

## REZOLVARE EXAMEN

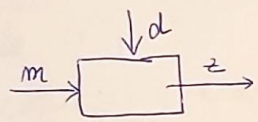
Se consideră sistemul de reglare automată cu schema bloc prezentată în figură, în care  $r(t)$  este referința,  $e(t)$  este eroarea de reglare și modelul de stare (MM-isi) al blocului P este:

$$\begin{cases} \dot{x}_1 = -2x_1 + 2x_2 + 40d \\ \dot{x}_2 = -0,5x_2 + 12,5m \\ z = x_1 \end{cases}$$

Sunt considerate 2 variante de regatoare (R) cu f.d.t:

$$R_1: H_R(s) = k_R \left( 1 + \frac{1}{T_i \cdot s} \right) = \frac{k_R (1 + sT_i)}{sT_i} (ET-Pi)$$

$$R_2: H_R(s) = \frac{k_R (1 + T_d s)}{1 + \frac{1}{T_f} s} (ET-PDT1)$$


$$\Rightarrow H_{zm}(s) = \left. \frac{z(s)}{m(s)} \right|_{d=0} \quad \text{și} \quad H_{zd}(s) = \left. \frac{z(s)}{d(s)} \right|_{m=0}$$

- trebuie găsite matricile A, B și C pt. a se găsi matricea de transfer  $H(s) = C \cdot (sI - A)^{-1} \cdot B = C \cdot M^{-1} \cdot B$

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 2 \\ 0 & -0,5 \end{bmatrix}}_A \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix}}_B \cdot \begin{bmatrix} m \\ d \end{bmatrix} \\ z = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_C \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

$$\text{deci } A = \begin{bmatrix} -2 & 2 \\ 0 & -0,5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$H(s) = C \cdot (sI - A)^{-1} \cdot B = C \cdot M^{-1} \cdot B = \begin{bmatrix} H_{x_{1m}}(s) & H_{x_{1d}}(s) \end{bmatrix} = \begin{bmatrix} H_{zm}(s) & H_{zd}(s) \end{bmatrix}$$

$$M = (sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 0 & -0,5 \end{bmatrix} = \begin{bmatrix} s+2 & -2 \\ 0 & s+0,5 \end{bmatrix} \Rightarrow$$

$$M^T = M^t = \begin{bmatrix} s+2 & 0 \\ -2 & s+0,5 \end{bmatrix}, \quad \det(M) = (s+2)(s+0,5) = 2(1+0,5s) \cdot 0,5(1+2s) \Rightarrow$$

$$\det(M) = (1+0,5s)(1+2s)$$

$$M^{-1} = \frac{1}{\det(M)} \cdot M^*$$

$$M^* = \begin{bmatrix} (-1)^{1+1} (n+0,5) & (-1)^{1+2} (-2) \\ (-1)^{2+1} \cdot 0 & (-1)^{2+2} (n+2) \end{bmatrix} = \begin{bmatrix} n+0,5 & 2 \\ 0 & n+2 \end{bmatrix}$$

$$M^{-1} = \frac{1}{(1+0,5n)(1+2n)} \begin{bmatrix} n+0,5 & 2 \\ 0 & n+2 \end{bmatrix}$$

$$H(n) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \frac{1}{(1+0,5n)(1+2n)} \begin{bmatrix} n+0,5 & 2 \\ 0 & n+2 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} =$$

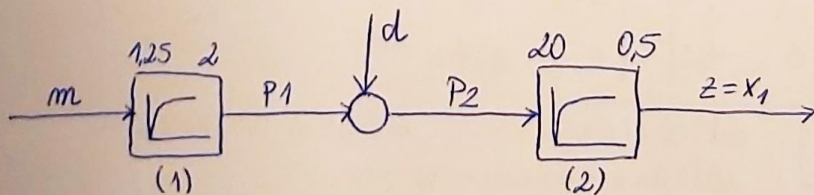
$$= \frac{1}{(1+0,5n)(1+2n)} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} n+0,5 & 2 \\ 0 & n+2 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} = \frac{1}{(1+0,5n)(1+2n)} \begin{bmatrix} n+0,5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix}$$

