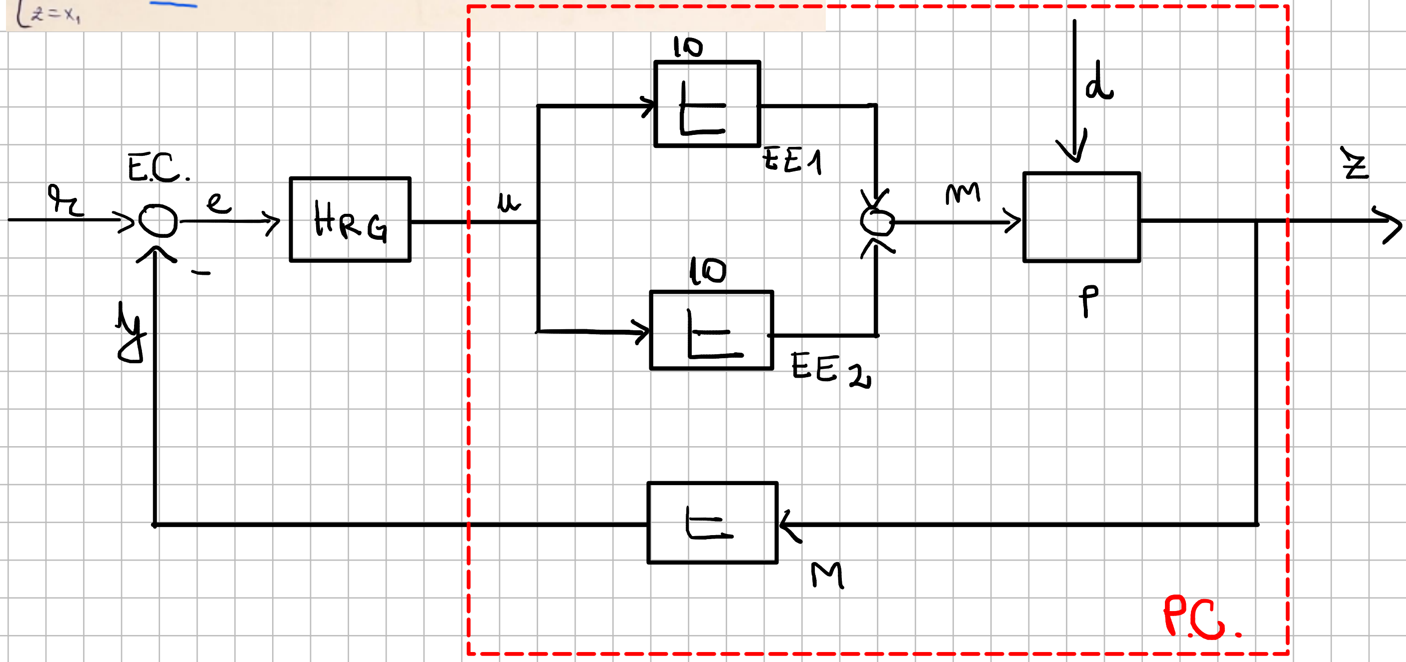
Subiect 2023

Se consideră sistemul de reglare automată cu schema bloc prezentată în figură, în care r(t) este referința, e(t) este eroarea de reglare și modelul de stare (MM-ISI) al blocului P este:

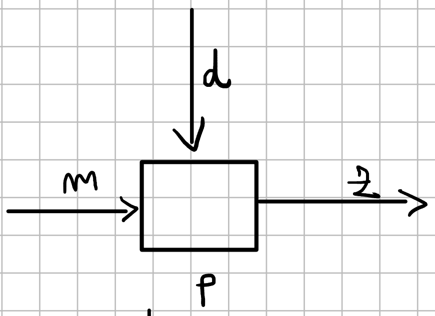
$$\begin{cases} \dot{x}_1 = -2x_1 + 2x_2 + 40d \\ \dot{x}_2 = -0{,}5x_2 + 12{,}5m \\ z = x_1 \end{cases}$$



Regulatoarele sunt de 2 tipuri:

R_1: ET-PI: $H_{RG}(s) = \dfrac{k_{RG}(1 + s \cdot T_i)}{s \cdot T}$

R_2: ET-PDT1: $H_{RG}(s) = \dfrac{k_{RG}(1 + T_d s)}{1 + s \cdot T_f}$



2 intrări

0 ieșire

$$\begin{bmatrix} m \\ d \end{bmatrix} = u(t)$$

marimi de intrare

$\Rightarrow H_{zm}\big|_{d=0} = ?$ și $H_{zd}\big|_{m=0} = ?$

$$\begin{cases} \dot{x}(t) = A \cdot x(t) + B \cdot u(t) \\ y(t) = c^T \cdot x(t) \end{cases}$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\begin{cases} \dot{x}_1 = -2 \cdot x_1 + 2x_2 + 0 \cdot m + 40d \\ \dot{x}_2 = 0 \cdot x_1 - 0{,}5x_2 + 12{,}5m + 0d \end{cases}$$

Daca analizam ecuatiile scrise cu rosu de mai sus, putem deduce urmatoarea relatie:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 2 \\ 0 & -0{,}5 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 40 \\ 12{,}5 & 0 \end{bmatrix}}_{B} \begin{bmatrix} m \\ d \end{bmatrix}$$

Pentru matricea A, luam coeficientii de la vectorul de stare (x - x1 si x2) si ii asezam in matrice pe linia 1 - de la $\dot{x}1$ si pe linia 2 - de la $\dot{x}2$

Asemanator procedam si pentru vectorul de intrare (m, d), doar ca pentru matricea B

$$z = 1 \cdot x_1 + 0 \cdot x_2$$

$$z = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$H(s) = C\underbrace{(s \cdot I - A)}_{M}^{-1} \cdot B$$

$$M = s \cdot I - A = s \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 0 & -0{,}5 \end{bmatrix} = \begin{bmatrix} s+2 & -2 \\ 0 & s+0{,}5 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

formulă curs

(mai usor decat sa faci inversa clasic)

$$M^{-1} = \frac{1}{(s+2)(s+0,5) - 0(-2)} \begin{bmatrix} s+0,5 & 0 \\ 2 & s+2 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+0,5 & 0 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} \cdot \frac{1}{(s+2)(s+0,5)}$$

$$= \frac{1}{(s+2)(s+0,5)} \cdot \begin{bmatrix} s+0,5 & 0 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} =$$

$$= \frac{1}{(s+2)(s+0,5)} \cdot \begin{bmatrix} 25 & 40(s+0,5) \end{bmatrix} =$$

$$= \begin{array}{cc} m & d \\ \end{array}$$

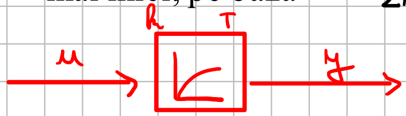
$$= \begin{bmatrix} \frac{25}{(s+2)(s+0,5)} & \frac{40}{s+2} \end{bmatrix} z \qquad \text{vector intrare} \begin{bmatrix} m \\ d \end{bmatrix}$$

$$\Rightarrow \quad H_{zm}\Big|_{d=0} = \frac{25}{(s+2)(s+0,5)}$$

$$H_{zd}\Big|_{m=0} = \frac{40}{s+2}$$

Verificarea MM-isi-ului

Daca observam, in blocul P, avem intrarea d. Putem desparti astfel blocul P in blocuri mai mici, pe baza H_{zm} si H_{zd}



$$H(s) = \frac{K}{sT+1}$$

$$H_{zm}\Big|_{d=0} = \frac{25}{(s+2)(s+0,5)} = \frac{25}{2(1+0,5s)\,0,5(1+2s)} = \frac{25}{(1+0,5s)(1+2s)}$$

$$H_{zd}\Big|_{m=0} = \frac{40}{s+2} = \frac{40}{2(1+0,5s)} = \frac{20}{1+0,5s}$$

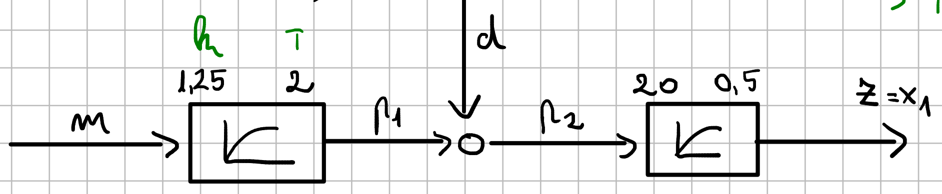
H_{zm} îl conține și pe H_{zd}

$\Rightarrow H_{zm} = H \cdot H_{zd}$

$$H = \frac{\cancel{1+0,5s}}{20} \cdot \frac{25}{\cancel{(1+0,5s)}(1+2s)} = \frac{1,25}{1+2s}$$

