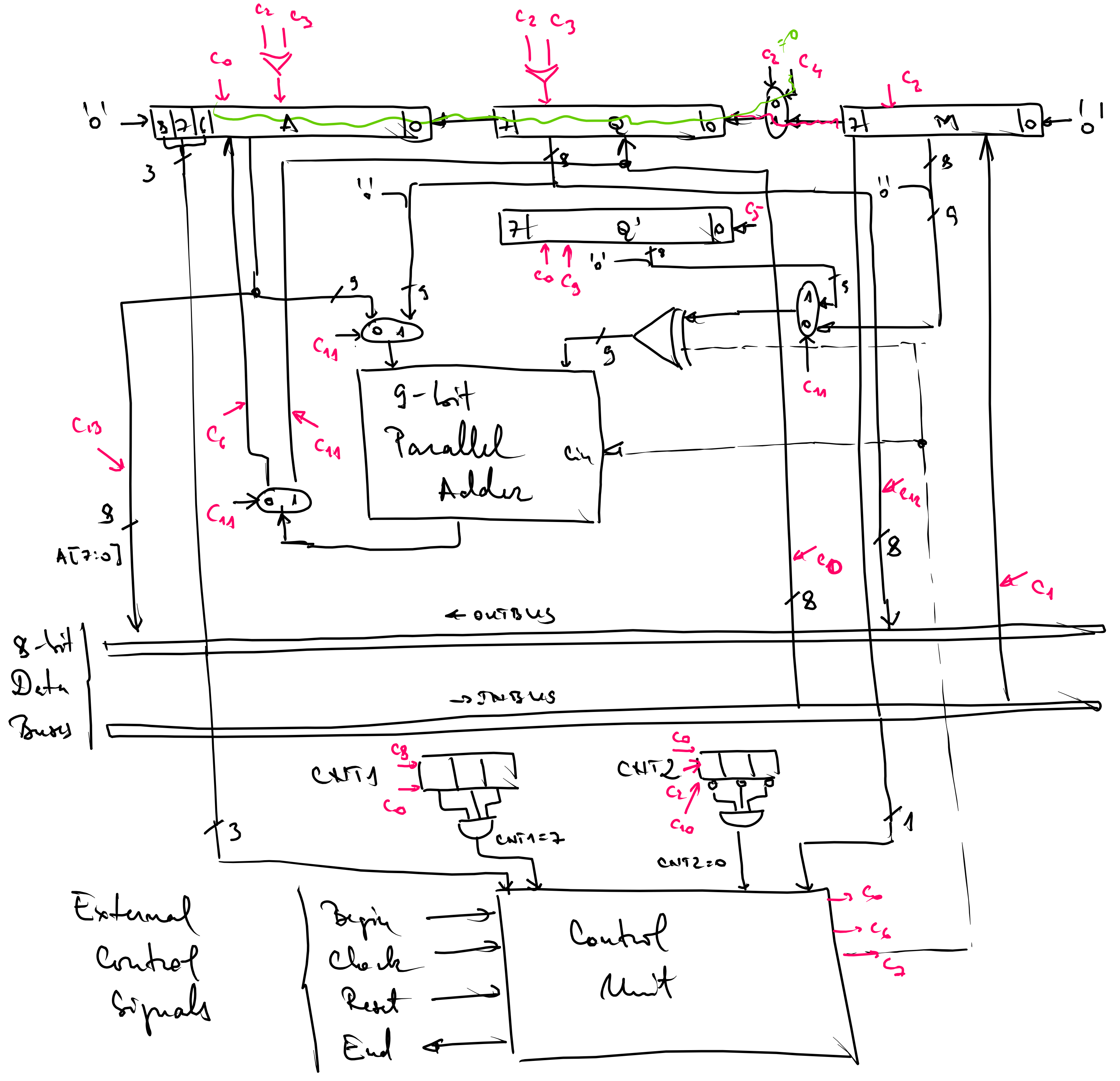
A	0	1	0
A'	0	0	1
	0	1	$\bar{1}$

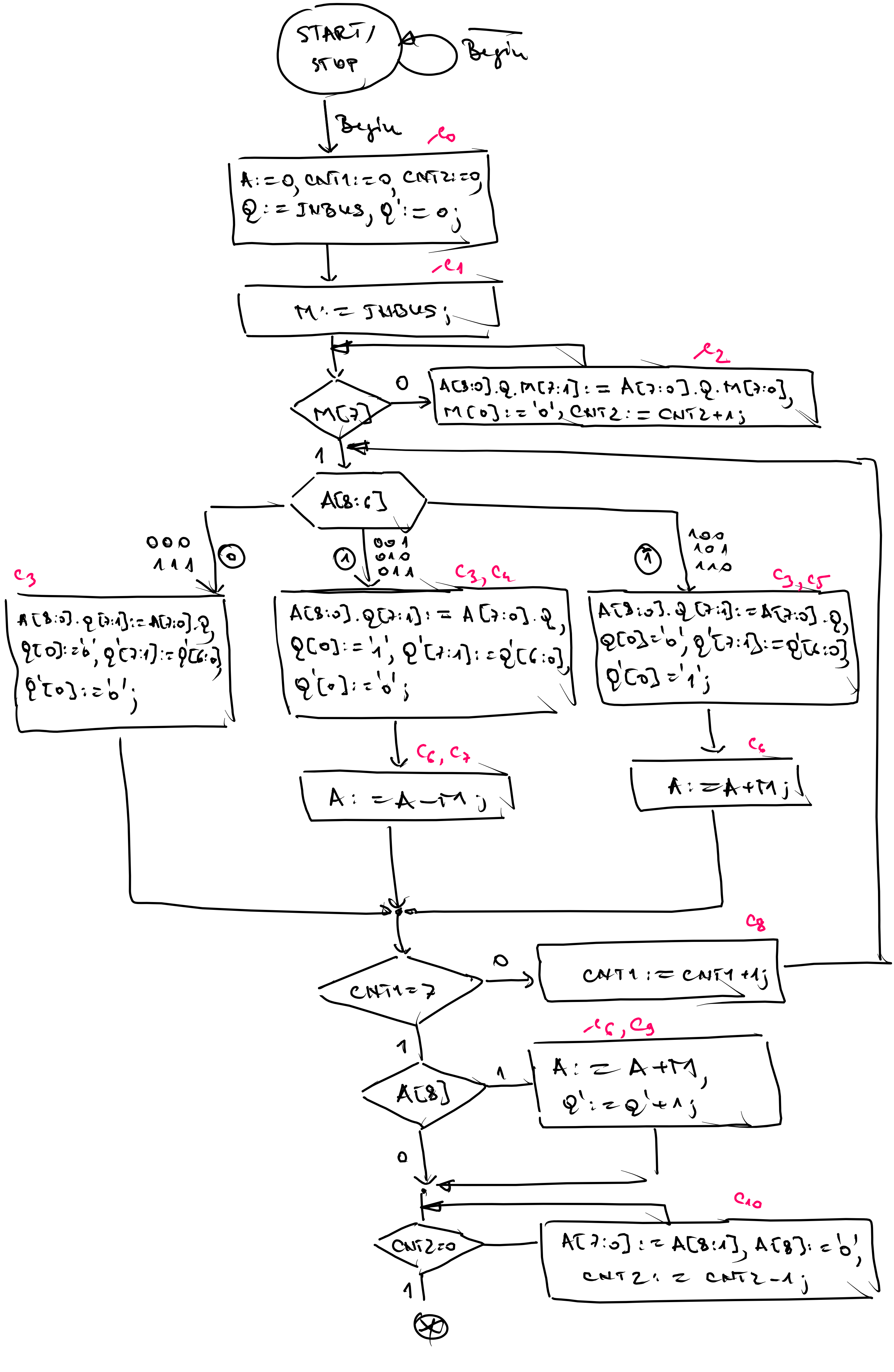
A 0010 0001
A' 0000 0100

$\underbrace{0001\ 1101}_{29_{ten}}$

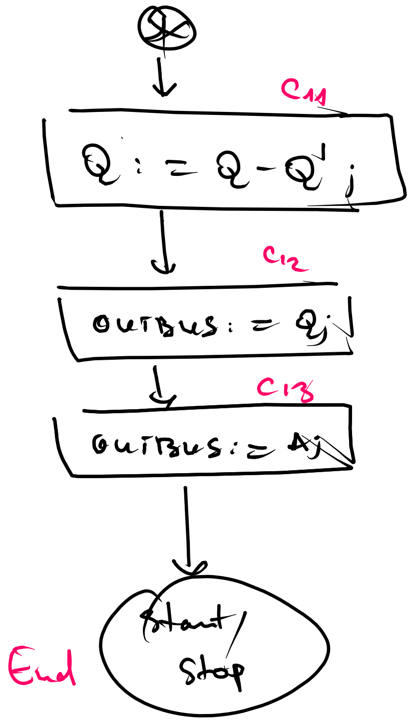
1.6.1 Implementing the SRT radix-2 algorithm

– Hayes hardware platform


'0'
A
Q
M
Q'
9-bit Parallel Adder
cin
← OUTBUS
→ INBUS
8-bit Data Buses
A[7:0]
CNT1
CNT2
CNT1=7
CNT2=0
External Control Signals
Begin
Clock
Reset
End
Control Unit



