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Вар. 30 (513020124)

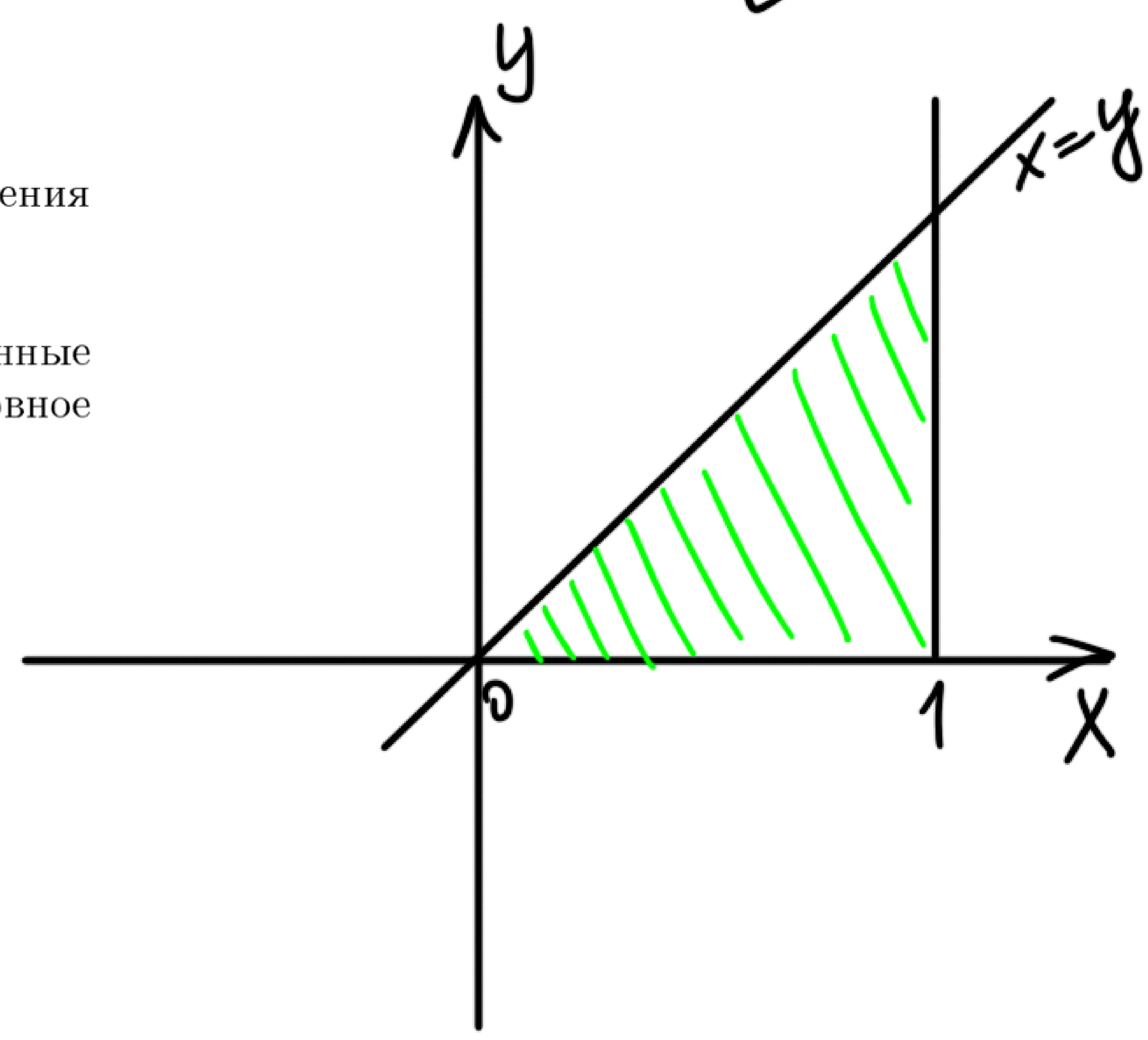
Случайная величина (ξ, η) имеет равномерное распределение в области

