

Se consideră SRA cu schema bloc alăturată, în care $r(t)$ este referință, $e(t)$ este eroarea de reglare și MM-151-ul blocului P este:

$$\begin{cases} \dot{X}_1 = -2X_1 + 2X_2 + 40d \\ \dot{X}_2 = -0,5X_2 + 12,5m \\ z = X_1 \end{cases}$$

$$R_1: H_R(s) = k_R \left(1 + \frac{1}{T_i s}\right) = \frac{k_R (1 + T_i s)}{T_i s} \quad (ET-Pi)$$

$$R_2: H_R(s) = \frac{k_R (1 + T_i s)}{1 + T_f s} \quad (ET-PDT_i)$$

Rezolvare: Căutăm P-ul $H_P(s)$:

$m \rightarrow \boxed{P} \rightarrow z$, sunt 2 intrări și 2 ieșiri \Rightarrow căutăm 2 f.d.t. $H_{zm}(s) = \frac{z(s)}{m(s)} \Big|_{d=0}$ și $H_{zd}(s) = \frac{z(s)}{d(s)} \Big|_{m=0}$

Căutăm matricele A, B, C, pt. a găsi matricea de transfer $H(s) = C \cdot (sI - A)^{-1} \cdot B$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 2 \\ 0 & -0,5 \end{bmatrix}}_A \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix}}_B \cdot \begin{bmatrix} m \\ d \end{bmatrix} \quad z = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_C \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Not. $M = sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 0 & -0,5 \end{bmatrix} = \begin{bmatrix} s+2 & -2 \\ 0 & s+0,5 \end{bmatrix}$, $M^{-1} = ?$ $M^{-1} = \frac{1}{\det M} \cdot M^*$

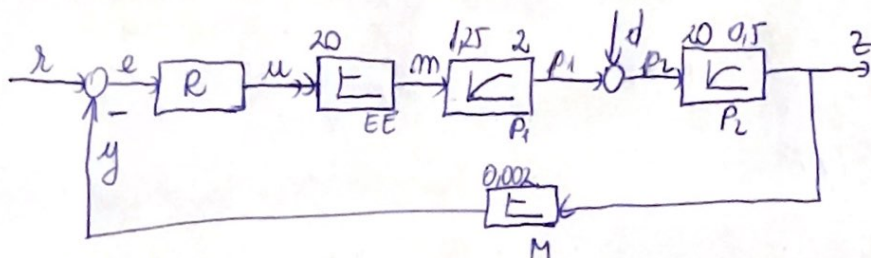
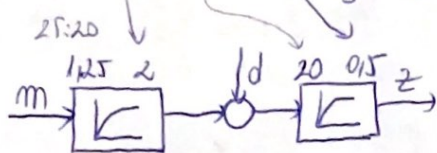
$$M^T = \begin{bmatrix} s+2 & 0 \\ -2 & s+0,5 \end{bmatrix}, \det M = \begin{vmatrix} s+2 & -2 \\ 0 & s+0,5 \end{vmatrix} = (s+2)(s+0,5) + 0 = (s+2)(s+0,5) = 2(0,5s+1) + 0,5(2s+1) = (1+0,5s)(1+2s) \quad (\text{mai util } (1+...))$$

$$M^* = \begin{bmatrix} (-1)^{1+1}(s+0,5) & (-1)^{1+2}(-2) \\ (-1)^{2+1}(-2) & (-1)^{2+2}(s+2) \end{bmatrix} = \begin{bmatrix} s+0,5 & 2 \\ 0 & s+2 \end{bmatrix} \Rightarrow M^{-1} = \begin{bmatrix} \frac{s+0,5}{(1+0,5s)(1+2s)} & \frac{2}{(1+0,5s)(1+2s)} \\ 0 & \frac{s+2}{(1+0,5s)(1+2s)} \end{bmatrix} \quad (\text{mai bine nu})$$

$$H(s) = C \cdot M^{-1} \cdot B = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \frac{1}{(1+0,5s)(1+2s)} \cdot \begin{bmatrix} s+0,5 & 2 \\ 0 & s+2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix} = \frac{1}{(1+0,5s)(1+2s)} \cdot \begin{bmatrix} 2s+0,5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 40 \\ 12,5 & 0 \end{bmatrix}$$

