А. Случайная величина ξ имеет $N(0,1)$ распределение

$$p_\xi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad x \in \mathbb{R}$$

Найти распределение $\eta = e^{\xi}$

$\operatorname{supp} \xi = \mathbb{R}$

$\operatorname{supp} \eta = \mathbb{R}^+$ (без $\{0\}$)

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \circledast, & x \in (0; +\infty) \end{cases}$$

$$\underline{F_\eta}(x) = P(\eta < x) =$$

$$= \underbrace{P(e^{\xi} < x)} = P(\xi < \ln x) =$$

$$= \underline{F_\xi(\ln x)}$$

$$\underbrace{p_\eta}(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ *, & x \in (0; +\infty) \end{cases}$$

$x \in (-\infty; +\infty)$

$$F_\xi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt =$$

$$F_\xi(\ln x) = \int_{-\infty}^{\ln x} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt$$

$\parallel$

$F_\eta(x)$ 0, потому, что $x \in (0; +\infty)$

$$= \int_{-\infty}^{\ln x} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt =$$

$$p_\eta = \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} e^{\frac{-\ln^2 x}{2}}$$

$$p_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{\ln^2 x}{2}}, & x \in (0; +\infty) \end{cases}$$

Проверочка:

$$\int_0^\infty \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} e^{\frac{-\ln^2 x}{2}} dx = \left[\begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \end{array} \right]$$

$x \in 0; \infty$

$t \in -\infty; \infty$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t^2}{2}} dt = 1$$

(Норм. распр)

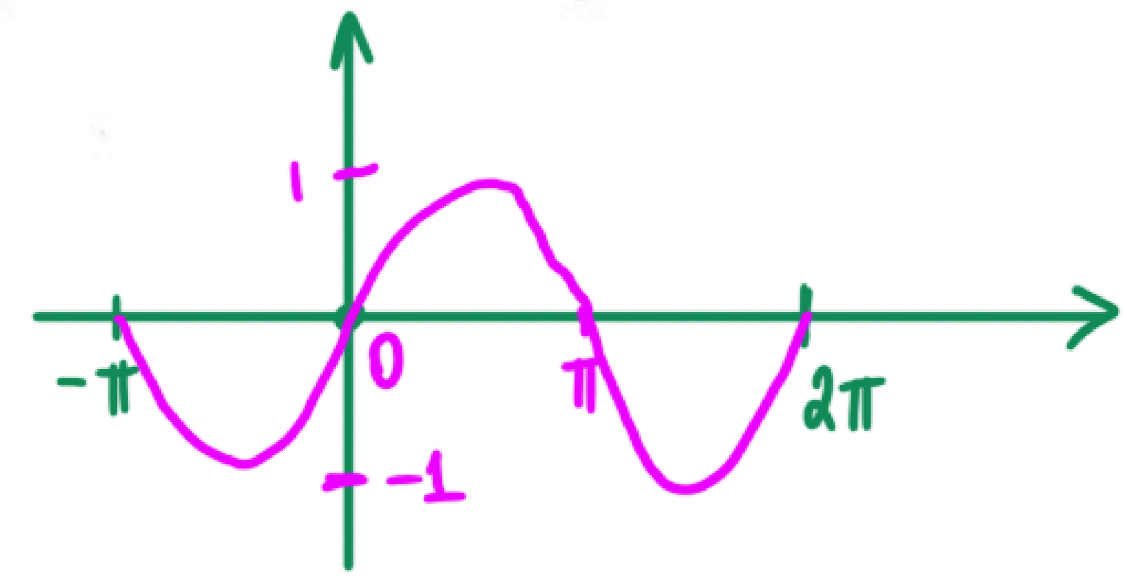
$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \int_0^x p_\eta(x) dx, & x \in (0; +\infty) \end{cases}$$

8. Случайная величина ξ имеет распределение с плотностью

$$p_\xi(x) = \begin{cases} \frac{1}{3}, & x \in [-1, 2] \\ 0, & \text{в ост случаях} \end{cases}$$

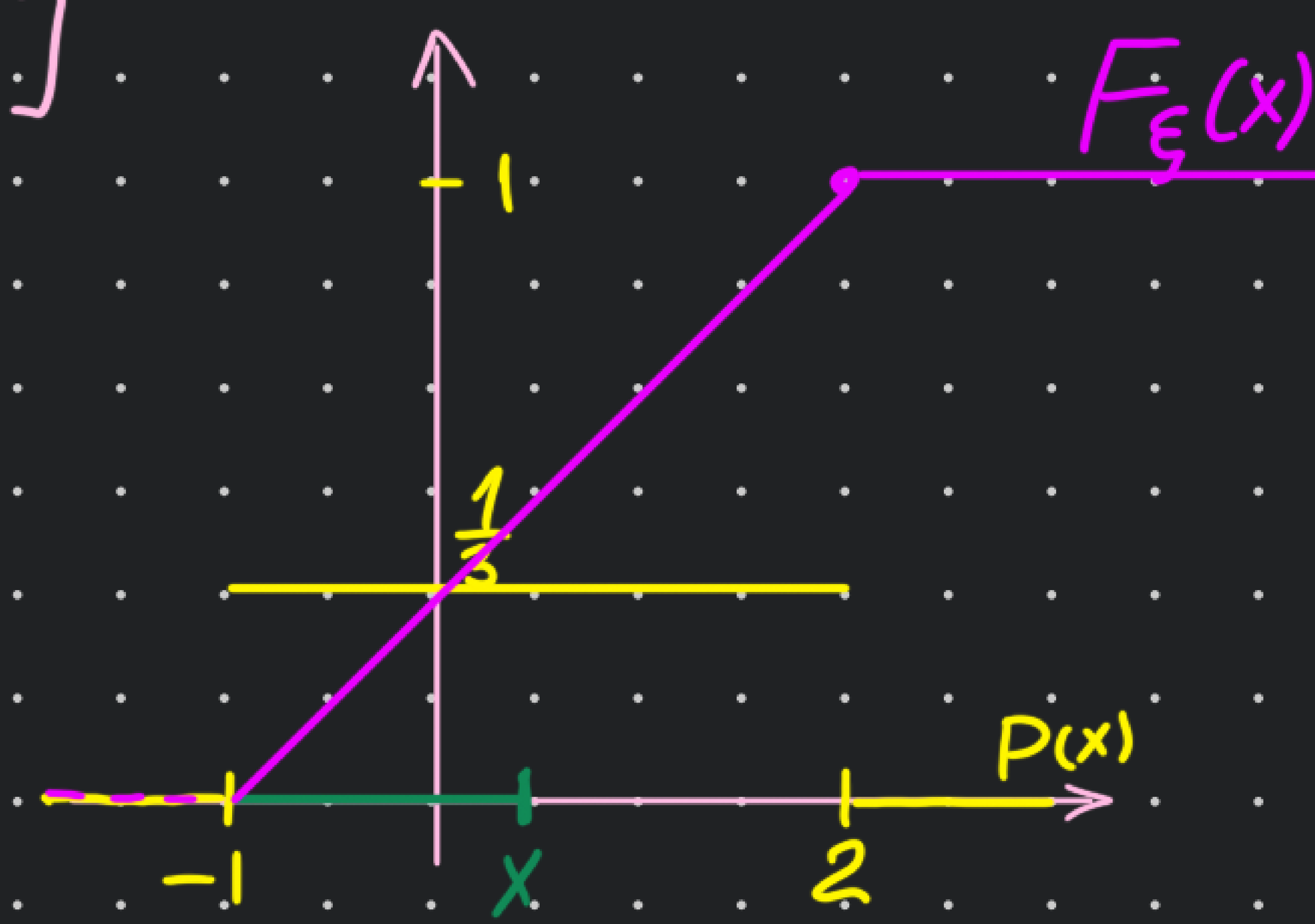
Найти распределение а) $\eta = \sin(\pi\xi)$