$$\begin{pmatrix} x - y \geq 0, \\ x \leq 1,\ y \geq 0 \end{pmatrix}$$

$\zeta = -1\xi^4 - 1,\ \nu = [5\eta],\ \mu = 2\xi - 2\eta.$

1. Найти $p_{\xi,\eta}$, функции и плотности распределения компонент. Будут ли компоненты независимыми?
2. Найти распределения с.в. ζ и ν; $E\zeta$, $E\nu$, $D\zeta$, $D\nu$.
3. Вычислить вектор мат. ожиданий и ковариационные характеристики вектора (ξ, η). Найти условное распределение ξ при условии η; $E(\xi|\eta)$, $D(\xi|\eta)$.
4. Найти распределение μ; $E\mu$; $D\mu$.

$$p_{\xi,\eta}(x,y) = \begin{cases} C, & x-y \geq 0 \\ & x \leq 1,\ y \geq 0 \\ 0, & \text{иначе} \end{cases}$$



① $$\iint\limits_{\triangle ABC} C\,dx\,dy = \int_0^1 dy \int_y^1 C\,dx = C \cdot S_{\triangle ABC} =$$

$$= C \cdot \frac{1}{2} = 1 \Rightarrow C = 2$$

$$p_{\xi,\eta}(x,y) = \begin{cases} 2, & x-y \geq 0 \\ & x \leq 1,\ y \geq 0 \\ 0, & \text{иначе} \end{cases}$$

$$p_\xi(x) = \int_0^x 2\,dy = 2y\Big|_0^x = 2x, \text{ при } x \in [0;1]$$

$$p_\eta(y) = \int_y^1 2\,dx = 2x\Big|_y^1 = 2 - 2y,\ y \in [0;1]$$

$$p_\xi(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ 2x, & x \in (0;1] \\ 0, & x \in (1; +\infty) \end{cases} \qquad F_\xi(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ x^2, & x \in (0;1] \\ 1, & x \in (1; +\infty) \end{cases}$$

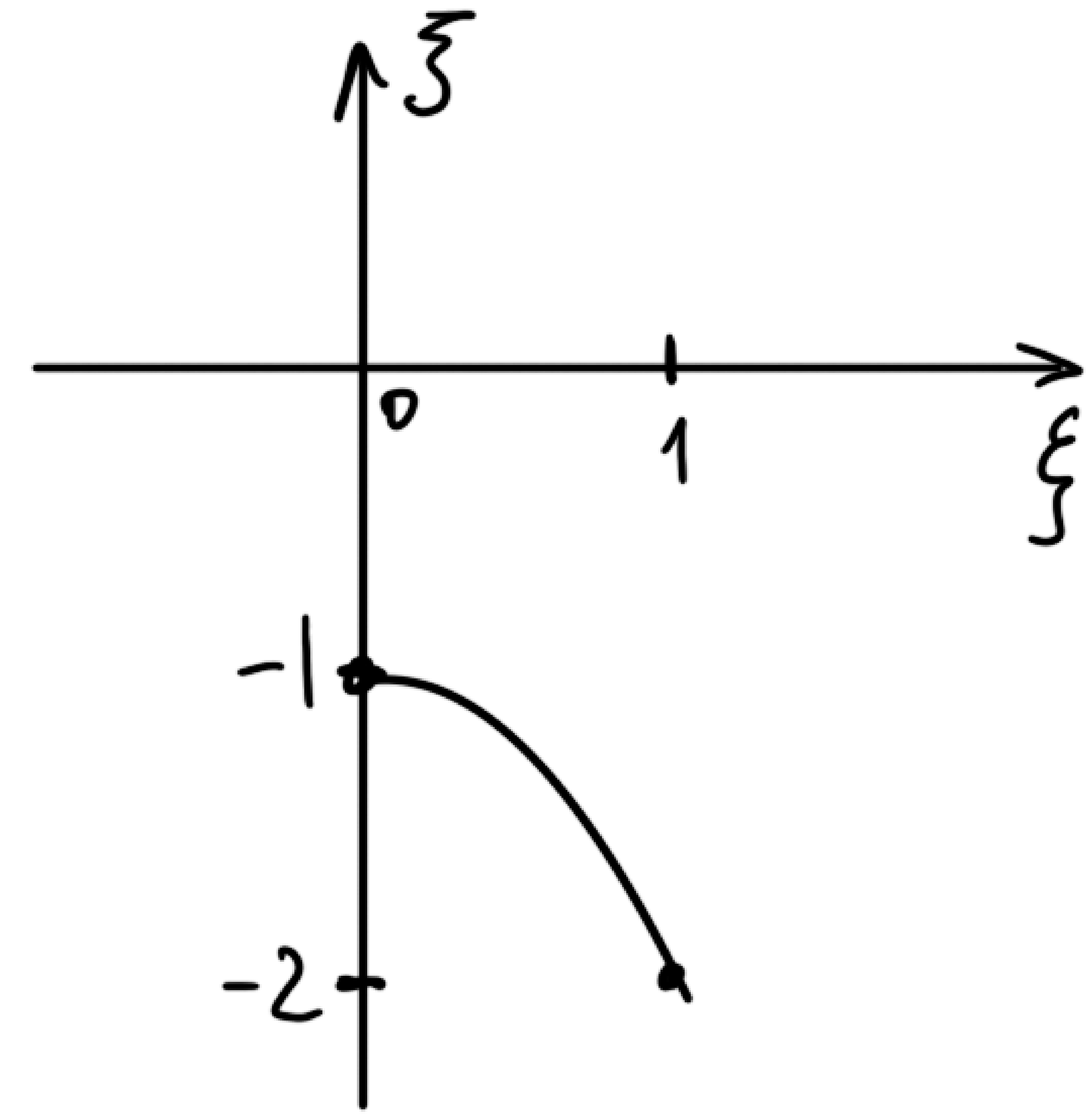
$$p_\eta(y) = \begin{cases} 0, & y \in (-\infty; 0] \\ 2-2y, & y \in (0;1] \\ 0, & y \in (1, +\infty) \end{cases} \qquad F_\eta(x) = \begin{cases} 0, & y \in (-\infty; 0] \\ 2y - y^2, & y \in (0;1] \\ 1, & y \in (1; +\infty) \end{cases}$$

