# Search for Sphalerons in Proton-Proton Collisions

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**Abstract**

In view of new possibilities becoming more realistic with FCC design and of recent promising results regarding $(B+L)$-violating processes detection we concentrated our research on generation and analysis of sphaleron transitions. The existence of instanton and sphaleron solutions which are associated with transitions between different vacuum states is well known since 1980s. However first calculations of instanton rate killed any hope to detect them even at very high energies while the calculation of sphaleron transitions rate is a tricky problem which continue being widely discussed. In our research we used HERBVI package to generate baryon- and lepton-number violating processes in proton-proton collisions at typical energies 14, 33, 40 and 100 TeV in order to estimate the upper limit on the sphaleron cross-section. We considered the background processes and determined the zero background regions.

## 1 Introduction

The non-Abelian nature of a Yang-Mills theory leads to a topologically nontrivial vacuum structure with an infinite number of ground states, which can be enumerated by Chern-Simons numbers $N_{CS}$. The transition between different vacuum states leads to the baryon and lepton number violation in electroweak Standard Model [1]. One can find the relation between lepton and baryon numbers variation $\Delta(B+L) = 6N_{CS},\ \Delta(B-L) = 0$.

The gauge-Higgs system is similar to a particle in the periodic potential (see Fig.1). An **instanton** is a localized, finite-action solution of the euclidean field equations, which describes the tunneling between vacua with different Chern-Simons numbers. There is also a second - static, unstable, finite-energy solution, known as **sphaleron** - which describes the transitions in a classical way possible only at high energies. The height of the barrier between different vacua at zero temperature is equal to the static energy of the sphaleron solution:

$$E_{sph} = \frac{2m_W}{\alpha_W} B\left(\frac{m_H}{m_W}\right) \approx 9 \text{ TeV} \tag{1}$$

where $m_W, m_H$ are W- and Higgs boson masses, $\alpha_W = \frac{g^2}{4\pi} = \frac{1}{29}$ - electroweak constant, B - a tabulated function.

Sphaleron can be considered as an unstable configuration of fields, which decays to the vacuum by emission of many particles (see Fig.2).

## 2 Sphaleron at colliders

In analogy to the quantum-mechanical results for a particle tunneling under the potential barrier between different states, one can estimate the instanton tunneling probability $\sigma_{inst} \propto \exp\left(-\frac{4\pi}{\alpha_W} \sim 10^{-170}\right)$, which is unobservably small. The amplitude is expected to be enhanced

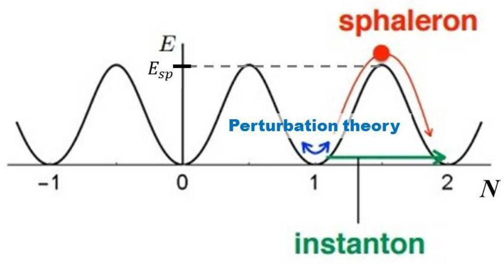


Figure 1: Energy density of the gauge field as a function of Chern-Simons numbers

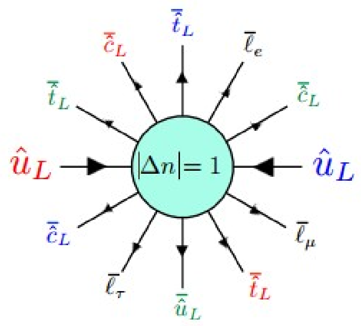


Figure 2: Graphical representation of an electroweak sphaleron transition

when the energies approach $E_{sph}$, since at energies above sphaleron the system can in principle evolve from one vacuum state to another in a classical way. At colliders we expect a sphaleron transition to give us the following Baryon and Lepton number violating (BLNV) process:

$$q + q \to 7\overline{q} + 3\overline{l} + n_B W(Z) + n_H H \tag{2}$$

The amplitude of it increases when it involves a large number of gauge and Higgs bosons. This process can not be described in terms of Feynman diagrams (is non-perturbative), but attempts to estimate its rate have been made within the instanton "perturbation theory" [3], which leads to the following expression for the BLNV cross section:

$$\sigma_{sph} \propto \exp\left[\frac{4\pi}{\alpha_W} S\left(\frac{E}{E_{sph}}\right)\right] \tag{3}$$