б) $\eta = 1/\xi$



$$p_\xi(x) = \begin{cases} \frac{1}{3}; & x \in [-1, 2] \\ 0, & \text{иначе} \end{cases}$$

$$F_\xi(x) = \begin{cases} 0, & x \in (-\infty; -1] \\ (x+1)\frac{1}{3}, & x \in (-1; 2] \\ 1, & x \in (2; +\infty) \end{cases}$$



а) $\eta = \sin(\pi\xi)$, $\quad \operatorname{supp} p\,\xi = [-1; 2]$

$\operatorname{supp} \eta = [-1; 1]$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; -1] \\ *, & x \in (-1; +1] \\ 1, & x \in (1; +\infty) \end{cases}$$

$$F_\eta(x) = P(\eta < x) = P(\sin(\pi\xi) < x)$$

$\sin(\pi\xi) < x$:

$x \leq -1$: $\quad P(\sin(\pi\xi) < 1) = 0$

$x \in (-1; 0]$: $\quad P(\sin(\pi\xi) < x) =$

$$\frac{-\operatorname{asin}(x) - \pi}{\pi} < \xi < \frac{\operatorname{asin}(x)}{\pi} \;//\; \frac{\pi - \operatorname{asin}(x)}{\pi} < \xi < \frac{\operatorname{asin}(x) + 2\pi}{\pi}$$

$x \in (0;1]:$ $\quad 0 < \xi < \frac{\operatorname{asin}(x)}{\pi}$

$$\frac{\pi - \operatorname{asin}(x)}{\pi} < \xi < 1$$

$x \in (-1;0]:$

$$F_\eta(x) = F_\xi\left(\frac{\operatorname{asin}(x)}{\pi}\right) - F_\xi\left(\frac{-\operatorname{asin}x - \pi}{\pi}\right)$$

$$+ F_\xi\left(\frac{\operatorname{asin}x + 2\pi}{\pi}\right) - F_\xi\left(\frac{\pi - \operatorname{asin}(x)}{\pi}\right) =$$

$=$

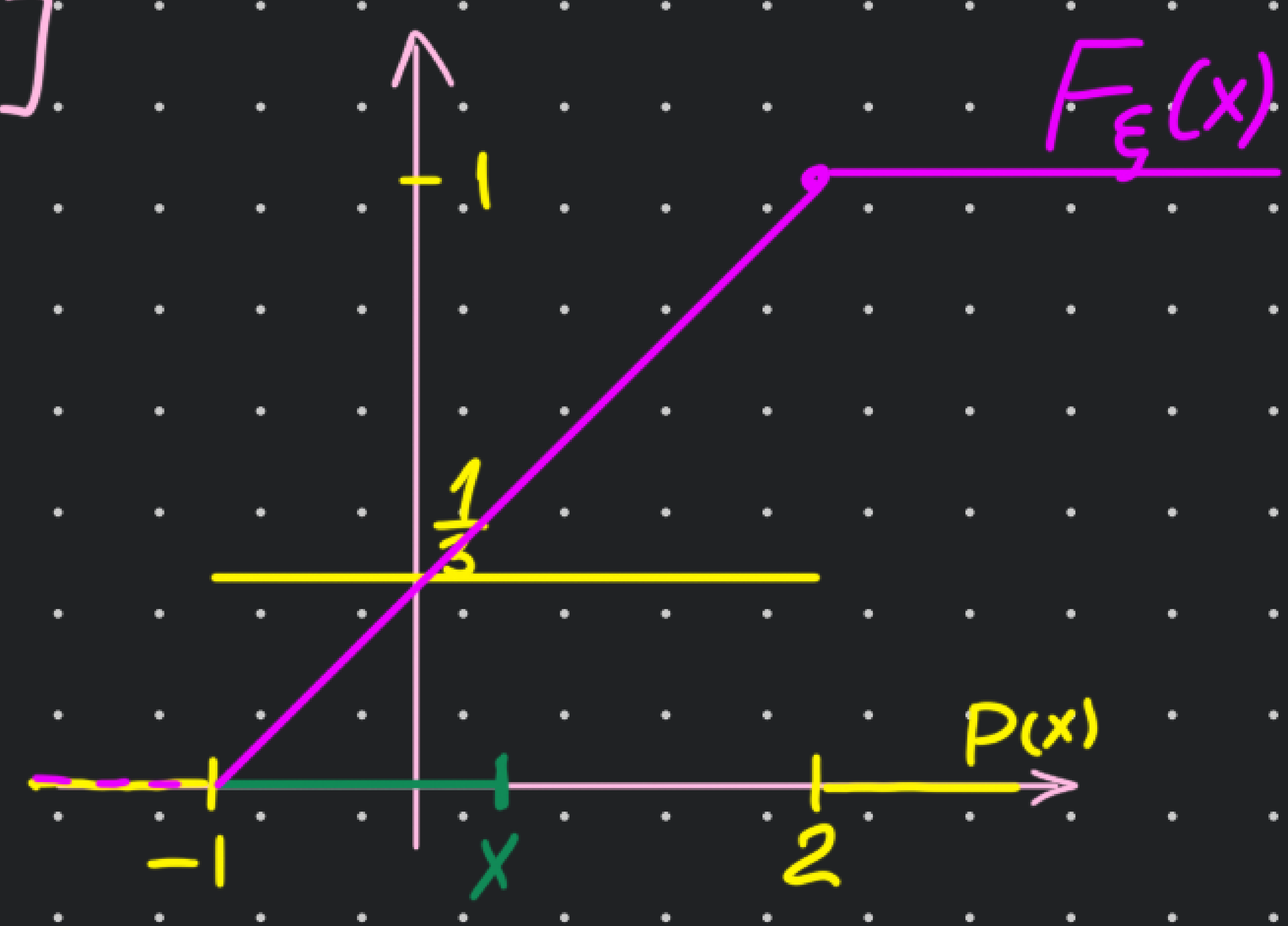
8. Случайная величина ξ имеет распределение с плотностью

$$p_\xi(x) = \begin{cases} \frac{1}{3}, & x \in [-1, 2] \\ 0, & \text{в ост. случаях} \end{cases}$$

