(R.-E. Precup, UPT, 2024)

1. DESCRIPTION AND GENERAL PROPERTIES OF SYSTEMS

1.1. Physical systems. Mathematical models. Dynamical systems

Generally speaking, a ***physical process*** (**PP**) or a ***physical-chemical process*** is a sequence of transformations and conversions that characterize sets of interconnected objects or phenomena, according to a certain structure, which are viewed in their temporal (or sometimes spatial) evolution. The progress of any PP involves matter or substance (mass and energy) transfer phenomena. A ***physical system*** (**PS**) is a system whose behavior changes over time, often in response to external stimulation or forcing, and it represents the material ensemble where a PP evolves.

The PS is characterized by two categories of variables from the point of view of its relation to the environment as illustrated in Fig. 1.1:

- ***input variables***, with the notation $\mathbf{u}(t)$, which externally influence the temporal evolution of the PS, and they represent the cause,
- ***output variables***, with the notation $\mathbf{y}(t)$, that are used in the characterization of the temporal evolution of the PS, and they represent the effect.

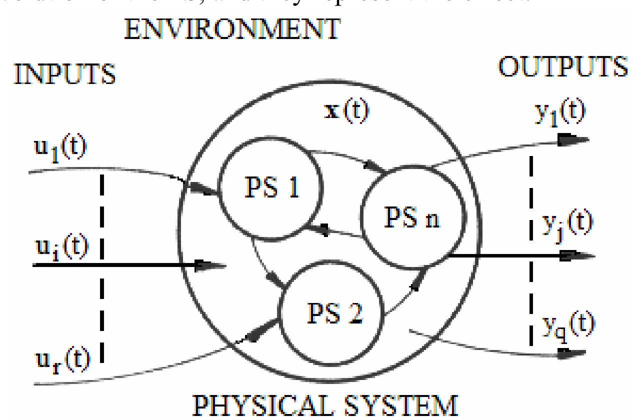


Fig. 1.1. *Representation of a physical system.*

If the evolution of the PS is considered in the finite time interval $[t_0, t_f]$, then

- the system will be in an initial state (situation) at the initial time moment t_0,
- the system will be in a final state (situation) at the final time moment t_f.

The quantitative and qualitative description of the phenomena in a PS makes always use of these categories of variables and of the concept of ***system state***.

The ***temporal evolution*** (the time is characterized by the independent variable *t*) of a system can be seen as the evolution in time of the two categories of variables (input and output ones) and of the system state. The temporal evolution of each system is subjected to the ***principle of causality***, which is interpreted as follows by means of two aspects:

- the evolution (in time) is always oriented towards past – present – future;
- the evolution of the outputs $\mathbf{y}(t)$ is always caused by the evolution of the inputs $\mathbf{u}(t)$ and of the initial state of the system and never in the backward sense.

Since total or partial storage (accumulation) and dissipation (of energy) phenomena always occur in physical processes, this transfer cannot take place instantaneous. In this context it is considered that the ***systems have*** a certain specific ***dynamics***. The way the system evolves is called the ***dynamics*** of the system.

The substance (mass and energy) transfer, storage, transformation and dissipation phenomena in a PS and, generally, the state in which the system is at a certain moment or at any time moment, can be described by means of some internal variables called ***state variables*** (this term is taken from macroscopic physics). The state variables are grouped in the state vector with the notation $\mathbf{x}(t)$. Generally speaking, the state of a dynamical system is a set of variables that completely summarizes the past history of the system, and allows us to predict its future evolution. In other words, the state variables of a PS are those variables (the set of variables) that allow, by their values at the current time moment *t*, to define system's situation or state at that time moment.

Knowing at a certain moment (for example, t_0) the state of a PS, with the notation $\mathbf{x}(t_0)$, allows us to predict the future evolution of the PS at the time moments $t > t_0$. Since the transition from a certain state of the PS to another one cannot take place instantaneous (because of the continuity of physical-chemical phenomena), the state variables of any PS should be continuous functions of time and defined on compacts.

The ensemble of input variables $\mathbf{u}(t)$, state variables $\mathbf{x}(t)$ and output variables $\mathbf{y}(t)$ concerning a PS is called ***characteristic variables of the PS*** and its notation is $\{\mathbf{u}(t), \mathbf{x}(t), \mathbf{y}(t)\}$.

A system generally has more than one input, output or state variable. Therefore, the characteristic variables are grouped in the following vectors (column matrices):

- the input vector $\mathbf{u}(t)$, with *r* components,
- the state vector $\mathbf{x}(t)$, with *n* components,
- the output vector $\mathbf{y}(t)$, with *q* components:

$$\mathbf{u}(t)=\begin{bmatrix} u_1(t) \\ u_2(t) \\ \dots \\ u_r(t) \end{bmatrix},\ \mathbf{x}(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_n(t) \end{bmatrix},\ \mathbf{y}(t)=\begin{bmatrix} y_1(t) \\ y_2(t) \\ \dots \\ y_q(t) \end{bmatrix}, \tag{1.1.1}$$

where r is the number of input variables, n is the number of state variables, and q is the number of output variables. The dimension n of the state vector $\mathbf{x}(t)\in\mathbf{R}^n$, which is also the number of state variables, is called ***the order of the system***.

Remarks: 1. The input variables of a system can only be input variables.

2. The output variables can also be state variables or contrarily.

3. The same PS can operate in several (technical) operating regimes. Therefore, its variables can be both input variables (for example, in the regime 1) and output variables (for example, in the regime 2). A simplified example of this situation is represented by electrical machines that can have:

- in the motor regime: the inputs represented by the armature (terminal) voltage and the excitation voltage, and the output represented by the speed;
- in the generator regime: the inputs represented by the speed and the excitation voltage, and the output represented by the armature voltage.

4. From the point of view of control, the output variables of a PS can be grouped functionally in:

- strictly speaking output variables, which are used to assess the control,
- measured output variables, which are used to achieve the control;

both categories of output variables are usually highlighted with different letters.

5. The clear difference between the vectors and the scalars will be carried out as follows using bold notations for the vectors. The same rule will be applied to matrices as well, but the matrices are highlighted with capital bold letters. A row vector will be considered as the matrix obtained by the transposition of a column vector; for example, we will use the notation $\mathbf{x}^T(t)=[x_1(t)\quad x_2(t)\quad \dots\quad x_n(t)]$.

The mathematical description and characterization of a PS requires the assignment of an adequate ***mathematical model*** (**MM**).

An important aspect concerning the derivation of the mathematical model assigned to a PS is represented by ***the choice of the state variables***. This choice should fulfill the following **requirements**:

- the variables should be continuous in time (with respect to time);
- the variables should characterize the matter transfer, storage, transformation and dissipation phenomena in the PS.

Table 1.1 (Preitl and Precup, 2001a; Preitl and Precup, 2001b) gives some recommendations concerning the choice of state variables in physical systems. As a rule of thumb, *the number of state variables is equal to the number of energy storage elements*. Several equivalents can be derived between the variables in different technical fields with direct impact on the choice of the state variables; such equivalents are presented in Table 1.2 (Preitl and Precup, 2001a; Preitl and Precup, 2001b).

Table 1.1. *Recommendations concerning the choice of state variables in physical systems with focus on technical systems.*

Type of transfer process	State variables	Examples of state equations	Examples of energy storage elements
Substance	Mass m	$m' = Q_m$, Q_m – mass flow	Hydraulic tank
Electrical	Electric charge q	$q' = i$, i – electric current (intensity)	Capacitor's armatures
Impulse	Translational impulse $p = m\,v$	$p' = \sum F_i$, F_i – force	Body subjected to translational or rotational motion
	Rotational impulse	$p_\omega ' = \sum M_i$, M_i – rotational torque	
Kinetic energy	Linear velocity (speed) v	$v' = (1/m)\sum F_i$, $v' = a$, a – linear acceleration	Body subjected to translational or rotational motion
	Angular velocity (speed) ω	$\omega' = (1/m)\sum M_i$, $\omega' = a_\omega$, a_ω – angular acceleration	
Potential energy	Linear position (space) s	$s' = v$, v – linear velocity (speed)	Body subjected to translational or rotational motion
	Angular position (rotational angle) α	$\alpha' = \omega$, ω – angular velocity (speed)	

Energy stored in an elastic system	Deformation force F	$F'=k_f v$, k_f – viscous friction coefficient (elastic coefficient)	Elastic spring or spiral spring subjected to deformation
	Deformation torque (moment) M	$M'=k_f \omega$, k_f – viscous friction coefficient (elastic coefficient)	
Thermal	Temperature θ	$\theta'=(1/C_\theta)q_\theta$ q_θ – thermal flow	Homogenous body of thermal capacitance C_θ
Electromagnetic energy: - in the electric field of a capacitor - în the magnetic field of an inductor	Voltage across the capacitor u_C	$u_C'=(1/C)i$	Capacitor of capacitance C
	Current through the inductor i_L	$i_L'=(1/L)u$	Inductor of inductance L

Table 1.2. *Equivalences between several variables and parameters involved in transfer processes of different technical fields.*

Series electrical circuit	Parallel electrical circuit	Translational mechanical system	Rotational mechanical system
Electric voltage u	Electric current i	Force F	Torque (moment) M
Electric load q	Magnetic flow Φ	Linear position (space) s	Angular position (rotational angle) α
Electric current (intensity) i	Electric voltage u	Linear velocity (speed) v	Angular velocity (speed)ă ω
Inductance L	Capacitance C	Mass m	Inertia moment J
Resistance R	Conductance $1/R$	Viscous friction coefficient μ	
Capacitance C	Inductance L	Elastic coefficient (spring constant) k_e	
Temperature difference Δθ	Pressure difference Δp		
Heat quantity Q_θ	Fluid quantity Q_f	Gas quantity Q_g	

Heat flow q_θ	Fluid flow q_f	Gas flow q_g
---	Hydraulic inertia coefficient	---
Thermal resistance R_θ	Hydraulic resistance R_h	Pneumatic resistance R_p
Thermal capacitance C_θ	Hydraulic capacitance C_h	Pneumatic capacitance C_p

A PS characterized by its mathematical model represents a ***dynamical system*** (**DS**) or an ***abstract system***. A dynamical model of a system is a set of mathematical laws explaining in a compact form and in quantitative way how the system evolves over time, usually under the effect of external excitations.