The function S is called the suppression factor, and its expansion is known only for low energies ($E << E_{sph}$) [4]. The estimation of the boson multiplicity leads to the following results:

$$\overline{n}_B \sim \frac{3}{2}\frac{\pi}{\alpha_W}\left(\frac{E}{E_{sph}}\right)^{4/3}, \quad \frac{n_H}{n_B} \sim \frac{1}{16} \tag{4}$$

The instanton approach breaks down at energies close to or higher than the sphaleron, eventually violating unitarity.
Another method to approximate sphaleron cross section [2] considers the sphaleron transitions modelled by a one-dimensional Schrödinger equation. It was shown that there is no further exponential suppression at energies higher than $E_{sph}$, the obtained cross section result still allows to calculate the value scaled by unknown constant. The main prediction of the research based on this approach [5] is the real possibility of sphaleron detection at FCC energies (maybe even at HL-LHC). Such a powerful statement renewed the interest toward the sphaleron study and stimulated our research.

# 3 Simulation of sphaleron process

In our study we used HERBVI [6] - hard-process Monte Carlo event generator interfacing to HERWIG, which is a general-purpose Monte Carlo event generator (version 6.5). HERBVI generates the BLNV process (2) corresponding to the transition between closest vacuum states: $\Delta B = \Delta L = -3$ initiated by valence quarks only ($u$ or $d$ quark) with a flat parton level cross section. We should emphasize the fact that HERBVI does not allow us to calculate the magnitude of sphaleron cross section, but still provides us with the possibility to investigate the properties of events. In order to continue with the qualitative and statistical analysis of generation results we changed the HERBVI output to the HepMC format. The advantage of using HERBVI+HERWIG is that one can control:

– parton distributions. Initially there were three internal HERWIG sets [7] of PDFs, but we also managed to interface it to LHAPDF library (the latest version - 6.1)

– boson multiplicity $n_B$. The default distribution is a fixed value, but it is also possible to apply a normal distribution modelled on the leading order BLNV matrix elements.

– process type - BLNV (sphaleron) process or multi-W (B+L conserved) process (consider only $qq$ interaction, since the $\bar{q}\bar{q}$, $q\bar{q}$ are much smaller in magnitude):

$$q_1 + q_2 \to q_3 + q_4 + n_B W(Z) + n_H H \tag{5}$$

# 4 Simulation results

As it was mentioned in the previous section, we can control in HERBVI parton and boson distributions. One can see on the Fig.3 that there is an evident difference between modern (LHAPDF) parton distributions and the old one (MSTW1998), when the difference between modern PDFs themselves is small enough to choose any of them without expecting any noticeable influence.

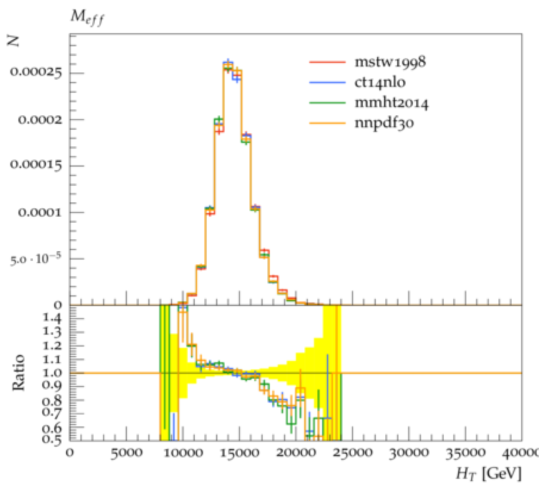


Figure 3: Distribution of the total energy of the events for different PDFs

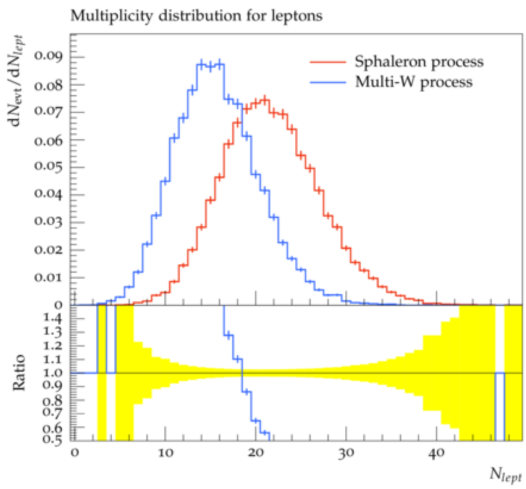


Figure 4: Distribution of the lepton multiplicity for two different HERBVI running modes

Another important question is how the boson multiplicity influences the generation, since there are two possible modes for it in HERBVI. First one is for a fixed number of bosons, since we don't have any theoretical prediction for sphaleron cross section, it's quite natural to use the simplest model we can. We expect a process with a huge number of produced bosons and we know the estimation of $n_B \sim 1/\alpha_W \approx 30$ ( see results on Fig.(6) ). The instanton approach leads to an energy dependence of the boson multiplicity (4), whereas its main idea is to use the perturbation theory for BLNV process (2) and to calculate the cross section and other parameters in leading order in the electroweak coupling $\alpha_W$. That's why the resulting model is called LOME (leadig order matrix element) (Fig. (5)). Since the boson multiplicity increases with the collision energy, it becomes easier to distinguish signal for higher energies, because there are not any other observable processes with such a big number of leptons in the end.

# 5 Cross section limits

The distributions we got allow us to carry out a qualitative analysis and put the limits on the sphaleron rate. We expect it to be possible to detect sphalerons at high energy proton-proton colliders, therefore we should know how the background looks like. The main sphaleron

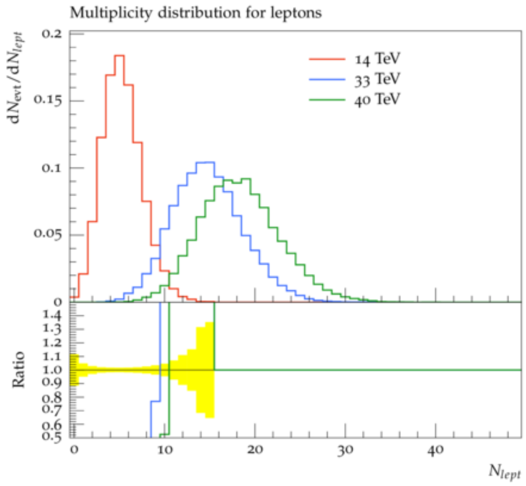


Figure 5: Distribution of the lepton multiplicity at different collision energies within LOME model for $n_B$

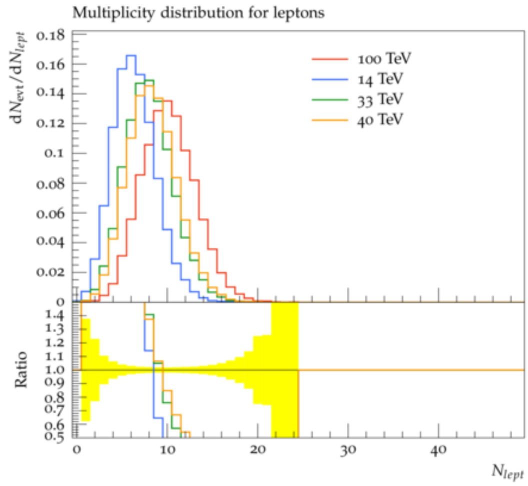


Figure 6: Distribution of the lepton multiplicity at different energies in case of fixed number of bosons ($n_B = 30$)

characteristic is the large number of bosons, resulting in a large number of leptons. The possible backgrounds are multi-W(Z) production (which can be generated with HERBVI (5)) and multiple top quark production processes.

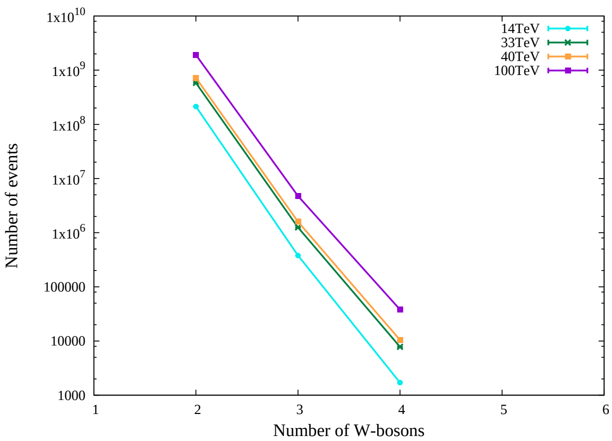


Figure 7: Dependence of multi-W process number of events on number of Ws for different energies. Integrated luminosity is assumed $3000fb^{-1}$.

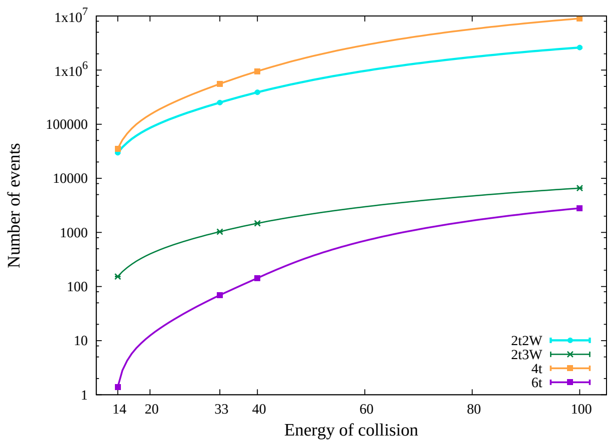


Figure 8: Dependence of different multiple top and multi-W process number of events on energy. Integrated luminosity is assumed $3000fb^{-1}$.

As we see (Fig. 4) the lepton distribution is similar for sphaleron and multi-W processes with a small shift to the region of higher lepton numbers for sphalerons. Being this difference small, it should be difficult to separate signal and background. It however is possible to calculate multi-W and multiple top quark cross-sections from ordinary perturbation theory. With the help of MadGraph we calculated in the leading order the multi-W cross section up to 4 W-bosons and multi-top cross sections up to 6 top quarks as well as 2t2W and 2t3W processes and then extrapolated the data. To continue calculations with a larger number of W-bosons and top quarks in proton-proton collisions we require much more computing resources than we have or the development of non-perturbative approach which faces with the same problems as one for sphalerons. We see (Fig. (7, 8)) that the cross section is decreasing with the growth of W-boson and top quarks number, that allows us to expect negligibly small cross section for such kind of processes at $n_B \sim 10$ (more detailed explanation can be found in [8, 9]).

In table 1 we indicate the lepton requirements applied at different energies and their corresponding acceptance (where the leptons satisfy $p_T > 25$ TeV, $|\eta| < 2.5$). The integrated luminosity is assumed to be $L = 3000 fb^{-1}$. We consider zero background, thus the 95% confidence level upper limit for number of events gives us $s_{upl} \sim 3$ evt (note: the estimation is made by frequentist approach without systematic uncertainties for simplicity). The last two columns of the table 1 include the cross section upper limit for sphalerons and the values calculated in [5], which are obviously excluded within our assumptions. In fact the magnitudes for sphaleron cross section in the last column can be substantially reduced, because they contain an unknown factor $p$, which was fixed by authors equal to 1, but nobody forbids it to be $\ll 1$.

Table 1: Upper limit for sphaleron cross section at different energies

| $E$ [TeV] | cuts | Acceptance | $\sigma_{upl}$, pb | $\sigma_{sph}$, pb [5] |
|---|---|---|---|---|
| 14 | 6 | 0.439 | $2.27 \cdot 10^{-6}$ | $41 \cdot 10^{-3}$ |
| 33 | 10 | 0.979 | $1.08 \cdot 10^{-6}$ | 300 |
| 40 | 10 | 0.998 | $1.013 \cdot 10^{-6}$ | - |
| 100 | 12 | 1 | $1.00 \cdot 10^{-6}$ | $141 \cdot 10^{3}$ |

# 6 Conclusions

Sphaleron transitions provide a source of $(B + L)$-violation in the Standard Model and are of a great interest for future high energy proton colliders, such as FCC-hh. While the cross-section for sphaleron production is unknown, it is still possible to study its properties. We used HERBVI for the sphaleron simulation, while the background was studied with MadGraph. It was shown that one can consider zero background for the region where the produced lepton number is higher than 6 at 14 TeV, 12 at 100 TeV. The upper limit obtained for sphaleron cross section is $2.27 \cdot 10^{-6}$ pb at 14 TeV and $1.00 \cdot 10^{-6}$ pb at 100 TeV, which excludes the cross section predictions made in [5].

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