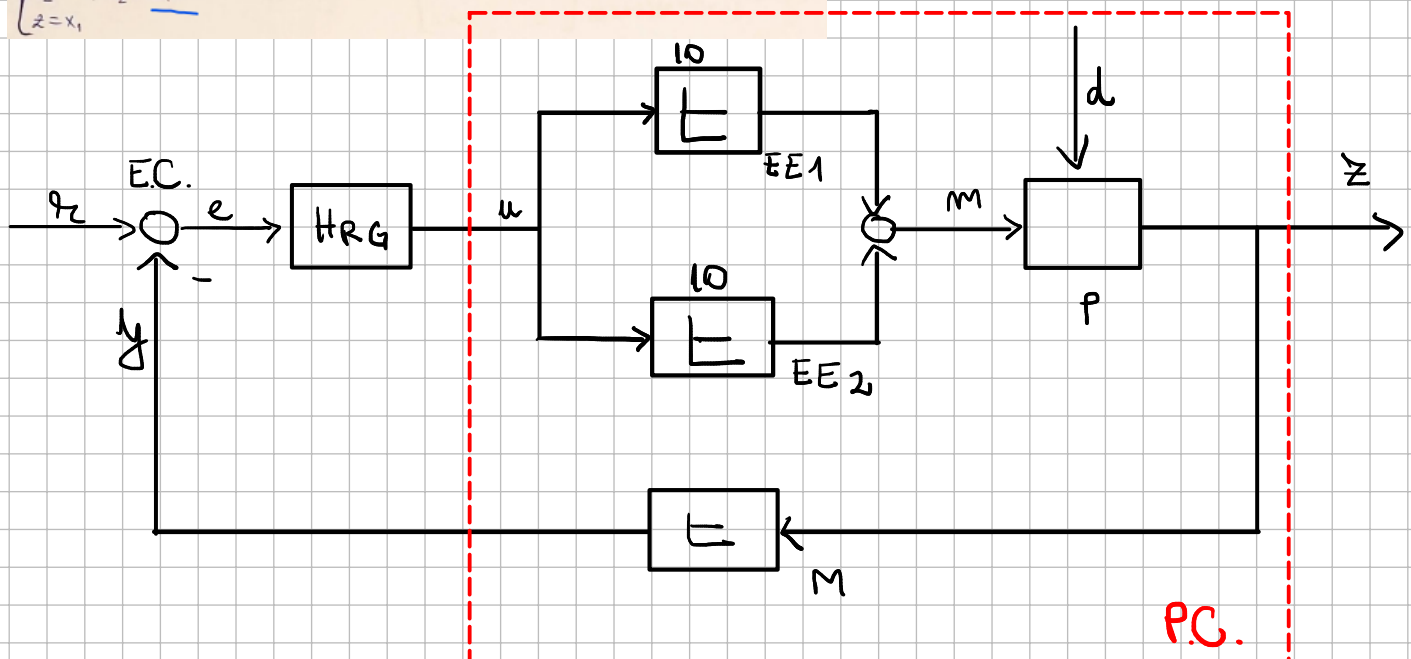


# Subiect 2023

Se consideră sistemul de reglare automată cu schema bloc prezentată în figură, în care  $r(t)$  este referința,  $e(t)$  este eroarea de reglare și modelul de stare (MH-isi) al blocului P este:

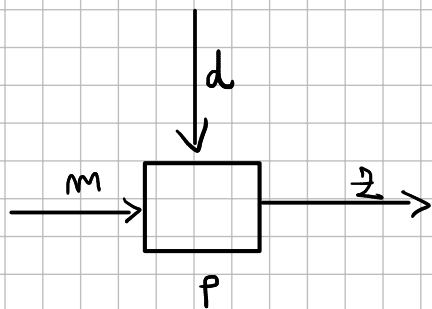
$$\begin{cases} \dot{x}_1 = -2x_1 + 2x_2 + 40d \\ \dot{x}_2 = -0,5x_2 + 12,5m \\ z = x_1 \end{cases}$$