Если случайные величины ξ и η независимы, то

$$p_{\xi,\eta}(x,y) = p_\xi(x) \cdot p_\eta(y)$$

У нас это равенство не выполняется, значит ξ и η – зависимы

2 a $\zeta = -\xi^4 - 1$



$\operatorname{supp} \xi = [0;1]$

$\operatorname{supp} \zeta = [-2;-1]$

$$F_\xi(x) = \begin{cases} 0, & x \in (-\infty;0] \\ x^2, & x \in (0;1] \\ 1, & x > 1 \end{cases}$$

$$F_\zeta(x) = \begin{cases} 0, & x \in (-\infty;-2] \\ *, & x \in (-2;-1] \\ 1, & x > -1 \end{cases}$$

$$F_\zeta(x) = P(\zeta < x) = P((-\xi^4 - 1) < x) =$$

$$P(-x < \xi^4 + 1) = P(\xi^4 > -x-1) = 1 - P(\xi^4 < -x-1)$$

$$= 1 - P(-\sqrt[4]{-x-1} < \xi < \sqrt[4]{-x-1}) = 1 + \underbrace{F_\xi(-\sqrt[4]{-x-1})}_{0} - F_\xi(\sqrt[4]{-x-1})$$

$$= 1 - (\sqrt{-x-1}) = 1 - \sqrt{-x-1}$$

$$F_\zeta(x) = \begin{cases} 0, & x \in (-\infty;-2] \\ 1 - \sqrt{-1-x}, & x \in (-2;-1] \\ 1, & x > -1 \end{cases}$$

$$p_\zeta(x) = \begin{cases} 0, & x \in (-\infty;-2] \\ \dfrac{1}{2\sqrt{-1-x}}, & x \in (-2;-1] \\ 0, & x > -1 \end{cases}$$

$$E\xi = \int_{-2}^{-1} x \cdot \frac{1}{2\sqrt{-1-x}}\,dx = \begin{bmatrix} -1-x=t & t\in[0;1] \\ x=-1-t & -dx=dt \end{bmatrix} =$$