~~$$= \frac{1}{(1+0,5n)(1+2n)} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} n+0,5 & 2 \\ 0 & n+2 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix}$$~~

$$= \frac{1}{(1+0,5n)(1+2n)} \begin{bmatrix} 25 & 40(n+0,5) \end{bmatrix} = \begin{bmatrix} \frac{25}{(1+0,5n)(1+2n)} & \frac{40 \cdot 0,5 \cdot (1+2n)}{(1+0,5n)(1+2n)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{25}{(1+0,5n)(1+2n)} & \frac{20}{1+0,5n} \end{bmatrix} = \begin{bmatrix} H_{zm}(n) & H_{zd}(n) \end{bmatrix}$$

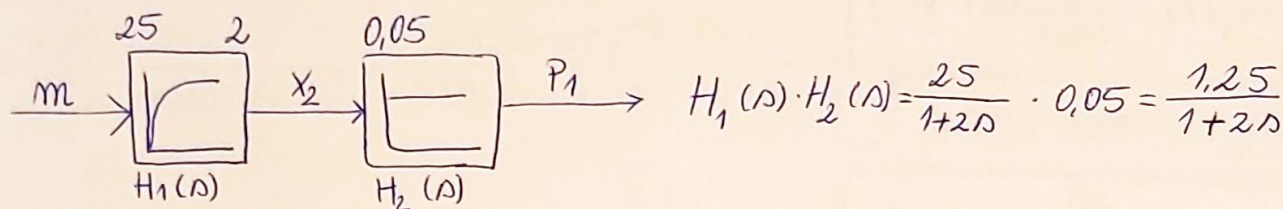
$$H_{zm}(n) = \frac{25}{(1+0,5n)(1+2n)} \quad \text{și} \quad H_{zd}(n) = \frac{20}{1+0,5n}$$



$\Rightarrow$  verificăm să vedem dacă obținem MM-isi-ul



$P_1(s) = \frac{1,25}{1+2s} m(s) \Rightarrow P_1(s) + 2s P_1(s) = 1,25 m(s) \Rightarrow$  trebuie să avem  
 $12,5 m(s) \Rightarrow$  despărțim  
 primul bloc în 2 blocuri (un ET-PT1 și un ET-P)



$$P_1(s) = 0,05 X_2(s) \Rightarrow p_1 = 0,05 X_2$$

$$X_2(s) = \frac{25}{1+2s} m(s) \Rightarrow X_2(s) + 2s X_2(s) = 25 m(s) \Rightarrow X_2 + 2\dot{X}_2 = 25m \Rightarrow$$

$$2\dot{X}_2 = -X_2 + 25m \Rightarrow \boxed{\dot{X}_2 = -0,5X_2 + 12,5m}$$

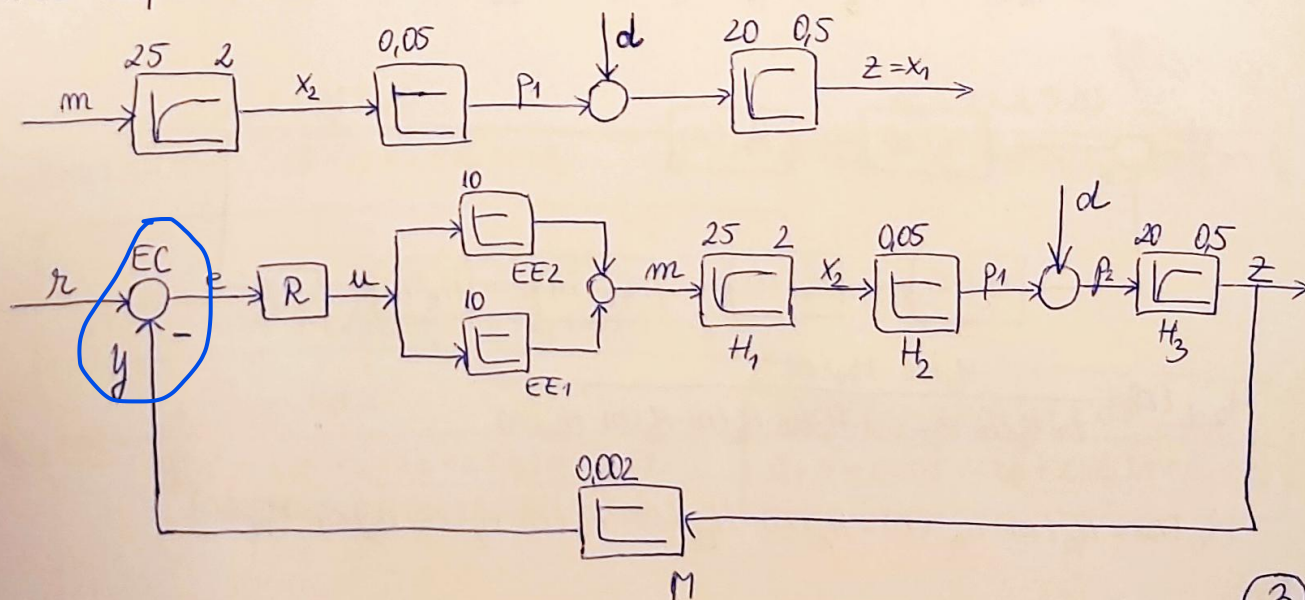
$$P_2(s) = p_1(s) + d(s) \Rightarrow p_2 = p_1 + d$$

