В. Вычислить математическое ожидание и дисперсию следующих абс. непрерывных распределений

а) Плотность распределения

$$p_\xi(x) = \begin{cases} \frac{1}{2}, & x \in [-1, 0] \\ e^{-x}/2, & x > 0 \\ 0, & x < -1 \end{cases}$$

б) Функция распределения

$$F_\xi(x) = \begin{cases} 0, & x \le 0 \\ \frac{x^3}{c}, & x \in (0, 2] \\ 1, & x > 2 \end{cases}$$ (найти c -?)

а) $$p_\xi(x) = \begin{cases} \frac{1}{2}, & x \in [-1; 0] \\ e^{-x}/2, & x > 0 \\ 0, & x < -1 \end{cases}$$

$$E_\xi = \int x p(x)\, dx = \int_{-1}^{0} \frac{x}{2}\, dx + \int_0^\infty \frac{e^{-x}}{2} x\, dx$$

$$= \frac{x^2}{4}\Big|_{-1}^{0} + \int_0^\infty x \frac{e^{-x}}{2}\, dx = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$$

$$\int_0^\infty x \frac{e^{-x}}{2}\, dx = \begin{bmatrix} e^{-x} = dv & x = u \\ v = -e^{-x} & dx = du \end{bmatrix} =$$

$$= \frac{1}{2}\left(x(-e^{-x})\Big|_0^\infty - \int_0^\infty -e^{-x}\, dx\right) = \frac{1}{2}\left(-e^{-x}\Big|_0^\infty\right) = \frac{1}{2}$$

$$D_\xi = \int x^2 p(x)\, dx - (E_\xi)^2 = \int_0^\infty \frac{e^{-x}}{2} x^2 +$$

$$+ \int_{-1}^{0} \frac{x^2}{2}\, dx - \left(\frac{1}{4}\right)^2 = \left[\text{аналогично 1, только 2 шага}\right] =$$

$$\int_0^\infty \frac{e^{-x}}{2} x^2\, dx = \begin{bmatrix} e^{-x} = dv & u = x^2 \\ v = -e^{-x} & du = x \end{bmatrix} =$$

$$= \frac{1}{2}\left(-e^{-x} \cdot x^2\Big|_0^\infty - \int_0^\infty -e^{-x} \cdot x\right) = [\ .\ ..\ ] =$$

$$= 1$$

$$D\xi = \underline{1+\frac{1}{6}} - \left(\frac{1}{4}\right)^2 = \frac{53}{48}$$

8) Функция распределения

$$F_\xi(x) = \begin{cases} 0, & x \le 0 \\ \frac{x^3}{c}, & x \in (0, 2] \\ 1, & x > 2 \end{cases} \quad \text{(найти c -?)}$$

$$p_\xi = \begin{cases} 0, & x \le 0 \\ \frac{3x^2}{8}, & x \in (0; 2] \\ 0, & x > 2 \end{cases}$$

$$\frac{x^3}{c}(2) = \frac{8}{c} = 1 \Rightarrow c = 8$$

$$E\xi = \int_0^2 x \cdot \frac{3x^2}{8}\,dx = \int_0^2 \frac{3x^3}{8}\,dx = \frac{3x^4}{32}\Big|_0^2 =$$

$$= \frac{48}{32} =$$

$$D\xi = \int_0^2 \frac{3x^4}{8}\,dx = \frac{3x^5}{40}\Big|_0^2 = \frac{3\cdot 32}{40} - \left(\frac{48}{32}\right)^2$$

в) Плотность распределения

$$p_\xi(x) = \begin{cases} c|\sin x|, & x \in [-\pi, \pi] \\ 0, & \text{в ост. случаях} \end{cases}$$

(найти c)

$$p_\xi(x) = \begin{cases} c|\sin x|, & x \in [-\pi, \pi] \\ 0, & \text{в ост. сл.} \end{cases}$$

$$\int_{-\pi}^{\pi} c \cdot |\sin x|\,dx = 2c\int_0^\pi \sin x\,dx = 2c(-\cos x)\Big|_0^\pi =$$

$$= 2c(1+1) = 4c = 1 \Rightarrow c = \frac{1}{4}$$

$$p_\xi(x) = \begin{cases} \frac{1}{4}|\sin x|, & x \in [-\pi; \pi] \\ 0, & \text{иначе} \end{cases}$$

$$E\xi = \int_{-\pi}^{\pi} x \cdot \frac{1}{4}|\sin x|\,dx = 0 \text{ по нечет-ти}$$