$$= \int_{0}^{1} (-1-t)\cdot\frac{1}{2\sqrt{t}}\,dt = -\int_{0}^{1} \frac{1}{2\sqrt{t}} + \frac{\sqrt{t}}{2}\,dt =$$

$$= -\left(\sqrt{t}\,\Big|_0^1 + \frac{t^{\frac{3}{2}}}{3}\,\Big|_0^1\right) = -\left(1 + \frac{1}{3}\right) = \frac{-4}{3}$$

$$D\xi = \int_{-2}^{-1} x^2 \cdot \frac{1}{2\sqrt{-1-x}}\,dx - \left(\frac{4}{3}\right)^2 =$$

$$= \begin{bmatrix} -1-x=t & \\ x^2=(t+1)^2 & t\in[0;1] \\ & -dx=dt \end{bmatrix} = \int_{0}^{1} (t+1)^2 \frac{1}{2\sqrt{t}}\,dt - \left(\frac{4}{3}\right)^2 =$$

$$= \int_{0}^{1} \frac{t^2}{2\sqrt{t}} + \frac{2t}{2\sqrt{t}} + \frac{1}{2\sqrt{t}}\,dt - \left(\frac{4}{3}\right)^2 = \int_{0}^{1} \frac{t^{\frac{3}{2}}}{2}\,dt + \int_{0}^{1} \sqrt{t}\,dt +$$

$$+ \int_{0}^{1} \frac{1}{2\sqrt{t}}\,dt - \left(\frac{4}{3}\right)^2 = \frac{1}{5} + \frac{2}{3} + 1 - \left(\frac{4}{3}\right)^2 = \frac{28}{15} - \frac{16}{9} =$$

$$= \frac{4}{45}$$

$\nu = [5\eta]$ $\quad$ $\operatorname{supp} \eta = [0;1]$

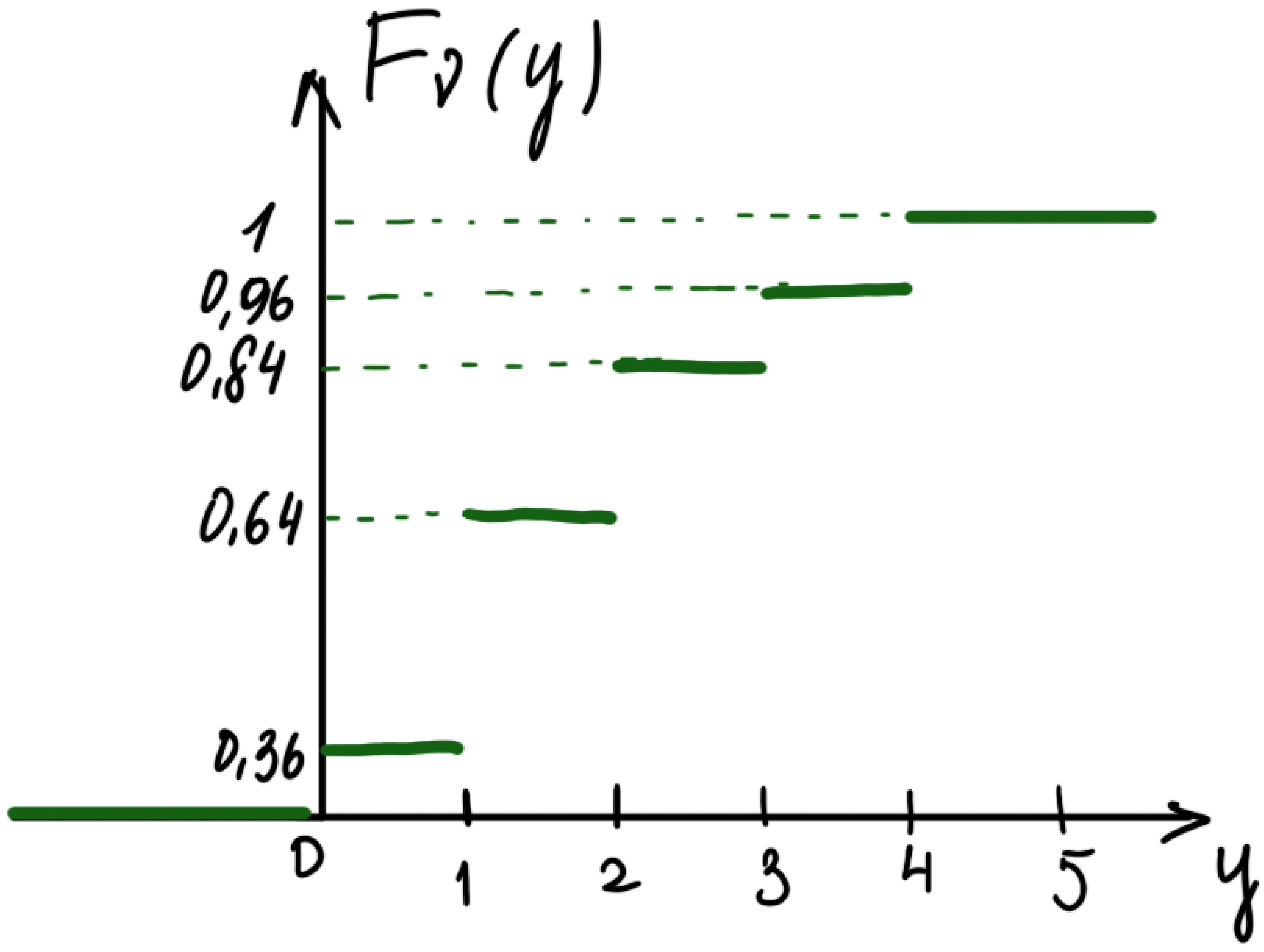
η	$[0;0{,}2)$	$[0{,}2;0{,}4)$	$[0{,}4;0{,}6)$	$[0{,}6;0{,}8)$	$[0{,}8;1)$	1
ν	0	1	2	3	4	5