Regulatorii sunt de 2 tipuri:

$$R_1: \text{ET-Pi}: H_{RG}(s) = \frac{h_{RG}(1+s \cdot T_i)}{s \cdot T}$$

$$R_2: \text{ET-PBT1}: H_{RG}(s) = \frac{h_{RG}(1+T_b s)}{1+s \cdot T_f}$$



2 intrări  
0 ieșiri

$$\Rightarrow H_{zm} \Big|_{d=0} = ? \quad \text{și} \quad H_{zd} \Big|_{m=0} = ?$$

$$\begin{bmatrix} m \\ d \end{bmatrix} = u(t)$$

marimi de intrare

$$\begin{cases} \dot{x}(t) = A \cdot x(t) + B \cdot u(t) \\ y(t) = C^T \cdot x(t) \end{cases}$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\begin{cases} \dot{x}_1 = -2 \cdot x_1 + 2x_2 + 0 \cdot m + 40d \\ \dot{x}_2 = 0 \cdot x_1 - 0,5x_2 + 12,5m + 0d \end{cases}$$

Daca analizam ecuatiile scrise cu rosu de mai sus, putem deduce urmatoarea relatie:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 2 \\ 0 & -0,5 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix}}_B \begin{bmatrix} m \\ d \end{bmatrix}$$

Pentru matricea A, luam coeficientii de la vectorul de stare ( $\dot{x}$  -  $x_1$  si  $x_2$ ) si ii asezam in matrice pe linia 1 - de la  $\dot{x}_1$  si pe linia 2 - de la  $\dot{x}_2$

Asemamator procedam si pentru vectorul de intrare (m, d), doar ca pentru matricea B

$$\begin{aligned} \dot{z} &= 1 \cdot x_1 + 0 \cdot x_2 \\ z &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned}$$

$$H(s) = C \underbrace{(sI - A)^{-1}}_M \cdot B$$

$$M = sI - A = s \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 0 & -0,5 \end{bmatrix} = \begin{bmatrix} s+2 & -2 \\ 0 & s+0,5 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

formula curs

(mai usor decat sa faci inversa clasic)

$$M^{-1} = \frac{1}{(s+2)(s+0,5) - 0(-2)} \begin{bmatrix} s+0,5 & 0 \\ 2 & s+2 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+0,5 & 0 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} \cdot \frac{1}{(s+2)(s+0,5)}$$

$$= \frac{1}{(s+2)(s+0,5)} \cdot \begin{bmatrix} s+0,5 & 0 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} =$$

$$= \frac{1}{(s+2)(s+0,5)} \cdot \begin{bmatrix} 25 & 40(s+0,5) \end{bmatrix} =$$

$$= \begin{bmatrix} \overset{m}{\frac{25}{(s+2)(s+0,5)}} & \overset{d}{\frac{40}{s+2}} \end{bmatrix} \quad \text{vector intrare} \quad \begin{bmatrix} m \\ d \end{bmatrix}$$

$$\Rightarrow H_{2m} \Big|_{d=0} = \frac{25}{(s+2)(s+0,5)}$$

$$H_{2d} \Big|_{m=0} = \frac{40}{s+2}$$

## Verificarea MM-isi-ului

Daca observam, in blocul P, avem intrarea d. Putem desparti astfel blocul P in blocuri

mai mici, pe baza  $H_{2m}$  si  $H_{2d}$



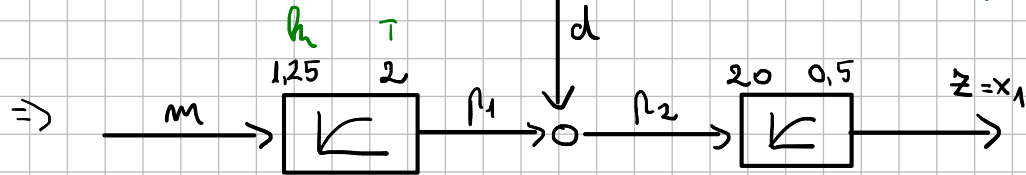
$$H_{2m} \Big|_{d=0} = \frac{25}{(s+2)(s+0,5)} = \frac{25}{2(1+0,5s)0,5(1+2s)} = \frac{25}{(1+0,5s)(1+2s)}$$

$$H_{2d} \Big|_{m=0} = \frac{40}{s+2} = \frac{40}{2(1+0,5s)} = \frac{20}{1+0,5s}$$