$$Z(s) = \frac{20}{1+0,5s} P_2(s) \xrightarrow{z=x_1} X_1(s) + 0,5X_1(s) \cdot s = 20P_2(s) \Rightarrow X_1 + 0,5\dot{X}_1 = 20P_2$$

$$\Rightarrow \left. \begin{aligned} 0,5\dot{X}_1 &= -X_1 + 20P_2 \\ P_2 &= p_1 + d \end{aligned} \right\} \Rightarrow \left. \begin{aligned} 0,5\dot{X}_1 &= -X_1 + 20p_1 + 20d \\ p_1 &= 0,05X_2 \end{aligned} \right\} \Rightarrow$$

$$0,5\dot{X}_1 = -X_1 + X_2 + 20d \quad | : 0,5 \Rightarrow \boxed{\dot{X}_1 = -2X_1 + 2X_2 + 40d}$$

$\Rightarrow$  se respectă MM-isi-ul



$$\left. \begin{aligned} H_{EE1}(s) &= 10(ET-P) \\ H_{EE2}(s) &= 10(ET-P) \end{aligned} \right\} \Rightarrow H_{EE}(s) = H_{EE1}(s) + H_{EE2}(s) \Rightarrow H_{EE}(s) = 20(ET-P)$$

(conexiune paralel)

$$H_1(s) = \frac{25}{1+2s} (ET-PT1)$$

$$H_2(s) = 0,05 (ET-P)$$

$$H_3(s) = \frac{20}{1+0,5s} (ET-PT1)$$

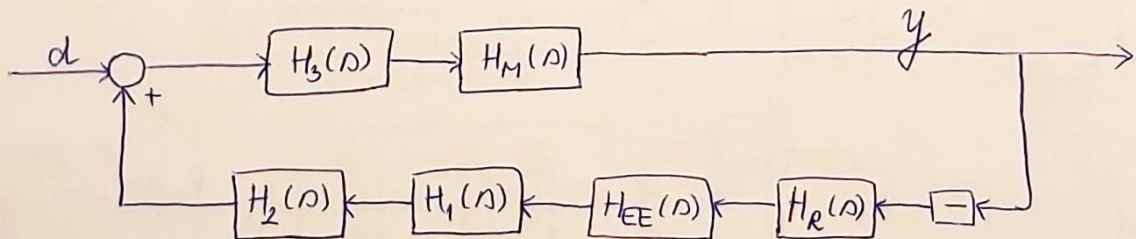
$$H_M(s) = 0,002 (ET-P)$$

① Calculați c.d.t. adică  $H_{y,r}(s)$ ,  $H_{y,d}(s)$  și f.d.t.  $H_0(s)$   
 f.d.t. a sistemului deschis

pentru:  $R1: T_i = 2,5 \text{ sec}$   
 $R2: T_d = 2,5 \text{ sec}, T_f = 0,1 \text{ sec}$