$$F_\eta(y) = \begin{cases} 0, & y \leqslant 0 \\ 2y - y^2, & y \in [0;1] \\ 1, & y > 1 \end{cases}$$

$$F_\nu(y) = P(\nu < y) =$$

$$P(\nu < 0) = 0$$
$$P(\nu < 1) = P(\eta < 0{,}2) = F_\eta(0{,}2) = 0{,}36$$
$$P(\nu < 2) = P(\eta < 0{,}4) = F_\eta(0{,}4) = 0{,}64$$
$$P(\nu < 3) = P(\eta < 0{,}6) = F_\eta(0{,}6) = 0{,}84$$
$$P(\nu < 4) = P(\eta < 0{,}8) = F_\eta(0{,}8) = 0{,}96$$
$$P(\nu < 5) = F_\eta(1) = 1$$

ν	0	1	2	3	4	5
P_ν	$0{,}36$	$0{,}28$	$0{,}2$	$0{,}12$	$0{,}04$	0



$$F_\nu(y) = \begin{cases} 0, & y \leqslant 0 \\ 0{,}36, & y \in (0;1] \\ 0{,}64, & y \in (1;2] \\ 0{,}84, & y \in (2;3] \\ 0{,}96, & y \in (3;4] \\ 1, & y > 4 \end{cases}$$

$$E\nu = 0{,}28 + 0{,}4 + 0{,}36 + 0{,}16 = 1{,}2$$

$$D\nu = 0{,}28 + 4 \cdot 0{,}2 + 9 \cdot 0{,}12 + 16 \cdot 0{,}04 -$$

$$-(1{,}2)^2 = 1{,}36$$

(3) $E\xi = \int_{-\infty}^{+\infty} x p_\xi(x)\,dx = \int_0^1 x\cdot 2x\,dx = \int_0^1 2x^2 dx =$

$= \frac{2}{3}x^3\Big|_0^1 = \frac{2}{3}$

$E\eta = \int_0^1 y\cdot(2-2y)\,dy = \frac{1}{3}$

$D\xi = \int_0^1 2x^3 dx - \frac{4}{9} = \frac{2x^4}{4}\Big|_0^1 - \frac{4}{9} = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

$D\eta = \int_0^1 y^2(2-2y)\,dy - \frac{1}{9} = \frac{1}{6} - \frac{1}{9} = \frac{1}{18}$