The dynamical system is a mathematical concept which is completely defined by **axioms** concerning:

- **categories of sets** and **classes of functions** employed in the description of the independent variable *t* and of the characteristic variables of the PS;
- **operators** (**functionals**) employed in the description of the structural relationships between the characteristic variables of the PS (Fig. 1.1 illustrates the relationships between the physical subsystems PS 1, PS 2 and PS n).

The connections between the PS and its associated MM (DS) for a single input-single output system are presented in Fig. 1.2.

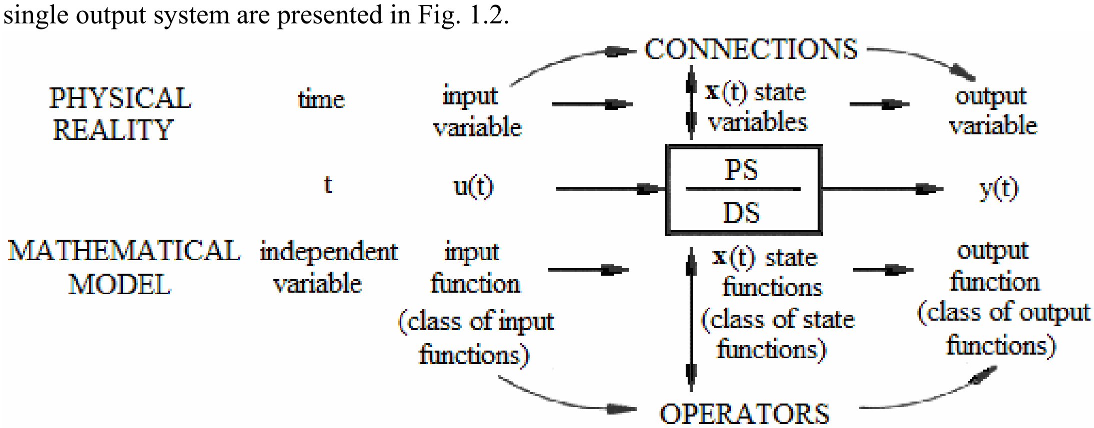


Fig. 1.2. *Connections between a PS and its associated DS.*

The main questions about a dynamical system are:

- to understand the system, i.e., to answer to questions like “How X and Y influence each other ?”;
- to simulate the system, i.e., to answer to questions like “What happens if I apply action Z on the system ?”;
- to design a control system for it, i.e., to answer to questions like “How to make the system behave the way I want ?”.

The qualitative models are also useful in non-technical domains that usually are treated by cybernetics. Some representative examples of such domains are politics, advertisement and psychology.

The necessity of models comes from the fact that experiments provide answers, but they have limitations such as:

- they may be too expensive (example: launch a space shuttle),
- they may be too dangerous (example: experiments on an operating nuclear plant),
- they may be impossible (example: the system does not exist yet being in the phase of design).

In contrast, mathematical models allow us to:

- capture the main phenomena that take place in the system (example: Newton’s law – a force on a mass produces an acceleration),
- analyze the system (get relations among dynamical variables),
- simulate the system (i.e., make predictions) about how the system behaves under certain conditions and excitations (in analytical form, or on a computer in terms of digital simulation of system’s behavior).

The mathematical modeling of a PS is exemplified as follows by the following application in the field of electric circuits (Preitl and Precup, 2001b).

Example 1.1: Let us consider the electric quadrupole PS with the electric diagram given in Fig. 1.3. The input is the voltage u_i and the output is the voltage u_e. A load resistor with the very large resistance R_s, $R_s >> R_1 > R_2$ is connected at the output, such that the output current i_e is negligible with respect to the currents i_{C2} and i_2, $i_e << i_{C2} < i_2$. Therefore, the quadrupole is considered to operate in a no-load operating regime according to this first approximation. The mathematical model that characterizes the macroscopic phenomena in this PS will be derived as follows.

Solution: Kirchhoff’s voltage and current laws characterize the phenomena in this PS and they result in the following equations, where the time argument *t* is omitted for the sake of simplicity:

$$u_i = R_1 i_1 + \frac{1}{C_1}\int_0^t i_{C1}(\tau)d\tau,\ u_{C1} = \frac{1}{C_1}\int_0^t i_{C1}(\tau)d\tau,$$
$$u_{C1} = R_2 i_2 + \frac{1}{C_2}\int_0^t i_{C2}(\tau)d\tau,\ u_{C2} = \frac{1}{C_2}\int_0^t i_{C2}(\tau)d\tau, \tag{1.1.2}$$
$$i_1 = i_{C1} + i_2,\ i_{C2} \approx i_2,\ i_e \approx 0,$$
$$u_e = u_{C2}.$$

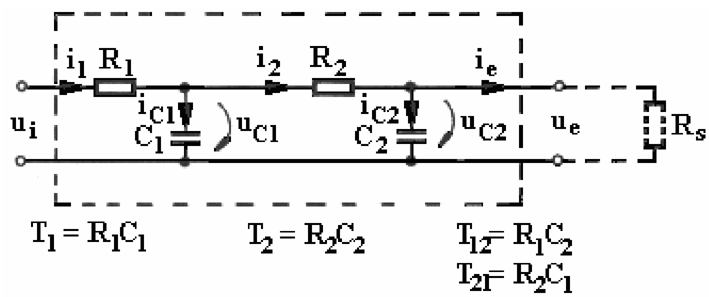


Fig. 1.3. *Electric diagram of the circuit in the example 1.1.*

The voltages u_{C1} and u_{C2} are continuous in time and they characterize the energy storages in the system. Therefore, these two voltages will be considered as state variables. Consequently, equations (1.1.2) are organized as the following MM:

$$\dot{u}_{C1} = -\frac{1}{T_1}u_{C1} + 0 \cdot u_{C2} + \frac{1}{T_1}u_i,$$
$$\dot{u}_{C2} = -\frac{1}{T_{12}}u_{C1} + \frac{1}{T_2}u_{C2} + 0 \cdot u_i, \tag{1.1.3}$$
$$u_e = u_{C2},$$

where $T_1 = R_1C_1, T_2 = R_2C_2, T_{12} = R_1C_2$.

Equations (1.1.3) describe the following composed relationship:

$$\text{input}\,[u_i] \to \text{states}\begin{bmatrix} u_{C1} \\ u_{C2} \end{bmatrix} \to \text{output}\,[u_e],$$

that corresponds to a typical mathematical model in systems theory and in automation called ***state-space mathematical model*** (**SS-MM**).

The general expression of an SS-MM of a continuous-time linear DS with a single input and a single output is

$$\begin{aligned} &\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,u(t) \; - \; \text{the state equation,} \\ &y(t) = \mathbf{c}^T\,\mathbf{x}(t) + d\,u(t) \; - \; \text{the output equation.} \end{aligned} \qquad (1.1.4)$$

This representation, which highlights the internal structure of the PS (the evolution of the internal process in the PS), is called ***structural representation of the DS (of the PS)***.

Removing in (1.2.2) or in (1.2.3) the state variables u_{C1} and u_{C2} (considered as internal variables) leads to the relationship

$$\text{input}\,[u_i] \;\rightarrow\; \text{output}\,[u_e],$$

that corresponds to another typical mathematical model called ***input-output mathematical model*** (**IO-MM**):

$$T_1 T_2 \ddot{u}_e + (T_1 + T_2 + T_{12})\dot{u}_e + u_e = u_i\,. \qquad (1.1.5)$$

This representation has the following general expression of an IO-MM for continuous-time linear time invariant (with constant parameters) systems with a single input and a single output:

$$\sum_{\nu=0}^{n} a_\nu y^{(\nu)}(t) = \sum_{\mu=0}^{m} b_\mu u^{(\mu)}(t)\,, \qquad (1.1.6)$$

where $m \le n$ is the ***causality condition***. A DS is **causal** if $y(t)$ does not depend on future inputs $u(\tau)\ \forall \tau > t$; a DS is **strictly causal** if $y(t)$ does not depend on $u(\tau)\ \forall \tau \ge t$, and the ***strictly causality condition*** is $m < n$. The representation (1.1.6) is also called ***functional representation of a DS*** (***of a PS***).