Найти распределение а) $\eta = \sin(\pi \xi)$

б) $\eta = 1/\xi$

$$p_\xi(x) = \begin{cases} \frac{1}{3}, & x \in [-1, 2] \\ 0, & \text{иначе} \end{cases}$$



$$F_\xi(x) = \begin{cases} 0, & x \in (-\infty; -1] \\ (x+1)\frac{1}{3}, & x \in (-1; 2] \\ 1, & x \in (2; +\infty) \end{cases}$$

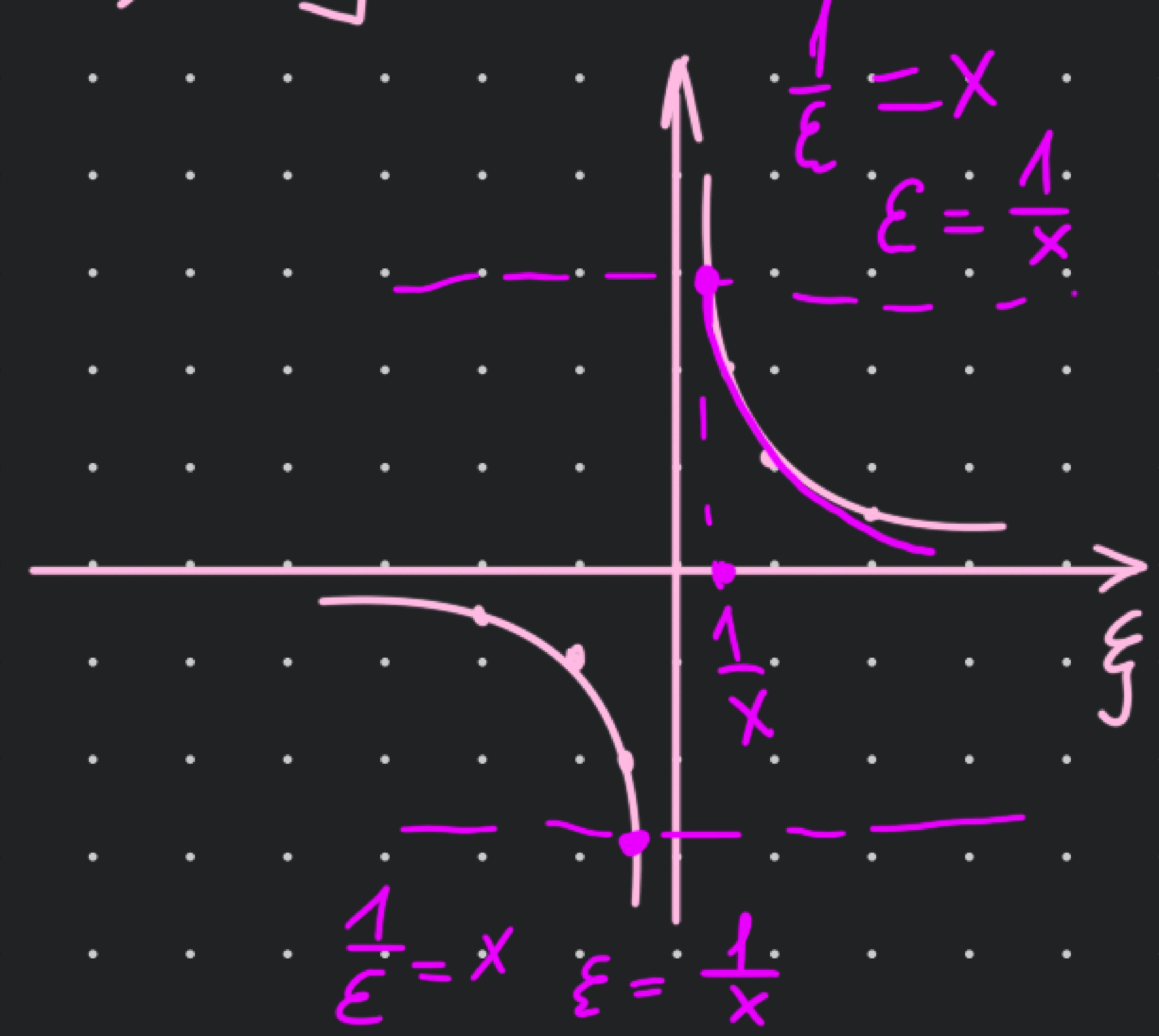
$$\operatorname{supp} \xi = [-1; 2]$$

$$\operatorname{supp} \eta = [1; 0) \cup (0; \tfrac{1}{2}]$$

$$F_\eta = \begin{cases} 0, & x \in (-\infty; -1] \\ *, & x \in (-1; 0) \cup (0; \frac{1}{2}] \\ 1, & x \in (\frac{1}{2}; +\infty) \end{cases}$$

$$F_\eta = \underline{P(\eta < x)} = P\left(\frac{1}{\xi} < x\right) =$$

$$= \left[\begin{array}{l} x > 0 : \ P\left(\xi > \frac{1}{x}\right) \\ x < 0 : \ P\left(\frac{1}{x} < \xi < 0\right) \end{array} \right] =$$



$x>0: \quad P\left(\xi > \frac{1}{x}\right) = F\left(\frac{1}{x}\right)$

$x>0 \rightarrow \frac{1}{x} >$

$x \in (0; 0{,}5) \rightarrow \frac{1}{x} \in (2; +\infty) \rightarrow F\left(\frac{1}{x}\right) = 1$

$x \in (0{,}5; +\infty) \rightarrow \frac{1}{x} \in (0; 2) \rightarrow F\left(\frac{1}{x}\right) = \frac{1}{3}\left(\frac{x+1}{x}\right)$