$H_{2m}$  îl conține și pe  $H_{2d}$

$$\Rightarrow H_{2m} = H \cdot H_{2d}$$

$$H = \frac{1+0,5s}{20} \cdot \frac{25}{(1+0,5s)(1+2s)} = \frac{1,25}{1+2s} \rightarrow h$$



$$p_1(s) = \frac{1,25}{1+2s} m(s) \quad | \quad (1+2s)$$

$$\Rightarrow p_1(s) + 2s \cdot p_1(s) = 1,25 m(s)$$

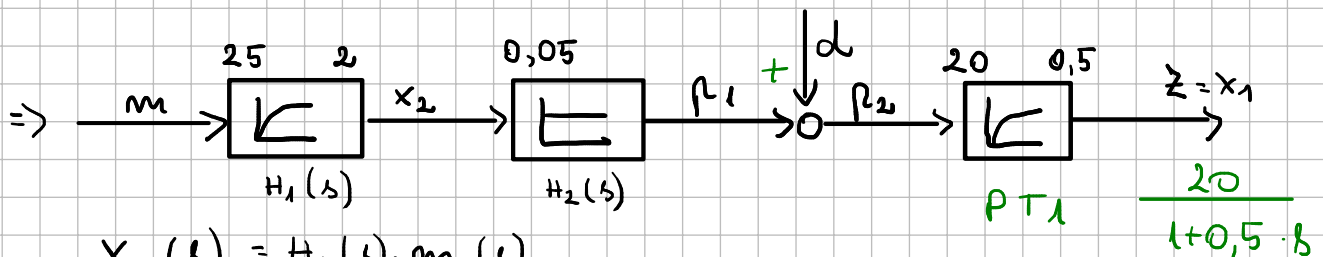
Dacă ne uităm la ecuația lui  $\dot{x}_1 \Rightarrow$  avem 12,5 m(s)  $\Rightarrow$  trebuie să despartim primul bloc

din cele 2 de mai sus în alte 2 blocuri  $\longrightarrow$  în serie

$$\Rightarrow H_1(s) \cdot H_2(s) = \frac{1,25}{1+2s} \quad \text{pt } H_1 \text{ putem lua de la } H_{2m}$$

$$\frac{25}{1+2s} \cdot H_2(s) = \frac{1,25}{1+2s}$$

$$H_2(s) = \frac{1,25}{25} = 0,05$$



$$x_2(s) = H_1(s) \cdot m(s)$$

$$x_2(s) = \frac{25}{1+2s} \cdot m(s) \quad | \cdot (1+2s)$$

$$x_2(s) + 2s x_2(s) = 25 m(s) \quad | :2$$

$$0,5 \cdot x_2(s) + s \cdot x_2(s) = 12,5 m(s)$$

$$0,5 x_2(s) + \dot{x}_2(t) = 12,5 m(s)$$

$$\Rightarrow \dot{x}_2 = -0,5 x_2 + 12,5 m \longrightarrow \text{a doua ecuație de la început}$$

$$x^{(m)}(t) = s^m x(s)$$

$$p_1(s) = 0,05 m(s)$$

$$p_1 = 0,05 m$$

$$p_1(s) + d(s) = p_2(s) \Rightarrow p_1 + d = p_2$$

$$X_1(s) = Z(s) = H_{2d}(s) \cdot p_2(s) = \frac{20}{1+0,5s} \cdot p_2(s) \quad | \cdot (1+0,5s)$$