$\rightarrow k$ $\rightarrow T$

$\Rightarrow$



$$p_1(s) = \frac{1,25}{1+2s}\, m(s) \quad \Big| \cdot (1+2s)$$

$$\Rightarrow \quad p_1(s) + 2 \cdot s \cdot p_1(s) = 1,25\, m(s)$$

Daca ne uitam la ecuatia lui $\dot{x}1$ $\Longrightarrow$ avem 12,5 m(s) $\Longrightarrow$ trebuie sa despartim primul bloc din cele 2 de mai sus in alte 2 blocuri $\longrightarrow$ în serie

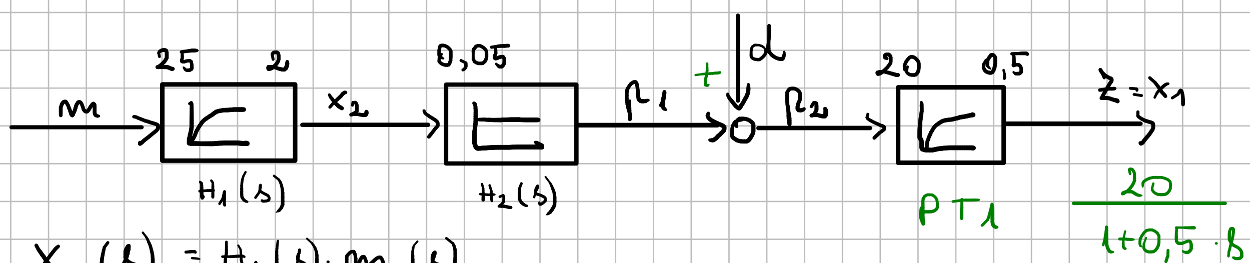
$$\Rightarrow \quad H_1(s) \cdot H_2(s) = \frac{1,25}{1+2s}$$

pt H_1 putem lua de la H_{zm}

$$\frac{25}{\cancel{1+2s}} \cdot H_2(s) = \frac{1,25}{\cancel{1+2s}}$$

$$H_2(s) = \frac{1,25}{25} = 0,05$$

$\Rightarrow$



$$x_2(s) = H_1(s) \cdot m(s)$$

$$x_2(s) = \frac{25}{1+2s} \cdot m(s) \quad \Big| \cdot (1+2s)$$

$$x_2(s) + 2\, s\, x_2(s) = 25\, m(s) \quad \Big| : 2$$

$$\boxed{x^{(m)}(t) = s^m x(s)}$$

$$0,5 \cdot x_2(s) + s \cdot x_2(s) = 12,5\, m(s)$$

$$0,5\, x_2(s) + \dot{x}_2(t) = 12,5\, m(s)$$

$$\Rightarrow \quad \dot{x}_2 = -0,5\, x_2 + 12,5\, m$$

$\longrightarrow$ a doua ecuație de la început

$$r_1(s) = 0{,}05\, m(s)$$

$$\boxed{r_1 = 0{,}05 \cdot m}$$

$$r_1(s) + d(s) = r_2(s) \Rightarrow \boxed{r_1 + d = r_2}$$

$$x_1(s) = z(s) = H_{zd}(s) \cdot r_2(s) = \frac{20}{1+0{,}5s} \cdot r_2(s) \quad \Big| (1+0{,}5s)$$