Remarks: 1. The concept of ***dynamical syste***m has been introduced here in a simple manner, rather intuitive than axiomatic but sufficiently conclusive to approach the specific automatic control problems.

2. Fig. 1.2 outlines that the systemic representation and characterization of a PS by MM makes use of:

- functions that describe the temporal evolution of the variables in the PS,
- functionals / operators that point out the connections in the PS.

1.2. Classifications of physical and dynamical systems

The definition and next classification of the dynamical systems will start with the classifications of the signals (variables) by which the systems are characterized.

1.2.1. Classification of signals and variables

A ***signal*** is a physical variable viewed as the **support to transmit the information concerning the evolution of a PS**.

The (deterministic) signals can be characterized analytically by functions *u*

$$u : T \rightarrow U \,, \tag{1.2.1}$$

that map each time moment $t \in T$ with *T* – the set of time moments, to an element $u(t) \in U$, where *U* is the set of values of the function *u*.

The ensemble of all possible input (*u*), state (**x**) or output (*y*) functions with the notation **U** exemplified for the input functions

$$\mathbf{U} = \{u \mid u : T \rightarrow U\} \tag{1.2.2}$$

represents the ***class of input, state or output functions***. Fig. 1.2 outlines these functions.

According to the character of the sets in (1.2.2), the signals are classified and divided as follows (Table 1.3):

A) According to the character of the set T of time moments, the signals are divided in:

- **signals that are continuous in time** (**continuous-time signals**), for which *T* is a compact set (i.e., a closed interval);
- **signals that are discontinuous in time**, with the expression

$$u(t) = \begin{cases} \neq 0 \ \text{ for } \ t \in [(k-1)T_e, (k-\alpha)T_e), \ k = 1,2,..., \\ \qquad\qquad 0 < \alpha < 1, \ \alpha = \text{const or not}, \\ \qquad\qquad\ = 0 \ \text{ otherwise}, \end{cases} \tag{1.2.3}$$

where T_e indicates the sampling period also denoted by T_s;

- **signals that discrete in time** (**discrete-time signals**), for which *T* is a set of discrete values.

This classification is illustrated by the three columns in Table 1.3.

B) According to the character of the set U, the signals are divided in:

- **signals with continuous values**, for which *U* is a compact set (the first row in Table 1.3);
- **signals with quantized values**, for which *U* is a countable set with discrete values (the second row in Table 1.3).

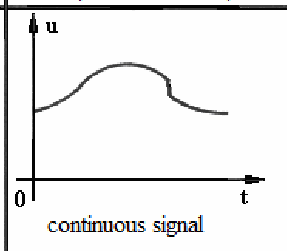
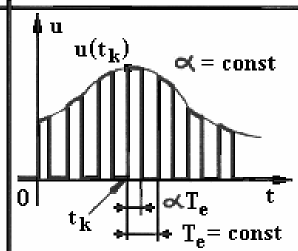
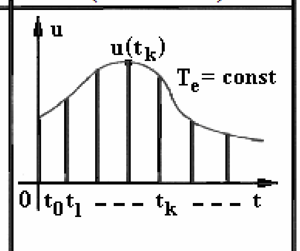
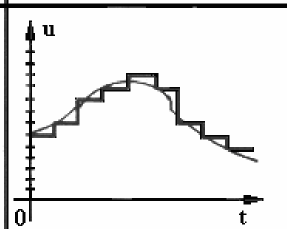
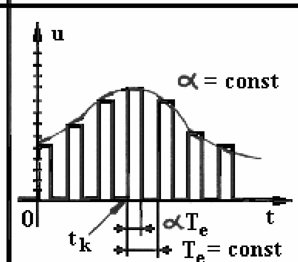
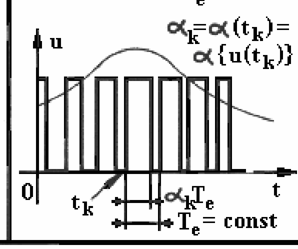
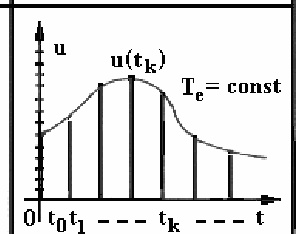
The signals that are continuous in time and with continuous values are called ***continuous signals***. These signals are specific to the PSs (to the technical processes) and to the control devices (CDs) with classical equipment (electric, electronics, mechanics, hydraulic, pneumatic, etc.). The control subject will be discussed in the next sections and chapters.

The signals that are discrete in time and with quantized (and next codified) are specific to discrete-time CDs, with numerical (digital) equipment.

The signals with continuous time and quantized values are used in control systems with discrete-time CDs as continuous extensions of discrete-time signals.

The other types of signals, namely the signals that are discontinuous in time, are used in some configurations of electronic CDs.

Table 1.3. *Classification of non-codified signals.*

U \ T		According to the character of T: Continuous in time (continuous-time)	Discontinuous in time	Discrete in time (discrete-time)
According to the character of U	With continuous values			
	With quantized values	 —	 	 —

C) According to the way of expression of the information carried by a signal, the signals are divided in:

- **non-codified signals**, for which the carrier of the information contents is signal's value at a certain moment, and these are the amplitude modulated signals, or its duration (width), and these are the pulse-width modulation (PWM) signals also called pulse-duration modulation signals (shown in Table 1.3);
- **codified signals**, for which the information contents is represented by

(1) either the number of pulses in a given time interval (the representation (1)),

(2) or a codification of the number of successive pulses at a given time moment (the parallel representation, (2-a)) or in a given time interval of width T_q (the serial representation, (2-b)).

Fig. 1.4 suggestive illustrates the ways of codification of the information carried in the sample at the time moment t_k, namely $u(t_k) = u_k$, of the continuous signal $u(t)$. $\bar{u}(t_k)$ stands for the quantized value of the sample, and $\bar{\bar{u}}(t_k)$ is the codified value of the sample.

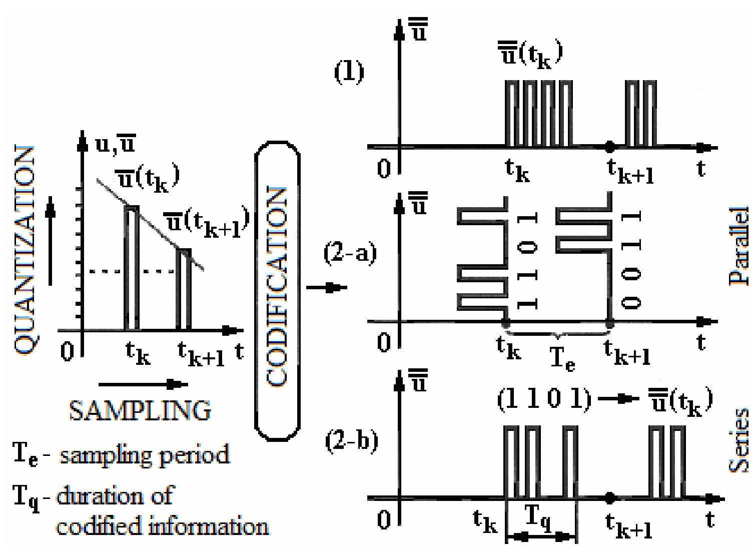


Fig. 1.4. *Examples of codification of signals in control systems with discrete-time CDs.*

The signals that are discrete in time and with quantized and codified values are specific to the digital signal processing in computers, digital signal processors and microcontrollers.

1.2.2. Additional details on the concept of dynamical system

The concept of DS is defined (Ionescu, 1975; Ionescu, 1985) in correlation with the set T of time moments. Consequently, the separate definition of the following concepts is carried out:

- ***continuous-time dynamical system*** (**C-DS**);
- ***discrete-time dynamical system*** (**D-DS**).

A) The continuous-time dynamical system (C-DS). The concept of C-DS can be defined by the dimensional generalization of the representation by SS-MM of a PS, that means by the generalization of equation (1.1.4) in terms of:

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{f}(t,\mathbf{x}(t),\mathbf{u}(t)) \ - \ \text{the state equation},\\&\mathbf{y}(t)=\mathbf{g}(t,\mathbf{x}(t),\mathbf{u}(t)) \ - \ \text{the output equation},\end{aligned} \quad (1.2.4)$$

where the initial conditions $\mathbf{x}(0)$ should be specified, **f** and **g** are functionals (vector ones of vector variables) that characterize the connections in the system, and **u**, **x** and **y** are the input, state and output vectors of the DS with the expressions given in (1.1.1):

$$\mathbf{u}(t)=\begin{bmatrix}u_1(t)\\u_2(t)\\\ldots\\u_r(t)\end{bmatrix},\ \mathbf{x}(t)=\begin{bmatrix}x_1(t)\\x_2(t)\\\ldots\\x_n(t)\end{bmatrix},\ \mathbf{y}(t)=\begin{bmatrix}y_1(t)\\y_2(t)\\\ldots\\y_q(t)\end{bmatrix},$$

where *r* is the number of input variables, *n* is the number of state variables, *q* is the number of output variables, *t* is the independent time argument (variable), $t\in T\subset\mathbf{R}$, $\mathbf{u}\in\mathbf{U}\subset\mathbf{R}^r$, $\mathbf{x}\in\mathbf{X}\subset\mathbf{R}^n$, $\mathbf{y}\in\mathbf{Y}\subset\mathbf{R}^q$.