$\mathrm{cov}_{\xi,\eta} = E\xi\eta - E\xi\cdot E\eta$

$E\xi\eta = \iint_{\triangle ABC} xy\, p_{\xi\eta}(x,y)\,dx\,dy = \int_0^1 y\,dy \int_y^1 2x\,dx =$

$= \int_0^1 y\cdot(1-y^2)\,dy = \int_0^1 y - y^3\,dy = \left(\frac{y^2}{2} - \frac{y^4}{4}\right)\Big|_0^1 =$

$= \frac{1}{4}$

$\mathrm{cov}_{\xi\eta} = \frac{1}{4} - \frac{2}{9} = \frac{9-8}{36} = \frac{1}{36}$

$E\begin{pmatrix}\xi\\ \eta\end{pmatrix} = \begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\end{pmatrix}$ $\quad Var\begin{pmatrix}\xi\\ \eta\end{pmatrix} = \begin{pmatrix}\frac{1}{18} & \frac{1}{36}\\ \frac{1}{36} & \frac{1}{18}\end{pmatrix}$

$\rho(\xi,\eta) = \frac{\frac{1}{36}}{\sqrt{\left(\frac{1}{18}\right)^2}} = \frac{\frac{1}{36}}{\frac{1}{18}} = \frac{1}{2}$

$P\ \xi|\eta$, $E\xi|\eta$, $D\xi|\eta$

$$P_{\xi|\eta} = \frac{P_{\xi\eta}(x,y)}{P_\eta(y)} = \begin{cases} \frac{1}{1-y}, & x \in [y;1] \\ 0, & \text{иначе} \end{cases}$$

$$E(\xi|\eta=y) = \int_y^1 x \cdot \frac{1}{1-y}\, dx = \frac{1}{1-y}\left(\frac{x^2}{2}\right)\Big|_y^1 =$$

$$= \frac{1}{1-y}(1-y^2)\cdot\frac{1}{2} = \frac{1+y}{2}$$

$$D(\xi|\eta=y) = \int_y^1 x^2 \cdot \frac{1}{1-y}\, dx - \left(E(\xi|\eta=y)\right)^2 =$$

$$= \frac{1}{3}(y^2+y+1) - \left(\frac{1+y}{2}\right)^2 = \frac{1}{3}y^2 + \frac{(-2y+1-3y^2)}{12} =$$