START / STOP
Begin
Begin
c0
A:=0, CNT1:=0, CNT2:=0,
Q:=INBUS, Q':=0;
c1
M:=INBUS;
M[7]
0
1
c2
A[8:0].Q.M[7:1]:=A[7:0].Q.M[7:0],
M[0]:='0', CNT2:=CNT2+1;
A[8:6]
000
111
0
001
010
011
1
100
101
110
1
c3
A[8:0].Q[7:1]:=A[7:0].Q,
Q[0]:='0', Q'[7:1]:=Q'[6:0],
Q'[0]:='0';
c3, c4
A[8:0].Q[7:1]:=A[7:0].Q,
Q[0]:='1', Q'[7:1]:=Q'[6:0],
Q'[0]:='0';
c3, c5
A[8:0].Q[7:1]:=A[7:0].Q,
Q[0]:='0', Q'[7:1]:=Q'[6:0]
Q'[0]:='1';
c6, c7
A:=A-M;
c6
A:=A+M;
CNT1=7
0
c8
CNT1:=CNT1+1;
1
A[8]
1
c6, c9
A:=A+M,
Q':=Q'+1;
0
CNT2=0
c10
A[7:0]:=A[8:1], A[8]:='0',
CNT2:=CNT2-1;
1



1.6.2 Speeding up division with radix-4 SRT

$q_i \in \{\bar{2}, \bar{1}, 0, 1, 2\}$ $\qquad \boxed{r_{i+1} = 4r_i - q_i b}$

$\boxed{|r_i| < b}$ $\qquad$ if $r_i = b \Rightarrow \boxed{r_{i+1} = 4b - q_i b} > 2b$

This condition cannot be met

$|r_i| < \frac{2}{3} b$ $\qquad$ if $r_i = \frac{2}{3} b \Rightarrow r_{i+1} = \frac{8b}{3} - q_i b$