From a mathematical point of view, the solution $\mathbf{x}(t)$ of the state equation (a system of *n* ordinary differential equations with *n* unknowns) in case of a Cauchy-type problem, describes the evolution of the state of the system, and $\mathbf{y}(t)$ describes the evolution of the output caused by the input $\mathbf{u}(t)$.

At a certain time moment *t*, the vector $\mathbf{x}(t)$ characterizes ***the state of the system***, and the 2-tuple (the ordered pair) $(t,\mathbf{x}(t))$ characterizes ***the phase of the system***. For the time interval $[t_0,t_f]$ the trajectory described by $\mathbf{x}(t)$ in the state space **X** is called ***state trajectory***, and the trajectory of $\mathbf{x}(t)$ in the space $T\times\mathbf{X}$ is called ***phase trajectory***. Similar, the trajectory of $\mathbf{y}(t)$ in the space $\mathbf{Y}$ is called ***output trajectory***.

Remark: The SS-MM described by (1.2.4) is specific to systems with several inputs and several outputs called ***Multi Input-Multi Output*** (**MIMO**) **systems**. The systems with one input and one output (see, for example, the SS-MM given in (1.1.3)) are called ***Single Input-Single Output*** (**SISO**) ***systems***.

B) A simple way to define the discrete-time dynamical system (D-DS) (Preitl and Precup, 2001a). The evolution in time of a PS can be observed by its characteristic variables. An external observer can see the evolution of these variables in continuous time, $t\in T\subset\mathbf{R}$, or at some discrete time moments t_k, where $k=0,1,2,\ldots$ or $k=\ldots,-1,0.1,\ldots$

These time moments t_k can be either equidistant at the time moment T_e or T_s called ***sampling period*** (Fig. 1.5) or not. T_e is usually constant for many theoretical and practical reasons.

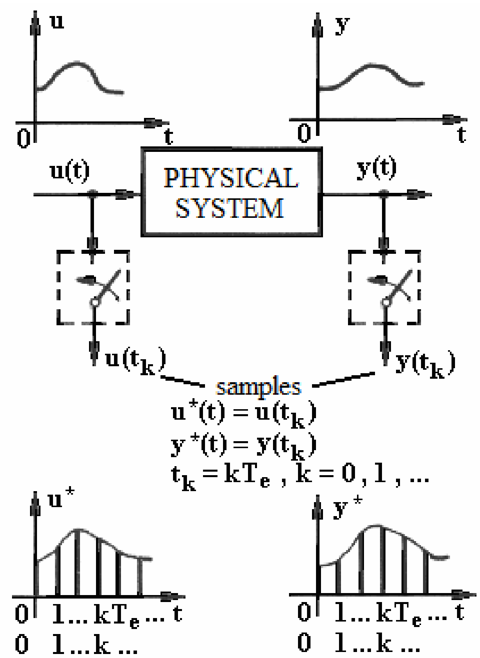


Fig. 1.5. *Illustration of sampling process.*

Remark: The external observer can be:

- a person,
- an equipment dedicated to the acquisition and processing of the information concerning the evolution of the PS (recording and monitoring of variables),
- a specialized device, dedicated to controlling the PS, which processes the information recorded from the process (system) in the numerical version (i.e., an analog-to-digital converter – ADC); the device that ensures the control of the PS is referred to as ***control device*** (**CD**).

The result of the above described process is the sequences of values $u^*(t)$, $y^*(t)$ and eventually $\mathbf{x}^*(t)$. The components $u(t_k) = u_k$ and $y(t_k) = y_k$ are called ***samples of the continuous signals*** $u(t)$ and $y(t)$.

A practical problem will be analyzed before the introduction of the concept of the discrete-time dynamical system. This problem is related to process control and the notions presented as follows do not have a conceptual character but a practical one.

The evolution of physical systems must be often subjected to a more or less rigorous control. This is important when a technological (technical) process takes place in the PS of laboratory, industry or non-industry (economical, biological) type, and the evolution of this process influences the quality of the final product.

The process controlled by a CD is called ***controlled process*** (**CP**). The ensemble {CD, CP} achieved with control focus is called ***control system*** (**CS**). The process control can be carried out in several ways as it will be shown in Sub-chapter 1.5.

Accepting that the CD finds out permanently the evolution of the CP by a ***feedback connection***, this leads to the ***closed-loop control system structure***.

In case of controlling a continuous CP – where $u(t)$ and $y(t)$ are continuous variables – by a digital (numerical) CD with w^*, y^* and u^* as discrete time variables that are quantized and coded (Fig. 1.6), two phenomena concerning the information processing at the interfacing points CD-CP and its mathematical treatment are described. These two rather opposite phenomena are 1 and 2:

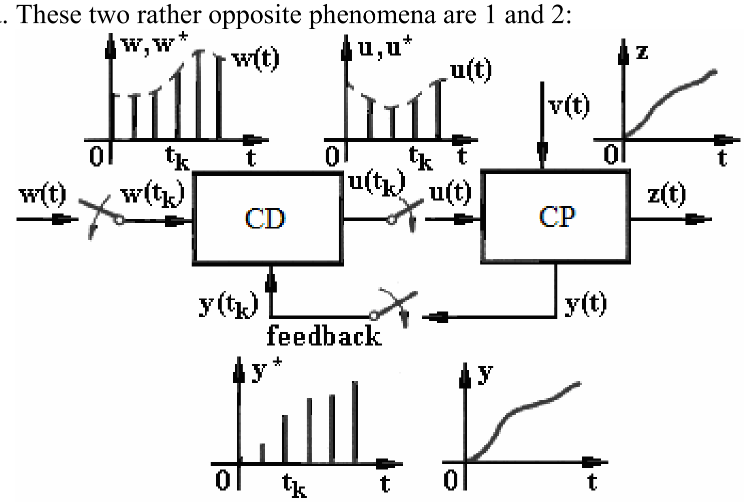


Fig. 1.6. *Representation of signals in case of controlling a continuous CP by a digital CD.*

- 1. The phenomenon described above takes place at the observation of the evolution of the CP by the CD in terms of $y(t)$, namely the sampling and the quantization are involved (the problem is similar for the reference input signal $w(t)$).
- 2. After transmitting the CP the control signal $u(t_k)$ elaborated by the CD at the discrete time moments $t_k = k\,T_e$, it is also necessary to carry out the construction of a continuous signal $u(t)$ on the basis of the samples u_k, and this signal will be actually applied to the CP.

The mathematical characterization of the PS where the information is available at discrete and equidistant time moments is carried out by means of the ***discrete-time dynamical system*** (**D-DS**) representation expressed as:

$$\mathbf{x}(k+1) = \mathbf{f}(k, \mathbf{x}(k), \mathbf{u}(k)) \ - \ \text{the state equation,} \tag{1.2.5}$$

$$\mathbf{y}(k) = \mathbf{g}(k, \mathbf{x}(k), \mathbf{u}(k)) \ - \ \text{the output equation,} \tag{1.2.6}$$

where $k \in \mathbf{Z}\,(\mathbf{N})$ is the counter of the discrete time moments $t_k = k\,T_e$.

Equation (1.2.5) is a **recurrent equation**, where each new value of the state vector $\mathbf{x}(k+1)$, at he time moment $t_k = (k+1)\,T_e$, is calculated on the basis of the actual values of the state vector $\mathbf{x}(k)$ and input vector $\mathbf{u}(k)$; the output equation next offers the actual value of the output vector $\mathbf{y}(k)$.