$$H_{y,r}(s) = \left. \frac{y(s)}{r(s)} \right|_{d=0} = \frac{H_R(s) \cdot H_{EE}(s) \cdot H_1(s) \cdot H_2(s) \cdot H_3(s) \cdot H_M(s)}{1 + H_R(s) \cdot H_{EE}(s) \cdot H_1(s) \cdot H_2(s) \cdot H_3(s) \cdot H_M(s)}$$

$$H_{y,d}(s) = \left. \frac{y(s)}{d(s)} \right|_{r=0} = \frac{H_3(s) \cdot H_M(s)}{1 - H_3(s) \cdot H_M(s) \cdot [(-) H_R(s) \cdot H_{EE}(s) \cdot H_1(s) \cdot H_2(s)]}$$



$$H_{y,d}(s) = \frac{H_3(s) \cdot H_M(s)}{1 + H_R(s) \cdot H_{EE}(s) \cdot H_1(s) \cdot H_2(s) \cdot H_3(s) \cdot H_M(s)}$$

$$H_0(s) = H_R(s) \cdot H_{PC}(s) = H_R(s) \cdot H_{EE}(s) \cdot H_1(s) \cdot H_2(s) \cdot H_3(s) \cdot H_M(s)$$

④



$$\boxed{R1:} \quad H_{y,r}(n) = \frac{\frac{k_R(1+2,5n)}{2,5n} \cdot 20 \cdot \frac{25}{1+2n} \cdot 0,05 \cdot \frac{20}{1+0,5n} \cdot 0,002}{1 + \frac{k_R(1+2,5n) \cdot 1}{2,5n(1+2n)(1+0,5n)}} =$$

$$= \frac{k_R(1+2,5n)}{2,5n(1+2n)(1+0,5n)} \cdot \frac{2,5n(1+2n)(1+0,5n)}{2,5n(1+2n)(1+0,5n) + k_R(1+2,5n)} =$$

$$= \frac{k_R(1+2,5n)}{2,5n(n^2 + 2,5n + 1) + 2,5k_Rn + k_R} = \frac{k_R(1+2,5n)}{2,5n^3 + 6,25n^2 + 2,5n + 2,5k_Rn + k_R}$$

$$\Rightarrow H_{y,r}(n) = \frac{k_R(1+2,5n)}{2,5n^3 + 6,25n^2 + 2,5n + 2,5k_Rn + k_R}$$

$$H_{y,d}(n) = \frac{\frac{20}{1+0,5n} \cdot 0,002}{\frac{2,5n^3 + 6,25n^2 + 2,5(1+k_R)n + k_R}{2,5n(1+2n)(1+0,5n)}} = \frac{0,1n(1+2n)}{2,5n^3 + 6,25n^2 + 2,5(1+k_R)n + k_R}$$

$$H_0(n) = \frac{k_R(1+2,5n)}{2,5n} \cdot \frac{1}{(1+0,5n)(1+2n)} \Rightarrow H_0(n) = \frac{k_R(1+2,5n)}{2,5n(1+0,5n)(1+2n)}$$

$$\boxed{R2:} \quad H_{y,r}(n) = \frac{\frac{k_R(1+2,5n)}{1+0,1n} \cdot \frac{1}{(1+0,5n)(1+2n)}}{1 + \frac{k_R(1+2,5n)}{(1+0,1n)(1+0,5n)(1+2n)}} = \frac{k_R(1+2,5n)}{(1+0,1n)(1+0,5n)(1+2n) + k_R(1+2,5n)}$$

$$= \frac{k_R(1+2,5n)}{(1+0,1n)(n^2 + 2,5n + 1) + 2,5k_Rn + k_R} \Rightarrow \frac{k_R(1+2,5n)}{n^2 + 2,5n + 1 + 0,1n^3 + 0,25n^2 + 0,1n + 2,5k_Rn + k_R}$$

$$\Rightarrow H_{y,r}(n) = \frac{k_R(1+2,5n)}{0,1n^3 + 1,25n^2 + (2,6 + 2,5k_R)n + k_R + 1}$$

$$H_{y,d}(n) = \frac{\frac{20}{1+0,5n} \cdot 0,002}{\frac{0,1n^3 + 1,25n^2 + (2,6 + 2,5k_R)n + k_R + 1}{\cancel{1+0,1n}(1+0,5n)(1+2n)}} = \frac{0,04(1+0,1n)(1+2n)}{0,1n^3 + 1,25n^2 + (2,6 + 2,5k_R)n + k_R + 1}$$

$$H_0(\lambda) = \frac{k_R(1+2,5\lambda)}{1+0,1\lambda} \cdot \frac{1}{(1+0,5\lambda)(1+2\lambda)} \Rightarrow \boxed{H_0(\lambda) = \frac{k_R(1+2,5\lambda)}{(1+0,1\lambda)(1+0,5\lambda)(1+2\lambda)}}$$

② Găsim valorile parametrului  $k_R > 0$  pt. care SRA este stabil.