if $q_i = 2$

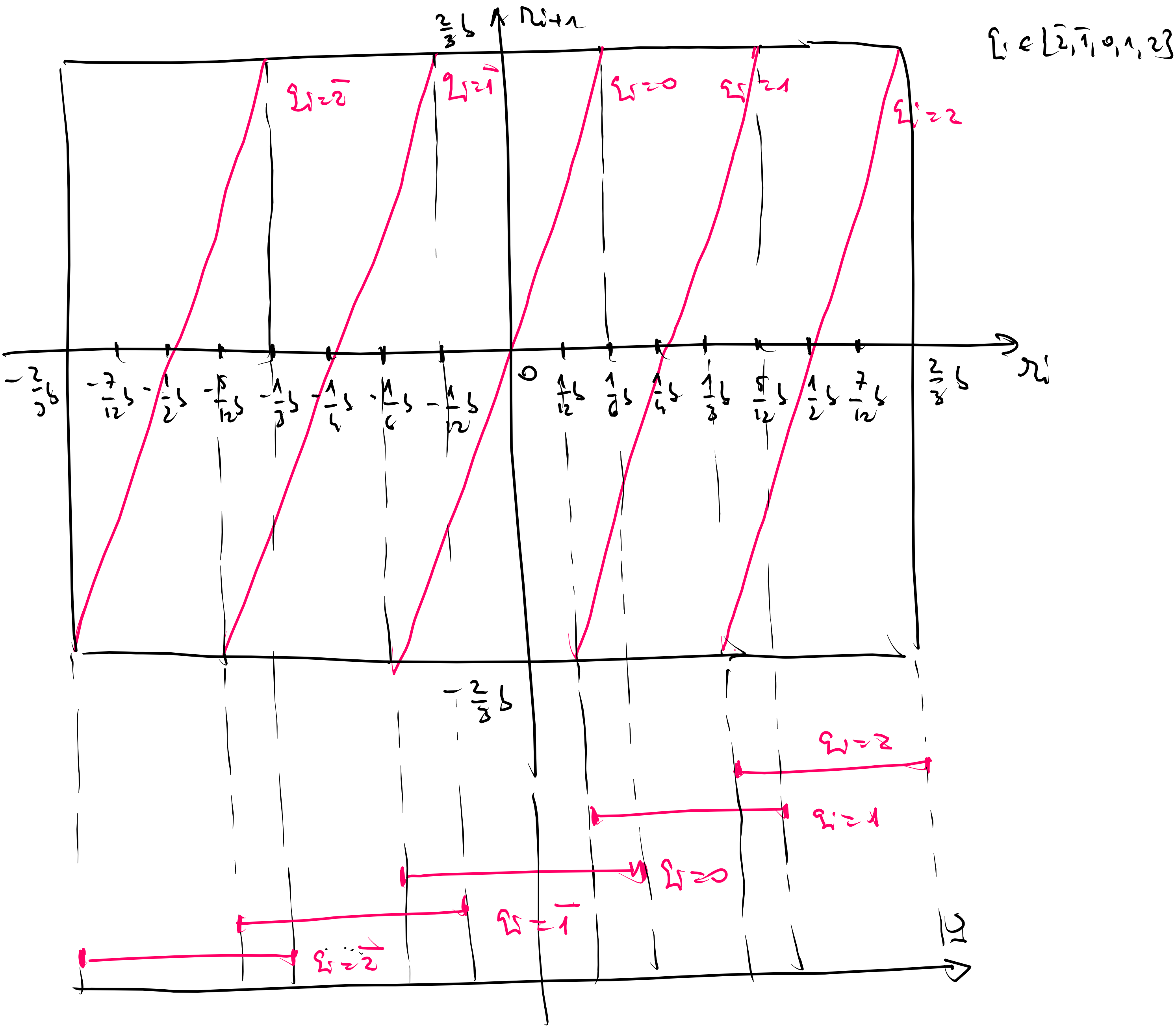
$r_{i+1} = \frac{8b}{3} - \frac{6b}{3} = \frac{2}{3} b$

$r_{i+1} = 4 \cdot \frac{3}{2} b - q_i b$

$6b - q_i b < \frac{3}{2} b$

$r_{i+1} = 4 \cdot \frac{2}{3} b - q_i b$

$= \frac{8}{3} b - \frac{6}{3} b = \frac{2}{3} b$



Use the first 6 bits of P ⇒
Value of b ⇒ } Quotient digit

$r_{i+1} = 4 \cdot r_i - q_i \cdot b \; ; \; q_i \in \{\bar{2}, \bar{1}, 0, 1, 2\}$

Figure J.33 Quotient selection for radix-4 division with quotient digits −2, −1, 0, 1, 2.

b	Range of P		q	b	Range of P		q
8	−12	−7	−2	12	−18	−10	−2
8	−6	−3	−1	12	−10	−4	−1
8	−2	1	0	12	−4	3	0
8	2	5	1	12	3	9	1
8	6	11	2	12	9	17	2
9	−14	−8	−2	13	−19	−11	−2
9	−7	−3	−1	13	−10	−4	−1
9	−3	2	0	13	−4	3	0
9	2	6	1	13	3	9	1
9	7	13	2	13	10	18	2
10	−15	−9	−2	14	−20	−11	−2
10	−8	−3	−1	14	−11	−4	−1
10	−3	2	0	14	−4	3	0
10	2	7	1	14	3	10	1
10	8	14	2	14	10	19	2
11	−16	−9	−2	15	−22	−12	−2
11	−9	−3	−1	15	−12	−4	−1
11	−3	2	0	15	−5	4	0
11	2	8	1	15	3	11	1
11	8	15	2	15	11	21	2

211 : 6 = 35
13
23
30
=1

211 −
128
= 83 −
64
19

111 011 $_{C_2}$
111 010 $_{C_1}$
100 101 $_{SM}$

COUNT	P	A	B
00	0 0000 0000 0 0001 1010	1101 0011 0110 0000	0000 0110 1100 0000
	$q_0 = 0$ 00 1101 001	10 00 00 00	b = 12 B = 01100 0000 2B = 11000 0000
01	$q_1 = 2$ 11010 0110 − 11000 0000 00 010 0110	00 000 00 10 00 00	
10	$q_2 = 1$ 01 0 01 10 00 − 0 11 00 0000 1 11 01 1000	00 00 10 01 00 00	
11	$q_3 = \bar{1}$		

k = 5

	P	A	
	1 0 1 1 0 0 0 0 0 + 0 1 1 0 0 0 0 0 0 → 0 0 0 1 0 0 0 0 0	$\boxed{\begin{matrix}00\\00\end{matrix}}$ 10 01 00 00 00 01	
shift	0 0 0 0 0 0 0 0 1	00 10 01 00 − 00 00 00 01 00 10 00 11	

$k = 5$

Remainder $= 1_{ten}$ Quotient $= 35_{ten}$

$q_i \in \{\bar{2}, \bar{1}, 0, 1, 2\}$

$0 \to \begin{matrix}00\\00\end{matrix}$ $\bar{1} \to \begin{matrix}00\\01\end{matrix}$

$1 \to \begin{matrix}01\\00\end{matrix}$ $\bar{2} \to \begin{matrix}00\\10\end{matrix}$

$2 \to \begin{matrix}10\\00\end{matrix}$

$1\,0\,\bar{2}\,\bar{1} = 1 \cdot 4^3 - 2 \cdot 4^1 - 1 \cdot 4^0 =$

$= 64 - 9 = 55_{ten}$

01 00 00 00 −
00 00 10 01

00 11 01 11 → 55_{ten}

16 +
7
= 23
32
= 55

$213 \,|\, 5 = 42$
20
= 13
10
= 3

213 −
128
= 85 −
64
= 21 −
16
= 5

111 001 $_{c2}$
111 000 $_{c1}$
100 111 $_{SM}$

Count	P	A	B
00	00000 0000 00001 1010 $q_0 = 1$ 00110 1010 − 01010 0000 11100 1010	1101 0101 1010 0000 1000 00 $\boxed{\begin{matrix}01\\00\end{matrix}}$	0000 0101 $\boxed{1010}$ 0000 b=10 B = 01010 0000 2B = 10100 0000
01	10010 1010 + 01010 0000 11100 1010	0000 $\boxed{\begin{matrix}01\\00\end{matrix}}$ $\begin{matrix}00\\01\end{matrix}$	
10	$q_2 = \bar{1}$ 10010 1000 + 01010 0000 11100 1000	00 $\boxed{\begin{matrix}01\\00\end{matrix}}$ 00 00 01 01	
11	$q_3 = \bar{1}$ 10010 0000 + 01010 0000 11100 0000	$\boxed{\begin{matrix}01\\00\end{matrix}}$ 00 0000 01 0101 +1	
cor	+ 01010 0000 00110 0000	01 00 0000 − 00 01 0110	Quotient 42_{ten}
shift	00000 0011	00 10 1010	

$k = 5$

Remainder 3_{ten}