If:

- the initial state (for $k = 0$) is known, $\mathbf{x}(0) = \mathbf{x}_0$,
- the sequence of input values is given, $\mathbf{u}(k)$, $k \in \mathbf{N}$,

then equations (1.2.5) and (1.2.6) will lead to:

$$\begin{aligned}
&\text{- for } k = 0:\ \mathbf{x}(0) \ \text{– known, } \mathbf{u}(k) \ \text{– given,}\\
&\text{- for } k = 1:\ \mathbf{x}(1) = \mathbf{f}(0, \mathbf{x}(0), \mathbf{u}(0)) \ \text{– calculated,}\\
&\text{- for } k = 2:\ \mathbf{x}(2) = \mathbf{f}(1, \mathbf{x}(1), \mathbf{u}(1)) = \mathbf{f}(1, \mathbf{f}(0, \mathbf{x}(0), \mathbf{u}(0)), \mathbf{u}(1)),\ \ldots
\end{aligned} \tag{1.2.7}$$

Remark: The counter of discrete time moments *k* is often noted with other symbols as, for example, even *t*. This counter should not be confused with the current time moment *t* used in C-DS. Using this notation, equations (1.2.5) and (1.2.6) are re-written as

$$\begin{aligned}
&\mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)),\\
&\mathbf{y}(t) = \mathbf{g}(t, \mathbf{x}(t), \mathbf{u}(t)).
\end{aligned} \tag{1.2.8}$$

The formal comparisons of equations (1.2.4) and (1.2.8) leads to the possibility of **the unified formal representation** of an MM of a system, namely:

$$\begin{aligned}
&\mathbf{x}'(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)),\\
&\mathbf{y}(t) = \mathbf{g}(t, \mathbf{x}(t), \mathbf{u}(t)),
\end{aligned} \tag{1.2.9}$$

where $\mathbf{x}'(t)$ stands for

$$\mathbf{x}'(t) = \begin{cases} \dot{\mathbf{x}}(t),\ t \in T \subset \mathbf{R} \ \text{ for C-DS,}\\ \mathbf{x}(t+1),\ t \in \mathbf{Z}\,(\mathbf{N}) \ \text{ for D-DS.} \end{cases} \tag{1.2.10}$$

Remark: The values at the time moments t_k of the discrete time variables $\mathbf{u}(k)$, $\mathbf{x}(k)$ and $\mathbf{y}(k)$ are often expressed as it is exemplified as follows for a single input variable:

$$u(k) \Leftrightarrow u(t_k) \Leftrightarrow \left. u(t) \right|_{t=t_k}.$$

1.2.3. Principles for classification of dynamical systems

On the basis of the information presented in the previous sections, the classification of dynamical systems should account for the types of signals processed by the systems and for the types of the functionals (operators) **f** and **g** that characterize the system structure. In the framework of the general form given in (1.2.9), the C-DS and D-DS concepts makes differences only in the way they treat the characteristic variables of the SD, namely:

- in case of C-DS: $t \in T \subset \mathbf{R}$ – continuous time,
- in case of D-DS: $t(k) \in \mathbf{Z}\,(\mathbf{N})$ – discrete time.

Practically, the PSs have certain particular features which should be reflected in the particular forms of the functionals **f** and **g** in (1.2.9). Some useful details related to this problem are presented in (Ionescu, 1975), (Belea, 1985), (Ionescu, 1985), (Preitl, 1992). A significant particular case is related to linear functionals **f** and **g**. In addition, the next chapters will mainly deal with **the class of linear systems with constant coefficients**.

In this context, a C-SD or a D-DS is called ***linear time-invariant*** (**LTI**) if the functionals **f** and **g** are linear and the parameters in these functionals are constant (time-invariant). These systems are subjected to the superposition theorem that means additivity and homogeneity.

The general SS-MM representation of an LTI system is

$$\begin{aligned} \mathbf{x}'(t) &= \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t), \\ \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t), \end{aligned} \tag{1.2.11}$$

where the matrices **A**, **B**, **C** and **D** have coefficients which are constant (invariant) with respect to time and their dimensions are

$$\dim \mathbf{A} = n \times n,\ \dim \mathbf{B} = n \times r,\ \dim \mathbf{C} = q \times n,\ \dim \mathbf{D} = q \times r, \tag{1.2.12}$$

where **A** is the system matrix and these matrices describe structural properties of the systems.

Remarks: 1. The physical system will be also considered as an LTI system in terms of the characterization (1.2.11).

2. The PSs are **inertial**. Therefore, the instantaneous transfer of information from the input to the output is not possible. Accordingly, this results in $\mathbf{D}=\mathbf{0}$. That is the reason why the form (1.2.11) of SS-MM is replaced by the particular form

$$\begin{aligned}&\mathbf{x}'(t)=\mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\,\mathbf{u}(t),\\&\mathbf{y}(t)=\mathbf{C}\,\mathbf{x}(t).\end{aligned} \tag{1.2.13}$$

However, the representation of PSs by simplified mathematical models (that are valid in certain conditions) makes use of $\mathbf{D}\neq\mathbf{0}$. This aspect results from the mathematical modeling (the system representation) of the resistive circuit given in Fig. 1.7 (a). The input is the voltage $u_i(t)$ and the output is the voltage $u_e(t)$ on the load resistor R_c. If the transfer phenomena $u_i\rightarrow u_e$ take place at low frequencies, the behavior of the system will be characterized by equation

$$u_e(t)=\frac{R_c}{R+R_c}u_i(t), \tag{1.2.14}$$

that highlights a non-inertial behavior from the input u_i to the output u_e, and the connection is direct:

$$y(t)=d\,u(t), \tag{1.2.15}$$

where $y=u_e$, $u=u_i$, $d=\dfrac{R_c}{R+R_c}=\mathbf{D}$, $\dim\mathbf{D}=1\times1$.

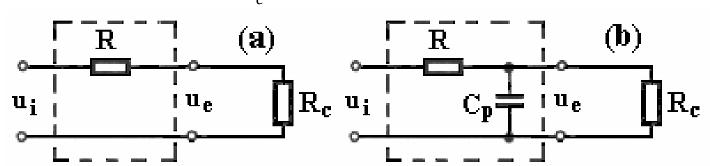


Fig. 1.7. *Resistive circuit (a) and equivalent diagram for very fast input variations (b).*

For very fast input variations (very high frequencies in the signal), certain approximations lead to the equivalent diagram presented in Fig. 1.7 (b), where C_p is a parasitic capacitance. Therefore, the dynamic behavior of the circuit is described by

$$\frac{R\,R_c}{R+R_c}C_p\dot{u}_e(t)+u_e(t)=\frac{R\,R_c}{R+R_c}u_i(t), \tag{1.2.16}$$

that points out the inertial character of the transfer processes.

This problem is treated in a similar manner in case of D-DSs.

The linear dependencies between the variables of PSs are practically rare and with limited usage. If the variations of the characteristic variables of a PS are restricted to domains where the linearity property is kept (in a more or less rigorous way), then the use of linear MMs can be justified. However, attaching to a nonlinear PS a linear (or linearized) MM is not always possible as it will be shown in the next chapter.

A remarkable frequently used particular case of SS-MM representation of LTI systems is that of ***Single Input-Single Output*** (**SISO**) ***systems***, with one input u ($r=1$) and one output y ($q=1$) which is characterized by the following form of (1.2.11):

$$\begin{aligned} &\mathbf{x}'(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,u(t), \\ &y(t) = \mathbf{c}^T\ \mathbf{x}(t) + d\,u(t), \end{aligned} \tag{1.2.17}$$

where the dimensions of the matrices **A**, **b**, $\mathbf{c}^T$ and d are given in (1.2.12) and adapted accordingly:

$$\dim \mathbf{A} = n \times n,\ \dim \mathbf{b} = n \times 1,\ \dim \mathbf{c}^T = 1 \times n,\ d - \text{scalar}. \tag{1.2.18}$$

1.3. Input-output models

Any system can be seen only from the point of view of the causal dependence (with the direction past → present → future) between its inputs and outputs without pointing out distinctly the inner variables, viz. the state variables. This situation concerns an ***input-output characterization*** of the system by the ***input-output mathematical model*** (**IO-MM**). The rigorous definition of this characterization can be formalized as shown, for example, in (Ionescu, 1985; Preitl, 1992). Only IO-MMs for SISO systems (Fig. 1.8) will be presented as follows.

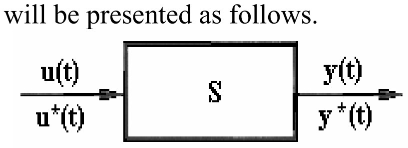


Fig. 1.8. *Single input-Single Output (SISO) system.*

A) For a linear and invariant C-DS, the IO-MM is a differential equation (linear with constant coefficients) expressed as

$$\sum_{\nu=0}^{n} a_\nu y^{(\nu)}(t) = \sum_{\mu=0}^{m} b_\mu u^{(\mu)}(t),\ t \in \mathbf{R}, \tag{1.3.1}$$

with

$$m \le n. \tag{1.3.2}$$

B) ***For a linear and invariant D-DS,*** the IO-MM is a linear recurrent equation (i.e., with discrete time) with constant coefficients expressed as

$$\sum_{\nu=0}^{n} a_\nu y(k+\nu) = \sum_{\mu=0}^{m} b_\mu u(k+\mu),\ k \in \mathbf{N}, \tag{1.3.3}$$

with

$$m \le n. \tag{1.3.4}$$

The condition (1.3.2) or (1.3.4) is referred to as ***the causality condition***. Its interpretation is straightforward for discrete-time systems: supposing $n=0$ and $m=0$, at $k=0$ (the initial time moment) the IO-MM (1.3.3) is $y(0) = (b_0 / a_0)u(0) + (b_1 / a_0)u(1)$, meaning that the actual output $y(0)$ depends on the future input $u(1)$, which is practically impossible.

