FA2 4. Hausaufgabe zur Vorlesung

(H7)

a) $\sum_{n\geq 0} \frac{(-2)^n}{3^{n+1}} = \sum_{n\geq 0} \frac{(-2)^n}{3^n \cdot 3} = \frac{1}{3} \sum_{n\geq 0} \left(\frac{-2}{3}\right)^n$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{n+1}} = \frac{1}{3} \sum_{n=0}^{\infty} \underbrace{\left(\frac{-2}{3}\right)}_{q}^n = \frac{1}{3} \cdot \frac{1-\left(-\frac{2}{3}\right)^1}{1+\frac{2}{3}} = \frac{1}{3} \cdot \frac{1+\frac{2}{3}}{1+\frac{2}{3}} = \frac{1}{3}$$

geometrische Reihe $q \in (-1, 1)$

b) $\sum_{n\geq 1} \frac{1}{\sqrt{n}} = \sum_{n\geq 1} \frac{1}{n^{\frac{1}{2}}}$

n^α, wo $\alpha = \frac{1}{2} < 1$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}} = \infty$$

c) $\sum_{n\geq 2} \frac{3n^2+3n+1}{n^3(n+1)^3} = \sum_{n\geq 2} \frac{n^3+3n^2+3n+3-n^3}{n^3(n+1)^3} =$

$= \sum_{n\geq 2} \frac{(n+1)^3-n^3}{n^3(n+1)^3} = \sum_{n\geq 2} \frac{1}{n^3} - \frac{1}{(n+1)^3}$

sei $\frac{1}{n^3} - \frac{1}{(n+1)^3} = x_n \to x_n = a_n - a_{n+1}$

wo $a_n = \frac{1}{n^3}$ und $\frac{1}{(n+1)^3}$, Teleskopreihe

$\to \sum_{n=2}^{\infty} = \sum_{n=2}^{\infty} a_n - a_{n+1} = a_2 - \lim_{n\to\infty} a_n = \frac{1}{8} - 0 = \frac{1}{8}$

d) $\sum_{n\geq 1} \frac{1}{(5n+1)(5n+6)} = \sum_{n\geq 1} \frac{1}{5}\left(\frac{1}{5n+1} - \frac{1}{5n+6}\right) = \sum_{n\geq 1} \frac{5n+6-5n-1}{(5n+1)(5n+6)} =$

$= \frac{1}{5} \sum_{n\geq 1} \left(\frac{1}{5n+1} - \frac{1}{5n+6}\right)$

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Sei $\frac{1}{(5n+1)(5n+6)} = x_n$ so $x_n = a_n - a_{n+1}$

$$\frac{1}{5}\sum_{n=1}^{\infty} a_n - a_{n+1} = \frac{1}{5}\cdot\left(6 - \lim_{n\to\infty}\frac{1}{5n+1}\right) = \frac{6}{5} - 0 = \frac{6}{5}$$

e) $$\sum_{n\geq 0}\left(\frac{(-3)^{n+1}}{5^{n+2}} - \frac{5}{(n+2)!}\right) = \sum_{n\geq 0}\frac{(-3)^{n+1}}{5^{n+2}} - \sum_{n\geq 0}\frac{5}{(n+2)!}$$

$$\sum_{n=0}^{\infty}\frac{(-3)^{n+1}}{5^{n+2}} = -\frac{3}{25}\sum_{n=0}^{\infty}\left(\frac{-3}{5}\right)^{n} = \frac{-3}{25}\cdot\frac{1}{1+\frac{3}{5}} = -\frac{3}{25}\cdot\frac{1}{\frac{8}{5}} =$$

geometrische Reihe

$\sum_{n=k}^{\infty}\frac{q^k}{1-q}$, wo $q = -\frac{3}{5}$ u. $k=0$

$$= -\frac{3}{25}\cdot\frac{5}{8} = -\frac{3}{8}. \quad (1)$$

$$\sum_{n=0}^{\infty}\frac{5}{(n+2)!} = 5\sum_{m=2}\left(\frac{1}{m!} - \frac{1}{0!} - \frac{1}{1!}\right) = 5\cdot(e-2) = 5e-10$$

Sei $m = n+2$

e-Reihe

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