Teoria sistemelor
Curs 2

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DESCRIPTION AND GENERAL PROPERTIES OF SYSTEMS

Outline

- **Physical systems. Mathematical models. Dynamical systems**
- Classifications of physical and dynamical systems
- Input-output models
- Graphical modeling by block diagrams
- Introduction to process control
- References

Physical systems. Mathematical models. Dynamical systems

- A physical process (PP) or a physical-chemical process is a sequence of transformations and conversions that characterize sets of interconnected objects or phenomena, according to a certain structure, which are viewed in their temporal (or sometimes spatial) evolution
- A physical system (PS) is a system whose behavior changes over time, often in response to external stimulation or forcing, and it represents the material ensemble where a PP evolves
- The PS is characterized by two categories of variables from the point of view of its relation to the environment (Fig. 1.1):

  • **input variables**, with the notation $\mathbf{u}(t)$, which externally influence the temporal evolution of the PS, and they represent the cause

  • **output variables**, with the notation $\mathbf{y}(t)$, which are used in the characterization of the temporal evolution of the PS, and they represent the effect

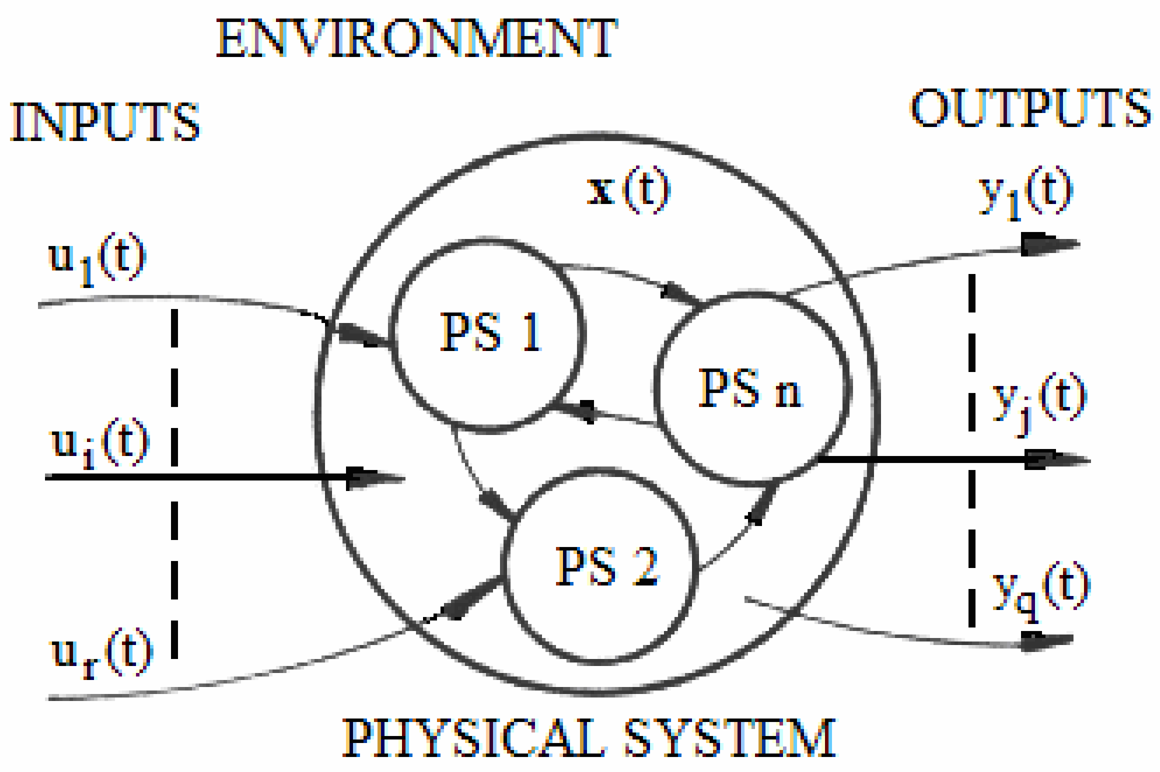

Fig. 1.1. Representation of a physical system.

- If the evolution of the PS is considered in the finite time interval $[t_0, t_f]$, then

    - the system will be in an initial state (situation) at the initial time moment t_0
    - the system will be in a final state (situation) at the final time moment t_f

- The **temporal evolution** (t indicates the independent time variable) of a system can be seen as the evolution in time of the two categories of variables (input and output ones) and of the system state. The temporal evolution of each system is subjected to the **principle of causality**, interpreted by means of:

    - the evolution (in time) is always oriented towards past – present – future
    - the evolution of the outputs $\mathbf{y}(t)$ is always caused by the evolution of the inputs $\mathbf{u}(t)$ and of the initial state of the system and never in the backward sense

- The substance (mass and energy) transfer, storage, transformation and dissipation phenomena in a PS and, generally, the state in which the system is at a certain moment or at any time moment, can be described by means of some internal variables called **state variables** (from macroscopic physics) – they are grouped in the state vector $\mathbf{x}(t)$
- The state of a dynamical system is a set of variables that completely summarizes the past history of the system, and allows us to predict its future evolution. In other words, the state variables of a PS are those variables (the set of variables) that allow, by their values at the current time moment t, to define system's situation or state at that time moment
- Knowing at a certain moment (e.g., t_0) the state of a PS, with the notation $\mathbf{x}(t_0)$, allows us to predict the future evolution of the PS at the time moments $t > t_0$
- The ensemble of input variables $\mathbf{u}(t)$, state variables $\mathbf{x}(t)$ and output variables $\mathbf{y}(t)$ concerning a PS is called **characteristic variables** of the PS, with the notation $\{\mathbf{u}(t), \mathbf{x}(t), \mathbf{y}(t)\}$

- A system generally has more than one input, output or state variable → the characteristic variables are grouped in the vectors (column matrices):
  - the input vector $\mathbf{u}(t)$, with r components,
  - the state vector $\mathbf{x}(t)$, with n components,
  - the output vector $\mathbf{y}(t)$, with q components:

$$\mathbf{u}(t)=\begin{bmatrix} u_1(t) \\ u_2(t) \\ \dots \\ u_r(t) \end{bmatrix},\ \mathbf{x}(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_n(t) \end{bmatrix},\ \mathbf{y}(t)=\begin{bmatrix} y_1(t) \\ y_2(t) \\ \dots \\ y_q(t) \end{bmatrix}$$

- r is the number of input variables, n is the number of state variables, q is the number of output variables
- The dimension n of the state vector $\mathbf{x}(t)\in\mathbf{R}^n$, which is also the number of state variables, is called **the order of the system**

- The input variables of a system can only be input variables
- The output variables can also be state variables or contrarily
- The same PS can operate in several (technical) operating regimes. Therefore, its variables can be both input variables (for example, in the regime 1) and output variables (for example, in the regime 2). Example: electrical machines:

  • in the motor regime: the inputs = the armature (terminal) voltage and the excitation voltage, the output = the speed

  • in the generator regime: the inputs = the speed and the excitation voltage, the output = the armature voltage
- From the point of view of control, the output variables of a PS can be grouped functionally in:
  • strictly speaking output variables – to assess the control
  • measured output variables – to achieve the control

Different letters are used to highlight both categories of output variables

- The clear difference between the vectors and the scalars will be carried out as follows using bold notations for the vectors
- The same rule will be applied to matrices as well, but the matrices are highlighted with capital bold letters
- A row vector will be considered as the matrix obtained by the transposition of a column vector. Example:

$$\mathbf{x}^T(t) = [x_1(t) \quad x_2(t) \quad \dots \quad x_n(t)]$$

- The mathematical description and characterization of a PS requires the assignment of an adequate **mathematical model (MM)**
- The derivation of the mathematical model assigned to a PS is related to **the choice of the state variables – requirements**:
  - the variables should be continuous in time (with respect to time)
  - the variables should characterize the matter transfer, storage, transformation and dissipation phenomena in the PS
- Table 1.1 gives some recommendations concerning the choice of state variables in physical systems
- Rule of thumb: **the number of state variables is equal to the number of energy storage elements**
- Table 1.2 gives several equivalents between the variables in different technical fields with direct impact on the choice of the state variables

Type of transfer process	State variables	Examples of state equations	Examples of energy storage elements
Substance	Mass m	$m' = Q_m$ Q_m - mass flow	Hydraulic tank
Electrical	Electric charge q	$q' = i$, i – electric current (intensity)	Capacitor's armatures
Impulse	Translational impulse $p = m\,v$	$p' = \sum F_i$ F_i - force	Body subjected to translational or rotational motion
	Rotational impulse	$p_\omega{}' = \sum M_i$ M_i - rotational torque	
Kinetic energy	Linear velocity (speed) v	$v' = (1/m) \sum F_i$ $v' = a$, a – linear acceleration	Body subjected to translational or rotational motion
	Angular velocity (speed) ω	$\omega' = a_\omega$ a_ω – angular acceleration	

Table 1.1. Recommendations concerning the choice of state variables in physical systems with focus on technical systems.

Table 1.1. Recommendations concerning the choice of state variables in physical systems with focus on technical systems (cont'd).

Potential energy	Linear position (space) s	$s' = v$, v – linear velocity (speed)	Body subjected to translational or rotational motion
	Angular position (rotational angle) α	$\alpha' = \omega$, ω – angular velocity (speed)	
Energy stored in elastic system	Deformation force F	$F' = k_f\, v$, k_f – viscous friction coefficient (elastic coefficient)	Elastic spring or spiral spring subjected to deformation
	Deformation torque (moment) M	$M' = k_f\, \omega$, k_f – viscous friction coefficient (elastic coefficient)	
Thermal	Temperature θ	$\theta' = (1/C_\theta)\, q_\theta$ q_θ – thermal flow	Homogenous body of thermal capacitance C_θ
Electromagnetic energy: - In the electric field of a capacitor - In the magnetic field of an inductor	Voltage across the capacitor u_c	$u_c' = (1/C)\, i$	Capacitor of capacitance C
	Current through the Inductor u_c	$i_L' = (1/L)\, u$	Inductor of inductance L

Table 1.2. Equivalences between several variables and parameters involved in transfer processes of different technical fields.

Series electrical circuit	Parallel electrical circuit	Translational mechanical system	Rotational mechanical system
Electric voltage u	Electric current i	Force F	Torque (moment) M
Electric load q	Magnetic flow Φ	Linear position (space) s	Angular position (rotational angle) α
Electric current (intensity) i	Electric voltage u	Linear velocity (speed) v	Angular velocity (speed) ω
Inductance L	Capacitance C	Mass m	Inertia moment J
Resistance R	Conductance $1/R$	Viscous friction coefficient μ	
Capacitance C	Inductance L	Elastic coefficient (spring constant) k_e	
Temperature difference $\Delta\theta$	Pressure difference Δp		
Heat quantity Q_θ	Fluid quantity Q_f	Gas quantity Q_s	
Heat flow q_θ	Fluid flow q_f	Gas flow q_g	
---	Hydraulic inertia coefficient	---	
Thermal Resistance R_θ	Hydraulic Resistance R_h	Pneumatic resistance R_p	
Thermal Capacitance C_θ	Hydraulic Capacitance C_h	Pneumatic capacitance C_p	

- A PS characterized by its mathematical model represents a **dynamical system (DS)** or an abstract system
- A dynamical model of a system is a set of mathematical laws explaining in a compact form and in quantitative way how the system evolves over time, usually under the effect of external excitations
- The dynamical system is a mathematical concept which is completely defined by **axioms** concerning:

  • **categories of sets and classes of functions** employed in the description of the independent variable *t* and of the characteristic variables of the PS

  • **operators (functionals)** employed in the description of the structural relationships between the characteristic variables of the PS

- Illustration of the connections between the PS and its associated MM (DS) for a single input-single output system:

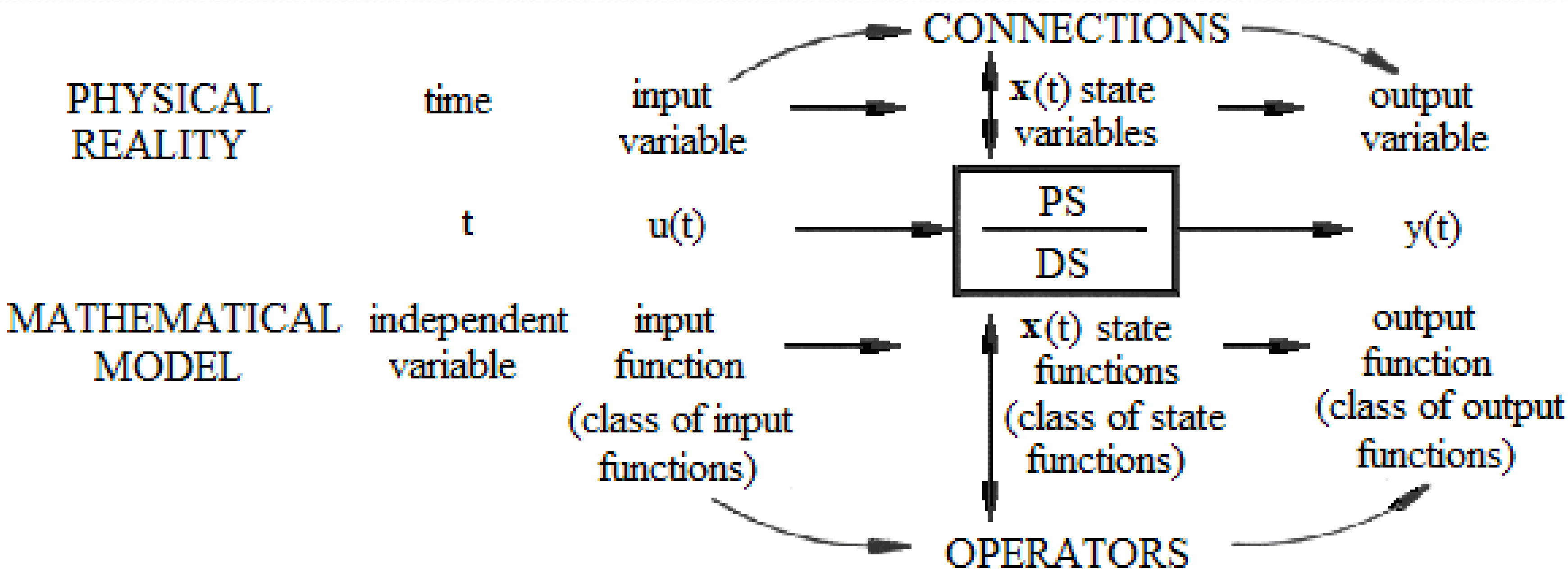

Fig. 1.2. Connections between a PS and its associated DS.

- Main questions about a dynamical system:
  - to understand the system, i.e., to answer to questions like “How X and Y influence each other ?”
  - to simulate the system, i.e., to answer to questions like “What happens if I apply action Z on the system ?”
  - to design a control system for it, i.e., to answer to questions like “How to make the system behave the way I want ?”
- The necessity of models comes from the fact that experiments provide answers, but they have limitations such as:
  - they may be too expensive (e.g., launch a space shuttle)
  - they may be too dangerous (e.g., experiments on an operating nuclear plant)
  - they may be impossible (e.g., the system does not exist yet being in the phase of design)

- In contrast, mathematical models allow us to:
  - capture the main phenomena that take place in the system (e.g., Newton's law– a force on a mass produces an acceleration)
  - analyze the system (get relations among dynamical variables),
  - simulate the system (i.e., make predictions) about how the system behaves under certain conditions and excitations (in analytical form, or on a computer in terms of digital simulation of system's behavior)
- Example of mathematical modeling of a PS – application in the field of electric circuits – please read the associated lecture material – Kirchhoff's voltage and current laws lead to equations that are next manipulated such that to lead to the composed relationship:

$$\text{input} \rightarrow \text{states} \rightarrow \text{output}$$

- This corresponds to a typical mathematical model in systems theory and in automation called **state-space mathematical model (SS-MM)**

- The general expression of an SS-MM of a continuous-time linear DS with a single input and a single output:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,u(t) \;-\; \text{the state equation}$$

$$y(t) = \mathbf{c}^T\,\mathbf{x}(t) + d\,u(t) \;-\; \text{the output equation}$$

- This representation, which highlights the internal structure of the PS (the evolution of the internal process in the PS), is called **structural representation of the DS (of the PS)**
- Removing the state variables in the state equations → another relationship called **input-output mathematical model (IO-MM)**:

$$\text{input} \rightarrow \text{output}$$

- The general expression of an IO-MM for continuous-time linear time invariant (with constant parameters) systems with a single input and a single output:

$$\sum_{\nu=0}^{n} a_{\nu} y^{(\nu)}(t) = \sum_{\mu=0}^{m} b_{\mu} u^{(\mu)}(t)$$

- It is also called **functional representation of a DS (of a PS)**
- $m \leq n$ is the **causality condition**.
- A DS is **causal** if $y(t)$ does not depend on future inputs $u(\tau)\ \forall \tau > t$
- A DS is **strictly causal** if $y(t)$ does not depend on $u(\tau)\ \forall \tau \geq t$, and the **strictly causality condition** is $m < n$

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Classification of signals and variables

- A signal is a physical variable viewed as the **support to transmit the information concerning the evolution of a PS**
- The (deterministic) signal can be characterized by: $u : T \to U$
- The ensemble of all possible input (u), state (**x**) or output (y) functions with the notation **U** exemplified for the input function: $\mathbf{U} = \{u \mid u : T \to U\}$

  represents the **class of input, state or output functions**

Classification of signals and variables (cont'd 1)

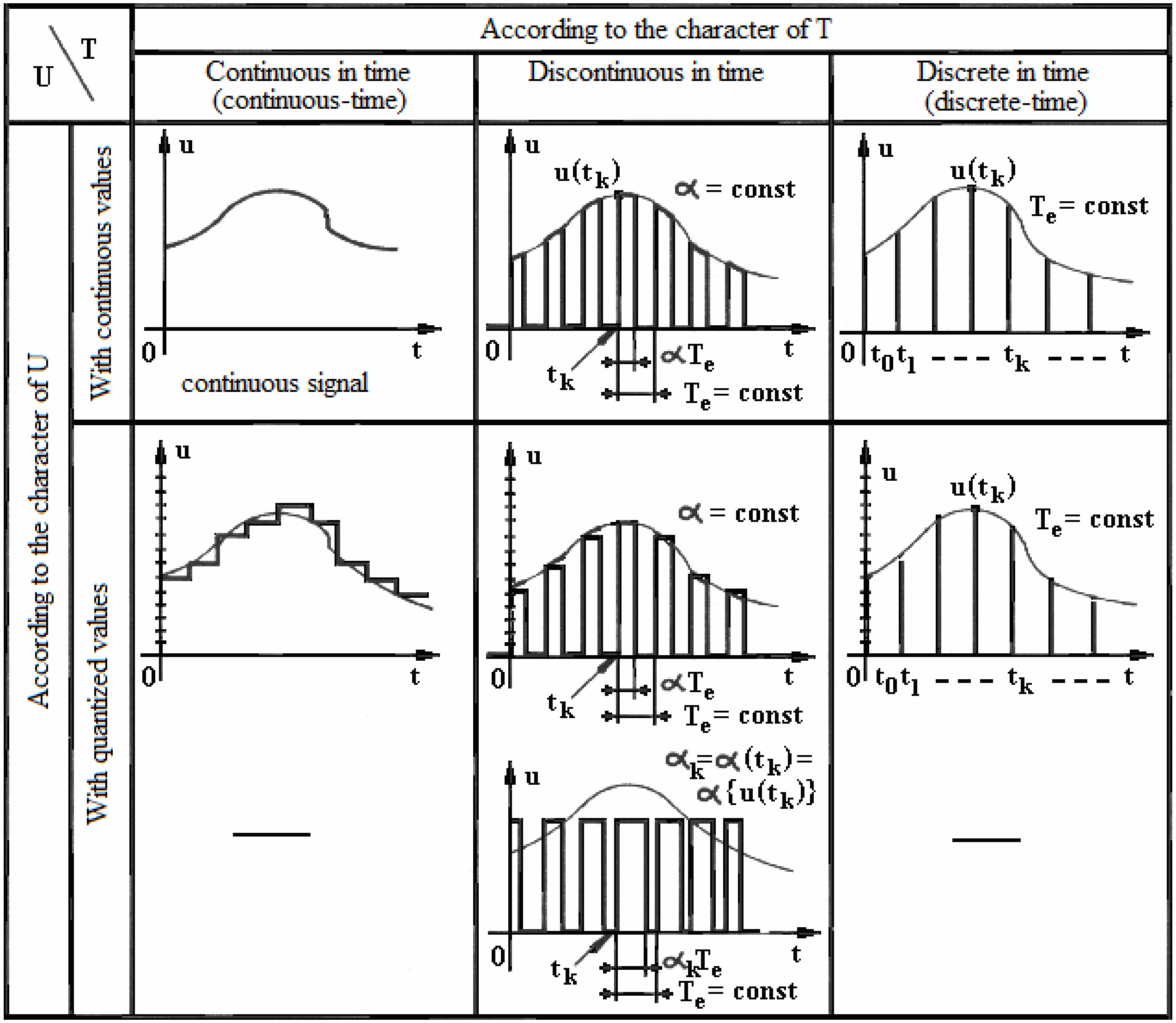
- Classification of signals according to the character of the two sets:

  A. ***According to the character of the set T of time moments***, the signals are divided in:
     - **signals that are continuous in time (continuous-time signals).**
     - **signals that are discontinuous in time**
     - **signals that discrete in time (discrete-time signals)**

  B. ***According to the character of the set U***, the signals are divided in:
     - **signals with continuous values**
     - **signals with quantized values**

  C. ***According to the way of expression of the information carried by a signal***, the signals are divided in:
     - **non-codified signals**
     - **codified signals**

Table 1.3. Classification of non-codified signals.



Classification of signals and variables (cont'd 2)

- The signals that are continuous in time and with continuous values are called **continuous signals** – specific to the PSs (to the technical processes) and to the control devices (CDs) with classical equipment (electric, electronics, mechanics, hydraulic, pneumatic, etc.) – in the next sections and chapters.
- The signals that are discrete in time and with quantized (and next codified) are specific to discrete-time CDs, with numerical (digital) equipment
- The signals with continuous time and quantized values are used in control systems with discrete-time CDs as continuous extensions of discrete-time signals
- The other types of signals (discontinuous in time) – used in specific configurations of electronic CDs

Classification of signals and variables (cont'd 3)

- The signals that are discrete in time and with quantized and codified values are specific to the digital signal processing in computers, digital signal processors and microcontrollers

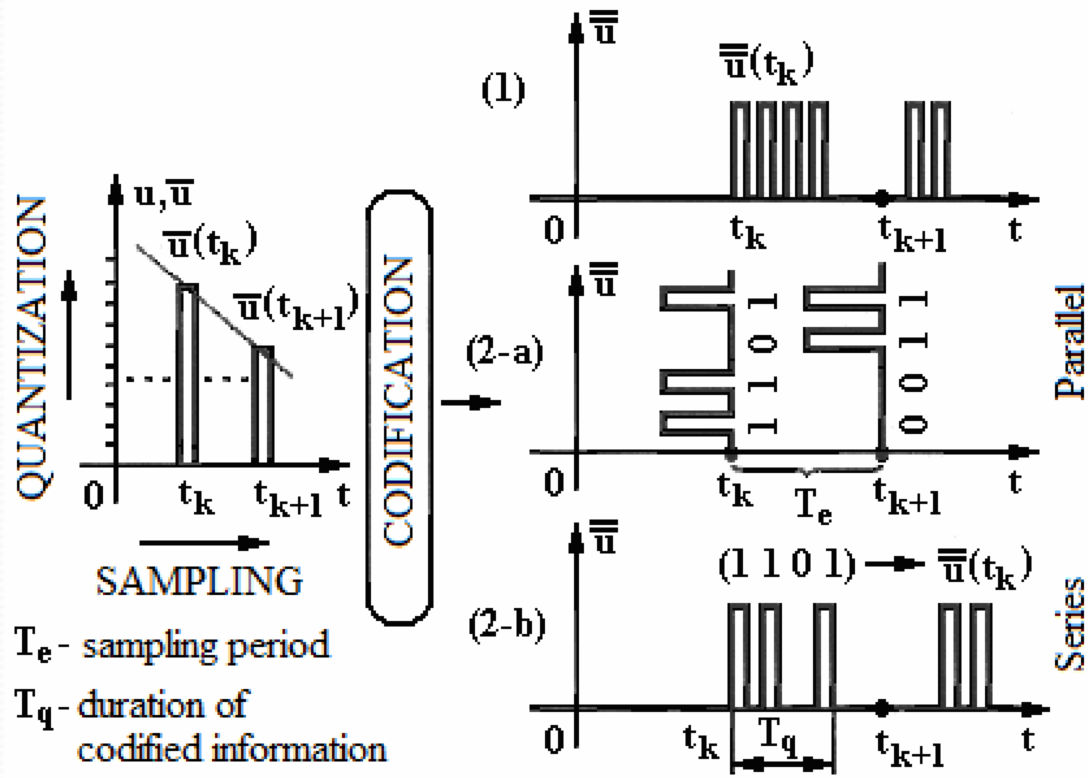


Fig. 1.4. Examples of codification of signals in control systems with discrete-time CDs.

Continuous-time dynamical system (C-DS)

- The concept of C-DS can be defined by the dimensional generalization of the representation by SS-MM of a PS:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)) \ - \ \text{the state equation}$$

$$\mathbf{y}(t) = \mathbf{g}(t, \mathbf{x}(t), \mathbf{u}(t)) \ - \ \text{the output equation}$$

- The initial conditions $\mathbf{x}(t_0)$ should be specified
- **f** and **g** are functionals (vector ones of vector variables) that characterize the connections in the system

Continuous-time dynamical system (C-DS, cont'd 1)

- **u**, **x** and **y** are the input, state and output vectors of the DS:

$$\mathbf{u}(t)=\begin{bmatrix} u_1(t) \\ u_2(t) \\ \dots \\ u_r(t) \end{bmatrix},\ \mathbf{x}(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_n(t) \end{bmatrix},\ \mathbf{y}(t)=\begin{bmatrix} y_1(t) \\ y_2(t) \\ \dots \\ y_q(t) \end{bmatrix}$$

- *r* is the number of input variables, *n* is the number of state variables, *q* is the number of output variables, *t* is the independent time argument (variable), $t \in T \subset \mathbf{R}$,

$$\mathbf{u} \in \mathbf{U} \subset \mathbf{R}^r,\ \mathbf{x} \in \mathbf{X} \subset \mathbf{R}^n,\ \mathbf{y} \in \mathbf{Y} \subset \mathbf{R}^q$$

- A system of *n* ordinary differential equations with *n* unknowns – a Cauchy-type problem

Continuous-time dynamical system (C-DS, cont'd 2)

