

AMCS 390 Fall 2017 Homework 1

Come prepared to present your solutions on Thursday, August 24th.

1. Choose one of the following two problems and methods:
 - (a) The harmonic oscillator $u''(t) = -u$, and the forward Euler method.
 - (b) The test problem with decay: $u'(t) = -u$ and the explicit midpoint method.

Write a program that uses the given method to approximate the solution of the problem up to time $T = 10$. Run your program with different choices of the step size and comment on what you observe.

Is the numerical method stable? Does the numerical solution converge to the exact solution? Discuss how your answers relate to the region of absolute stability of the method.

2. Characterize all solutions of the initial value problem

$$u'(t) = \sqrt{u}$$

with initial value $u(0) = 0$. What can you say about the more general problem $u'(t) = u^{1/\alpha}$ (with $u(0) = 0$) for any $\alpha > 1$?

Bonus: what about $u'(t) = u^\alpha$ with $u(0) = 1$ for $\alpha > 1$? See Hundsdorfer & Verwer section I.2.1 for another example like these.