[R1:]  $\Delta(\lambda) = 1 + H_0(\lambda) = 2,5\lambda^3 + 6,25\lambda^2 + 2,5(1+k_R)\lambda + k_R = a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$

Sunt impuse conditiile necesare specificate în T3:

$$a_3 = 2,5 > 0$$

$$a_2 = 6,25 > 0$$

$$\left. \begin{aligned} a_1 = 1+k_R > 0 &\Rightarrow k_R > -1 \Rightarrow k_R \in (-1; \infty) \\ a_0 = k_R > 0 &\Rightarrow k_R \in (0; \infty) \end{aligned} \right\} \Rightarrow \boxed{k_R \in (0; +\infty)} (*)$$

$$m=3 \Rightarrow H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 6,25 & k_R & 0 \\ 2,5 & 1+k_R & 0 \\ 0 & 6,25 & k_R \end{bmatrix}$$

Sunt impuse conditiile de stabilitate:

$$\det(H_1) = 6,25 > 0$$

$$\det(H_2) = 6,25(k_R+1) - 2,5k_R = 3,75k_R + 6,25 > 0 \Rightarrow k_R > -1,6667 \Rightarrow$$

$$k_R \in (-1,6667; \infty) (1)$$

$$\det(H_3) = a_0 \det(H_2) = k_R(3,75k_R + 6,25) > 0 \Rightarrow k_R \in (0; +\infty) \cap (-1,6667; +\infty)$$

$$\Rightarrow k_R \in (0; +\infty) (2)$$

$$\xrightarrow{(1) \cap (2)} \boxed{k_R \in (0; +\infty)} (**)$$

$$\text{din } (*) \cap (**) \Rightarrow \boxed{k_R \in (0; +\infty)}$$

[R2:]  $\Delta(\lambda) = 1 + H_0(\lambda) = 0,1\lambda^3 + 1,25\lambda^2 + (2,6+2,5k_R)\lambda + k_R+1 = a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$

$$a_3 = 0,1 > 0$$

$$a_2 = 1,25 > 0$$

$$\left. \begin{aligned} a_1 = 2,6+2,5k_R > 0 &\Rightarrow k_R > -1,04 \Rightarrow k_R \in (-1,04; \infty) \\ a_0 = k_R+1 > 0 &\Rightarrow k_R > -1 \Rightarrow k_R \in (-1; \infty) \end{aligned} \right\} \Rightarrow \boxed{k_R \in (0; +\infty)} (*)$$

$$m=3 \Rightarrow H = \begin{bmatrix} 1,25 & k_R+1 & 0 \\ 0,1 & 2,5k_R+2,6 & 0 \\ 0 & 1,25 & k_R+1 \end{bmatrix} \Rightarrow \det(H_1) = 1,25 > 0$$

$$\det(H_2) = 1,25(2,5k_R+2,6) - 0,1(k_R+1)$$

$$= 3,125k_R + 3,25 - 0,1k_R - 0,1$$

$$= 3,025k_R + 3,15 > 0 \Rightarrow k_R > -1,041$$

$$\Rightarrow k_R \in (-1,041; +\infty) (1)$$



$$\det(H_3) = a_0 \det(H_2) = (k_R + 1)(3,025k_R + 3,15) > 0 \Rightarrow k_R \in (-1; +\infty) \cap (-1,041; +\infty) \Rightarrow k_R \in (-1; +\infty) \quad (2)$$

$$\xrightarrow{\text{din (1) și (2)}} k_R \in (-1; \infty) \Rightarrow k_R \in (0; +\infty) \cap (-1; +\infty) \Rightarrow k_R \in (0; +\infty) \quad (**)$$

$$\xrightarrow{\text{din (*) și (**)}} \boxed{k_R \in (0; +\infty)}$$

③ Considerând ieșirea  $y(t)$ , acceptând că sistemul este stabil și alegând o valoare a lui  $k_R > 0$ , găsiți valoarea statismului natural  $\gamma_m^*(y)$ . Acceptând valorile nominale  $d_m = 50$  și  $y_m = 100$ , găsiți valoarea statismului natural în unități raportate în procente  $\gamma_m^*(y)^{\%}$ .