- $\mathbf{x}(t)$ – **the state** of the system, $(t, \mathbf{x}(t))$ – **the phase** of the system
- **State trajectory**, **phase trajectory**, **output trajectory** (for $\mathbf{y}(t)$)
- This SS-MM is specific to systems with several inputs and several outputs called **Multi Input-Multi Output (MIMO)** systems
- The systems with one input and one output are called **Single Input-Single Output (SISO) systems**

Discrete-time dynamical system (D-DS)

- The evolution in time of a PS can be observed by its characteristic variables in continuous time, $t \in T \subset \mathbf{R}$, or at some discrete time moments t_k
- t_k can be either equidistant at the time moment T_e (or T_s) called **sampling period** or not
- T_e is usually constant for many theoretical and practical reasons
- $u(t_k) = u_k$ and $y(t_k) = y_k$ – **samples of the continuous signals** $u(t)$ and $y(t)$

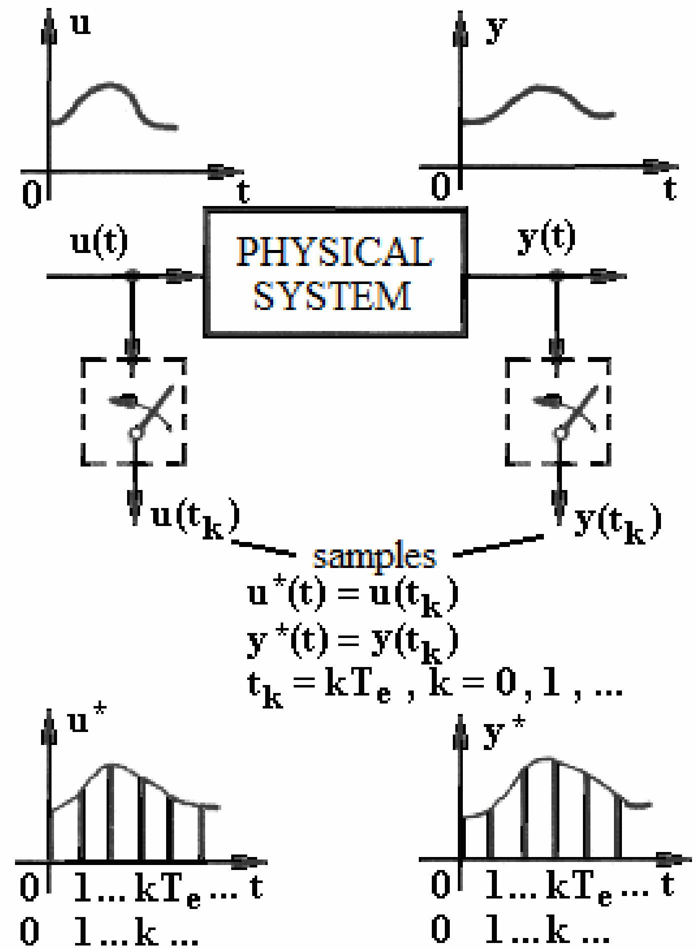


Fig. 1.5. Illustration of sampling process.

Discrete-time dynamical system (D-DS, cont'd 1)

- The process controlled by a CD is called **controlled process (CP)**
- The ensemble {CD, CP} achieved with control focus is called **control system (CS)**
- Accepting that the CD finds out permanently the evolution of the CP by a **feedback connection**, this leads to the **closed-loop control system structure**
- In case of controlling a continuous CP – where $u(t)$ and $y(t)$ are continuous variables – by a digital (numerical) CD with w^* , y^* and u^* as discrete time variables that are quantized and coded (Fig. 1.6), rather two phenomena concerning the information processing at the interfacing points CD-CP and its mathematical treatment are:

Discrete-time dynamical system (D-DS, cont'd 2)

- The observation of the evolution of the CP by the CD in terms of $y(t)$ – the sampling and the quantization are involved (similar for the reference input signal $w(t)$)
- After transmitting the CP the control signal $u(t_k)$ elaborated by the CD at the discrete time moments $t_k=k\,T_e$, it is needed to carry out the construction of a continuous signal $u(t)$ on the basis of the samples u_k, and this signal will be actually applied to the CP

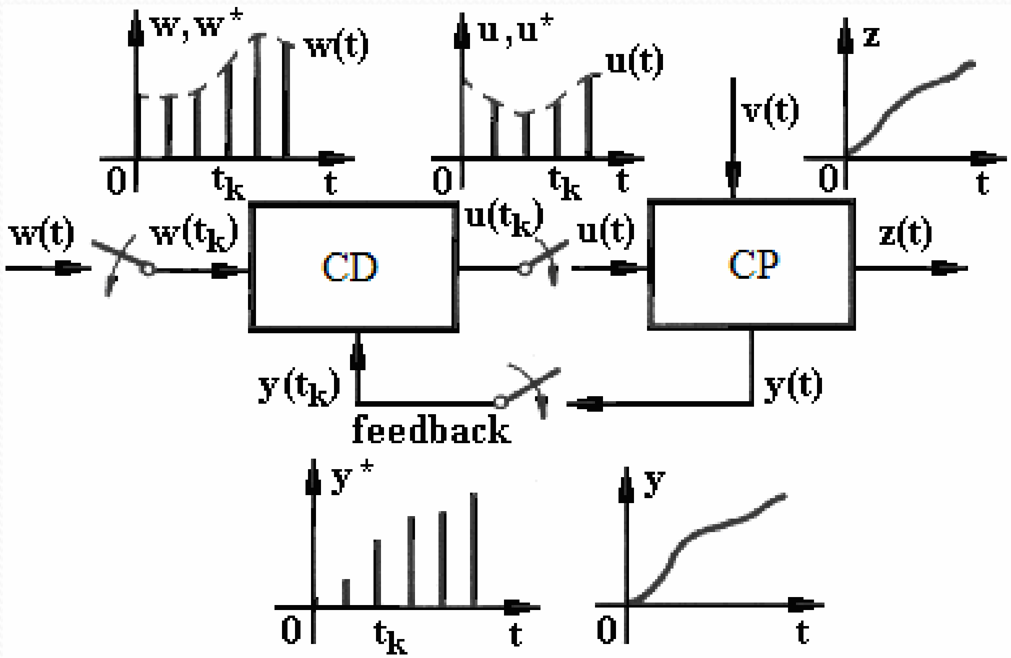


Fig. 1.6. Representation of signals in case of controlling a continuous CP by a digital CD.

