AMCS 390 Fall 2017 Homework 5

Come prepared to present your solutions on Thursday, November 16th.

1. Consider the two-point BVP

$$\begin{aligned} \epsilon u'' &= au' + b(t)u + q(t) \\ u(0) &= b_1 \\ u(1) &= b_2, \end{aligned}$$

where $a \neq 0$ is a constant and b, q are continuous functions, all $\mathcal{O}(1)$.

(a) Write the ODE in first-order form for the vairables $y_1 = u$ and $y_2 = \epsilon u' - au$.

(b) Letting $\epsilon \to 0$, show that the limit system is an index-1 DAE.

(c) Show that only one of the boundary conditions is needed to determine the solution of the DAE. Which one? Why?

2. Consider the DAE

$$\begin{aligned} y_1' &= y_3 y_2' - y_2 y_3' \\ 0 &= y_2 \\ 0 &= y_3. \end{aligned}$$

(a) Show that the DAE has index 1.

(b) Show that if we add to the right-hand side the (small) perturbation

$$\delta(t) = (0, \epsilon \sin(\omega t), \epsilon \cos(\omega t))^T$$

then in the perturbed solution $y_1'(t) = \epsilon^2 \omega$, which is unbounded as $\omega \to \infty$. DAEs are unstable in the sense that small perturbations to the algebraic equations can induce large perturbations in the solution.