[R1]: Regulatorul este de tip PI, are componentă integratoare  $\Rightarrow$

$$\text{statismul natural este nul} \Rightarrow \boxed{\gamma_m^*(y) = 0}$$

$$\gamma_m^*(y)^{\%} = \gamma_m^*(y) \cdot \frac{d_m}{y_m} \cdot 100\% \Rightarrow \boxed{\gamma_m^*(y)^{\%} = 0} \quad (\text{pe baza relației din L4, pag. 3}) \rightarrow \text{relația 28}$$

$$[R2]: \gamma_m^*(y) = \frac{y_{\infty}}{d_{\infty}} \Big|_{r_{\infty}=0} \text{ sau } \gamma_m^*(y) = \frac{k_N(y)}{1+k_0}, \quad k_0 = k_R \cdot k_{PC}$$

luăm  $k_R = 3$  și alegem a 2-a opțiune deoarece nu avem valoarea lui  $d_{\infty}$

$$\gamma_m^*(y) = \frac{k_N(y)}{1+k_0} = \frac{0,04}{1+3} \Rightarrow \boxed{\gamma_m^*(y) = 0,01}; \quad \gamma_m^*(y)^{\%} = 0,01 \cdot \frac{50}{100} \cdot 100\% = 0,5$$

$$\boxed{\gamma_m^*(y)^{\%} = 0,5} \quad (\text{rel. 28})$$

$$k_N = 20 \cdot 0,002 = 0,04$$

$$k_0 = k_R \cdot k_{PC}$$

$$k_R = 3$$

$$k_{PC} = 20 \cdot 25 \cdot 0,05 \cdot 20 \cdot 0,002 = 1$$

$$\left. \begin{array}{l} k_0 = k_R \cdot k_{PC} \\ k_R = 3 \\ k_{PC} = 20 \cdot 25 \cdot 0,05 \cdot 20 \cdot 0,002 = 1 \end{array} \right\} \Rightarrow k_0 = 3$$

④ Acceptând că sistemul este stabil și alegând o valoare a lui  $k_R > 0$ , pentru  $d_{\infty} = 50$  și  $z_{\infty} = 5000$  calculați VRSC  $\{r_{\infty}, e_{\infty}, u_{\infty}, m_{\infty}, y_{\infty}\}$

$$[R1]: \text{RG-PI} \Rightarrow \left. \begin{array}{l} e_{\infty} = 0 \\ e_{\infty} = r_{\infty} - y_{\infty} \end{array} \right\} \Rightarrow r_{\infty} = y_{\infty} \Rightarrow \boxed{r_{\infty} = 10}$$

$$y_{\infty} = 0,002 \cdot z_{\infty} = 10 \Rightarrow \boxed{y_{\infty} = 10}$$

$$m_{\infty} = 20 u_{\infty} \Rightarrow \boxed{m_{\infty} = 160}$$

$$x_{2\infty} = 25 m_{\infty} = 500 u_{\infty} \Rightarrow \boxed{x_{2\infty} = 4000}$$



$$p_{100} = 0,05 x_{200} = 25 u_{00} \Rightarrow p_{100} = 200$$

$$p_{200} = p_{100} + d_{00} = 25 u_{00} + 50 \Rightarrow p_{200} = 250$$

$$z_{00} = x_{100} = 20 p_{200} = 500 u_{00} + 1000 = 5000 \Rightarrow 500 u_{00} = 4000 \Rightarrow u_{00} = 8$$