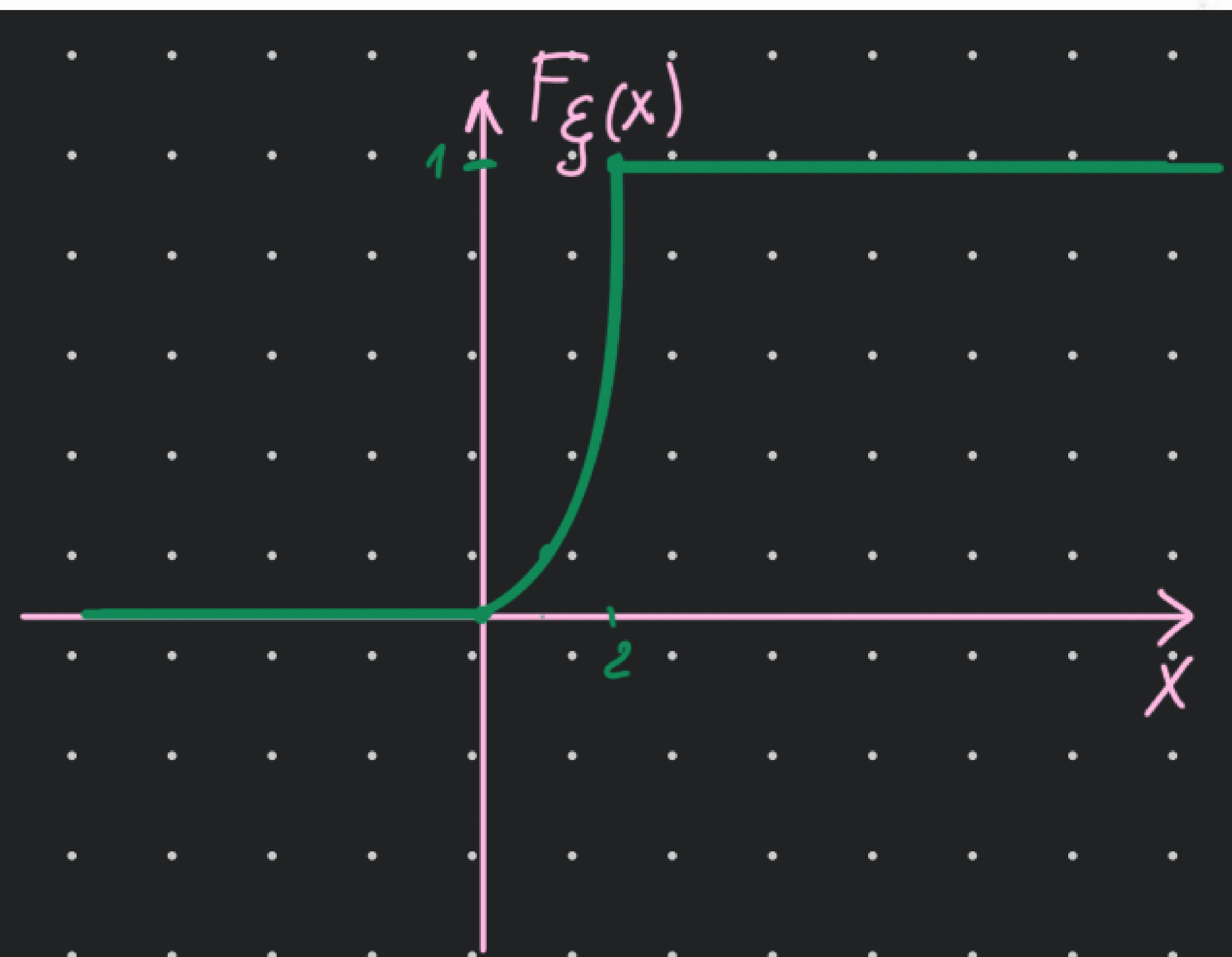
С. Функция распределения случайной величины ξ

$$F_\xi(x) = \begin{cases} 0, & x \le 0 \\ x^3/8, & x \in [0,2] \\ 1, & x > 2 \end{cases}$$

Найти распределение а) $\eta = \xi^2$

б) $\eta = (\xi - 1)^2$

в) $\eta = \xi^3$



$supp\, P_\xi = [0;2]$

ξ

а) $\eta = \xi^2$, $supp\, \eta = [0;4]$

$$F_\eta = \begin{cases} 0, & x \in (-\infty; 0] \\ *, & x \in (0;4] \\ 1, & x \in (4; +\infty) \end{cases}$$

$$F_\eta = P(\eta < x) = P(\xi^2 < x) =$$

$$= P(\overset{0}{-\sqrt{x}} < \xi < \sqrt{x}) = F_\xi(\sqrt{x}) - F_\xi(\quad) =$$

$$= \frac{(\sqrt{x})^3}{8} - \frac{(0)^3}{8} = \frac{(\sqrt{x})^3}{8} = \frac{\sqrt{x}^3}{8}$$

$$F_\eta = \begin{cases} 0, & x \in (-\infty; 0] \\ \sqrt{x}^3/4, & x \in (0;4] \\ 1, & x \in (4; +\infty) \end{cases} \qquad p_\eta = \begin{cases} 0, & x \in (-\infty; 0] \\ \frac{3}{2}\frac{\sqrt{x}}{4}, & x \in (0;4] \\ 0, & x \in (4; +\infty) \end{cases}$$

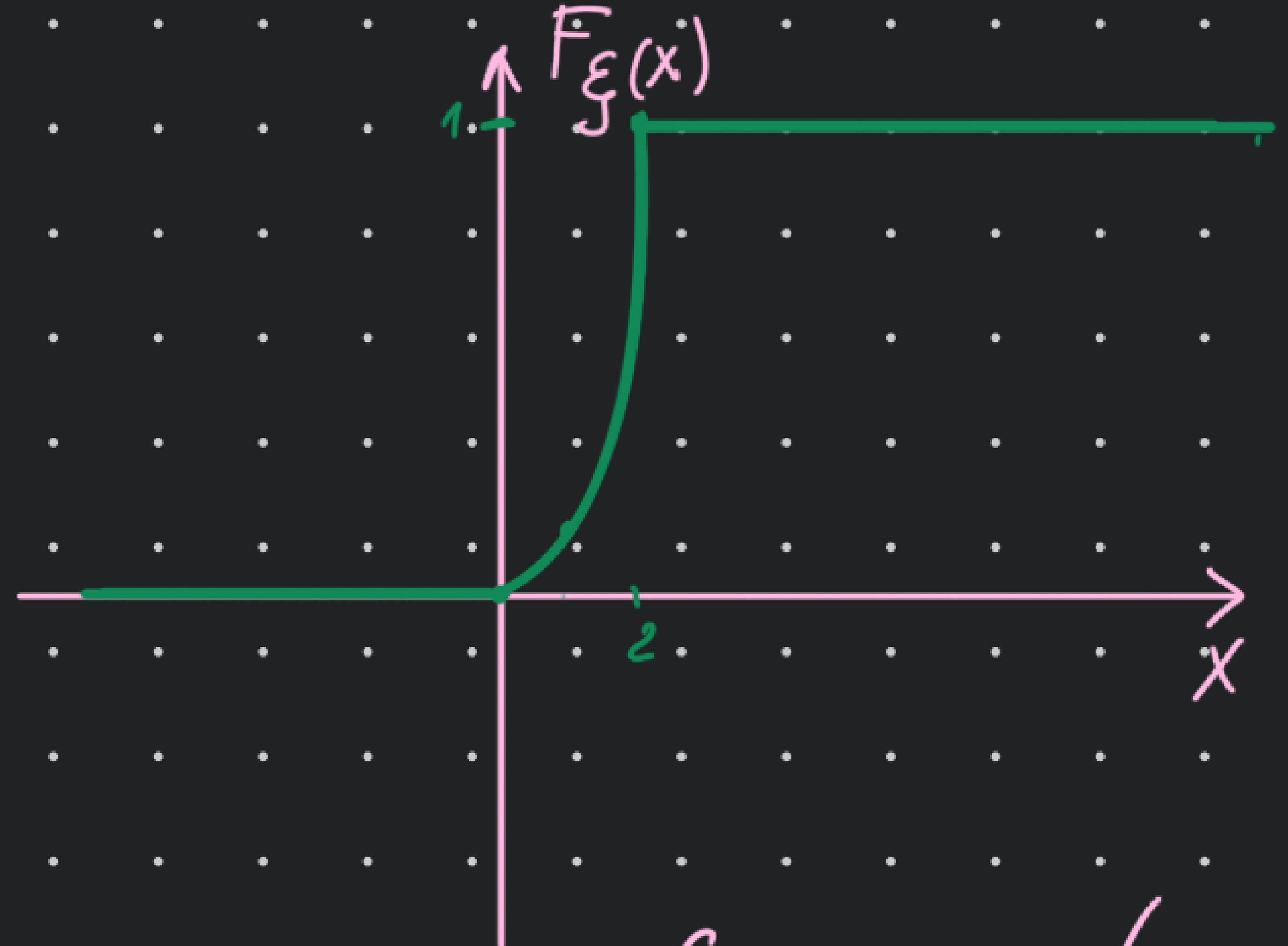
С. Функция распределения случайной величины ξ

$$F_\xi(x) = \begin{cases} 0, & x \le 0 \\ x^3/8, & x \in [0, 2] \\ 1, & x > 2 \end{cases}$$