$$X_1(s) + 0,5 \cdot s \cdot X_1(s) = 20 p_2(s)$$

$$s \cdot x(s) = \dot{x}(t)$$

$$X_1(s) + 0,5 \cdot \dot{X}_1(s) = 20 p_2(s) \quad | \cdot 2$$

$$s^2 x(s) = x^{(2)}(t)$$

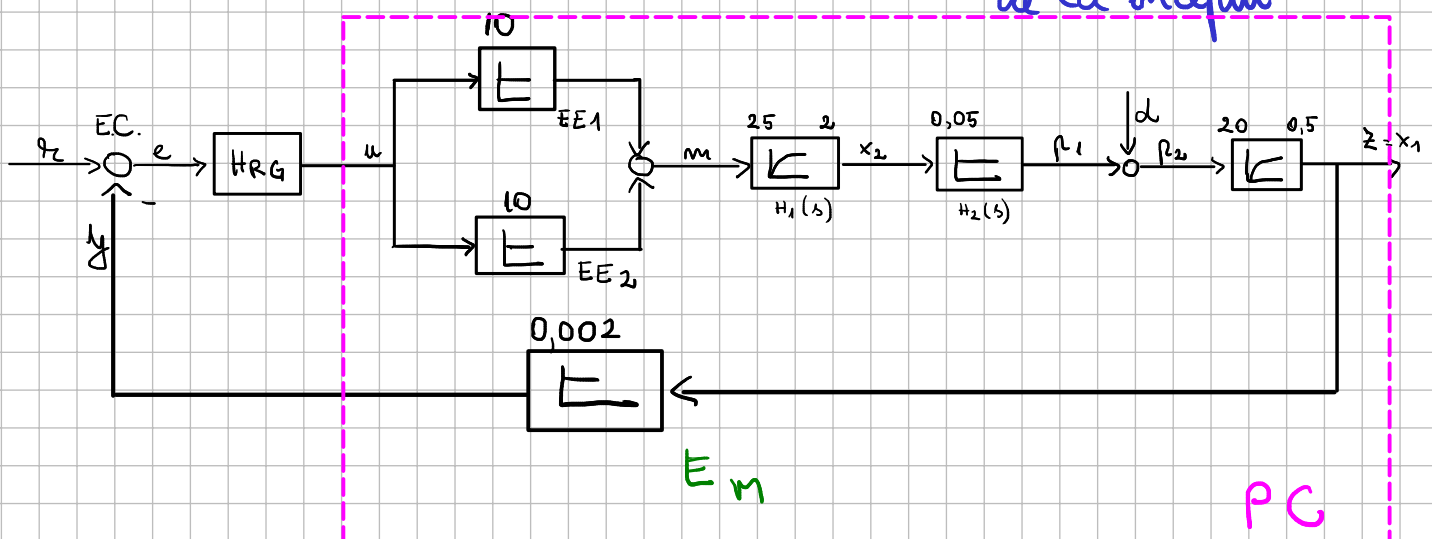
$$2 X_1(s) + \dot{X}_1(s) = 40 p_2(s)$$

$$\dot{X}_1 = -2 X_1 + 40 p_2$$

$$\Rightarrow \dot{X}_1 = -2 X_1 + 40 (p_1 + d)$$

$$\Rightarrow \dot{X}_1 = -2 X_1 + 40 \cdot 0,05 \cdot X_2 + 40 d$$

$$\Rightarrow \dot{X}_1 = -2 X_1 + 2 X_2 + 40 d \longrightarrow \text{prima ecuație de la început}$$



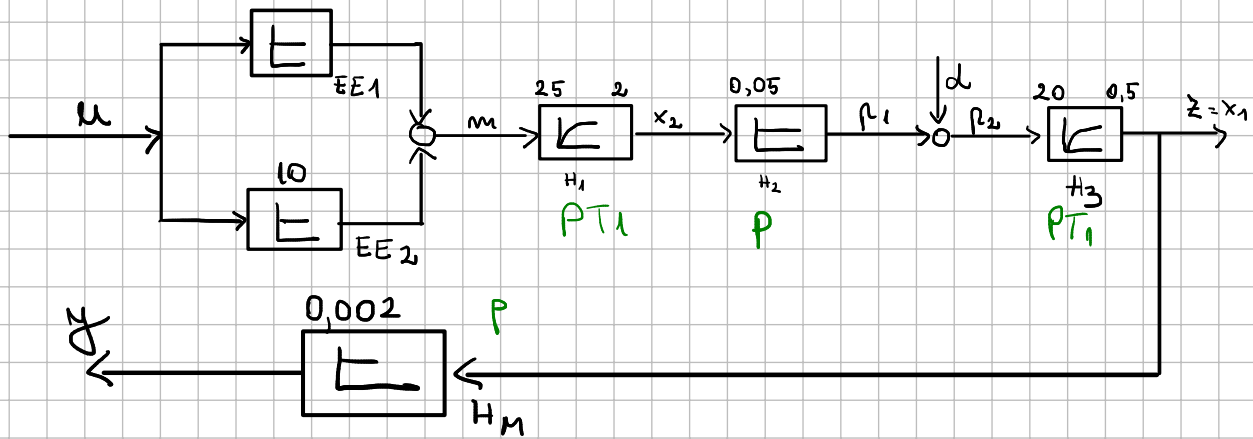
Calculați c.d.t.  $H_{yr}(s)$ ,  $H_{yd}(s)$ ,  $H_o(s)$

