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7. Hausaufgabe zur Vorlesung:

(H11)

1) $f:\mathbb{R}\to\mathbb{R}$ mit $f(-3)=2$, $f'(-3)=0$, $f''(-3)=f^{(3)}(-3)=-6$

$f^{(4)}(-3)=72$

$$f(x)=f(-3)+\overbrace{\frac{f'(-3)}{1!}\cdot(x+3)}^{0}+\frac{f''(-3)}{2!}(x+3)^2+\frac{f^{(3)}(-3)}{3!}(x+3)^3+$$

$$+\frac{f^{(4)}(-3)}{4!}(x+3)^4=2-3(x+3)^2-(x+3)^3+3(x+3)^4$$

2) $f,g:(-1,\infty)\to\mathbb{R}$, $f(x)=\frac{1}{\sqrt{x+1}}$, $g(x)=f(x)\cdot\sin x=\frac{\sin x}{\sqrt{x+1}}$

a) $$T_2(x,0)=\sum_{k=0}^{2}\frac{f^{(k)}(0)}{k!}x^k=f(0)+\frac{f'(0)}{1!}\cdot x+\frac{f''(0)}{2!}x^2=$$

$$=1-\frac{1}{2}x+\frac{3}{4}x^2=\frac{3}{4}x^2-\frac{1}{2}x+1$$

$$f'(x)=-\frac{\frac{1}{2\sqrt{x+1}}}{x+1}=-\frac{1}{2(x+1)^{\frac{3}{2}}}\qquad f''(x)=-\frac{1}{2}\cdot(x+1)^{-\frac{3}{2}}=\frac{3}{4}\cdot(x+1)^{-\frac{5}{2}}$$

b) $R_2(x,0)$ $\quad x\in(-1,\infty)\setminus\{0\}$

Sei $x\in(-1,\infty)\setminus\{0\}$ Nach $\boxed{Th7}\Rightarrow\exists\, c$ zwischen x und 0 sodass $R_2(x,0)=\frac{f^{(3)}(c)}{3!}x^3$

$$f''(x)=\frac{3}{4}\cdot(x+1)^{-\frac{5}{2}}\Rightarrow$$

$$\Rightarrow f^{(3)}(x)=\frac{3}{4}\cdot\left(-\frac{5}{2}\right)\cdot(x+1)^{-\frac{7}{2}}=-\frac{15}{8}(x+1)^{-\frac{7}{2}}\Rightarrow$$

$$\Rightarrow R_2(x,0)=\frac{-\frac{15}{8}(0+1)^{-\frac{7}{2}}}{3!}x^3=-\frac{15\cdot 3}{4}x^3$$

c) $f^{(n)}(x)=?$, $x\in(-1,\infty)$ u. $n\in\mathbb{N}$

$f^{(n)}(x)=n!\cdot a_n$

d) $g^{(n)}(0)=g!\cdot a_n$