Search for Sphalerons in Proton-Proton Collisions

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Abstract

In view of new possibilities becoming more realistic with FCC design and of recent promising results regarding $(B+L)$-violating processes detection we concentrated our research on generation and analysis of sphaleron transitions. Since the Standard Model still remains valid for most fundamental interactions the sphaleron study is of current importance providing the missing explanation of baryon-antibaryon asymmetry. The existence of instanton and sphaleron solutions which are associated with transitions between different vacuum states is well known since 1980s [1,2]. However first calculations of instanton rate killed any hope to detect them even at very high energies (it occurs to be exponentially suppressed) while the calculation of sphaleron transitions rate is a tricky problem which continue being widely discussed. In our research we considered the baryon- and lepton-number violating processes in proton-proton collisions at FCC energies in order to estimate the upper limit on the sphaleron cross-section.

1 Introduction

It is well known that both baryon (B) and lepton (L) numbers are not conserved in the standard electroweak theory. Within the Standard Model the electromagnetic and the weak interactions are unified to the electroweak interaction and the corresponding symmetry is a composition of an Abelian $U(1)$ and a non-Abelian $SU(2)$ group. The non-Abelian nature of a Yang-Mills theory leads to a topologically nontrivial vacuum structure with an infinite number of ground states ("topological charges"). In order to understand the origin of instanton/sphaleron solutions [3] one can consider the simplified form of the electroweak Lagrangian:

$$L = -\frac{1}{2}Tr[F_{\mu\nu}F^{\mu\nu}] + \frac{1}{2}(D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\lambda}{4}(\Phi^\dagger\Phi - v^2)^2 + i\overline{\Psi}_L^{(i)}\gamma^\mu D_\mu\Psi_L^{(i)} \tag{1}$$

There should exist $n_L = 12$ (three families of quarks and leptons) conserved global $U(1)$ currents at the classical level, which correspond to the fermion numbers conservation. However, this conservation is broken, therefore the transition between different vacuum states (the change of fermion numbers) becomes possible. These states can be numerated by Chern-Simons numbers - integers:

$$N = \frac{g^2}{16\pi^2}\int d^4x Tr[F_{\mu\nu}F^{\mu\nu}] \tag{2}$$

Thus one can easily find the relation between lepton and baryon numbers variation:

$$\Delta N_e = \Delta N_\mu = \Delta N_\tau = N \tag{3}$$

$$\Delta B = 3N \tag{4}$$

$$\Delta(B+L) = 6N, \quad \Delta(B-L) = 0 \tag{5}$$

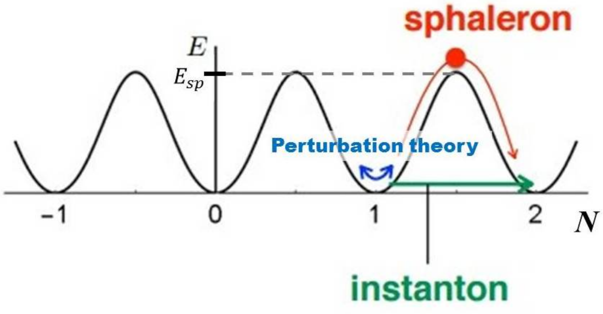

Figure 1: Energy density of the gauge field as a function of Chern-Simons numbers

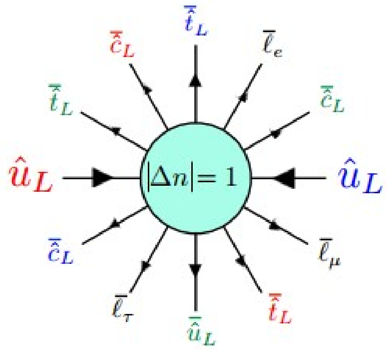

Figure 2: Graphical representation of the Stadard Model electroweak process, referred to as a sphaleron. The simplest type of sphaleron, pictured, changes baryon number by 3 units

2 Electroweak instanton and sphaleron theory

The gauge-Higgs system is similar to a particle in the periodic potential (see Fig.1). An **instanton** is a localized, finite-action solution of the Euclidean field equations, which describes the tunneling event between vacua with different Chern-Simons numbers. There is also the second - static, unstable, finite-energy solution, known as **sphaleron** - which describes the transitions in a classical way possible only at high energies. The height of the barrier between different vacua at zero temperature is equal to the static energy of the sphaleron solution:

$$E_{sph} = \frac{2m_W}{\alpha_W} B\left(\frac{m_H}{m_W}\right) \tag{6}$$

where m_W, m_H are W- and Higgs boson masses, $\alpha_W = \frac{g^2}{4\pi} = \frac{1}{29}$ - electroweak constant.
In analogy to the quantum-mechanical results for a particle tunneling under the potential barrier between different states, one can estimate the instanton tunneling amplitude, since the action is known:

$$A_{inst} \propto \exp(-S_{inst}), \quad S_{inst} = \frac{8\pi^2}{g^2} \tag{7}$$

The instanton process probability is unobservably small: $\sigma_{inst} \propto \exp\left(-\frac{4\pi}{\alpha_W} \sim 10^{-170}\right)$, that's why nobody expects to detect them. The amplitude is expected to be enhanced when the energies approach E_{sph}, since at energies above sphaleron the system can in principle evolve from one vacuum state to another in a classical way. Sphaleron corresponds to an unstable configuration of fields, which, after a small perturbation, decays to the vacuum by emission of many particles. Consider the baryon- and lepton-number violating process (BLNV process) which corresponds to $\Delta N = -1$ (it is natural to expect the large number of gauge and Higgs bosons in the end) (see Fig.2):

$$q + q \to 7\overline{q} + 3\overline{l} + n_B W(Z) + n_H H \tag{8}$$

The BLNV process (8) can not be described in terms of Feynman diagrams (is non-perturbative), but still it is possible to estimate its rate within the instanton approach [5] - the "perturbation theory" for instanton solutions. It was found that the cross section of the sphaleron process grows exponentially with boson multiplicity and parton-parton centre of mass energy. The general expression for the BLNV cross section is the following:

$$\sigma_{sph} \propto \exp\left[\frac{4\pi}{\alpha_W} S\left(\frac{E}{E_{sph}}\right)\right] \tag{9}$$

The function S is called the suppression factor, whereas its expansion is known only for low energies ($E << E_{sph}$) [6]:

$$S = -1 + \frac{9}{8}\left(\frac{E}{E_{sph}}\right)^{4/3} - \frac{9}{16}\left(\frac{E}{E_{sph}}\right)^{2} + O\left(\left(\frac{E}{E_{sph}}\right)^{8/3}\right) \tag{10}$$

The estimation of boson multiplicity in the same region of energies (8) leads to the following results:

$$n_B \propto \frac{1}{\alpha}, \quad \frac{n_H}{n_B} \sim \frac{1}{16} \tag{11}$$

The unresolved problem is how to calculate the BLNV processes cross section for energies close to or higher then the sphaleron energy E_{sph}. There are several problems in the instanton approach. First one is about the suppresion factor expansion, when $\frac{E}{E_{sph}}$ is no longer small we should include all powers in the sum, but we don't have an a priori model for it. Another problem is concerned with the unitarity violation due to the exponential growth.
Another attempt to calculate sphaleron cross section was made recently by Jonh Ellis and Kazuki Sakurai [7]. Their calculations were based on Tye and Wong (TW) approach [4], which consider the sphaleron transitions modelled by a one-dimensional Schrödinger equation of the form (see the definitions in [4]):

$$\left(-\frac{1}{2m}\frac{\partial^2}{\partial^2 Q} + V(Q)\right)\Psi(Q) = E\Psi(Q) \tag{12}$$

The effective potential $V(Q)$ is periodic, that's why the expected solution is Bloch wave and the band structure of energies can be easily obtained. There is no any further exponential suppression since the energies become higher than E_{sph}, the obtained cross section result still allows to calculate the value scaled by unknown constant. The correctness of the theory is widely discussed now, but anyway the main prediction of the research made by Ellis and Sakurai is the real possibility of sphaleron detection at FCC energies (maybe even at HL-LHC). Such a powerful statement renewed the interest toward sphaleron study and stimulated our research, since we want to have a look at these processes within numerical methods based on instanton approach.

3 Baryogenesis

To argue the actuality of this research it is worth to mention the baryogenesis in the Standard Model. There are three required components of all baryogenesis models, known as Sakharov conditions:

- C and CP violation
- Thermal nonequilibrium
- Baryon number (B) violation

The first condition is introduced in the SM via Cabibbo-Kobayashi-Maskawa (CKM) matrix, the second one can be explained within the standard cosmology model ΛCDM, what about the last one? It has been shown that during the electroweak phase transition the sphalerons can induce a baryon asymmetry [2].

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