$$x_1(s) + 0{,}5 \cdot s \cdot x_1(s) = 20\, r_2(s)$$

$$s \cdot x(s) = \dot{x}(t)$$

$$x_1(s) + 0{,}5 \cdot \dot{x}_1(t) = 20\, r_2(s) \quad \Big| \cdot 2$$

$$s^2 x(s) = x^{(2)}(t)$$

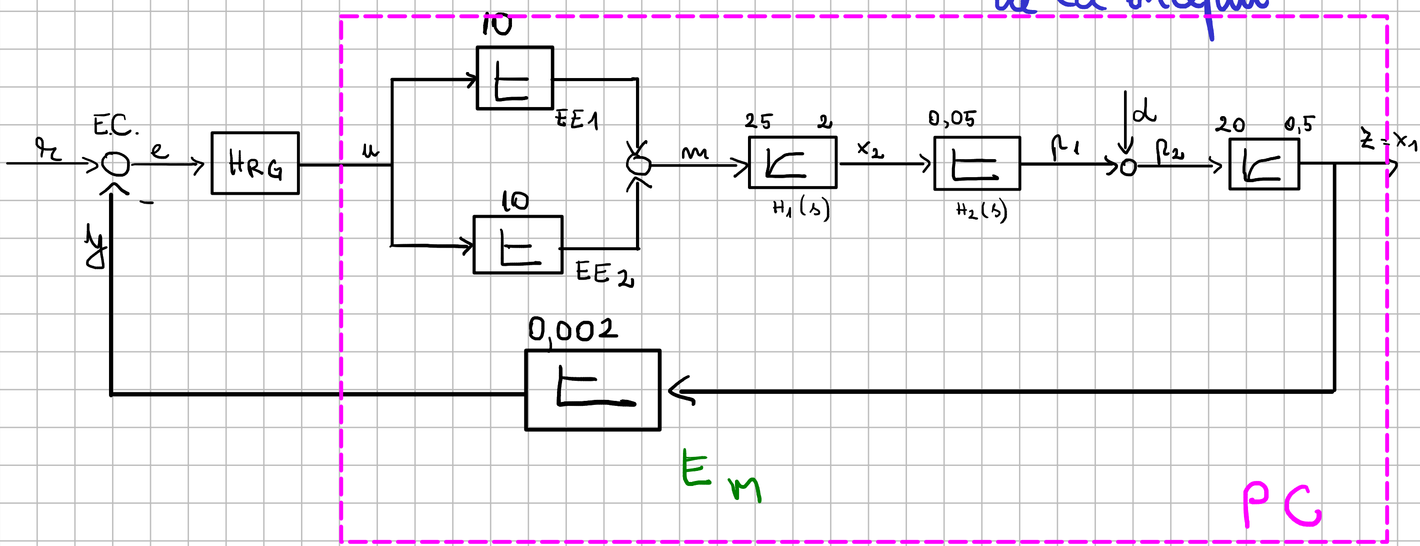
$$2x_1(s) + \dot{x}_1(t) = 40\, r_2(s)$$

$$\dot{x}_1 = -2x_1 + 40\, r_2$$

$$\Rightarrow \dot{x}_1 = -2x_1 + 40(r_1 + d)$$

$$\Rightarrow \dot{x}_1 = -2x_1 + 40 \cdot 0{,}05 \cdot x_2 + 40d$$

$$\Rightarrow \dot{x}_1 = -2x_1 + 2x_2 + 40d \longrightarrow \text{prima ecuație de la început}$$



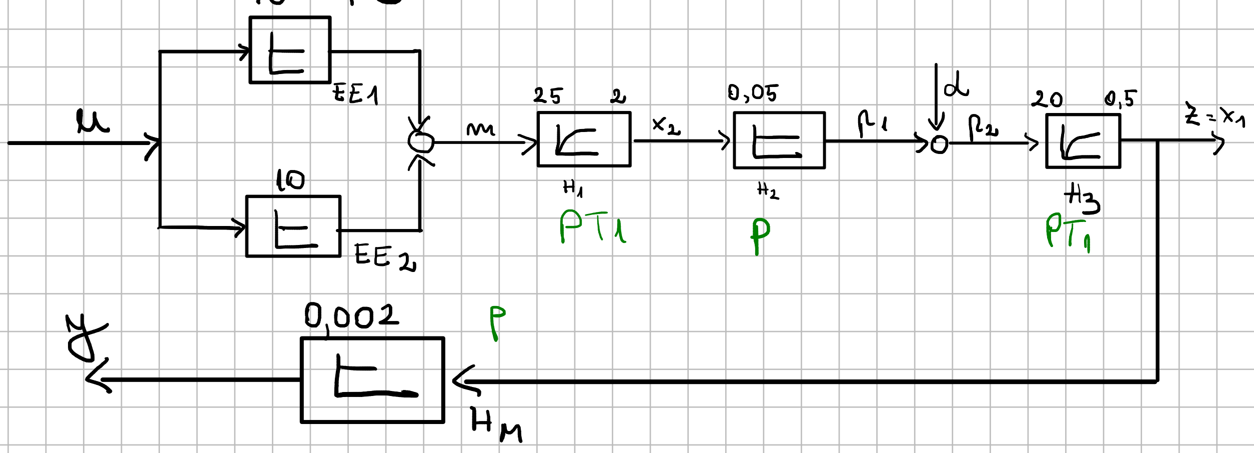
Calculați c.d.t. $H_{yr}(s)$, $H_{yd}(s)$, $H_0(s)$

$H_0(s)$ → f.d.t a sistemului deschis

pt R_1: $T_i = 2{,}5$ sec.

pt. R_2: $T_d = 2{,}5$ sec, $T_f = 0{,}1$ sec.

Calculăm H_{PC}

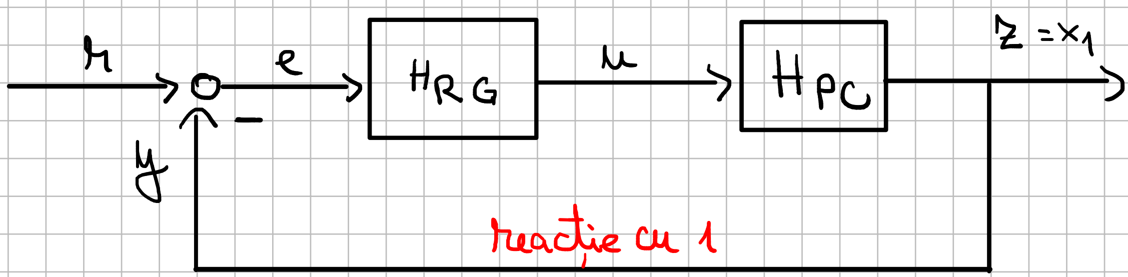


H_{EE_1}, H_{EE_2} - paralel $\Rightarrow H_{EE} = H_{EE_1} + H_{EE_2} = 10 + 10 = 20$

$H_{EE}, H_1, H_2, H_3, H_M$ - serie

$$\Rightarrow H_{PC} = 20 \cdot \frac{25}{1+2s} \cdot 0{,}05 \cdot \frac{20}{1+0{,}5s} \cdot 0{,}002 =$$

$$= \frac{1}{(1+2s)(1+0{,}5s)} = H_{yu}\Big|_{d=0}$$



H_{RG}, H_{PC} - serie $\Rightarrow H_o = H_{RG} \cdot H_{PC}$

$H_s, 1$ - reacție $\Rightarrow H_\alpha = \dfrac{H_o}{1 + H_o \cdot 1}$

$$\Rightarrow H_{yr} = H_\alpha = \frac{H_o}{1+H_o}$$

R_1: $T_i = 2{,}5$ sec $\Rightarrow H_{RG} = \dfrac{k_R(1+2{,}5s)}{2{,}5s} \rightarrow$ tip PI

$$H_{PC} = \frac{\overset{\rightarrow k_{PC}}{1}}{(1+\underbrace{0{,}5}_{T_2}s)(1+\underbrace{2}_{T_1}s)}$$

$$\left.\begin{aligned} k_{PC} &= 1 \\ T_1 &= 2 \\ T_2 &= 0{,}5 \end{aligned}\right\} \Rightarrow T_1 > T_2$$