Remarks concerning the introductory concepts in systems theory presented in the previous sections:

1. As a matter of principle, for the same PS the two mathematical characterizations – IO-MM and SS-MM – should have the same contents. This is valid (at least from a mathematical point of view) in highlighting the connection input → output. However, in certain particular conditions the two representations are not equivalent, the SS-MM being more comprehensive.
2. In case of complex systems, the mathematical treatment by the use of IO-MMs proves to be often more simple and intuitional than the SS-MMs. Moreover, in many practical applications the IO-MM characterization of PSs meets the requirements specific to the design of control systems using the process output. The input-output approach to control systems design corresponds mainly to the classical control theory.
3. In case of input-output representation of systems use is made of **the black bo**x concept. This is often related to the way to determine the IO-MM on the basis of processing the measurements concerning the evolution in dynamic regimes of the input and the output (**experimental identificatio**n or **data-driven identification**).

1.4. Graphical modeling by block diagrams

The intuitional and often detailed characterization of the functional and informational structure of a system – PS or DS – is carried out by several ***block diagrams***. The following two ***types of block diagrams*** are used in automation, and they can be detailed as function of their scope:

A) Functional block diagrams. These diagrams highlight, by means of several representations specific to different technical fields, the functional structure and partially the constructive structure of the system. Using such schemes, the specialists can derive the functional and operating principles of the system and sometimes the specialists can derive a MM of the system.

B) Informational block diagrams. These diagrams highlight, by representations specific to automation, the informational structure of the system. The informational block diagrams are important in the analysis and development of control systems.

The informational block diagrams employ several types of sub-systems with specific representations, where:

- the characteristic variables are pointed out by oriented segments in the sense of the information flow,
- the structural dependencies (the functionals) are pointed out by blocks associated with several ways to specify the properties of these blocks.

A part of these blocks is synthesized in Table 1.4.

The informational block diagrams are built on the basis of the first principle equations that describe the phenomena in the system. The diagrams should highlight:

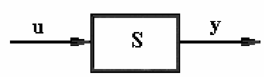
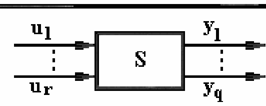
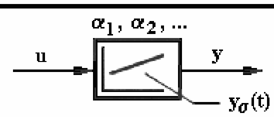
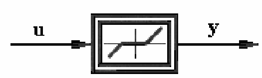
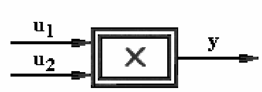
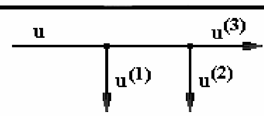
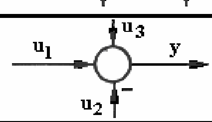
- the characteristic variables of the system (**u**, **x**, **y**);
- if possible, the variables that can be measured and that can be used in control;
- in case of control systems, the variables that are processed in the CD.

Example 1.2 (Preitl, 1992): Let us consider the technical process that deals with the heating of a room of thermal capacity C_i. The heating is carried out by electrical heating elements placed in the floor as shown in Fig. 1.9 (a). The power dissipated in the heating resistors (R), with the notation p_e, can be modified – by means of a power amplifier which consists of the grid command ensemble ACG and the thyristor bridge PTr – by the modification of the control voltage u_c. It is required to derive the MMs of the PS accepting the following simplifying assumptions:

1. The system is considered with concentrated parameters. The room and the environment are homogenous and isotropic and the heating change takes place by convection and radiation.
2. At each moment, the heat exchange between the two environments (internal-external) is proportional to the temperature difference between these two environments.
3. The heat change between the room and the neighboring rooms is negligible.
4. The power p_e dissipated in the resistors R is independent of the processes in the room and it is transferred fully to the heating block BI of thermal capacity C_m and from the BI to the room.

The *input variables* in the PS are the dissipated power p_e by which the desired temperature is ensured in the room (with the role of control signal) and the external temperature θ_e (with the role of disturbance input). The heat change between the two environments affects seriously the value of the temperature in the room θ_i from the desired value θ_{i0} ($\theta_e < \theta_{i0}$).

Table 1.4. *Representation of systems by informational block diagrams.*

CONTENTS OF REPRESENTATION	BLOCK DIAGRAM REPRESENTATION	REMARKS
SISO SYSTEM		**general representation**
MIMO SYSTEM **r - number of inputs** **q - number of outputs**		**general representation** $u = \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$ $y = \begin{bmatrix} y_1 \\ \vdots \\ y_q \end{bmatrix}$
		
LINEAR SISO SYSTEM $y = f(u)$ f( ) - **operator characterized by the parameters** $\alpha_1, \alpha_2, \ldots$		$y_\sigma(t)$ - **system response to a particular input variation (step)**
		
NONLINEAR SISO SYSTEM $y = f(u)$ **or** **NONLINEAR MISO SYSTEM** $y = f(u_1, u_2)$		**inner notation:** - **nonlinearity character** f(u) - **nonlinear operation:** ×, :, √ , ...
		
TAKEOFF POINT		$u = u^{(1)} = u^{(2)} = u^{(3)}$ $(\cdot)$ - **branching directions**
SUMMING POINT		$y = u_1 - u_2 + u_3$

The *assessed output* is the temperature in the room θ_i, and the *measured output* is the current i_θ given by the temperature sensor TT1 (the measuring element M1). The external temperature θ_e is measured by the temperature sensor TT2 (the measuring element M2) that gives the current $i_{\theta e}$.

The following two tasks are required:

1. Derive the IO-MM and the SS-MM that characterize the phenomena in the room.
2. Build the block diagram of the process.

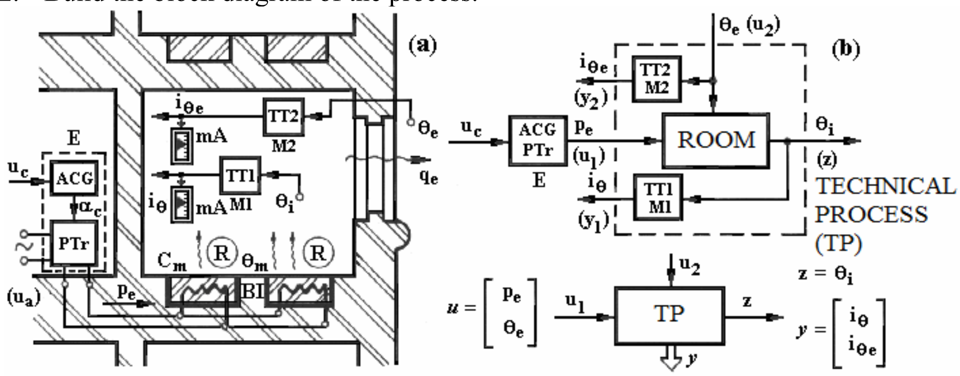


Fig. 1.9. *Functional block diagram (a) and informational block diagram (b) for the technical process that deals with the heating of a room.*

Solution: The thermal balance equations of the process can be expressed in the simplified form

$$\begin{aligned}
&C_m\dot{\theta}_m = p_e - K_m(\theta_m - \theta_i),\\
&C_i\dot{\theta}_i = K_m(\theta_m - \theta_i) - K_p(\theta_i - \theta_e),\\
&i_\theta = K_{M1}\theta_i,\\
&i_{\theta e} = K_{M2}\theta_{ie}.
\end{aligned} \tag{1.4.1}$$

We introduce the notations T_m and T_i for the time constant of the heating block and of the room, respectively:

$$T_m = C_m / K_m,\ T_i = C_i / K_p. \tag{1.4.2}$$

The state variables are chosen as θ_m and θ_i. Equations (1.4.1) are next organized in order to obtain:

- The SS-MM of the process:

$$\begin{aligned}
&\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u},\\
&\mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u} - \text{for the measured outputs},\\
&z = \mathbf{f}^T\,\mathbf{x} - \text{for the assessed output},
\end{aligned} \tag{1.4.3}$$

where

$$\mathbf{u}=\begin{bmatrix}p_e\\ \theta_e\end{bmatrix},\ \mathbf{x}=\begin{bmatrix}\theta_m\\ \theta_i\end{bmatrix},\ \mathbf{y}=\begin{bmatrix}i_\theta\\ i_{\theta e}\end{bmatrix},$$
$$\mathbf{A}=\begin{bmatrix}-1/T_m & 1/T_m\\ K_mK_p/T_i & -(1+K_mK_p)/T_i\end{bmatrix},$$
$$\mathbf{B}=\begin{bmatrix}1/(T_mK_m) & 0\\ 0 & 1/T_i\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}0 & K_{M1}\\ 0 & 0\end{bmatrix}, \tag{1.4.4}$$
$$\mathbf{D}=\begin{bmatrix}0 & 0\\ 0 & K_{M2}\end{bmatrix},\ \mathbf{f}^T=[0\quad 1].$$

- The IO-MM of the process can be expressed with respect to the two inputs by eliminating the state variables in the SS-MM, and the result is

$$T_mT_i\ddot{\theta}_i+[T_i+T_m(1+K_m/K_p)]\dot{\theta}_i+\theta_i=p_e/K_p+\theta_e+T\dot{\theta}_e. \tag{1.4.5}$$

When in the system (considered as stable) the time variations of all variables are balanced out (analytically, for $t\rightarrow\infty$, when all variables are constant), this corresponds to the ***steady-state regime*** (**SSR**). The following relationship between the ***steady-state values*** (**SSV**, pointed out with the subscript 0) can be expressed:

$$\theta_{i0}=p_{e0}/K_p+\theta_{e0}. \tag{1.4.6}$$

The block diagram of the process is presented in Fig. 1.9 (b) along with details concerning the input and output vectors.