pt  $R_1$ :  $T_i = 2,5 \text{ sec.}$

pt.  $R_2$ :  $T_d = 2,5 \text{ sec}$ ,  $T_f = 0,1 \text{ sec.}$

f.d.t a sistemului deschis

## Calculăm $H_{PC}$

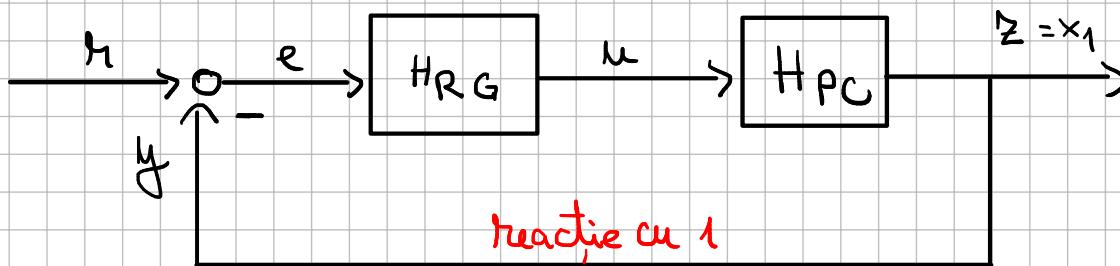


$$H_{EE1}, H_{EE2} - \text{paralel} \Rightarrow H_{EE} = H_{EE1} + H_{EE2} = 10 + 10 = 20$$

$$H_{EE}, H_1, H_2, H_3, H_M - \text{serie}$$

$$\Rightarrow H_{PC} = 20 \cdot \frac{25}{1+25} \cdot 0,05 \cdot \frac{20}{1+0,55} \cdot 0,002 =$$

$$= \frac{1}{(1+25)(1+0,55)} = H_{yH} \Big|_{d=0}$$



$$H_{RG}, H_{PC} - \text{serie} \Rightarrow H_0 = H_{RG} \cdot H_{PC}$$

$$H_s, 1 - \text{reactie} \Rightarrow H_\alpha = \frac{H_0}{1 + H_0 \cdot 1}$$

$$\Rightarrow H_{yH} = H_\alpha = \frac{H_0}{1 + H_0}$$

$$R_1: T_i = 2,5 \text{ sec} \Rightarrow H_{RG} = \frac{k_R (1+2,5s)}{2,5s} \rightarrow \text{tip PI}$$

$$H_{PC} = \frac{1}{(1+0,5s)(1+25s)}$$

$\downarrow \quad \downarrow$   
 $T_2 \quad T_1$

$$\left. \begin{array}{l} k_{PC} = 1 \\ T_1 = 2 \\ T_2 = 0,5 \end{array} \right\} \Rightarrow T_1 > T_2$$