Discrete-time dynamical system (D-DS, cont'd 3)

- The mathematical characterization of the PS where the information is available at discrete and equidistant time moments is carried out by means of the **discrete-time dynamical system (D-DS)** representation:

$$\mathbf{x}(k+1)=\mathbf{f}(k,\mathbf{x}(k),\mathbf{u}(k)) \ - \ \text{the state equation}$$

$$\mathbf{y}(k)=\mathbf{g}(k,\mathbf{x}(k),\mathbf{u}(k)) \ - \ \text{the output equation}$$

- $k \in \mathbf{Z}\,(\mathbf{N})$ is the counter of the discrete time moments $t_k=k\,T_e$
- The state equation is a **recurrent equation**
- Other notation for $k=t$:

$$\mathbf{x}(t+1)=\mathbf{f}(t,\mathbf{x}(t),\mathbf{u}(t))$$

$$\mathbf{y}(t)=\mathbf{g}(t,\mathbf{x}(t),\mathbf{u}(t))$$

- **The unified formal representation** of an MM of a system:

$$\mathbf{x}^{'}(t)=\mathbf{f}(t,\mathbf{x}(t),\mathbf{u}(t))$$
$$\mathbf{y}(t)=\mathbf{g}(t,\mathbf{x}(t),\mathbf{u}(t))$$

$$\mathbf{x}^{'}(t)=\begin{cases}\dot{\mathbf{x}}(t),\ t\in T\subset\mathbf{R}\ \ \text{for C - DS}\\ \mathbf{x}(t+1),\ t\in\mathbf{Z}\,(\mathbf{N})\ \ \text{for D - DS}\end{cases}$$

Principles for classification of dynamical systems

- Depends on types of signals processed by the systems and the types of the functionals (operators) **f** and **g** that characterize the system structure
- Continuous-time versus discrete-time systems
- The general SS-MM representation of a **linear time-invariant (LTI) system** (the functionals **f** and **g** are linear + their parameters are constant, i.e., time-invariant):

$$\mathbf{x}'(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)$$

- The matrices **A**, **B**, **C** and **D** have coefficients which are constant (invariant) with respect to time and their dimensions are:

$$\dim \mathbf{A} = n \times n,\ \dim \mathbf{B} = n \times r,\ \dim \mathbf{C} = q \times n,\ \dim \mathbf{D} = q \times r$$

Principles for classification of dynamical systems (cont'd 1)

- **A** is the system matrix and these matrices describe structural properties of the system
- The PSs are **inertial** → the instantaneous transfer of information from the input to the output is not possible → **D** = **0** → the general expression of the SS-MM of an LTI is replaced by the particular form

$$\mathbf{x}'(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t)$$

- Example of resistive circuit – in the associated lecture material

Principles for classification of dynamical systems (cont'd 2)

- A remarkable frequently used particular case of SS-MM representation of LTI systems is that of **Single Input-Single Output (SISO) systems**, with one input u (r=1) and one output y (q=1):

$$\mathbf{x}'(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,u(t)$$
$$y(t) = \mathbf{c}^T\,\mathbf{x}(t) + d\,u(t)$$

- Dimensions of matrices:

$$\dim \mathbf{A} = n \times n,\ \dim \mathbf{b} = n \times 1,\ \dim \mathbf{c}^T = 1 \times n,\ d - \text{scalar}$$

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- Any system can be seen only from the point of view of the causal dependence (with the direction past → present → future) between its inputs and outputs without pointing out distinctly the inner variables, viz. the state variables
- This situation concerns an input-output characterization of the system by the **input-output mathematical model (IO-MM)**

Fig. 1.8. Single input-Single Output (SISO) system.

- ***For a linear and invariant C-DS,*** the IO-MM is a differential equation (linear with constant coefficients):

$$\sum_{\nu=0}^{n} a_\nu y^{(\nu)}(t) = \sum_{\mu=0}^{m} b_\mu u^{(\mu)}(t),\ t \in \Re \qquad m \le n$$

- ***For a linear and invariant D-DS,*** the IO-MM is a linear recurrent equation (i.e., with discrete time) with constant coefficients:

$$\sum_{\nu=0}^{n} a_\nu y(k+\nu) = \sum_{\mu=0}^{m} b_\mu u(k+\mu),\ k \in \mathbf{N} \qquad m \le n$$

- $m \le n$ is **the causality condition**

- For the same PS the two mathematical characterizations – IO-MM and SS-MM – should have the same contents – valid (mathematically) in highlighting the connection input → output. However, in certain particular conditions the two representations are not equivalent, the SS-MM being more comprehensive
- In case of complex systems, the mathematical treatment by the use of IO-MMs proves to be often more simple and intuitional than the SS-MMs + in many practical applications the IO-MM characterization of PSs meets the requirements specific to the design of control systems using the process output. The input-output approach to control systems design corresponds mainly to the classical control theory

- In case of input-output representation of systems use is made of the **black box** concept
- This is often related to the way to determine the IO-MM on the basis of processing the measurements concerning the evolution in dynamic regimes of the input and the output (**experimental identification** or **data-driven identification**)

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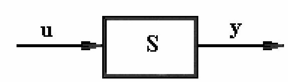
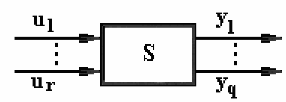
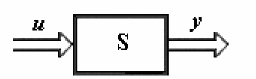
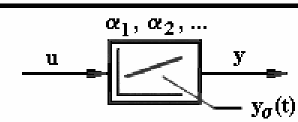
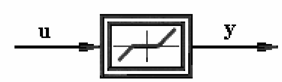
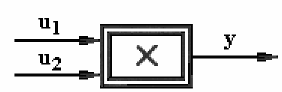
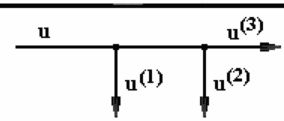
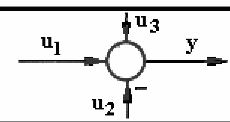
- Two types of block diagrams are used in automation depending on their scope:

    A) Functional block diagrams: they highlight, by means of several representations specific to different technical fields, the functional structure and partially the constructive structure of the system. Using such schemes, the specialists can derive the functional and operating principles of the system and sometimes the specialists can derive an MM of the system

    B) Informational block diagrams: they highlight, by representations specific to automation, the informational structure of the system. They are important in the analysis and development of control systems

- The informational block diagrams employ several types of sub-systems with specific representations, where:
  - the characteristic variables, pointed out by oriented segments (arrows) in the sense of the information flow
  - the structural dependencies (the functionals), pointed out by blocks associated with several ways to specify their properties (Table 1.4)
- The informational block diagrams are built on the basis of the first principle equations that describe the phenomena in the system. They should highlight:
  - the characteristic variables of the system (**u**, **x**, **y**)
  - if possible, the variables that can be measured and that can be used in control
  - in case of control systems, the variables that are processed in the CD
- Examples (in the associated lecture material): room heating + student dynamics

Table 1.4. Representation of systems by informational block diagrams.

