11. Hausaufgabe zur Vorlesung:

(H20)

$$f(x,y) = 2y^3 + x^2y + x^2 + 5y^2$$

① Sei $(x,y) \in \mathbb{R}^3 \Rightarrow \frac{\partial f}{\partial x}(x,y) = 2xy + 2x$

$\frac{\partial f}{\partial y}(x,y) = 6y^2 + x^2 + 10y$

② $\begin{cases} 2xy + 2x \\ 6y^2 + x^2 + 10y \end{cases} \Leftrightarrow \begin{cases} y = -1 \\ 6y^2 + x^2 + 10y \end{cases} \longrightarrow$

$\Rightarrow 6 + x^2 - 10 = x^2 - 4 \Rightarrow x = \pm 2 \Rightarrow x \in \{-2, 2\} \Rightarrow (-2,-1)$ und $(2,-1)$ stationäre Punkte von f

③ $\frac{\partial^2 f}{\partial x^2} = 2y + 2$ $\qquad \frac{\partial^2 f}{\partial x \partial y} = 2x$

$\frac{\partial^2 f}{\partial y^2} = 12y + 10$ $\qquad \frac{\partial^2 f}{\partial y \partial x} = 2x$ $\Rightarrow$

$\Rightarrow H_f(x,y) = \begin{pmatrix} 2y+2 & 2x \\ 2x & 12y+10 \end{pmatrix}, \forall (x,y) \in \mathbb{R}^2$

④ $(-2,-1), (2,-1) \rightarrow$ stationäre P

$H_f(-2,-1) = \begin{pmatrix} 0 & -4 \\ -4 & -2 \end{pmatrix}$

$\Delta_{H_f}(-2,-1) = -16 < 0 \Rightarrow H_f(-2,-1)$ ist indefinit $\Rightarrow$

$\Rightarrow (-2,-1)$ ist keine lokale Extremstelle

$H_f(2,-1) = \begin{pmatrix} 0 & 4 \\ 4 & -2 \end{pmatrix}$

$\Delta(2,-1) = -16 < 0 \Rightarrow (2,-1)$ keine lokale Extremst

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(H 2.1)

$f:\mathbb{R}^3 \to \mathbb{R}$

$$f(x,y,z) = x^3+y^3+z^3+12xy-3z$$

a)

$$\frac{\partial f}{\partial x} = 3x^2+12y \qquad \frac{\partial f}{\partial y} = 3y^2+12x \qquad \frac{\partial f}{\partial z} = 3z^2-3$$

$$\frac{\partial^2 f}{\partial x^2} = 6x \qquad \frac{\partial^2 f}{\partial y \partial x} = 12, \qquad \frac{\partial^2 f}{\partial z \partial x} = 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = 12 \qquad \frac{\partial^2 f}{\partial y^2} = 6y \qquad \frac{\partial^2 f}{\partial z \partial y} = 0$$

$$\frac{\partial^2 f}{\partial x \partial z} = 0 \qquad \frac{\partial^2 f}{\partial y \partial z} = 0 \qquad \frac{\partial^2 f}{\partial z^2} = 6z$$

$$Hf(x,y,z) = \begin{pmatrix} 6x & 12 & 0 \\ 12 & 6y & 0 \\ 0 & 0 & 6z \end{pmatrix}$$

b)

$$\begin{cases} 3x^2+12y \\ 3y^2+12x \\ 3z^2-3 \end{cases} \Rightarrow \begin{cases} 3(x^2+4y) \\ 3(y^2+4x) \\ 3(z^2-1) \end{cases} \Rightarrow \begin{cases} x^2+4y \\ y^2+4x \\ z^2-1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x^2 = -4y \\ y^2 = -4x \\ z = \pm 1 \end{cases} \Rightarrow x = y \Rightarrow x, y \in \{-4, 0\}, z \in \{-1, +1\}$$

$$\Rightarrow (-4,-4,-1), (-4,-4,1), (0,0,-1), (0,0,1)$$

d) $$Hf(0,0,1) = \begin{pmatrix} 0 & 12 & 0 \\ 12 & 0 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$\det Hf(0,0,1) = 0$$

$$Hf(-4,-4,-1) = \begin{pmatrix} -24 & 12 & 0 \\ 12 & -24 & 0 \\ 0 & 0 & -6 \end{pmatrix}$$

$\det Hf(-4,-4,-1) = -6 \cdot 24 \cdot 24 + 6 \cdot 12 \cdot 12 < 0 \Rightarrow$ negativ definit