Example 1.3: Given a four years undergraduate course, the simplifying assumption that the percentages of students promoted, repeaters, and dropouts are roughly constant, the direct enrollment in the second, third and fourth years is not allowed and students can not enroll for more than four years, the first principle equations of the fourth order linear discrete-time dynamic model can be expressed as a discrete-time SS-MM:

$$\begin{aligned}
&x_1(k+1)=\beta_1x_1(k)+u(k),\\
&x_2(k+1)=\alpha_1x_1(k)+\beta_2x_2(k),\\
&x_3(k+1)=\alpha_2x_2(k)+\beta_3x_3(k),\\
&x_4(k+1)=\alpha_3x_3(k)+\beta_4x_4(k),\\
&y(k)=\alpha_4x_4(k),
\end{aligned} \tag{1.4.7}$$

where k is the year, $x_i(k)$ is the number of students enrolled in year i at year k, $u(k)$ is the number of freshmen at year k, $y(k)$ is the number of graduates at year k, α_i is the promotion rate during year *i*, and β_i is the failure rate during year *i*, $i=1...4$. Zero initial conditions are assumed in this example as in the example 1.2.

This is an extension of the example for the three years undergraduate course given in (Bemporad, 2011).

Defining the state vector $\mathbf{x}=[x_1 \quad x_2 \quad x_3 \quad x_4]^T$, the matrix form of the discrete-time SS-MM (1.4.7) is

$$\begin{aligned} &\mathbf{x}(k+1)=\mathbf{A}\,\mathbf{x}(k)+\mathbf{b}\,u(k), \\ &y(k)=\mathbf{c}^T\ \mathbf{x}(t), \end{aligned} \tag{1.4.8}$$

with the matrices

$$\mathbf{A}=\begin{bmatrix} \beta_1 & 0 & 0 & 0 \\ \alpha_1 & \beta_2 & 0 & 0 \\ 0 & \alpha_2 & \beta_3 & 0 \\ 0 & 0 & \alpha_3 & \beta_4 \end{bmatrix},\ \mathbf{b}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ \mathbf{c}^T=[0 \quad 0 \quad 0 \quad \alpha_4]. \tag{1.4.9}$$

The IO-MM is derived as follows by the elimination of the state variables in (1.4.7) or (1.4.8):

$$\begin{aligned} &y(k)-(\beta_1+\beta_2+\beta_3+\beta_4)y(k-1)+(\beta_1\beta_2+\beta_1\beta_3+\beta_1\beta_4+\beta_2\beta_3 \\ &+\beta_2\beta_4+\beta_3\beta_4)y(k-2)-(\beta_1\beta_2\beta_3+\beta_1\beta_2\beta_4+\beta_1\beta_3\beta_4+\beta_2\beta_3\beta_4)y(k-3) \\ &+\beta_1\beta_2\beta_3\beta_4 y(k-4)=\alpha_1\alpha_2\alpha_3\alpha_4 u(k-4). \end{aligned} \tag{1.4.10}$$

1.5. Introduction to process control

The higher and higher complexity of ***technical / technological processes*** (**TPs**) and of the biological, economical ones, etc., the more and more demanding quality requirements and the more and more strong connection between processes and between processes and the environment have lead to the ***necessity of process control***.

The processes **can be controlled** (Fig. 1.10):

- ***manually*** (a), by the often continuous and direct intervention of a human operator, and
- ***automatically*** (b), by the use of specialized equipment dedicated to carry out the control operations, and such equipment is called ***automation equipment*** (**AE**).

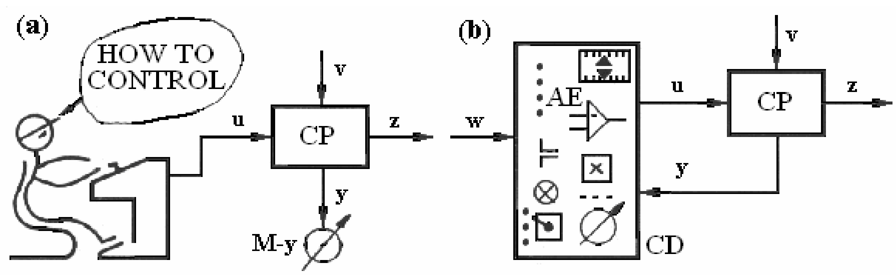


Fig. 1.10. *Manual control (a) and automatic control (b) of a process.*

The ensemble of **AE interconnected constructively and functionally** and dedicated to the control of a process called ***controlled process*** (**CP**) is referred to as ***control device*** (**CD**) or ***automatic control device*** (**ACD**).

CD and CP are in strong interconnection in the process control. The constructive-functional ensemble that consists of CD and CP and achieved for process control is called ***control system*** (**CS**) or ***automatic control system*** (**ACS**).

A simplified form of the operations involved in the manual or in the automatic control of a CP highlights two ways to deal with (Fig. 1.11):

(a) Without the direct tracking of the CP progress by the CD, this is the case of ***open-loop control***. The CS built in this way is called ***open-loop control system*** (**OL-CS**) or ***system with command*** (***manual***, ***automatic***) with the block diagram presented in Fig. 1.11 (a).

The main **shortcomings** of an OL-CS are:

- since the disturbances that act on the CP are unknown, their effect cannot be anticipated;
- even if an operator tracks with great attention the CP progress, the action of disturbances will affect the CP progress from the desired progress and operation;
- the impossibility to control unstable processes.

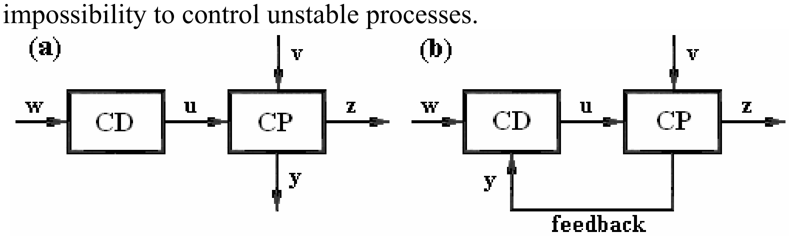


Fig. 1.11. *Open-loop control system (a) and closed-loop control system (b).*

These shortcomings have lead to the transfer to another category of CS described as follows.

(b) With the direct and often continuous tracking of the CP progress by the CD, Fig. 1.11 (b). This is the case of ***closed-loop control*** or of ***feedback control***. The CS built in this way is called ***closed-loop control system*** (**CL-CS**).

The complex CSs are designed such that the CDs should ensure both closed-loop control (in the automatic regime) and – with some constraints – open-loop control (for example, in the manual control regime).

The main ***control tasks*** of a CD in a CL-CS are:

- to ensure the memorizing of the information concerning the way that the CP should evolve by means of the reference input $w(t)$;
- to ensure the tracking of the CP evolution (progress) by means of the measured output $y(t)$ and to compare the effective evolution ($y(t)$) with the desired one ($w(t)$) by the computation of the difference $e(t) = w(t) - y(t)$ called ***control error***;
- to make the according decision of intervention in the PC progress by the elaboration of the ***control signal*** $u(t)$ and its transmission to the CP.

The following nomenclature is used for the variables in Fig. 1.10 and Fig. 1.11: w – reference input (also indicated by r), u – control signal, z – assessed output by which the quality of the progress of the process is appreciated (it is often the controlled variable of the CP), y – measured output (measured by the block M-y) which is accessed in order to build an ACS with respect to output, v – disturbance input (also indicated by d).

The oriented segments illustrated in Fig. 1.11 point out the sense of information flow concerning those variables. These variables are also information carrying support related to the phenomena in the ACS.

An operational ACS requires that the variables u and y – that interface CD and CP – should be understood by the blocks they enter. This involves ***the same physical nature, the same variation domain and the same energy level***.

In many situations the measured output (of the process) $y(t)$ is exactly a measure of the assessed output $z(t)$ obtained by a ***measuring element*** (**M**), Fig. 1.12 (a). This variable can also be the measure of other variables in the process (z_a), Fig. 1.12 (b).