$$= \frac{1}{(1+0,5s)(1+2s)} \cdot \begin{bmatrix} 25 & 40s+20 \end{bmatrix} = \begin{bmatrix} \frac{25}{(1+0,5s)(1+2s)} & \frac{40(1+0,5s)}{(1+0,5s)(1+2s)} \end{bmatrix} = \begin{bmatrix} \frac{25}{(1+0,5s)(1+2s)} & \frac{40 \cdot 0,5(1+2s)}{(1+0,5s)(1+2s)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{25}{(1+0,5s)(1+2s)} & \frac{20}{(1+0,5s)} \end{bmatrix} = \begin{bmatrix} H_{zm}(s) & H_{zd}(s) \end{bmatrix}$$



$$H_{EE}(s) = H_{EE_1}(s) + H_{EE_2}(s) = 10 + 10 = 20(ET-P)$$

$$H_{P_1}(s) = \frac{1,25}{1+2s}(ET-PT_1)$$

$$H_{P_2}(s) = \frac{20}{1+0,5s}(ET-PT_1)$$

$$H_M(s) = 0,002(ET-P)$$

① Calc. c.d.t. adică $H_{y,r}(s)$, $H_{y,d}$, $H_d(s)$

$$\begin{aligned} \textcircled{P_1} H_{y,r}(s) &= \frac{y(s)}{r(s)} \Big|_{d=0} = \frac{H_{EE}(s) \cdot H_{P_1}(s) \cdot H_{P_2}(s) \cdot H_M(s) \cdot H_R(s)}{1 + H_{EE}(s) \cdot H_{P_1}(s) \cdot H_{P_2}(s) \cdot H_M(s) \cdot H_R(s) \cdot 1} = \\ &= \frac{20 \cdot \frac{1,25}{1+2s} \cdot \frac{20}{1+0,5s} \cdot 0,002 \cdot \frac{k_R(1+sT_i)}{\Delta T_i}}{1 + 20 \cdot \frac{1,25}{1+2s} \cdot \frac{20}{1+0,5s} \cdot 0,002 \cdot \frac{k_R(1+sT_i)}{\Delta T_i}} = \\ &= \frac{k_R(1+sT_i)}{(1+2s)(1+0,5s)\Delta T_i} \cdot \frac{(1+2s)(1+0,5s)\Delta T_i}{(1+2s)(1+0,5s)\Delta T_i + k_R(1+sT_i)} = \\ &= \frac{k_R + sT_i k_R}{(s^2 + 2,5s + 1)\Delta T_i + k_R + k_R sT_i} = \\ &= \frac{k_R + sT_i k_R}{s^2 \Delta T_i + s(2,5\Delta T_i + s(T_i + k_R)) + k_R} \\ \text{Pt } T_i &= 1,5 \text{ sec} \\ &= \frac{k_R + sT_i k_R}{2,5s^3 + 6,25s^2 + 2,5s + 2,5k_R s + k_R} \end{aligned}$$

$$H_{y,d}(\Delta) = \frac{y(\Delta)}{d(\Delta)} \Big|_{z=0} = \frac{H_R(\Delta) \cdot H_M(\Delta)}{1 + H_R(\Delta) \cdot H_M(\Delta) \cdot H_R^{(1)} H_{EE}^{(1)} H_{P_1}(\Delta) \cdot (-)} =$$

$$= \frac{\frac{20}{1+0,5\Delta} \cdot 0,002}{1 + \frac{20}{1+0,5\Delta} \cdot 0,002 \cdot \frac{k_R(1+2,5\Delta)}{2,5\Delta} \cdot 20 \cdot \frac{1,25}{1+2\Delta}} = \frac{0,04}{1+0,5\Delta}$$

$$= \frac{0,04}{1+0,5\Delta} \cdot \frac{2,5\Delta(1+2\Delta)(1+0,5\Delta)}{2,5\Delta(1+2\Delta)(1+0,5\Delta) + k_R(1+2,5\Delta)} = \frac{0,1\Delta(1+2\Delta)}{2,5\Delta(\Delta^2 + 2,5\Delta + 1) + k_R + 2,5\Delta k_R} =$$