$$\begin{aligned} \text{R2: } \text{RG-PDT1} \Rightarrow \left. \begin{aligned} e_{00} &\neq 0 \\ e_{00} &= x_{00} - y_{00} \end{aligned} \right\} \Rightarrow e_{00} = x_{00} - 10 \Rightarrow 4 = x_{00} - 10 \Rightarrow \\ \boxed{x_{00} = 14} \end{aligned}$$

$$y_{00} = 0,002 z_{00} = 10 \Rightarrow y_{00} = 10$$

$$u_{00} = 2 e_{00} \quad (k_R = 2) \Rightarrow u_{00} = 8$$

$$m_{00} = 20 u_{00} = 40 e_{00} \Rightarrow m_{00} = 160$$

$$x_{200} = 25 m_{00} = 1000 e_{00} \Rightarrow x_{200} = 4000$$

$$p_{100} = 0,05 x_{200} = 50 e_{00} \Rightarrow p_{100} = 200$$

$$p_{200} = p_{100} + d_{00} = 50 e_{00} + 50 \Rightarrow p_{200} = 250$$

$$z_{00} = 20 p_{200} \Rightarrow 1000 e_{00} + 1000 = 5000 \Rightarrow 1000 e_{00} = 4000 \Rightarrow e_{00} = 4$$

⑤ Det. valorile parametrului  $\kappa$  care garantează stabilitatea sistemului linear în timp discret.

$$\text{R1: } H(z) = \frac{3z^2 - 4z + 1}{z^3 - 2z^2 + (\kappa + 1,3)z - 0,1}$$

$$\begin{aligned} \Delta(z) = z^3 - 2z^2 + (\kappa + 1,3)z - 0,1 &= a_3 z^3 + a_2 z^2 + a_1 z + a_0 \Rightarrow \begin{aligned} a_3 &= 1 \\ a_2 &= -2 \\ a_1 &= \kappa + 1,3 \\ a_0 &= -0,1 \end{aligned} \\ \text{cu } n=3 \text{ și } a_3 &= 1 > 0 \end{aligned}$$

Sunt testate primele 3 condiții de stabilitate:

$$\Delta(1) = 1 - 2 + \kappa + 1,3 - 0,1 = \kappa + 0,2 > 0 \Rightarrow \kappa > -0,2 \Rightarrow \kappa \in (-0,2; +\infty), (1)$$

$$\Delta(-1) = -1 - 2 - (\kappa + 1,3) - 0,1 = -3 - \kappa - 1,3 - 0,1 = -\kappa - 4,4 < 0 \quad (n \text{ este impar}) \Rightarrow$$

$$\kappa + 4,4 > 0 \Rightarrow \kappa > -4,4 \Rightarrow \kappa \in (-4,4; +\infty), (2)$$

$$\left. \begin{aligned} a_0 &= -0,1 \Rightarrow |a_0| = 0,1 \\ a_3 &= 1 \end{aligned} \right\} \Rightarrow 0,1 < 1 \Rightarrow |a_0| < a_3$$

$$b_0 = \begin{vmatrix} a_0 & a_3 \\ a_3 & a_0 \end{vmatrix} = \begin{vmatrix} -0,1 & 1 \\ 1 & -0,1 \end{vmatrix} = (-0,1)^2 - 1 = -0,99$$

$$b_1 = \begin{vmatrix} a_0 & a_2 \\ a_3 & a_1 \end{vmatrix} = \begin{vmatrix} -0,1 & -2 \\ 1 & \kappa + 1,3 \end{vmatrix} = -0,1(\kappa + 1,3) + 2 = -0,1\kappa - 0,13 + 2 = 1,87 - 0,1\kappa$$



$$b_2 = \begin{vmatrix} a_0 & a_1 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} -0,1 & c+1,3 \\ 1 & -2 \end{vmatrix} = 0,2 - (c+1,3) = 0,2 - c - 1,3 = -(c+1,1)$$