$$h_R = \frac{1}{2 \text{ mpc} \cdot T_2} = \frac{1}{2 \cdot 1 \cdot 0,5} = 1$$

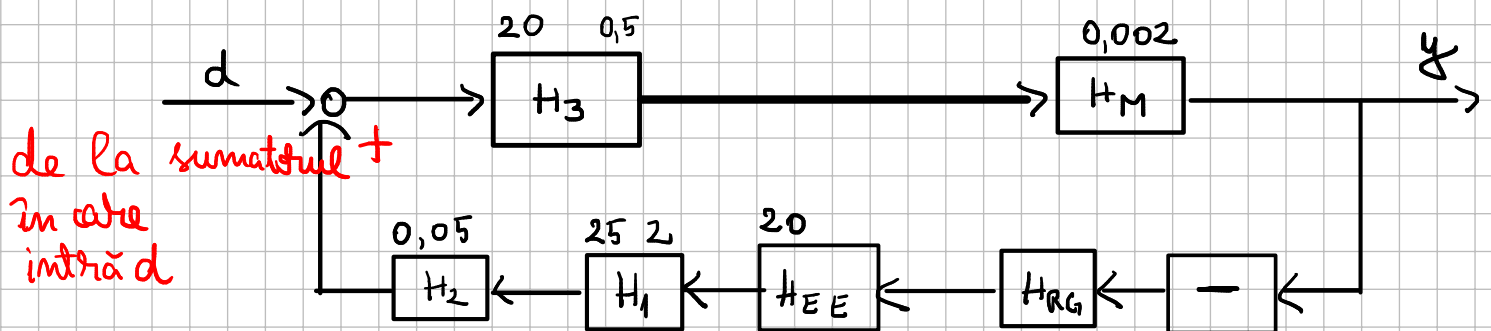
$T_R = \text{constanta de timp mai mare} = 2$

$$h_R = h_R \cdot T_R = 2 \cdot 1 = 2$$

$$\Rightarrow H_{RG}(s) = \frac{2}{2,5 \cdot s} (1 + 2,5s)$$

$$H_0(s) = H_{RG}(s) \cdot H_{PC}(s) = \frac{\boxed{2} (1 + 2,5s)}{2,5 \cdot s \cdot (1 + 2s) \cdot (1 + 0,5s)}$$

$$H_{yR}(s) \Big|_{d=0} = \frac{H_0(s)}{1 + H_0(s)} = \frac{\frac{2 \cdot (1 + 2,5s)}{2,5 \cdot s \cdot (1 + 2s) \cdot (1 + 0,5s)}}{1 + \frac{2 \cdot (1 + 2,5s)}{2,5 \cdot s \cdot (1 + 2s) \cdot (1 + 0,5s)}}$$



$H_3, H_m$  - serie

$$H_{3m} = H_3 \cdot H_m = 0,002 \cdot \frac{20}{1 + 0,5s} = \frac{0,04}{1 + 0,5s}$$

↓  
de la sumatorul  
în care intră  
y

toate cele de pe ramura de jos - serie

$$H_S = - H_{RG} \cdot H_{EE} \cdot H_1 \cdot H_2 =$$

$$= - \frac{2 (1 + 2,5s)}{2,5s} \cdot 20 \cdot \frac{25}{1 + 2s} \cdot 0,05 =$$

$$= \frac{50 (1 + 2,5s)}{2,5s \cdot (1 + 2s)}$$

$H_s, H_{3M}$  - reactie

$$\Rightarrow H_{yd(s)} \Big|_{h=0} = \frac{H_{3M}}{1 - H_{3M} \cdot H_s} = \frac{\frac{0,04}{1+0,5s}}{1 - \frac{0,04}{1+0,5s} \cdot \frac{50(1+2,5s)}{2,5 \cdot s \cdot (1+2s)}}$$

$R_2: T_d = 2,5 \text{ sec}, T_f = 0,1 \text{ sec} \Rightarrow H_{RG}(s) = \frac{K_R (1+2,5s)}{1+2,5s}$

aceleasi calculu



② Găsiți  $h_R > 0$  pt care SRA - stabil

→ matricea Hurwitz

$H(s) \rightarrow$  denominator

$$H_0 \rightarrow \Delta(s) = 1 + H_0$$

$$H_0(s) = \frac{h_R \cdot (1 + 2,5s)}{2,5s \cdot (1 + 2s) \cdot (1 + 0,5s)}$$

$$\Delta(s) = 1 + \frac{h_R \cdot (1 + 2,5s)}{2,5s \cdot (1 + 2s) \cdot (1 + 0,5s)} = 0$$