Найти распределение а) $\eta = \xi^2$

б) $\eta = (\xi - 1)^2$

в) $\eta = \xi^3$



$$\operatorname{supp} \xi = [0; 2]$$

б) $\eta = (\xi - 1)^2$

$$\operatorname{supp} \eta = [0; 1]$$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ *, & x \in (0; 1] \\ 1; & x \in (1; +\infty) \end{cases}$$

$$F_\eta(x) = F((\xi - 1)^2 < x) =$$

$$= F(-\sqrt{x} + 1 < \xi < \sqrt{x} + 1) = F(\sqrt{x} + 1) - F(-\sqrt{x} + 1) =$$

$$= \frac{(\sqrt{x} + 1)^3}{8} - \frac{(1 - \sqrt{x})^3}{8}$$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \frac{(\sqrt{x}+1)^3}{8} - \frac{(1-\sqrt{x})^3}{8}, & x \in [0; 1] \\ 1; & x \in (1; +\infty) \end{cases}$$

$$P_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ 6(\sqrt{x}+1)^2 \cdot \frac{1}{2\sqrt{x}}, & x \in [0; 1] \\ 0, & x \in (1; \infty) \end{cases}$$

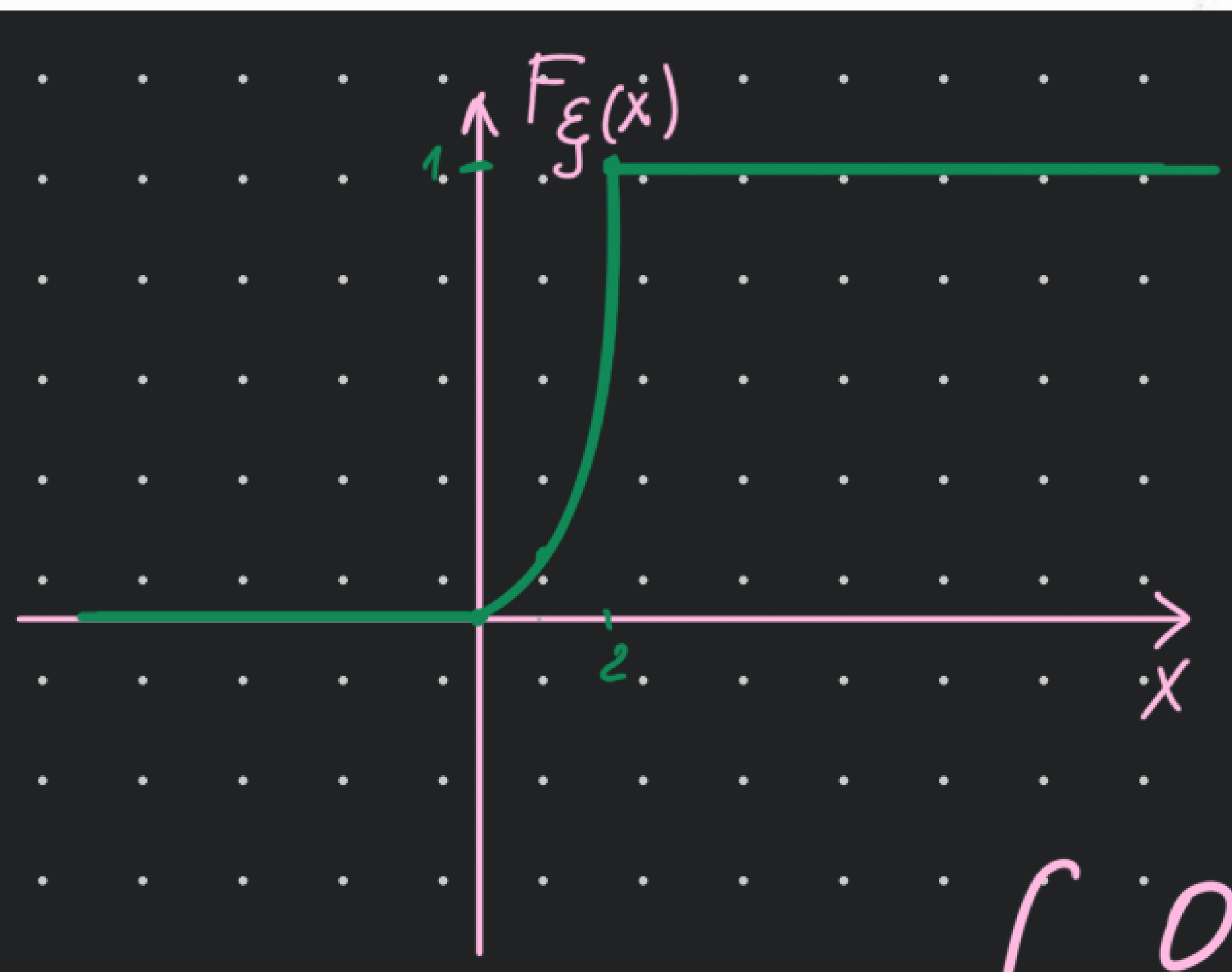
С. Функция распределения случайной величины ξ

$$F_\xi(x) = \begin{cases} 0, & x \le 0 \\ x^3/8, & x \in [0,2] \\ 1, & x > 2 \end{cases}$$

Найти распределение а) $\eta = \xi^2$

б) $\eta = (\xi - 1)^2$

в) $\eta = \xi^3$



$\operatorname{supp} P_\xi = [0;2]$

$\operatorname{supp} \eta = [0;8]$

$$F_\eta = \begin{cases} 0, & x \in (-\infty;0] \\ *, & x \in (0;8] \\ 1, & x \in (8;+\infty) \end{cases}$$

$$F_\eta(x) = P(\xi^3 < x) = P(\xi < \sqrt[3]{x}) = \frac{x}{8}$$

$$F_\eta = \begin{cases} 0, & x \in (-\infty;0] \\ \frac{x}{8}, & x \in (0;8] \\ 1, & x \in (8;+\infty) \end{cases} \qquad p_\eta(x) = \begin{cases} 0, & x \in (-\infty;0] \\ \frac{1}{8}, & x \in (0;8] \\ 0, & x \in (8;+\infty) \end{cases}$$