Limie	$z^0$	$z^1$	$z^2$	$z^3$
1	$-0,1$ ( $a_0$ )	$c+1,3$ ( $a_1$ )	$-2$ ( $a_2$ )	$1$ ( $a_3$ )
2	$1$ ( $a_3$ )	$-2$ ( $a_2$ )	$c+1,3$ ( $a_1$ )	$-0,1$ ( $a_0$ )
3	$-0,99$ ( $b_0$ )	$1,87-0,1c$ ( $b_1$ )	$-(c+1,1)$ ( $b_2$ )	—
4	$-(c+1,1)$ ( $b_2$ )	$1,87-0,1c$ ( $b_1$ )	$-0,99$ ( $b_0$ )	—

$$\left. \begin{aligned} |b_0| &= 0,99 \\ |b_2| &= |-(c+1,1)| = c+1,1 \end{aligned} \right\} \Rightarrow |b_0| > |b_2| \Rightarrow 0,99 > c+1,1 \Rightarrow c < -0,11$$

$$\Rightarrow c \in (-\infty; -0,11) \quad (3)$$

$$\text{Dim (1), (2) și (3)} \Rightarrow c \in (-0,2; -0,11)$$

$$\boxed{R2}: H_2(z) = \frac{6z^2 - 3z + 95}{z^3 + 2z^2 + (c-1,3)z + 0,1}$$

$$\Delta(z) = z^3 + 2z^2 + (c-1,3)z + 0,1 = a_3 z^3 + a_2 z^2 + a_1 z + a_0 \Rightarrow$$

$$a_3 = 1$$

$$a_2 = 2$$

$$a_1 = c-1,3$$

$$a_0 = 0,1$$

$$\text{cu } n=3 \text{ și } a_3=1 > 0$$

Sunt testate primele 3 condiții de stabilitate:

$$\Delta(1) = 1 + 2 + c - 1,3 + 0,1 = c + 1,8 > 0 \Rightarrow c > -1,8 \Rightarrow c \in (-1,8; \infty) \quad (1)$$

$$\Delta(-1) = -1 + 2 + (c-1,3)(-1) + 0,1 = 1 - c + 1,3 + 0,1 = 2,4 - c < 0 \Rightarrow c > 2,4$$

$$\Rightarrow c \in (2,4; \infty) \quad (2)$$

$$a_0 = 0,1 \Rightarrow |a_0| = 0,1 \left. \right\} \Rightarrow |a_0| < a_3 \Rightarrow 0,1 < 1$$

$$a_3 = 1$$

$$b_0 = \begin{vmatrix} a_0 & a_3 \\ a_3 & a_0 \end{vmatrix} = \begin{vmatrix} 0,1 & 1 \\ 1 & 0,1 \end{vmatrix} = 0,1^2 - 1 = -0,99$$

$$b_1 = \begin{vmatrix} a_0 & a_2 \\ a_3 & a_1 \end{vmatrix} = \begin{vmatrix} 0,1 & 2 \\ 1 & c-1,3 \end{vmatrix} = 0,1(c-1,3) - 2 = 0,1c - 0,13 - 2 = 0,1c - 2,13$$

$$b_2 = \begin{vmatrix} a_0 & a_1 \\ a_3 & a_2 \end{vmatrix} = \begin{vmatrix} 0,1 & c-1,3 \\ 1 & 2 \end{vmatrix} = 0,2 - (c-1,3) = 0,2 - c + 1,3 = -c + 1,5 = -(c-1,5)$$

Linie	$z^0$	$z^1$	$z^2$	$z^3$
1	0,1 ( $a_0$ )	$c-1,3$ ( $a_1$ )	2 ( $a_2$ )	1 ( $a_3$ )
2	1 ( $a_3$ )	2 ( $a_2$ )	$c-1,3$ ( $a_1$ )	0,1 ( $a_0$ )
3	-0,99 ( $b_0$ )	$0,1c-2,13$ ( $b_1$ )	$-(c-1,5)$ ( $b_2$ )	-
4	$-(c-1,5)$ ( $b_2$ )	$0,1c-2,13$ ( $b_1$ )	-0,99 ( $b_0$ )	-

$$\left. \begin{aligned} |b_0| &= 0,99 \\ |b_2| &= |-(c-1,5)| = c-1,5 \end{aligned} \right\} \Rightarrow |b_0| > |b_2| \Rightarrow 0,99 > c-1,5 \Rightarrow c < 2,49$$

$$\Rightarrow c \in (-\infty; 2,49), (3)$$

$$\dim(1), (2), (3) \Rightarrow c \in (2,4; 2,49)$$

SCHEMA DE LA ÎNCEPUTUL EXAMENULUI:

