Search for Sphalerons in Proton-Proton Collisions

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Abstract

In view of new possibilities becoming more realistic with FCC design and of recent promising results regarding $(B+L)$-violating processes detection we concentrated our research on generation and analysis of sphaleron transitions. Since the Standard Model still remains valid for most fundamental interactions the sphaleron study is of current importance providing the missing explanation of baryon-antibaryon asymmetry. The existence of instanton and sphaleron solutions which are associated with transitions between different vacuum states is well known since 1980s [1, 2]. However first calculations of instanton rate killed any hope to detect them even at very high energies (it occurs to be exponentially suppressed) while the calculation of sphaleron transitions rate is a tricky problem which continue being widely discussed. In our research we considered the baryon- and lepton-number violating processes in proton-proton collisions at high energies in order to estimate the upper limit on the sphaleron cross-section.

1 Introduction

It is well known that both baryon (B) and lepton (L) numbers are not conserved in the standard electroweak theory.The non-Abelian nature of a Yang-Mills theory leads to a topologically nontrivial vacuum structure with an infinite number of ground states. The existence of instanton/sphaleron solutions [3] even in case of the simplified form of the electroweak Lagrangian (1)

$$L = -\frac{1}{2}Tr[F_{\mu\nu}F^{\mu\nu}] + \frac{1}{2}(D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\lambda}{4}(\Phi^\dagger\Phi - v^2)^2 + i\overline{\Psi}_L^{(i)}\gamma^\mu D_\mu\Psi_L^{(i)} \tag{1}$$

leads to the fermion numbers conservation breakdown, therefore the transition between different vacuum states (the change of fermion numbers) becomes possible. These states can be numerated by Chern-Simons numbers - integers:

$$N = \frac{g^2}{16\pi^2}\int d^4x Tr[F_{\mu\nu}F^{\mu\nu}] \tag{2}$$

Thus one can easily find the relation between lepton and baryon numbers variation:

$$\Delta N_e = \Delta N_\mu = \Delta N_\tau = N \tag{3}$$

$$\Delta B = 3N \tag{4}$$

$$\Delta(B+L) = 6N, \quad \Delta(B-L) = 0 \tag{5}$$

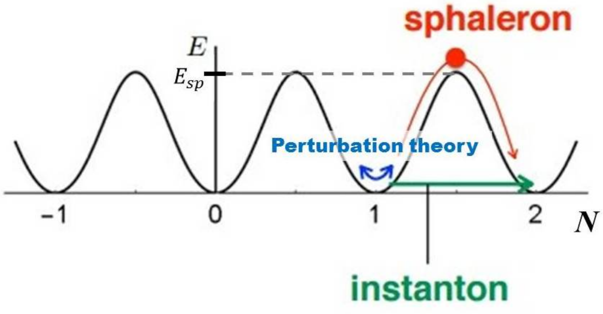

Figure 1: Energy density of the gauge field as a function of Chern-Simons numbers

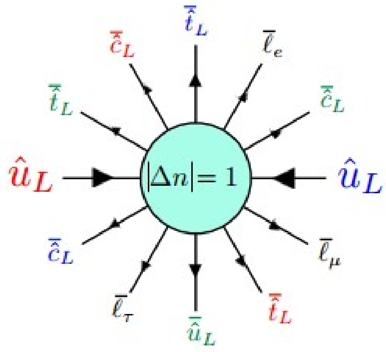

Figure 2: Graphical representation of the Stadard Model electroweak process, referred to as a sphaleron. The simplest type of sphaleron, pictured, changes baryon number by 3 units

2 Electroweak instanton and sphaleron theory

The gauge-Higgs system is similar to a particle in the periodic potential (see Fig.1). An **instanton** is a localized, finite-action solution of the Euclidean field equations, which describes the tunneling event between vacua with different Chern-Simons numbers. There is also the second - static, unstable, finite-energy solution, known as **sphaleron** - which describes the transitions in a classical way possible only at high energies. The height of the barrier between different vacua at zero temperature is equal to the static energy of the sphaleron solution:

$$E_{sph} = \frac{2m_W}{\alpha_W} B\left(\frac{m_H}{m_W}\right) \tag{6}$$

where m_W, m_H are W- and Higgs boson masses, $\alpha_W = \frac{g^2}{4\pi} = \frac{1}{29}$ - electroweak constant, B - a tabulated function.
In analogy to the quantum-mechanical results for a particle tunneling under the potential barrier between different states, one can estimate the instanton tunneling probability $\sigma_{inst} \propto \exp\left(-\frac{4\pi}{\alpha_W} \sim 10^{-170}\right)$, which is unobservably small, that's why nobody expects to detect them. The amplitude is expected to be enhanced when the energies approach E_{sph}, since at energies above sphaleron the system can in principle evolve from one vacuum state to another in a classical way. Sphaleron corresponds to an unstable configuration of fields, which decays to the vacuum by emission of many particles (see Fig.2). Similarly to Feynman approach to perturbative processes we can treat a sphaleron at collider in the following way (it is natural to expect the large number of gauge and Higgs bosons in the end):

$$q + q \to 7\overline{q} + 3\overline{l} + n_B W(Z) + n_H H \tag{7}$$

The Baryon and Lepton number violating (BLNV) process (7) can not be described in terms of Feynman diagrams (is non-perturbative), but still it is possible to estimate its rate within the instanton approach [5] - the "perturbation theory" for instanton solutions. It was found that the cross section of the sphaleron process grows exponentially with boson multiplicity and parton-parton centre of mass energy. The general expression for the BLNV cross section is the following:

$$\sigma_{sph} \propto \exp\left[\frac{4\pi}{\alpha_W} S\left(\frac{E}{E_{sph}}\right)\right] \tag{8}$$

The function S is called the suppression factor, whereas its expansion is known only for low energies ($E << E_{sph}$) [6]. The estimation of boson multiplicity in the same region of energies leads to the following results:

$$\overline{n_B} \sim \frac{3}{2}\frac{\pi}{\alpha_W}\left(\frac{E}{E_{sph}}\right)^{4/3}, \quad \frac{n_H}{n_B} \sim \frac{1}{16} \tag{9}$$

There are several problems in the instanton approach which leads to the theory breakdown at energies close or higher than the spahleron. First one is about the suppresion factor expansion, when $\frac{E}{E_{sph}}$ is no longer small we should include all powers in the sum, but we don't have an a priori model for it. Another problem is concerned with the unitarity violation due to the exponential growth.
Another attempt to calculate sphaleron cross section was made recently by Jonh Ellis and Kazuki Sakurai [7]. Their calculations were based on Tye and Wong (TW) approach [4], which consider the sphaleron transitions modelled by a one-dimensional Schrödinger equation.They showed that there is no any further exponential suppression since the energies become higher than E_{sph}, the obtained cross section result still allows to calculate the value scaled by unknown constant. The correctness of the theory is widely discussed now, but anyway the main prediction of the research made by Ellis and Sakurai is the real possibility of sphaleron detection at FCC energies (maybe even at HL-LHC). Such a powerful statement renewed the interest toward sphaleron study and stimulated our research, since we want to have a look at these processes within numerical methods based on instanton approach.

3 Simulation of sphaleron process

In order to simulate sphaleron process we used HERBVI [8] - hard-process Monte Carlo event generator interfacing to HERWIG, which is a general-purpose Monte Carlo event generator (version 6.5). The BLNV process generated by HERBVI corresponds to the case: $\Delta B = \Delta L = -3$ and is cosidered in terms of reaction (7). In fact there are two types of BLNV processes: qq and $\bar{q}\bar{q}$, but the contribution of the last one is two to three orders of magnitude smaller, therefore we suppose it to be negligible. The parton level cross section is assumed to be flat, the matrix elements in cross section expression are constant, that means the distributions of produced particles being determined by phase space only. We consider a steeply rising cross section, which is cut off at some energy in order to satisfy unitarity constraints. We should emphasize the fact that HERBVI does not allow us to calculate the magnitude of sphaleron cross section, but still it provides us possibility to investigate the properties of events. The advantage of this package is that one can control:

- parton distributions, using either internal HERWIG sets [9] or LHAPDF library (the latest version - 6.1)
- boson multiplicity n_B - either a fixed value or a normal distribution modelled on the leading order BLNV matrix element
- process type - BLNV (sphaleron) process or non-perturbative multi-W (B+L conserved) process (consider only qq interaction, since the $\bar{q}\bar{q}$, $q\bar{q}$ are much smaller in magnitude):

$$q_1 + q_2 \to q_3 + q_4 + n_B W(Z) + n_H H \tag{10}$$

4 Simulation results

It was important to check the correctness of HERBVI generation as well as distinction in the results one can get applying different conditions. Hence we would like to discuss in this section basic HERBVI running modes. As it was mentioned earlier, we can control in HERBVI parton and boson distributions. One can see on the Fig.3 that there is an evident difference between modern (LHAPDF) parton distributions and the old one (MSTW1998), when the difference between modern PDFs themselves is small enough to choose any of them without expecting any noticeable influence.

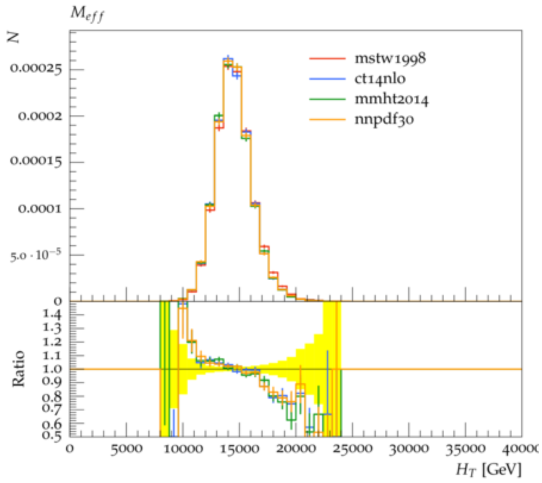

Figure 3: Distribution of the total energy of the events for different PDFs

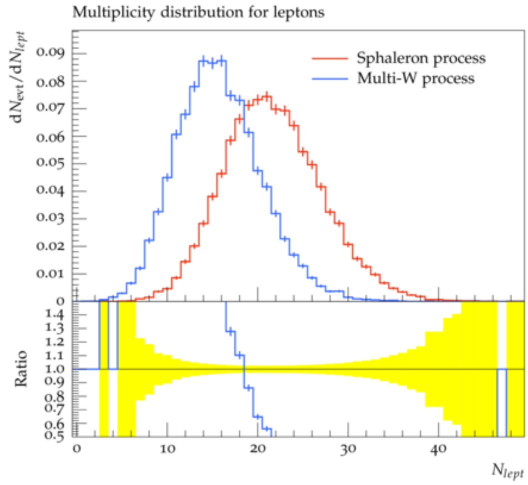

Figure 4: Distribution of lepton multiplicity in events for two different HERBVI running modes

Another important question is how the boson multiplicity influences the generation, because two modes are possible in HERBVI. First one is for fixed number of bosons, since we don't have any theoretical prediction for sphaleron cross section, it's quite natural to use the simplest model we can. We expect a process with a huge number of produced bosons and we know the estimation of $n_B \sim 1/\alpha_W \approx 30$ (see results on Fig.(6)). On the other hand the instanton approach leads to the energy dependence of boson multiplicity (9). The main idea of instanton approach is to use the perturbation theory for BLNV process (7) and to calculate the cross section and other parameters in leading order in the electroweak coupling α_W, that's why the resulting model is called LOME (leadig order matrix element) (Fig. (5)). Since the boson multiplicity increases with the collision energy, it becomes easier to distinguish signal for higher energies, because there are not any other observable processes with such a big number of leptons in the end.

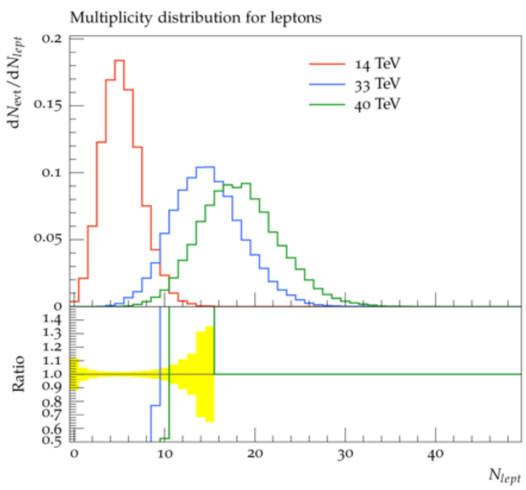

Figure 5: Distribution of lepton multiplicity in events for differen energies in case of LOME number of bosons

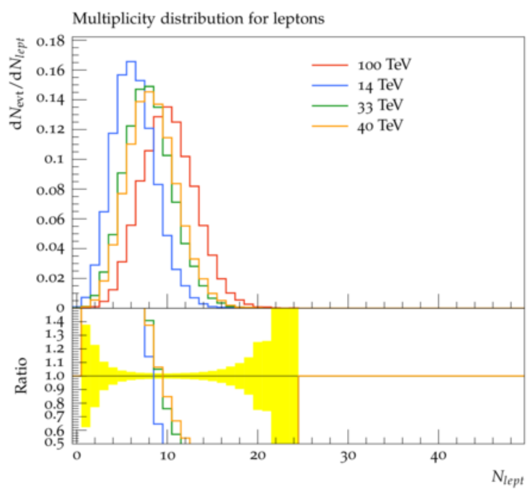

Figure 6: Distribution of lepton multiplicity in events for differen energies in case of fixed number of bosons ($n_B = 30$)

5 Cross section limits

The generation of BLNV process with HERBVI still doesn't give the information about the cross section magnitude. The distributions we get allow us to carry out a qualitative analysis and put the limits on the spahleron rate. We expect it to be possible to detect sphalerons at high energy proton-proton colliders, therefore we should know how the background looks like. The main sphaleron characteristic is the huge number of bosons resulting in huge number of leptons. The possible backgrounds are multi-W(Z) production and multiple top quark production processes. HERBVI can generate non-perturbative multi-W processes (10) and as we see (Fig. 4) the distributions of final particles (for example leptons) are similar for sphaleron and multi-W processes with a small shift in the region of higher lepton numbers for sphalerons, but the difference is so small, that it should be very difficult to separate signal and background. On the other hand it is possible to calculate multi-W production and multiple top quark in terms of perturbation theory for numbers ≤ 6 and then extrapolate the data. We see (Fig. (7, 8)) that the cross section is decreasing with the growth of W and top number, that allows us to expect negligible small cross section for such kind of processes at $n_B \sim 10$.

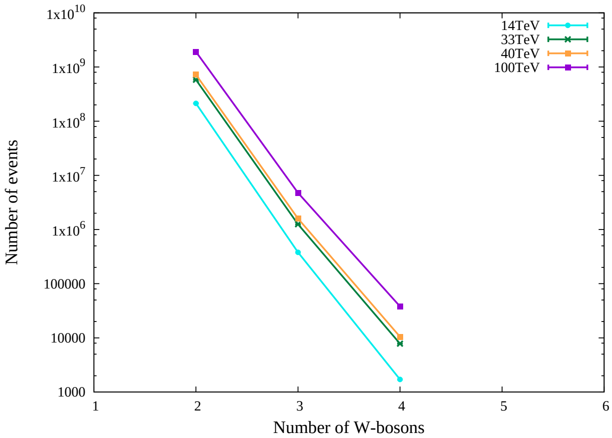

Figure 7: Dependence of multi-W process cross sections on number of Ws for different energies

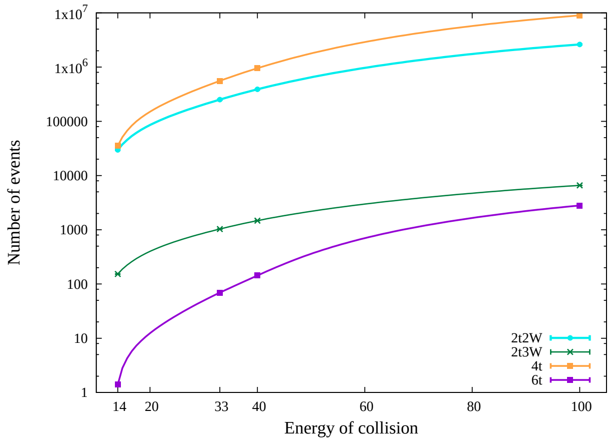

Figure 8: Dependence of different multi-top process cross sections on energy

6 Conclusions

- Sphalerons provide a great interest for future collider physics, since they can only be observed in high energy proton-proton collisions. If these sphaleron processes are detected, we'll get:
  - a truly remarkable breakthrough in understanding non-perturbative EW dynamics
  - clarification of baryogenesis.
- We tested the sphaleron generator HERBVI, examined different running modes, studied the influence of parton and boson distributions on the sphaleron decay products.
- The output was modified to the format common for collider physics (HepMC), hence it is easy to continue detector simulation and further statistical analysis of the HERBVI results.
- The background simulation was performed in order to estimate sensitivity limits (our next step).
- The qualitative analysis of sphaleron process was carried out; the magnitude of the sphaleron cross section, however, remains undefined

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