D. Плотность распределения величины ξ задана формулой

$$p_\xi(x) = \begin{cases} e^x/2, & x \le 0 \\ \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, & x > 0 \end{cases}$$

Найти распределение

а) $\eta = \xi^2$

б) $\eta = e^\xi$

$$F_\xi(x) = \begin{cases} x \le 0: \int_{-\infty}^{x} \frac{e^t}{2} dt \\ x > 0: \int_{-\infty}^{0} \frac{e^t}{2} dt + \int_{0}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \end{cases}$$

$$\int_{-\infty}^{x} \frac{e^t}{2} dt = \frac{e^x}{2} - \frac{e^{-\infty}}{2} = \frac{e^x}{2}$$

$$F_\xi(x) = \begin{cases} \frac{e^x}{2}, & x \le 0 \\ \frac{1}{2} + \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt, & x > 0 \end{cases}$$

$\operatorname{supp} \xi = \mathbb{R}$ а) $\operatorname{supp} \xi^2 = \mathbb{R}^+$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ *, & x \in (0; +\infty] \end{cases}$$

$$F_\eta(x) = P(\eta < x) = P(\xi^2 < x) =$$

$$P(-\sqrt{x} < \xi < x) = F_\xi(\sqrt{x}) - F_\xi(-\sqrt{x}) =$$

$$= \frac{1}{2} + \int_0^{\sqrt{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} - \frac{e^{-\sqrt{x}}}{2}$$

$$F_\eta(x) = \begin{cases} 0, x \in (-\infty; 0] \\ \frac{1}{2} + \int_0^{\sqrt{x}} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt - \frac{e^{-\sqrt{x}}}{2}, x \in (0; +\infty) \end{cases}$$

$$p_\eta(x) = \begin{cases} 0, x \in (-\infty; 0] \\ \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x}{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \cdot \frac{e^{-\sqrt{x}}}{2} \end{cases}$$

$\parallel$ $\quad x \in (0; +\infty)$

$$\frac{1}{2\sqrt{x}} \left(\frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x}{2}} + \frac{e^{-\sqrt{x}}}{2} \right) ?$$

б) $\eta = e^{\xi}$ $\qquad$ $\operatorname{supp} \eta = (0; +\infty)$

$$F_\eta(x) = \begin{cases} 0, x \in (-\infty; 0] \\ *, x \in (0; +\infty) \end{cases}$$

$$F_\eta(x) = P(e^{\xi} < x) = P(\xi < \ln x) =$$

$$= F_\xi(\ln x)$$

$\ln x \le 0$: $\quad F_\xi = \frac{e^{\ln x}}{2_{\ln x}}$

$x \le 1$

$x \geq 1$: $\quad F_\xi = \frac{1}{2} + \int_0 \frac{1}{\sqrt{2\pi}} e^{\frac{\cancel{t}^2}{2}} dt$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \dfrac{e^{\ln x}}{2}, & x \in (0; 1] \\ \dfrac{1}{2} + \displaystyle\int_0^{\ln x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt, & x > 1 \end{cases}$$

$$P_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \dfrac{1}{x} \dfrac{e^{\ln x}}{2}, & x \in (0; 1] \\ \dfrac{1}{x} \left(\dfrac{1}{\sqrt{2\pi}} e^{-\frac{\ln^2 x}{2}} \right), & x > 1 \end{cases}$$

$$\int_0^1 \frac{1}{x} \frac{e^{\ln x}}{2} dx = \frac{1}{2}$$

$$\int_1^\infty \frac{1}{x} \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{\ln^2 x}{2}} \right) dx = \begin{bmatrix} t = \ln x \\ dt = \dfrac{dx}{x} \end{bmatrix}$$

$x \in [1; \infty)$

$\ln x \in (0; \infty)$

$$= \underbrace{\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt}_{\text{Нормальное}} = \frac{1}{2}$$

E. Плотность распределения ξ задана формулой

$$p_\xi(x) = \begin{cases} |x|, & x \in [-1, 1] \\ 0, & \text{в ост. сл.} \end{cases}$$

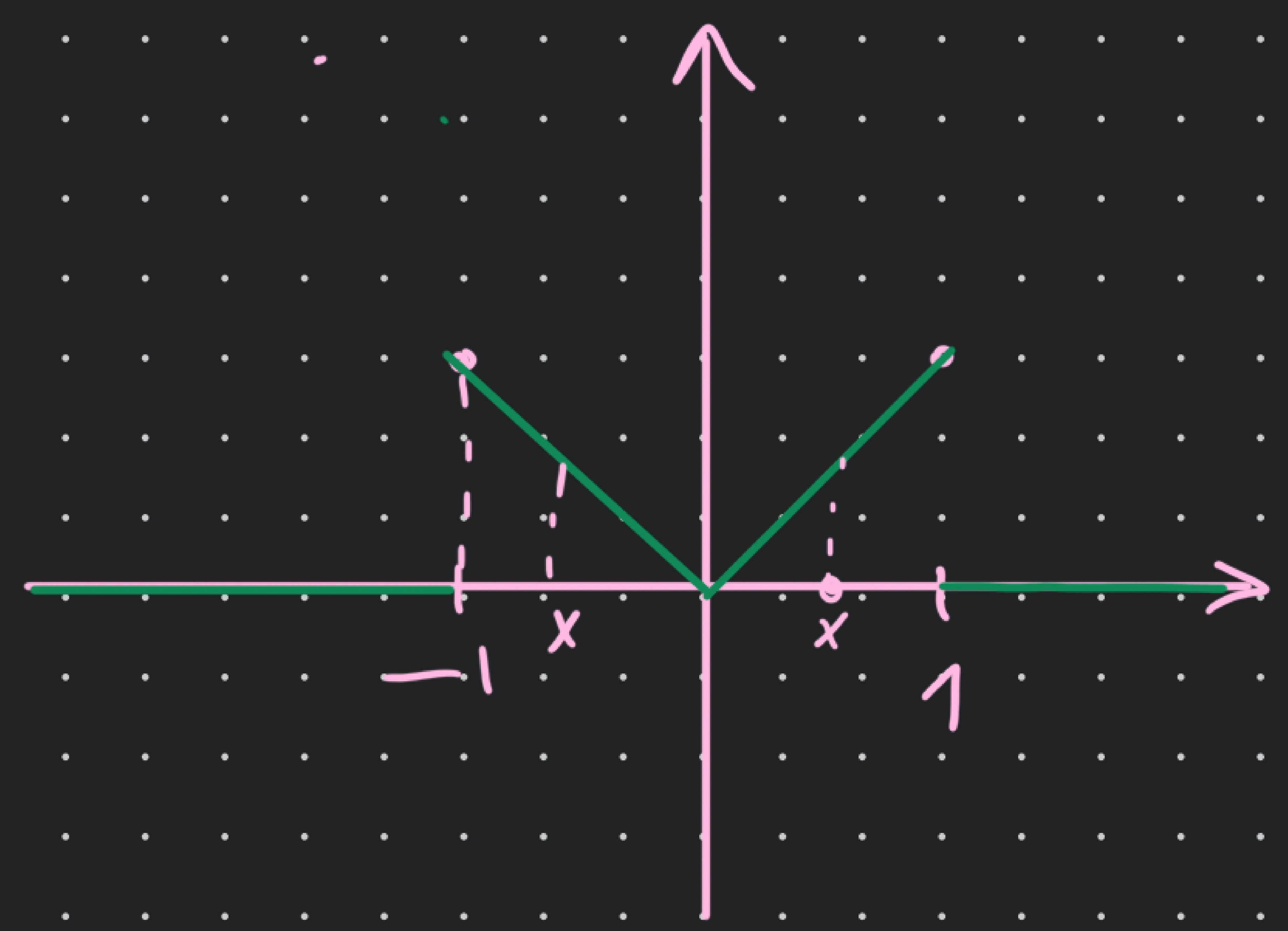
Найти распределения

а) $\eta = \xi^4$

б) $\eta = (\xi - 1)^2$

$\operatorname{supp} \xi = [-1;1]$

$$F_\eta(x) = \begin{cases} 0, & x \le -1 \\ \frac{1-x^2}{2}, & x \in (-1, 0] \\ \frac{1}{2} + \frac{x^2}{2}, & x \in (0;1] \\ 1, & x \in (1;+\infty) \end{cases}$$



a) $\operatorname{supp} \eta = [0;1]$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty;0] \\ *, & x \in (0;1] \\ 1, & x \in (1,+\infty) \end{cases}$$

$$F_\eta(x) = P(\xi^4 < x) = P(-\sqrt[4]{x} < \xi < \sqrt[4]{x}) =$$

$$= -F_\xi(-\sqrt[4]{x}) + F_\xi(\sqrt[4]{x}) =$$

$$= -\frac{1-(-\sqrt[4]{x})^2}{2} + \frac{1}{2} + \frac{(\sqrt[4]{x})^2}{2} =$$

$$= \frac{\sqrt[2]{x} - 1}{2} + \frac{1}{2} + \frac{\sqrt[2]{x}}{2} = \sqrt{x}$$

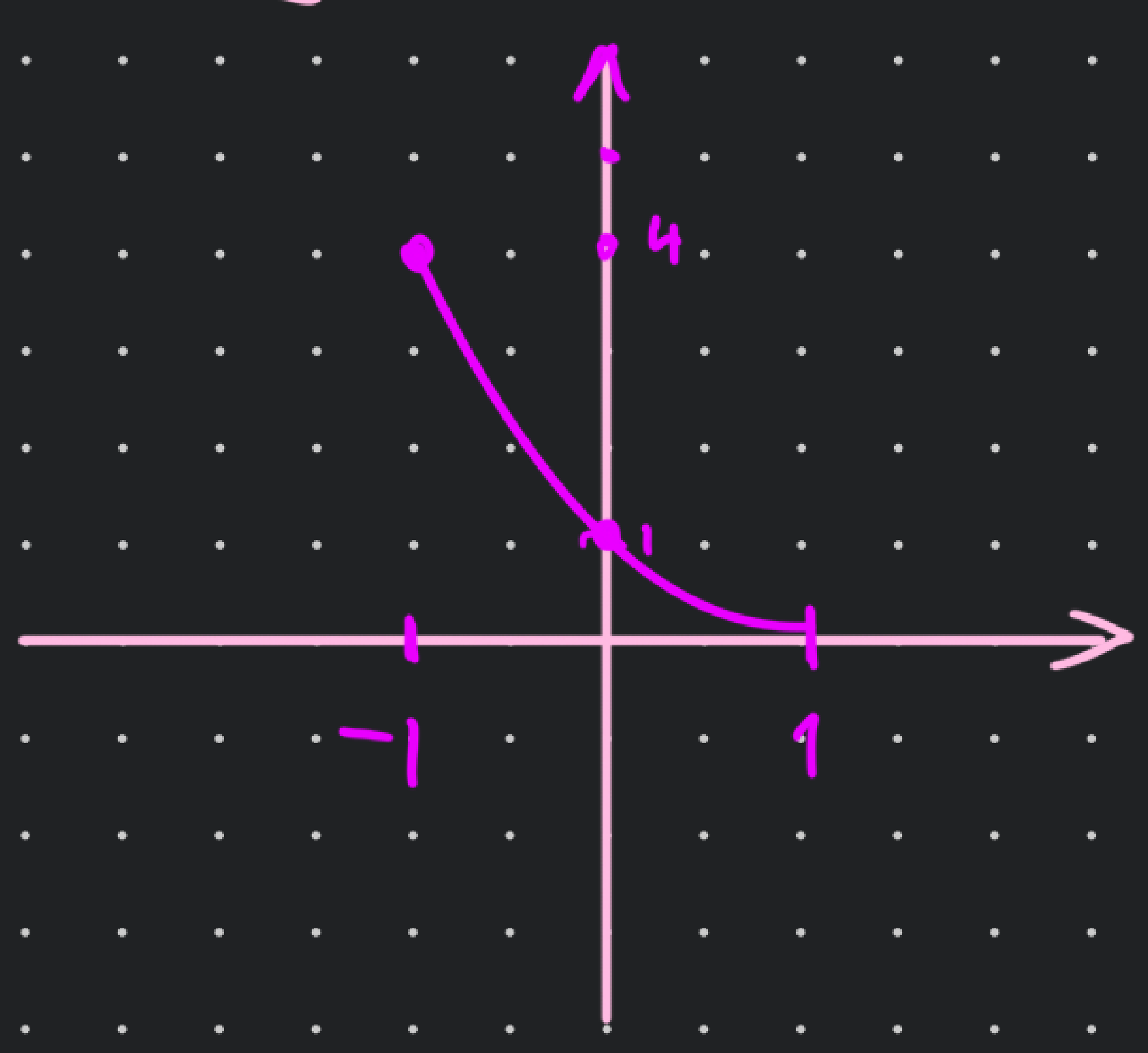
$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \sqrt{x}, & x \in (0; 1] \\ 1, & x \in (1, +\infty) \end{cases}$$

$$P_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \frac{1}{2\sqrt{x}}, & x \in (0; 1] \\ 1, & x \in (1, +\infty) \end{cases} \qquad \int_0^1 \frac{1}{2\sqrt{x}} = \sqrt{x}\Big|_0^1 = 1$$

б) $\eta = (\xi - 1)^2$ $\qquad$ $\operatorname{supp} \xi = [-1; 1]$

$\operatorname{supp} \eta = [0; 4]$

$$F_\xi(x) = \begin{cases} 0, & x \le -1 \\ \frac{1-x^2}{2}, & x \in (-1, 0] \\ \frac{1}{2} + \frac{x^2}{2}, & x \in (0; 1] \\ 1, & x \in (1; +\infty) \end{cases}$$



$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ *, & x \in (0, 4] \\ 1, & x \in (4, +\infty) \end{cases}$$

$$F_\eta(x) = P\left((\xi - 1)^2 < x\right) =$$

$$= P\left(-\sqrt{x} + 1 < \xi < \sqrt{x} + 1\right) = F_\xi(\sqrt{x} + 1) - F_\xi(-\sqrt{x} + 1)$$

$$= \quad [x \in (1;4] : \quad (-\sqrt{x}+1) < 0$$

$$= \quad 1 - \left(\frac{1-(1-\sqrt{x})^2}{2}\right) =$$

$$= \frac{2-1+1-2\sqrt{x}+x}{2} =$$

$$= 1-\sqrt{x}+\frac{x}{2}$$

$$x \in [0;1]$$

$$1 - \left(\frac{1}{2} + \frac{(1-\sqrt{x})^2}{2}\right) =$$

$$= 1 - \frac{1}{2} - \left(\frac{1-2\sqrt{x}+x}{2}\right) =$$

$$= \frac{1}{2} - \frac{1}{2} + \sqrt{x} - \frac{x}{2} = \sqrt{x} - \frac{x}{2}$$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty;0] \\ \sqrt{x}-\frac{x}{2}, & x \in (0;1] \\ 1-\sqrt{x}+\frac{x}{2}, & x \in (1;4] \\ 1, & x > 4 \end{cases}$$

$$p_\eta(x) = \begin{cases} 0, & x \in (-\infty;0] \\ \frac{1}{2\sqrt{x}}-\frac{1}{2}, & x \in (0;1] \\ -\frac{1}{2\sqrt{x}}+\frac{1}{2}, & x \in (1;4] \\ 0, & x > 4 \end{cases}$$

$$\int_0^1 \frac{1}{2\sqrt{x}} - \frac{1}{2}\,dx = \frac{1}{2}$$

$$\int_1^4 -\frac{1}{2\sqrt{x}} + \frac{1}{2}\,dx = \frac{1}{2}$$

сошлос

F. Функция распределения ξ имеет вид.

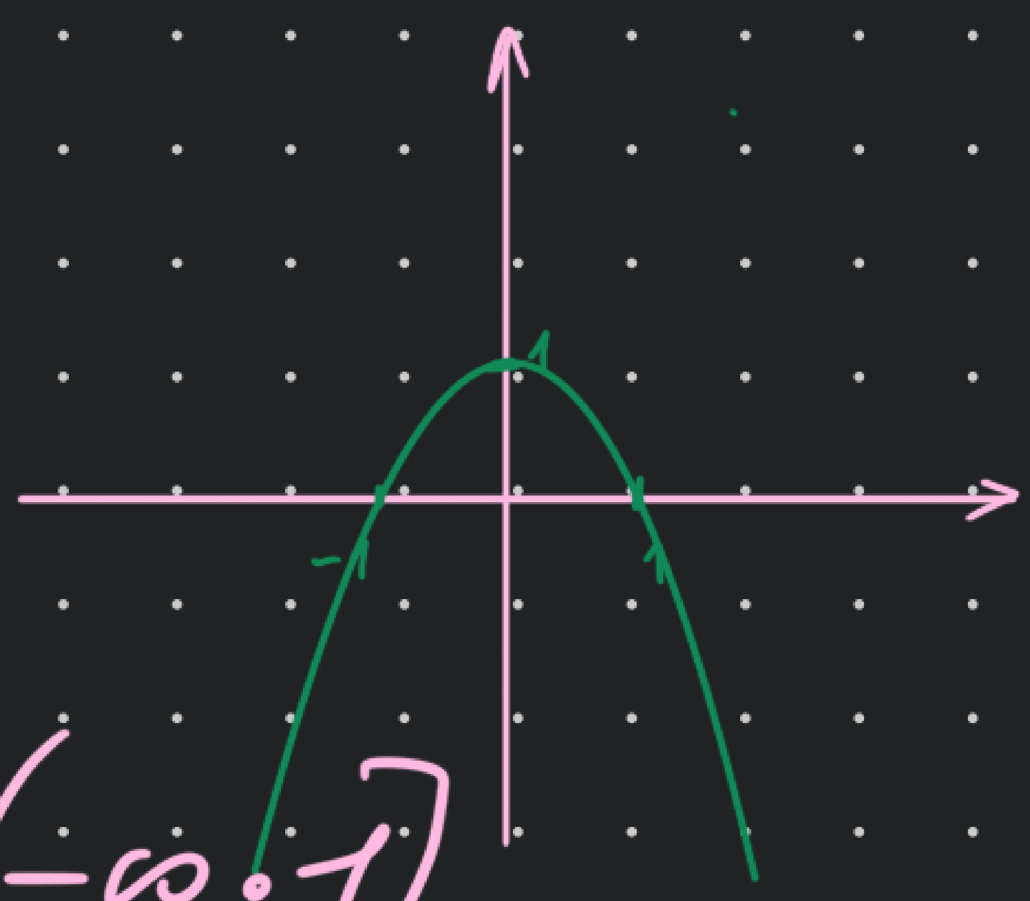
$$F_\xi(x) = \begin{cases} 0, x \le 0 \\ 1-e^{-x}, x > 0 \end{cases}$$

Найти распределение

а) $\eta = 1 - \xi^2$

б) $\eta = \cos(\pi \xi)$

а) $supp\, \xi = \mathbb{R}^+$ $\quad supp\, \eta = [-\infty; 1]$



$$F_\eta = \begin{cases} *, & x \in (-\infty; 1] \\ 1, & x \in (1; +\infty) \end{cases}$$

$$F_\eta = P(1-\xi^2 < x) = P(1-x < \xi^2)$$

$$= P(-\sqrt{1-x} < \xi < +\sqrt{1-x}) =$$

$$= F_\xi(\sqrt{1-x}) - F_\xi(-\sqrt{1-x}) =$$

$$= 1 - e^{-\sqrt{1-x}} - 0 = 1 - e^{-\sqrt{1-x}}$$

б) $\eta = \cos(\pi\xi)$ $\quad \operatorname{supp}\eta = [-1;1]$

$$F_\eta = \begin{cases} 0, & x \in (-\infty; -1] \\ *, & x \in [-1;1] \\ 1, & x \in (1;+\infty) \end{cases}$$

$$F_\eta = P(\cos(\pi\xi) < \cancel{x})$$

$$\frac{\cos^{-1}(x) + 2\pi n}{\pi} < \xi < \frac{-\cos^{-1}(x) + 2\pi n + 2\pi}{\pi}$$

$$1 - e^{-\left(\frac{\cos^{-1}(x) + 2\pi n}{\pi}\right)} - 1 + e^{-\left(\frac{-\cos^{-1}(x) + 2\pi n + 2\pi}{\pi}\right)}$$

$$= -e^{-\left(\frac{\cos^{-1}(x) + 2\pi n}{\pi}\right)} + e^{-\left(\frac{-\cos^{-1}x + 2\pi(n+1)}{\pi}\right)}$$