$$= \frac{0,1\Delta(1+2\Delta)}{2,5\Delta^3 + 6,25\Delta^2 + \Delta(2,5 + 2,5\Delta(1+k_R)) + k_R} \quad (\text{dacă } H_{y,r} \text{ și } H_{y,d} \text{ au același numitor, e simetric la } \Delta)$$

$$H_0(\Delta) = H_R(\Delta) \cdot H_{PC}(\Delta)$$

$$H_{PC}(\Delta) = H_{EE}(\Delta) \cdot H_P(\Delta) \cdot H_R(\Delta) \cdot H_M(\Delta) = 20 \cdot \frac{1,25}{1+2\Delta} \cdot \frac{20}{1+0,5\Delta} \cdot 0,002 = \frac{1}{(1+2\Delta)(1+0,5\Delta)}$$

$$H_0(\Delta) = \frac{k_R(1+2,5\Delta)}{2,5\Delta} \cdot \frac{1}{(1+2\Delta)(1+0,5\Delta)} = \frac{k_R(1+2,5\Delta)}{2,5\Delta(1+2\Delta)(1+0,5\Delta)}$$

(R2) $H_{y,d}$, $H_{y,r}$ și H_0 au același numitor, diferă doar funcția regulatorului

$T_d = 2,5 \text{ sec}$
 $T_f = 0,1 \text{ sec}$

$$H_{y,r}(\Delta) = \frac{20 \cdot \frac{1,25}{1+2\Delta} \cdot \frac{20}{1+0,5\Delta} \cdot 0,002 \cdot \frac{k_R(1+2,5\Delta)}{1+0,1\Delta}}{1 + 20 \cdot \frac{1,25}{1+2\Delta} \cdot \frac{20}{1+0,5\Delta} \cdot 0,002 \cdot \frac{k_R(1+2,5\Delta)}{1+0,1\Delta}} =$$

$$= \frac{k_R(1+2,5\Delta)}{(1+2\Delta)(1+0,5\Delta)(1+0,1\Delta)} \cdot \frac{(1+2\Delta)(1+0,5\Delta)(1+0,1\Delta)}{(1+2\Delta)(1+0,5\Delta)(1+0,1\Delta) + k_R(1+2,5\Delta)} = \frac{k_R(1+2,5\Delta)}{(\Delta^2 + 2,5\Delta + 1)(1+0,1\Delta) + k_R + 2,5\Delta k_R}$$

$$= \frac{k_R(1+2,5\Delta)}{(\Delta^2 + 2,5\Delta + 1 + 0,1\Delta^3 + 0,25\Delta^2 + 0,1\Delta + k_R + 2,5\Delta k_R)} = \frac{k_R(1+2,5\Delta)}{0,1\Delta^3 + 1,25\Delta^2 + \Delta(2,6 + 2,5\Delta k_R) + k_R}$$

$$H_{y,d}(s) = \frac{\frac{20}{1+0,5s} \cdot 0,002}{1 + \frac{20}{1+0,5s} \cdot 0,002 \cdot \frac{1,25}{1+2s} \cdot 20 \cdot \frac{k_R(1+2,5s)}{1+0,1s}} = \frac{0,04 \cdot (1+0,5s)(1+2s)(1+0,1s)}{1+0,5s(1+2s)(1+0,1s) + k_R(1+2,5s)}$$

$$= \frac{0,04(s^2 + 2,5s + 1)}{s^2 + 2,5s + 1 + 0,1s^3 + 0,15s^2 + 0,1s + k_R + 2,5sk_R} = \frac{0,04(s^2 + 2,5s + 1)}{0,1s^3 + 1,25s^2 + 2,6s + 2,5sk_R + k_R}$$

$$H_0(s) = H_R(s) \cdot H_{RC}(s) = \frac{k_R(1+2,5s)}{1+0,1s} \cdot \frac{1}{(1+0,5s)(1+2s)} = \frac{k_R(1+2,5s)}{(1+0,1s)(1+0,5s)(1+2s)}$$

② Găsim $k_R > 0$ pt. care SRA stabil

Ⓡ₁ Folosim criteriul de stabilitate a lui Hurwitz

$$\Delta(s) = 1 + H_0(s) = 1 + \frac{k_R(1+2,5s)}{(1+2s)(1+0,5s) \cdot 2,5s} = \frac{2,5s(1+2s)(1+0,5s) + k_R(1+2,5s)}{2,5s(1+2s)(1+0,5s)}$$