$$(2,5s + 5s^2)(1 + 0,5s) + h_R(1 + 2,5s) = 0$$

$$2,5s + 1,25s^2 + 5s^2 + 2,5s^3 + h_R + 2,5s \cdot h_R = 0$$

$$\Delta(s) = \underbrace{2,5s}_{a_3 > 0} + \underbrace{6,25s^2}_{a_2 > 0} + \underbrace{2,5s(1 + h_R)}_{a_1} + \underbrace{h_R}_{a_0} = 0$$

$$a_1 = 2,5(1 + h_R) > 0 \Rightarrow 1 + h_R > 0 \quad h_R > -1$$

$$a_0 = h_R > 0$$

$$\Rightarrow h_R > 0 \Rightarrow h_R \in (0, +\infty)$$

$$H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 6,25 & h_R & 0 \\ 2,5 & 2,5(1+h_R) & 0 \\ 0 & 6,25 & h_R \end{bmatrix}$$

$$H_1 = 6,25 > 0$$

$$H_2 = 15,625(1 + h_R) - 2,5h_R = 15,625 + 13,125h_R > 0$$

$$h_R > - \frac{15,625}{13,125}$$

$$H_3 = a_0 \det(H_2) = \underset{>0}{h_R} \cdot (15,625 + \underset{>0}{13,125 h_R})$$

$$\Rightarrow h_R > 0$$

$\Rightarrow$  sistem stabil

③ Considerând expresia  $y(t)$ , acceptând că sistemul este stabil și alegând o valoare a lui  $k_R > 0$ , găsim valoare statismului natural  $f_m(y)$ . Acceptând valorile nominale  $d_m = 50$  și  $y_m = 100$ , găsim valoare statismului natural în unități raportate în procente  $f_m(y) \%$ .

Statismul natural

$$f_m = \frac{y_\infty}{V_\infty} \Big|_{w_\infty=0} = \frac{k_N}{1+k_0}$$

$$f_m = \frac{z_\infty}{V_\infty} \Big|_{w_\infty=0} = \frac{k_H}{1+k_0}$$

•  $i, p_i, p_{ib} \longrightarrow e_\infty = 0$ ;  $f_m = 0$  (fără statism)  
astatic

•  $P, P_{T1}, P_{BT1} \longrightarrow e_\infty \neq 0$  și  $f_m \neq 0$  (cu statism)

$$k_0 = k_R \cdot k_P$$

$$R_1: ET-P_i: H_{RG}(s) = \frac{k_{RG}(1+s \cdot T_i)}{s \cdot T}$$

$\longrightarrow$  regulator de tip  $P_i$

$$\Rightarrow e_\infty = 0 \text{ și } f_m(y) = 0$$

$$d_m = 50, \quad y_m = 100$$

$$f_m(y) \% = \underbrace{f_m(y)}_0 \cdot \frac{d_m}{y_m} \cdot 100 \% = 0$$

$$R_2: \quad \text{ET-PBT1: } H_{RG}(s) = \frac{h_{RG}(1+T_b s)}{1+s \cdot T_f}$$

$$\Rightarrow \gamma_m(y) \neq 0, \quad e_\infty \neq 0$$

$$H_{PC} = \frac{\textcircled{1} \rightarrow PC}{(1+0,5s)(1+2s)}$$

$\downarrow T_2 \quad \quad \downarrow T_1$

$$\left. \begin{array}{l} h_{PC} = 1 \\ T_1 = 2 \\ T_2 = 0,5 \end{array} \right\} \Rightarrow T_1 > T_2$$

$$h_R = 3$$

$$h_0 = h_R \cdot h_{PC} = 3 \cdot 1 = 3$$

avem 2 blocuri

$y \leftrightarrow d$

$$\gamma_m(y) = \frac{h_M(y)}{1+h_0} = \frac{20 \cdot 0,002}{1+3} = \frac{0,04}{4} = 0,01$$

$$\gamma_m(y)\% = 0,01 \cdot \frac{50}{100} \cdot \frac{100}{100} = 0,5\%$$