$$= \frac{4y^2-2y+1-3y^2}{12} = \frac{y^2-2y+1}{12} = \frac{(y-1)^2}{12}$$

$\eta = \xi - 2$

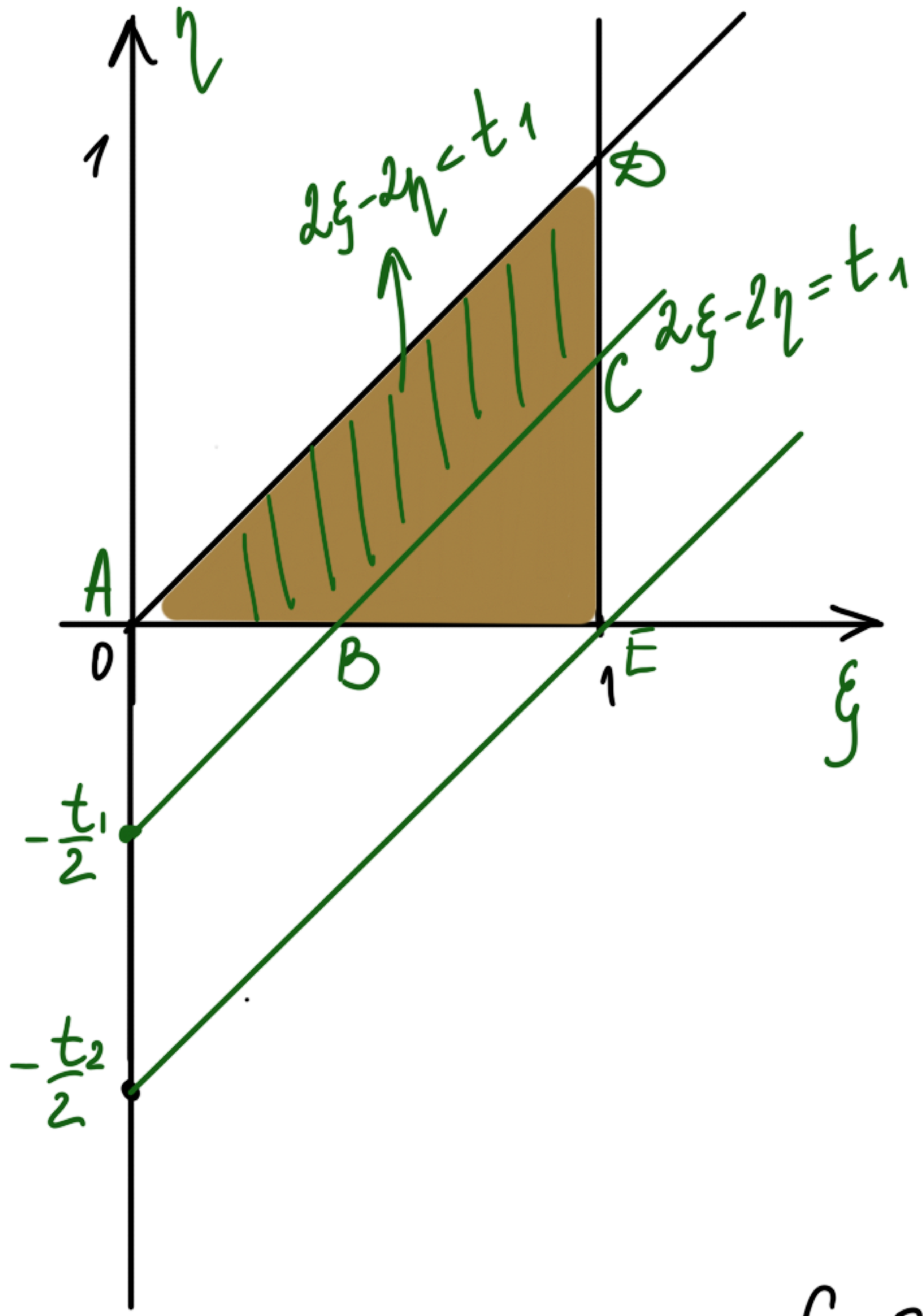
4 $\mu = 2\xi - 2\eta$

$t = 2\xi - 2\eta$

$$F_\mu(t) = P(\mu < t)$$

$$\operatorname{supp} \mu = [0;2] \qquad F_\mu = \begin{cases} 0, & t \in (-\infty;0] \\ \circledast, & t \in (0;2] \\ 1, & t > 2 \end{cases}$$

$$F_\mu(t) = P(2\xi - 2\eta < t) = P\left(\xi < \frac{t}{2} + \eta\right) =$$



$$2\xi - 2\eta < t$$

$$\xi - \eta < \frac{t}{2}$$

$$\eta > \xi - \frac{t}{2}$$

$$P(2\xi - 2\eta < t) =$$

$$= \frac{S_{ABCD}}{S_{AED}} = \frac{\frac{1}{2} - \frac{\left(1-\frac{t}{2}\right)^2}{2}}{\frac{1}{2}}$$

$$= 1 - \left(1 - \frac{t}{2}\right)^2 = t - \frac{t^2}{4}$$

$$t \in [0;2]$$

$$F_\mu(t) = \begin{cases} 0, & t \in (-\infty; 0] \\ t - \frac{t^2}{4}, & t \in (0;2] \\ 1, & t \in (2; +\infty) \end{cases}$$

$$P_\mu(t) = \begin{cases} 0, & t \in (-\infty; 0] \\ 1 - \frac{t}{2}, & t \in (0;2] \\ 0, & t \in (2; +\infty) \end{cases}$$

$$E\mu = \int_0^2 t\left(1 - \frac{t}{2}\right) dt = \frac{2}{3}$$

$$D\mu = \int_0^2 t^2 \left(1 - \frac{t}{2}\right) dt - \frac{4}{9} = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$$