AMCS 390 Fall 2017 Homework 2

Come prepared to present your solutions on Monday, August 28th.

1. A circulant matrix is any $m \times m$ matrix that can be written as $A = \sum_{k=0}^{m-1} a_{k+1} C^k$, where C is the cyclic shift matrix:

$$C = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & & \vdots \\ 0 & & \ddots & \ddots & 0 \\ 0 & & \cdots & 0 & 1 \\ 1 & 0 & & \cdots & 0 \end{bmatrix}$$

   Show that the eigenvectors of any such matrix are discrete Fourier modes with entries $v_j = e^{ij\xi}$. What are the appropriate values of ξ?

2. Determine the numerical dispersion relation for the heat equation semi-discretized with second-order centered differences. How does this differ from the true dispersion relation?