Luăm $\Delta(s)$ ca fiind numărătorul

$$\Delta(s) = 2,5s(s^2 + 2,5s + 1) + k_R + 2,5sk_R = 2,5s^3 + s^2(6,25 + 6,25k_R) + s(2,5 + 2,5k_R) + k_R = 2,5s^3 + 6,25s^2 + 2,5s(1+k_R) + k_R$$

Impunem condițiile necesare

$$a_3 = 2,5 > 0$$

$$a_2 = 6,25 > 0$$

$$a_1 = 1 + k_R > 0 \Rightarrow k_R > -1 \Rightarrow k_R \in (-1, +\infty)$$

$$a_0 = k_R > 0 \Rightarrow k_R \in (0, +\infty)$$

$$\left. \begin{array}{l} a_3 > 0 \\ a_2 > 0 \\ a_1 > 0 \\ a_0 > 0 \end{array} \right\} \Rightarrow k_R \in (-1; +\infty) \cap (0; +\infty) \Rightarrow k_R \in (0; +\infty) \quad \text{①}$$

$$n=3 \Rightarrow \text{matricea lui Hurwitz } H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 6,25 & k_R & 0 \\ 2,5 & 1+k_R & 0 \\ 0 & 6,25 & k_R \end{bmatrix}$$

Impunem condițiile suficiente

$$\det(H_1) = a_2 = 2,5 > 0$$

$$\det(H_2) = a_2 a_1 - a_3 a_0 = 6,25 + 6,25k_R - 2,5k_R > 0 \Leftrightarrow 6,25 + 3,75k_R > 0 \Rightarrow$$

$$\Rightarrow 3,75k_R > -6,25 \Rightarrow k_R \in (0, +\infty) \Rightarrow k_R > -1,66 \Rightarrow k_R \in (-1,66; +\infty)$$

$$\det(H_3) = \underbrace{k_R}_{>0} \cdot \det(H_2) \Rightarrow k_R \in (0; +\infty)$$

$\left. \begin{array}{l} \det(H_1) > 0 \\ \det(H_2) > 0 \\ \det(H_3) > 0 \end{array} \right\} \Rightarrow$

③ Considerând esecia $y(t)$, sist. stabil, alegeți o val. pt. $k_P > 0$, găsiți val. statismului natural $s_m^e(y)$. Acceptând val. nominale $d_m = 50$ și $y_m = 100$, găsiți valoarea statismului nat. în emitați raportată în procente $s_m^e(y)\%$.

(R₁) Avem regulator ~~$\pi = \pi = 1$~~ $s_m^e(y) = 0$ de tip PI, are componentă integratoare
 \Rightarrow statismul natural este nul $s_m^e(y) = 0$

$$s_m^e(y)\% = s_m^e(y) \cdot \frac{d_m}{y_m} \cdot 100\% = 0$$

(R₂) $s_m^e(y) = \frac{y_{\infty}}{d_{\infty}}$ (cm lung) sau $s_m^e(y) = \frac{k_N}{1+k_0}$, $k_0 = k_P \cdot k_{PC}$

$$k_N = 20 \cdot 0,002 = 0,04$$

$$k_P = 3 \text{ (abs)} > 0$$

$$k_{PC} = 20 \cdot 1,25 \cdot 20 \cdot 0,002 = 1$$

$$\left. \begin{array}{l} k_0 = 3 + 1 = 4 \end{array} \right\} \Rightarrow s_m^e(y) = \frac{0,04}{4} = 0,01$$

$$s_m^e(y)\% = s_m^e(y) \cdot \frac{d_m}{y_m} \cdot 100\% = 0,01 \cdot \frac{50}{100} \cdot 100\% = \frac{0,5}{100} \cdot 100\% = 0,5$$

④ Acceptând sist. stabil, alegeți o val. pt. $k_P > 0$, pt. $d_{\infty} = 50$ și $Z_{\infty} = 5000$, calc. VRSC
 $\{r_{\infty}, l_{\infty}, u_{\infty}, m_{\infty}, y_{\infty}\}$

