Topological Anomalies and Modular Bootstrap in 2D CFTs with Discrete Symmetries

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Two-dimensional *conformal field theories* (CFTs) play a central role in describing critical phenomena and the ultraviolet or infrared limits of many quantum theories. In the thesis we analyze the structure of the conformal group in arbitrary dimensions and then focus on the two-dimensional case. In two dimensions the algebra becomes infinite-dimensional and is called the Virasoro algebra. In this context we focus on the study of CFTs defined on the torus, on this surface it is possible to exhibit an additional symmetry, known as modular invariance. This further symmetry is at the center of our analysis: the modular invariance of the partition function is translated into consistency constraints. These constraints are the basis of the *modular bootstrap* program, a non-perturbative method that allows us to infer bounds on the spectrum of the theory. This method is particularly useful because it does not use either the Lagrangian of the theory or the operator content but only uses how partition functions are mixed under modular transformation. A central aspect of the work concerns the study of discrete symmetries and *Topological Defect Lines* (TDL) which implement on the torus the action of the global symmetry group and can give rise to topological anomalies. The presence of a discrete symmetry in a 2-dimensional CFT introduces new partition functions associated with the topological defects and eigenspaces of the symmetry and this modifies the modular constraints that the theory must respect.
Starting with the abelian cases $\mathbb{Z}_2$ and $\mathbb{Z}_3$, for which the modular transformation matrices of the partition function vector and the numerical results of the bounds are known, the analysis was extended to the non-abelian case of the semidirect product $\mathbb{Z}_3 \rtimes \mathbb{Z}_2 = S_3$. In this context, the symmetry group representations are no longer one-dimensional and the defect fusion rules become more complicated than in previous cases. This extension represents the original theoretical contribution of the thesis, since it allows to explicitly formulate the equations of *modular bootstrap* for theories endowed with non-abelian symmetry, defining the space of partition functions closed under the action of the modular group and allowing us to infer new bounds associated to a CFT with symmetry S_3. Finally, the method was implemented numerically through scripts in *Mathematica* and *Python*, which translate the modular invariance conditions into a semidefinite programming problem solved through the *SDPB* packet through the use of the *linear function* α. Such implementation allowed obtaining the upper *bound* on the conformal dimension of the lightest primary admitted in a theory consistent with modular invariance and symmetry S_3. The upper bound, $\Delta^*_N(c)$, was calculated as the central charge, c, varied and the maximum derivative order, N, of the functional α varied.

Keywords: conformal field theory, modular bootstrap, discrete symmetries, topological defects, anomalies, $\mathbb{Z}_2$, $\mathbb{Z}_3$, S_3, semidefinite programming.