$$D\xi = \int_{-\pi}^{\pi} x^2 \cdot \frac{1}{4}|\sin x|\,dx = \frac{1}{2}(\pi^2-4)$$

$$= 2\int_{0}^{\pi} x^2 \cdot \frac{1}{4}\sin x\,dx =$$

$$= \frac{1}{2}\int_{0}^{\pi} x^2 \cdot \sin x\,dx = \begin{bmatrix} x^2 = v & \sin x = du \\ 2x = dv & u = -\cos x \end{bmatrix}$$

$$= \frac{1}{2}\left(-v\cos x \Big|_0^{\pi} + \int_0^{\pi} 2x\cos x\,dx\right) =$$

$$= \begin{bmatrix} 2x = v & \cos x = du \\ 2dx = dv & \sin x = u \end{bmatrix} =$$

$$= \frac{1}{2}\left(-x^2\cos x\Big|_0^{\pi} + 2\left(x\cdot\sin x\Big|_0^{\pi} - \int_0^{\pi}\sin x\,dx\right)\right)$$

$$= \frac{1}{2}\left(-x^2\cos x\Big|_0^{\pi} + 2x\sin x\Big|_0^{\pi} - 2(-\cos x)\Big|_0^{\pi}\right) =$$

$$= \frac{1}{2}\left(-x^2\cos x\Big|_0^{\pi} + 2x\sin x\Big|_0^{\pi} + 2\cos x\Big|_0^{\pi}\right) =$$

$$= \frac{1}{2}\left(\pi^2 + (-2) + (-2)\right) = \frac{1}{2}(\pi^2 - 4)$$

) Функция распределение

$$F_\xi(x) = \begin{cases} 0, & x \le 0 \\ \sin x; & x \in [0, \pi/2] \\ 1, & x > \pi/2 \end{cases}$$

$$p_\xi(x) = \begin{cases} 0, & x \le 0 \\ \cos x, & x \in (0; \frac{\pi}{2}] \\ 1, & x > \pi/2 \end{cases}$$

$$E\xi = \int_0^{\frac{\pi}{2}} \cos x \cdot x = \begin{bmatrix} x = v & ; & \cos x = du \\ dx = dv & ; & \sin x = u \end{bmatrix} =$$

$$= x \sin x \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx =$$

$$= \frac{\pi}{2} - (-\cos x)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2} - (-0 - (-1)) =$$

$$= \frac{\pi}{2} - 1.$$

$$D\xi = \int_0^{\frac{\pi}{2}} \cos x \cdot x^2 - (E\xi)^2 = [\text{аналогич}] =$$

$$= \frac{1}{4}(\pi^2 - 8) - \left(\frac{\pi}{2} - 1\right)^2$$

g) Плотность распределения

$$p_\xi(x) = \begin{cases} \frac{a^p}{\Gamma(p)} x^{p-1} e^{-ax}, & x \geq 0 \\ 0, & x \leq 0 \end{cases} \quad \begin{matrix} a>0 \\ p>0 \end{matrix}$$

$\Gamma(p)$ – гамма функция Эйлера

$\Gamma(p+1) = p\Gamma(p)$

$$p_\xi(x) = \begin{cases} \dfrac{a^p}{\int_0^\infty t^{(p-1)} e^{-t} dt} x^{p-1} e^{-ax} & x \geq 0 \\ 0, \; x \leq 0 \end{cases} \quad \begin{matrix} a>0 \\ p>0 \end{matrix}$$

$\Gamma(p)$ – не зависит от x

$$c = \frac{a^p}{\Gamma(p)} \quad x^{p-1} e^{-ax}$$

$$E\xi = c \int_0^\infty x^p e^{-ax} dx = \left[\begin{matrix} ax = t \\ adx = dt \end{matrix} \right] =$$

$$= \frac{c}{a} \int_0^\infty \left(\frac{t}{a}\right)^p e^{-t} dt = \frac{c}{a^{p+1}} \int_0^\infty t^p e^{-t} dt =$$

$$= \frac{c}{a^{p+1}} \Gamma(p+1) = \frac{c}{a^{p+1}} \cdot p \cdot \Gamma(p) =$$

$$= \frac{a^p p \cdot \cancel{\Gamma(p)}}{a^{p+1} \cancel{\Gamma(p)}} = \frac{p}{a}$$

$$D\xi = C\int_0^\infty x^{p+1} e^{-ax}\,dx = \begin{bmatrix} ax = t \\ a\,dx = dt \end{bmatrix} =$$