(R₁) Regulator tip PI

$$\left. \begin{array}{l} l_{\infty} = 0 \\ l_{\infty} = r_{\infty} - y_{\infty} \end{array} \right\} \Rightarrow r_{\infty} = y_{\infty}$$

$$m_{\infty} = 20 u_{\infty} = 160$$

$$p_{1\infty} = 1,25 m_{\infty} = 1,25 \cdot 20 \cdot u_{\infty} = 25 u_{\infty} = 200$$

$$p_{2\infty} = p_{1\infty} + d_{\infty} = 25 u_{\infty} + 50 = 250$$

$$Z_{\infty} = \cancel{20 p_{2\infty}} \quad 20 p_{2\infty} = 500 u_{\infty} + 1000 \quad \left. \begin{array}{l} 500 u_{\infty} + 1000 = 5000 \\ 500 u_{\infty} = 4000 \Rightarrow u_{\infty} = 8 \end{array} \right\} \Rightarrow$$

$$Z_{\infty} = 5000$$

$$y_{\infty} = 0,002 \cdot Z_{\infty} = 10 \Rightarrow r_{\infty} = 10$$

- (R₁) Regulator PD₁ ⇒ $e_{\infty} \neq 0$
 $e_{\infty} = r_{\infty} - y_{\infty}$

1 $m_{\infty} = 20 \cdot u_{\infty} = 160$

$p_{1\infty} = 1,25 m_{\infty} = 1,25 \cdot 20 u_{\infty} = 25 u_{\infty} = 200$

$p_{2\infty} = p_{1\infty} + d_{\infty} = 25 u_{\infty} + 50 = 250$

1 $\left. \begin{aligned} z_{\infty} &= 20 \cdot p_{2\infty} = 500 u_{\infty} + 1000 \\ z_{\infty} &= 5000 \end{aligned} \right\} \Rightarrow 500 u_{\infty} + 1000 = 5000$
 $u_{\infty} = 8$

1 $y_{\infty} = 0,002 \cdot 5000 = 10 \Rightarrow e_{\infty} = r_{\infty} - 10$

Algem $k_R = 2 \Rightarrow u_{\infty} = \frac{1}{k_R} k_R \cdot e_{\infty} = 2 e_{\infty} \Rightarrow e_{\infty} = 4 \Rightarrow 4 = r_{\infty} - 10$
 $u_{\infty} = 8 \qquad \qquad \qquad r_{\infty} = 14$

⑤ Det. val. lui κ care garantează stabilitatea sist. liniar în timp discret

(R₁) $H(z) = \frac{3z^2 - 4z + 1}{z^3 - 2z^2 + (c+1,3)z - 0,1}$

Luăm $\Delta(z)$ ca fiind numitorul

$\Delta(z) = z^3 - 2z^2 + (c+1,3)z - 0,1$ cu $m=3$ și $a_3=1 > 0$

↓
Sunt testate 4 condiții de stabilitate

- $\Delta(1) = 1^3 - 2 \cdot 1^2 + (c+1,3) \cdot 1 - 0,1 = 1 - 2 + c + 1,3 - 0,1 = 0,2 + c > 0 \Rightarrow c > -0,2$
 $\Rightarrow c \in (-0,2; +\infty)$ ①
- $\Delta(-1) = (-1)^3 - 2 \cdot (-1)^2 + (c+1,3) \cdot (-1) - 0,1 = -1 - 2 - c - 1,3 - 0,1 = -4,4 - c < 0$ m. impar.
 $\Rightarrow -c > 4,4 / (-1) \Rightarrow c < -4,4 \quad -c < 4,4 / (-1) \Rightarrow c > -4,4 \Rightarrow c \in (-4,4; +\infty)$ ②

• $|a_d| < a_m \Leftrightarrow |a_0| < a_3 \Leftrightarrow |-0,1| < 1 \Leftrightarrow 0,1 < 1$ ③

• $|b_0| > |b_{m-1}| \Leftrightarrow |b_0| > |b_2|$

$$b_0 = \begin{vmatrix} a_0 & a_3 \\ a_3 & a_0 \end{vmatrix} = a_0^2 - a_3^2 = (-0,1)^2 - 1^2 = 0,01 - 1 = -0,99$$

$$b_1 = \begin{vmatrix} a_0 & a_2 \\ a_3 & a_1 \end{vmatrix} = a_0 a_1 - a_2 a_3 = (-0,1) \cdot (c+1,3) - (-2) \cdot 1 = -0,1c - 0,13 + 2 = 1,87 - 0,1c$$

$$b_2 = \begin{vmatrix} a_0 & a_1 \\ a_3 & a_2 \end{vmatrix} = a_0 a_2 - a_1 a_3 = (-0,1) \cdot (-2) - (c+1,3) \cdot (-4) = 0,2 + 4c + 5,2 = 5,4 + 4c$$

$$|b_1| > |b_2| \Leftrightarrow |-0,99| > |5,4 + 4c| \Leftrightarrow 0,99 > 5,4 + 4c \Leftrightarrow c < -0,11 \Rightarrow c \in (-\infty; -0,11) \textcircled{3}$$

$$\text{Din } \textcircled{1}, \textcircled{2} \text{ și } \textcircled{3} \Rightarrow c \in (-0,2; +\infty) \cap (-4,4; +\infty) \cap (-\infty; -0,11) \Rightarrow c \in (-0,2; -0,11)$$

$$\textcircled{R_2} H(z) = \frac{6z^2 - 3z + 0,5}{z^3 + 2z^2 + (c-1,3)z + 0,1}$$

Luăm $\Delta(z)$ ca fiind numitorul

$$\Delta(z^3) = z^3 + 2z^2 + (c-1,3)z + 0,1 \quad \begin{matrix} a_3 = 1 \\ a_2 = 2 \\ a_1 = c-1,3 \\ a_0 = 0,1 \end{matrix} \quad \begin{matrix} m=3 \text{ și } a_3=1 > 0 \\ \Downarrow \\ \text{sunt testate 4 condiții de} \\ \text{stabilitate} \end{matrix}$$

$$\bullet \Delta(1) = 1^3 + 2 \cdot 1^2 + (c-1,3) \cdot 1 + 0,1 = 1 + 2 + c - 1,3 + 0,1 = 1,8 + c > 0 \Rightarrow c > -1,8 \Rightarrow c \in (-1,8; +\infty) \textcircled{1}$$

$$\bullet \Delta(-1) = (-1)^3 + 2 \cdot (-1)^2 + (c-1,3) \cdot (-1) + 0,1 = -1 + 2 - c + 1,3 + 0,1 = -c + 2,4 < 0 \text{ (n. impar)}$$

$$\Rightarrow 2,4 - c < 0 \mid \cdot (-1) \Rightarrow c > 2,4 \Rightarrow c \in (2,4; +\infty) \textcircled{2}$$

$$\bullet |a_0| < \prod_{m=1}^n |a_m| \Leftrightarrow |a_0| < |a_3| \Leftrightarrow 0,1 < 1 \textcircled{4}$$

$$\bullet |b_0| > |b_{n-1}| \Leftrightarrow |b_0| > |b_2|$$

$$b_0 = \begin{vmatrix} a_0 & a_3 \\ a_3 & a_0 \end{vmatrix} = a_0^2 - a_3^2 = 0,1^2 - 1^2 = 0,01 - 1 = -0,99$$

$$b_1 = \begin{vmatrix} a_0 & a_2 \\ a_3 & a_1 \end{vmatrix} = a_0 a_1 - a_2 a_3 = 0,1 \cdot (c - 1,3) - 2 = 0,1c - 0,13 - 2 = 0,1c - 2,13$$

$$b_2 = \begin{vmatrix} a_0 & a_1 \\ a_3 & a_2 \end{vmatrix} = a_0 a_2 - a_1 a_3 = 0,1 \cdot 2 - (c - 1,3) = 0,2 - c + 1,3 = -c + 1,5 = -(c - 1,5)$$

$$|b_0| > |b_2| \Leftrightarrow (b_0 - b_2)(b_0 + b_2) > 0 \quad 0,99 > c - 1,5 \Leftrightarrow c < 2,49 \Rightarrow c \in (-\infty; 2,49) \textcircled{3}$$

$$\text{Dim } \textcircled{1}, \textcircled{2}, \textcircled{3} \Rightarrow c \in \left(\mathbb{R} \setminus (-1,8; +\infty) \cap (2,4; \infty) \cap (-\infty; 2,49) \right) \\ \Rightarrow c \in (2,4; 2,49) \quad \dots$$