$$k_r = \frac{1}{2\,k_{pc} \cdot T_2} = \frac{1}{2 \cdot 1 \cdot 0{,}5} = 1$$

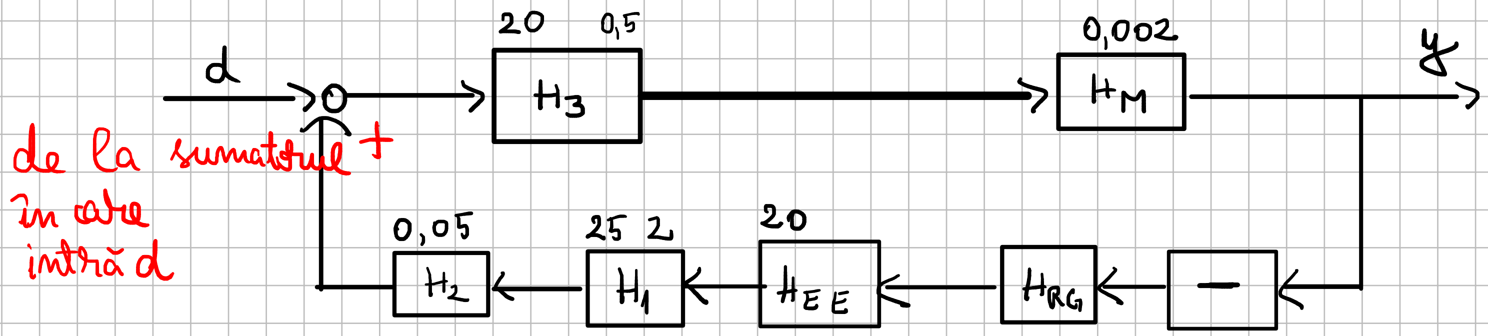
T_r = constanta de timp mai mare = 2

$$k_R = k_r \cdot T_r = 2 \cdot 1 = 2$$

$$\Rightarrow H_{RG}(s) = \frac{2}{2{,}5 \cdot s}\,(1 + 2{,}5\,s)$$

$$H_0(s) = H_{RG}(s) \cdot H_{pc}(s) = \frac{\overset{k_r}{\boxed{2}}\,(1+2{,}5\,s)}{2{,}5\,s \cdot (1+2s)\cdot(1+0{,}5\,s)}$$

$$H_{yr}(s)\Big|_{d=0} = \frac{H_0(s)}{1+H_0(s)} = \frac{\dfrac{2\cdot(1+2{,}5\,s)}{2{,}5\,s\cdot(1+2s)\cdot(1+0{,}5\,s)}}{1 + \dfrac{2\cdot(1+2{,}5\,s)}{2{,}5\,s\cdot(1+2s)\cdot(1+0{,}5\,s)}}$$



de la sumatorul în care intră y

H_3, H_M - serie

$$H_{3M} = H_3 \cdot H_M = 0{,}002 \cdot \frac{20}{1+0{,}5\,s} = \frac{0{,}04}{1+0{,}5\,s}$$

toate cele de pe ramura de jos - serie

$$H_s = -\,H_{RG} \cdot H_{EE} \cdot H_1 \cdot H_2 =$$

$$= -\frac{2\,(1+2{,}5\,s)}{2{,}5\,s} \cdot 20 \cdot \frac{25}{1+2\,s} \cdot 0{,}05 =$$

$$= \frac{50\,(1+2{,}5\,s)}{2{,}5 \cdot s \cdot (1+2\,s)}$$

H_s, H_{3M} - reacție

$$\Rightarrow H_{yd}(s)\Big|_{r=0} = \frac{H_{3M}}{1 - H_{3M} \cdot H_s} = \frac{\frac{0,04}{1+0,5s}}{1 - \frac{0,04}{1+0,5s} \cdot \frac{50(1+2,5s)}{2,5 s \cdot (1+2s)}}$$

R_2: $T_d = 2,5$ sec, $T_f = 0,1$ sec $\Rightarrow H_{RG}(s) = \frac{k_R(1+2,5s)}{1+2,5s}$

aceleași calcule

② Găsiți $k_R > 0$ pt care SRA – stabil

$\longrightarrow$ matricea Hurwitz

$H(s) \longrightarrow \Delta(s)$ numitor

$H_0 \longrightarrow \Delta(s) = 1 + H_0$

$$H_0(s) = \frac{k_R \cdot (1+2{,}5s)}{2{,}5s \cdot (1+2s)\cdot(1+0{,}5s)}$$

$$\Delta(s) = 1 + \frac{k_R \cdot (1+2{,}5s)}{2{,}5s \cdot (1+2s)\cdot(1+0{,}5s)} = 0$$

$$(2{,}5s + 5s^2)(1+0{,}5s) + k_R(1+2{,}5s) = 0$$

$$2{,}5s + 1{,}25s^2 + 5s^2 + 2{,}5s^3 + k_R + 2{,}5 \cdot s \cdot k_R = 0$$

$n = 3$

$$\Delta(s) = \underbrace{2{,}5}_{\substack{a_3 \\ >0}} s^3 + \underbrace{6{,}25}_{\substack{a_2 \\ >}} s^2 + \underbrace{2{,}5s(1+k_R)}_{a_1} + \underbrace{k_R}_{a_0} = 0$$