CONTENTS OF REPRESENTATION	BLOCK DIAGRAM REPRESENTATION	REMARKS
SISO SYSTEM		**general representation**
MIMO SYSTEM **r - number of inputs** **q - number of outputs**	 	**general representation** $u = \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$ $y = \begin{bmatrix} y_1 \\ \vdots \\ y_q \end{bmatrix}$
LINEAR SISO SYSTEM $y = f(u)$ $f(\ )$ - **operator characterized by the parameters** $\alpha_1, \alpha_2, \ldots$	 	$y_\sigma(t)$ - **system response to a particular input variation (step)**
NONLINEAR SISO SYSTEM $y = f(u)$ **or** **NONLINEAR MISO SYSTEM** $y = f(u_1, u_2)$	 	**inner notation:** - **nonlinearity character** $f(u)$ - **nonlinear operation:** $\times$, $:$, $\sqrt{\ }$, ...
TAKEOFF POINT		$u = u^{(1)} = u^{(2)} = u^{(3)}$ $(\cdot)$ - **branching directions**
SUMMING POINT		$y = u_1 - u_2 + u_3$

Outline

- Physical systems. Mathematical models. Dynamical systems
- Classifications of physical and dynamical systems
- Input-output models
- Graphical modeling by block diagrams
- **Introduction to process control**
- References

- The higher and higher complexity of technical / technological processes (TPs) and of the biological, economical ones, etc., the more and more demanding quality requirements and the more and more strong connection between processes and between processes and the environment have lead to the necessity of process control.
- The processes **can be controlled** (Fig. 1.10):
  - **manually** (a), by the often continuous and direct intervention of a human operator, and
  - **automatically** (b), by the use of specialized equipment dedicated to carry out the control operations, called **automation equipment (AE)**

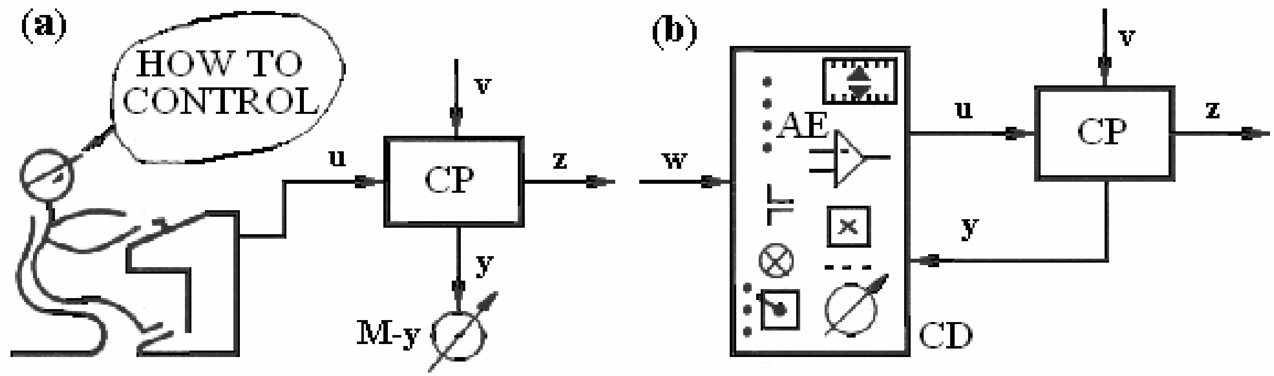


Fig. 1.10. Manual control (a) and automatic control (b) of a process.

- The ensemble of AE interconnected constructively and functionally and dedicated to the control of a process called **controlled process (CP)** is referred to as **control device (CD)** or **automatic control device (ACD)**
- CD and CP are in strong interconnection in the process control. The constructive functional ensemble that consists of CD and CP and achieved for process control is called **control system (CS)** or **automatic control system (ACS)**

- A simplified form of the operations involved in the manual or in the automatic control of a CP highlights two ways to deal with (Fig. 1.11):

  (a) Without the direct tracking of the CP progress by the CD, this is the case of **open-loop control**. The CS built in this way is called **open-loop control system (OL-CS)** or **system with command (manual, automatic)** (Fig. 1.11 (a))

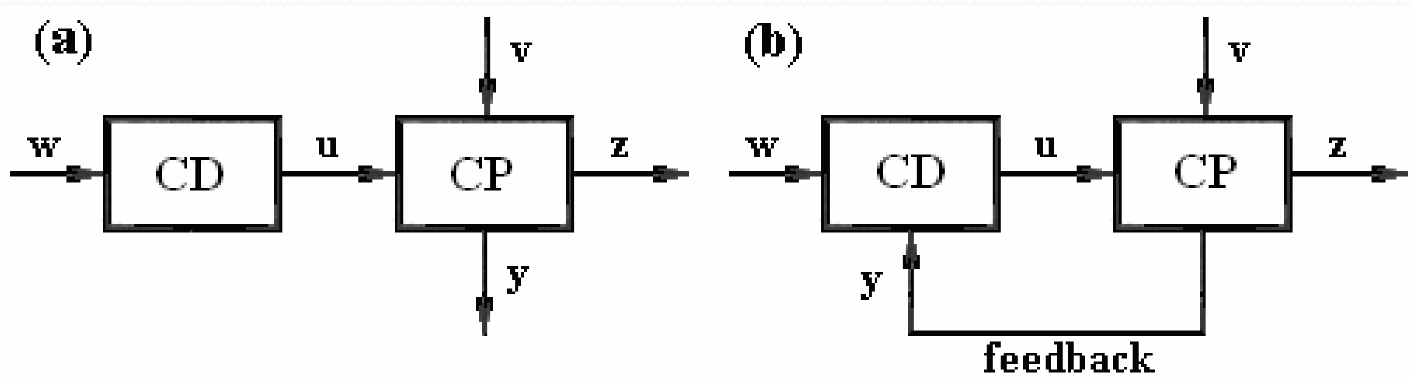


Fig. 1.11. Open-loop control system (a) and closed-loop control system (b).

- The main **shortcomings of an OL-CS**:
- since the disturbances that act on the CP are unknown, their effect cannot be anticipated (foreseen)
- even if an operator tracks with great attention the CP progress, the action of disturbances will affect the CP progress from the desired progress and operation
- the impossibility to control unstable processes

(b) With the direct and often continuous tracking of the CP progress by the CD, Fig. 1.11 (b). This is the case of **closed-loop control** or of **feedback control**. The CS built in this way is called closed-loop control system (CL-CS)

- The **main control tasks** of a CD in a CL-CS:
- to ensure the memorizing of the information concerning the way that the CP should evolve by means of the reference input $w(t)$
- to ensure the tracking of the CP evolution (progress) by means of the measured output $y(t)$ and to compare the effective evolution ($y(t)$) with the desired one ($w(t)$) by the computation of the difference $e(t) = w(t) - y(t)$ called **control error**
- to make the according decision of intervention in the CP progress by the elaboration of the control signal $u(t)$ and its transmission to the CP

Nomenclature: w – reference input (also r), u – control signal, z – assessed output by which the quality of the progress of the process is appreciated (it is often the controlled variable of the CP), y – measured output (measured by the block M-y) – assessed in order to build an ACS with respect to output, v – disturbance input (also d)