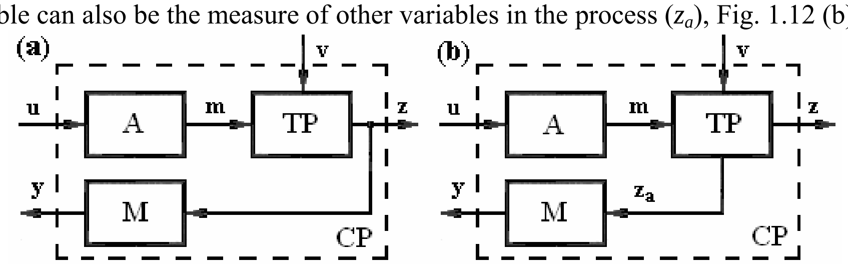


Fig. 1.12. *Detailed block diagrams of controlled process.*

The control signal $u(t)$ produced by the CD is converted and amplified by the ***actuator*** (**A**) for the intervention in the CP. The actuator ensures the necessary support (substance, stuff) required for the operation of the process.

Since often from a constructive point of vies the blocks A and M belong to the technological equipment where the process evolves, and they are dimensioned in strong relation with its functionality, the ensemble {A, TP, M} is called ***controlled process*** (**CP**) or ***fixed part*** of the system or ***controlled object***.

The process control involves the achievement of several tasks detailed at different hierarchical levels. The main **control tasks** at the lower process level are

- the achievement of the controls (logical, sequential and combinatorial) concerning the discrete time events;
- the achievement of the control of technological parameters (of the specific variables);
- the supervision of the system operation and the assurance of the safe operation of the process and of the control system.

Example 1.4 (Åström and Murray, 2008): The cruise control system of a car is a common ACS encountered in everyday life. The system attempts to maintain a constant velocity in the presence of disturbances primarily caused by changes in the slope of a road. The controller compensates for these unknowns by measuring the speed of the car and adjusting the throttle appropriately.

The process modeling starts with the block diagram given in Fig. 1.13. Let v be the speed (velocity) of the car and v_r the desired (reference) speed. The controller, which typically is of the proportional-integral (PI) type to be discussed in the next chapters, receives the signals v and v_r and generates a control signal u that is sent to an actuator that controls the throttle position. The throttle in turn controls the torque T delivered by the engine, which is transmitted through the gears and the wheels, generating a force F that moves the car. There are disturbance forces F_d due to variations in the slope of the road, the rolling resistance and aerodynamic forces. The cruise controller also has a human-machine interface that allows the driver to set and modify the desired speed. There are also functions that disconnect the cruise control when the brake is touched.

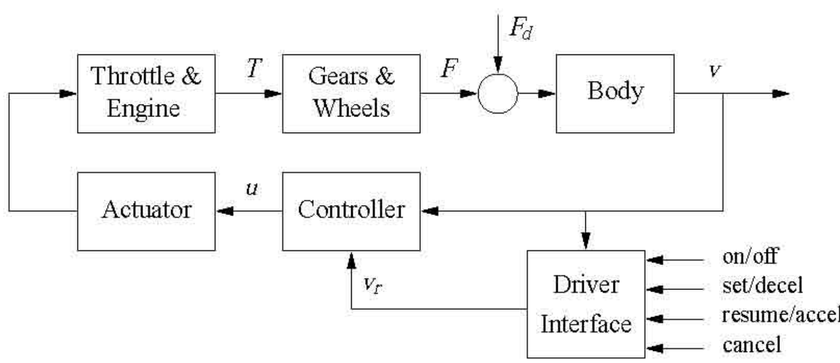


Fig. 1.13. *Detailed block diagram of an automotive cruise control system (Åström and Murray, 2008).*

The system has many individual components, i.e., actuator, engine, transmission, wheels and car body, and the development of a detailed model can be very complicated. In spite of this, the model required to design the cruise controller can be rather simple. The development of an MM is started with the force balance equation for the car body. Let v be the speed of the car, m the total mass (including passengers), F the force generated by the contact of the wheels with the road, and F_d the disturbance force due to gravity, friction and aerodynamic drag. The equation of motion of the car is

$$m\,\dot{v} = F - F_d\,. \tag{1.5.1}$$

The force F is generated by the engine, whose torque T is proportional to the rate of fuel injection, which is itself proportional to a control signal $0 \le u \le 1$ that controls the throttle position. The torque also depends on the engine speed ω. A simple representation of the torque at full throttle is given by the torque curve

$$T(\omega) = T_m[1 - \beta(\omega/\omega_m - 1)]^2\,, \tag{1.5.2}$$

where the maximum torque T_m is obtained at the engine speed ω_m. The typical values of the parameters in (1.5.2) are parameters are (Åström and Murray, 2008): T_m = 190 Nm, ω_m = 420 rad/s (about 4000 RPM) and β = 0.4. Let n be the gear ratio and r the wheel radius. The engine speed is related to the velocity through the expression

$$\omega = (n/r)v = \alpha_n v\,, \tag{1.5.3}$$

where $\alpha_n = n/r$, and the expression of the driving force is

$$F = (nu/r)T(\omega) = \alpha_n u T(\alpha_n v)\,. \tag{1.5.4}$$

Typical values of α_n for the gears 1 through 5 are (Åström and Murray, 2008): α_1 = 40, α_2 = 25, α_3 = 16, α_4 = 12 and α_5 = 10. The inverse of α_n has a physical interpretation as the effective wheel radius.

The disturbance force F_d has three major components: F_g – the forces due to gravity, F_r – the forces due to rolling friction, and F_a – the aerodynamic drag. Letting the slope of the road be θ, the gravity gives the force

$$F_g = m\,g\,\sin\theta\,, \tag{1.5.5}$$

where g = 9.8 m/s^2 is the gravitational constant. A relatively simple model of rolling friction is

$$F_r = m\,g\,C_r\,\mathrm{sgn}(v)\,, \tag{1.5.6}$$

where C_r is the coefficient of rolling friction with the typical value C_r = 0.01. The aerodynamic drag is proportional to the square of the speed:

$$F_a = 0.5\,\rho\, C_d A\, v^2\,, \tag{1.5.7}$$

where ρ is the density of air, C_d is the shape-dependent aerodynamic drag coefficient, and A is the frontal area of the car. Typical values of these parameters are ρ = 1.3 kg/m^3, C_d = 0.32 and A = 2.4 m^2.

Summarizing, the MM of the car used in cruise control is

$$m\,\dot{v} = \alpha_n u T(\alpha_n v) - m\,g\,C_r\,\mathrm{sgn}(v) - 0.5\,\rho\,C_d A\,v^2 - m\,g\,\sin\theta\,, \tag{1.5.8}$$

where the function T is given by equation (1.5.2). The model (1.5.8) is a dynamical system of first order. The state variable is the car velocity v, which is also the output. The input is the control signal u that controls the throttle position, and the disturbance input is the force F_d, which depends on the slope of the road. The system is nonlinear because of the torque curve, the gravity term and the nonlinear character of rolling friction and aerodynamic drag. There can also be variations in the parameters as, for example, the mass of the car depends on the number of passengers and the load being carried in the car.

This model can be extended with the model a feedback controller that attempts to regulate (control) the speed of the car in the presence of disturbances. The proportional-integral (PI) controller is the most widely used with this respect, and its model is

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau\,, \tag{1.5.9}$$

where $e(t) = v_r(t) - v(t)$ is the control error, k_p is the proportional gain, and k_i is the integral gain. k_p and k_i are the parameters of the PI controller, which have to be tuned appropriately such that to meet the performance specifications imposed to the automatic cruise control system. Even if the model (1.5.8) is very simple and obtained by several approximations and it is not perfectly accurate, it can be used to design a controller and make use of the feedback in the controller to manage the uncertainty in the system and to ensure the desired dynamics of the control system.

Some concluding remarks will be presented as follows in the final part of this chapter.

The phenomena in the automatic cruise CS evolve depending on the own dynamics. Therefore, the whole control system of the car will have a well stated ***dynamic behavior*** imposed by the control system discussed in the example 1.4 and by the other control systems in the car.

The CP and the CD as part of an ACS can be viewed as physical (sub)systems which are interconnected. The automation engineer has the task to adapt the dynamics of the CD to the CP such that the overall dynamics of the ACS to be adequate.

In order to develop the ACS, the quantitative and qualitative characterization of its subsystems should be carried out. That is the reason why use is made of ***mathematical models*** (**MM**s) associated to the PSs.

The MMs of each subsystem (CD and CP) and the characterization of the interconnections by means of interconnection equations are involved in the general mathematical characterization of the overall system, i.e., in the ***modeling of the ACS***.

The unified approach of the problematic of process control requires the structuring and organization of the MMs and of the tasks of each of the two subsystems, the CD and the CP. This structuring is based on the **principle of causality of physical phenomena**. The theoretical foundations of this structuring, of the analyses concerning the structures achieved, are offered by ***system theory*** or ***system science***.

An applicative part of system theory is related to process control (automation). Besides the mathematical characterization of systems, a special attention is given to the technical achievement and to the implementation of control structures with control equipment. The process informatics and the industrial informatics play an important role with this regard.

The process control essentially requires the assurance of the quality requirements also called the performance specifications concerning the process operation. The next chapters will offer general methods to study and to analyze specific to system theory, which are applied in order to develop and design ACSs.

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