$$\text{-}\ p^2/a^2$$

$$= \frac{C}{a}\int_0^\infty \left(\frac{t}{a}\right)^{p+1} \cdot e^{-t}\,dt =$$

$$\text{-}\ p^2/a^2$$

$$= \frac{C}{a^{p+2}}\int t^{p+1} e^{-t}\,dt = \frac{Cp^2\Gamma(p)}{a^{p+2}} =$$

$$\text{-}\ p^2/a^2 \qquad \text{-}\ p^2/a^2$$

$$= \frac{a^p}{\Gamma(p)} \cdot \frac{p(p+1)\Gamma(p)}{a^{p+2}} = \underline{\underline{\frac{p(p+1)}{a^2}}} - \frac{p^2}{a}$$

$$\text{-}\ p^2/a$$

е) Функция распределения

$$F_\xi(x) = \begin{cases} 0, & x \le 1 \\ 1 - 1/x, & x > 1. \end{cases}$$

$$p_\xi(x) = \begin{cases} 0, & x \le 1 \\ \frac{1}{x^2}, & x > 1. \end{cases}$$

$$E\xi = \int_1^{+\infty} \frac{1}{x}\,dx = \ln x \Big|_1^{\infty} \text{ — расх.}$$

$$D\xi = \int_1^{+\infty} 1\,dx - \left(\ln x \Big|_1^{\infty}\right)^2 =$$

$$= x\Big|_1^{+\infty} - \ln x\Big|_1^{+\infty} \text{ — расх}$$

C

а) $p_\xi(x) = \begin{cases} \frac{1}{2}, x \in [-1,1] \\ 0, \text{в ост. сл} \end{cases}$

$\eta = \xi^2$

$$p_\xi(x) = \begin{cases} \frac{1}{2}, x \in [-1;1] \\ 0, \text{ в ост.} \end{cases}$$

$$F_\xi(x) = \begin{cases} 0, & x \in (-\infty; -1] \\ \frac{x}{2} + \frac{1}{2}, & x \in [-1; 1] \\ 1, & \text{иначе} \end{cases}$$

$\operatorname{supp} \xi = [-1; 1]$ $\quad \operatorname{supp} \xi^2 = [0; 1]$

$$F_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \sqrt{x}, & x \in (0; 1] \\ 1, & x \in [1; +\infty) \end{cases}$$

$$F_\eta(x) = P(\eta < x) =$$

$$= P(\xi^2 < x) = P(-\sqrt{x} < \xi < \sqrt{x}) =$$

$$= F(\sqrt{x}) - F(-\sqrt{x}) = \left(\frac{\sqrt{x}}{2} + \frac{1}{2}\right) - \left(-\frac{\sqrt{x}}{2} + \frac{1}{2}\right)$$

$$= \sqrt{x}$$

$$p_\eta(x) = \begin{cases} 0, & x \in (-\infty; 0] \\ \frac{1}{2\sqrt{x}}, & x \in (0; 1] \\ 0, & x \in [1; +\infty) \end{cases}$$

$$E\xi = \frac{x^{3/2}}{3}\Big|_0^1 = \frac{1}{3}$$

$$D\xi = \frac{x^{5/2}}{5}\Big|_0^1 - \frac{1}{9} = \frac{1}{5} - \frac{1}{9}$$

8) $p_\xi(x) = \begin{cases} e^{-(x-1)}, & x \geq 1 \\ 0, & x < 1 \end{cases}$

$\eta = e^{\xi - 1}$

$$F_\xi(x) = \begin{cases} 0, & x < 1 \\ -e^{1-x} + 1, & x \geq 1 \end{cases}$$

$\operatorname{supp} \xi = [1, +\infty)$ $\quad \operatorname{supp} \eta = [1; +\infty)$

$$F_\eta = \begin{cases} 0, & x < 1 \\ -\frac{1}{x} + 1, & x \geq 1 \end{cases}$$

$$F_\eta = P(\eta < x) = P(e^{\xi - 1} < x) =$$

$$= P(\xi - 1 < \ln x) = P(\xi < \ln x + 1) =$$

$$= F(\ln x + 1) = -e^{-\ln x} + 1 = -\frac{1}{x} + 1$$

$$p_\xi = \begin{cases} 0, & x < 1 \\ \frac{1}{x^2}, & x \geq 1 \end{cases}$$

$$E\xi = \int_1^\infty x/x^2 = \log x\Big|_1^\infty$$ – расх

$$D\xi = \int_1^\infty 1\,dx - \log x\Big|_1^\infty =$$

$$= (x - \log x)\Big|_1^\infty$$ – расх