- An operational ACS requires that the variables *u* and *y* – that interface CD and CP – should be understood by the blocks they enter. This involves ***the same physical nature, the same variation domain and the same energy level***
- In many situations the measured output (of the process) $y(t)$ is exactly a measure of the assessed output $z(t)$ obtained by a ***measuring element*** (**M**), Fig. 1.12 (a). This variable can also be the measure of other variables in the process (z_a), Fig. 1.12 (b)

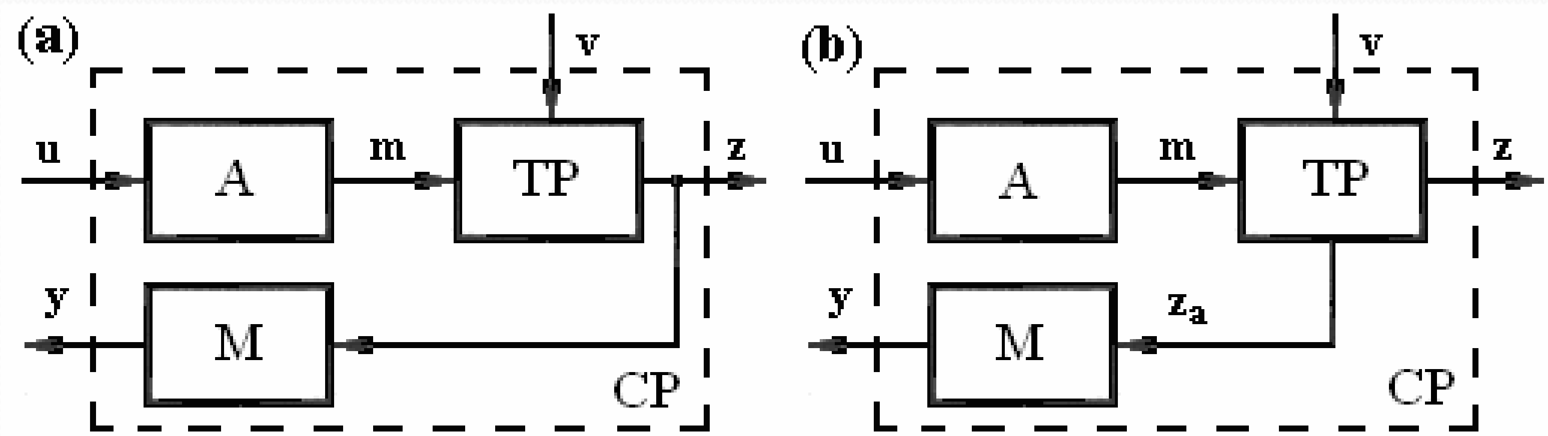


Fig. 1.12. Detailed block diagrams of controlled process.

- The control signal $u(t)$ produced by the CD is converted and amplified by the **actuator (A)** for the intervention in the CP. The actuator ensures the necessary support (substance, stuff) required for the operation of the process
- Since often from a constructive point of vies the blocks A and M belong to the technological equipment where the process evolves, and they are dimensioned in strong relation with its functionality, the ensemble {A, TP, M} is called **controlled process (CP)** or **fixed part** of the system or **controlled object**

- The process control involves the achievement of several tasks detailed at different hierarchical levels. The main **control tasks** at the lower process level:
  - the achievement of the controls (logical, sequential and combinatorial) concerning the discrete time events
  - the achievement of the control of technological parameters (of the specific variables)
  - the supervision of the system operation and the assurance of the safe operation of the process and of the control system

- Example (in the associated lecture material): the cruise control system of a car

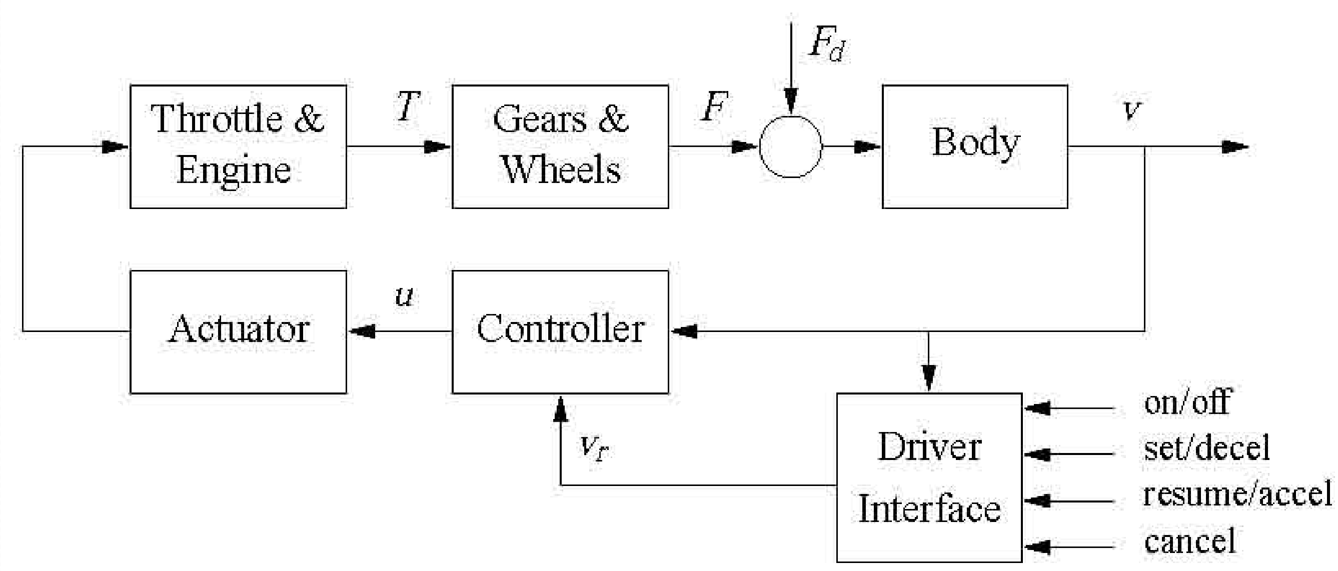


Fig. 1.13. Detailed block diagrams of an automotive cruise control system (Åström and Murray, 2008).

- The first-order nonlinear mathematical model can be extended with the model a feedback controller that attempts to regulate (control) the speed of the car in the presence of disturbances
- The proportional-integral (PI) controller is the most widely used with this respect, and its model is

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau$$

- $e(t) = v_r(t) - \mathrm{v}(t)$ is the control error, k_p is the proportional gain, k_i is the integral gain
- k_p and k_i are the parameters of the PI controller, which have to be tuned appropriately such that to meet the performance specifications imposed to the automatic cruise control system
- Even if the process model is very simple and obtained by several approximations and it is not perfectly accurate, it can be used to design a controller and make use of the feedback in the controller to manage the uncertainty in the system and to ensure the desired dynamics of the control system

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Thank you very much for your attention!