$$a_1 = 2{,}5(1+k_R) > 0 \Rightarrow 1 + k_R > 0 \quad k_R > -1$$

$$a_0 = k_R > 0$$

$$\Rightarrow k_R > 0 \Rightarrow k_R \in (0, +\infty)$$

$$H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 6{,}25 & k_R & 0 \\ 2{,}5 & 2{,}5(1+k_R) & 0 \\ 0 & 6{,}25 & k_R \end{bmatrix}$$

$$H_1 = 6{,}25 > 0$$

$$H_2 = 15{,}625(1+k_R) - 2{,}5k_R = 15{,}625 + 13{,}125k_R > 0$$

$$k_R > -\frac{15,625}{13,125}$$

$$H_3 = a_0 \det(H_2) = \underset{>0}{k_R} \cdot (\underset{>0}{15,625 + 13,125\, k_R})$$

$\Rightarrow k_R > 0$

$\Rightarrow$ sistem stabil

③ Considerând ieșirea $y(t)$, acceptând că sistemul este stabil și alegând o valoare a lui $k_R>0$, găsiți valoare statismului natural $\gamma_n(y)$. Acceptând valorile nominale $d_n=50$ și $y_n=100$, găsiți valoare statismului natural în unități raportate în procente $\gamma_n(y)\%$.

Statismul natural

$$\gamma_n = \left.\frac{y_\infty}{v_\infty}\right|_{w_\infty=0} = \frac{k_N}{1+k_0}$$

$$\gamma_n = \left.\frac{z_\infty}{v_\infty}\right|_{w_\infty=0} = \frac{k_N}{1+k_0}$$

- i, PI, PID $\longrightarrow$ $e_\infty=0$; $\gamma_n=0$ (fără statism / astatic)
- P, PT1, PDT1 $\longrightarrow$ $e_\infty\neq 0$ și $\gamma_n\neq 0$ (cu statism)

$$k_0 = k_R \cdot k_{PC}$$

R_1: ET-PI: $H_{RG}(s) = \dfrac{k_{RG}(1+s\cdot T_i)}{s\cdot T}$

$\longrightarrow$ regulator de tip PI

$\Rightarrow e_\infty = 0$ și $\gamma_n(y) = 0$

$d_n = 50$, $y_n = 100$

$$\gamma_n(y)\% = \underbrace{\gamma_n(y)}_{0} \cdot \frac{d_n}{y_n} \cdot 100\% = 0$$

R_2: ET-PbT1: $H_{RG}(s) = \frac{k_{RG}(1+T_s s)}{1+ s \cdot T_f}$

$\Rightarrow \varepsilon_m(y) \neq 0$, $e_\infty \neq 0$

$H_{PC} = \frac{1}{(1+0{,}5s)(1+2s)}$ $\rightarrow$ PC

T_2, T_1

$\left.\begin{array}{l} k_{PC} = 1 \\ T_1 = 2 \\ T_2 = 0{,}5 \end{array}\right\} \Rightarrow T_1 > T_2$

$k_R = 3$

$k_0 = k_R \cdot k_{PC} = 3 \cdot 1 = 3$

avem 2 blocuri

$y \leftrightarrow d$

$$\varepsilon_m(y) = \frac{k_N(y)}{1+k_0} = \frac{20 \cdot 0{,}002}{1+3} = \frac{0{,}04}{4} = 0{,}01$$

$$\varepsilon_m(y)^{\%} = 0{,}01 \cdot \frac{50}{1\cancel{0}\cancel{0}} \cdot \cancel{100} = 0{,}5\,\%$$

(4) Acceptând că sistemul este stabil şi alegând o valoare a lui $k_R > 0$, pentru $d_\infty = 50$ şi $z_\infty = 5000$ calculaţi valorile de regim staţionar constant $\{r_\infty,\ e_\infty,\ u_\infty,\ m_\infty,\ y_\infty\}$.

P, PT1, PDT1

$y_\infty = k \cdot u_\infty$

$u_\infty = \text{const.}$

$y_\infty = \text{const}$

D, PD, DT1

$u_\infty = \text{const}$

$y_\infty = (u_\infty) = 0$

I, PI, PID

$y_\infty = \text{const}$

$u_\infty = (y_\infty) = 0$