Probabilistic data structures. Part 4. Similarity

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PROBABILISTIC DATA STRUCTURES

ALL YOU WANTED TO KNOW BUT WERE AFRAID TO ASK

PART 4: SIMILARITY

Andrii Gakhov tech talk @ ferret

SIMILARITY

Agenda:

- Locality-Sensitive Hashing (LSH)
- MinHash
- ▶ SimHash

THE PROBLEM

To find clusters of similar documents from the document set

To find duplicates of the document in the document set

LOCALITY SENSITIVE HASHING

LOCALITY SENTSITIVE HASHING

- Locality Sentsitive Hashing (LSH) was introduced by Indyk and Motwani in 1998 as a family of functions with the following property:
 - similar input objects (from the domain of such functions) have a higher probability of colliding in the range space than non-similar ones
- LSH differs from conventional and cryptographic hash functions because
 it aims to maximize the probability of a "collision" for similar
 items.
- Intuitively, LSH is based on the simple idea that, if two points are close together, then after a "projection" operation these two points will remain close together.

LSH: PROPERTIES

 One general approach to LSH is to "hash" items several times, in such a way that similar items are more likely to be hashed to the same bucket than dissimilar items are.
 We then consider any pair that hashed to the same bucket for any of the hashings to be a candidate pair.

LSH: PROPERTIES

- Examples of LSH functions **are known** for *resemblance*, cosine and hamming distances.
- LSH functions **do not exist** for certain commonly used similarity measures in information retrieval, the *Dice coefficient* and the *Overlap coefficient* (Moses S. Charikar, 2002)

LSH: PROBLEM

- Given a query point, we wish to find the points in a large database that are closest to the query. We wish to guarantee with a high probability equal to 1δ that we return the nearest neighbor for any query point.
 - Conceptually, this problem is easily solved by iterating through each point in the database and calculating the distance to the query object. However, our database may contain billions of objects – each object described by a vector that contains hundreds of dimensions.
- LSH proves useful to identify nearest neighbors quickly even when the database is very large.

LSH: FINDING DUPLICATE PAGES ON THE WEB

How to select features for the webpage?

document vector from page content

 Compute "document-vector" by case-folding, stop-word removal, stemming, computing term-frequencies and finally, weighting each term by its inverse document frequency (IDF)

shingles from page content

- Consider a document as an sequence of words. A shingle is a hash value of a k-gram which is sub-sequence of k consequent words.
- Hashes for shingles can be computed using Rabin's fingerprints

connectivity information

The idea that similar web pages have several incoming links in common.

anchor text and anchor window

• The idea that similar documents should have similar anchor text. The words in the anchor text or anchor window are folded into the document-vector itself.

phrases

Idea is to identify phrases and compute a document-vector that includes phrases as terms.

LSH: FINDING DUPLICATE PAGES ON THE WEB

- The Web contains many duplicate pages, partly because content is duplicated across sites and partly because there is more than one URL that points to the same page.
- If we use shingles as features, each shingle represents a portion of a Web page and is computed by forming a histogram of the words found within that portion of the page.
- We can test to see if a portion of the page is duplicated elsewhere on the Web by looking for other shingles with the same histogram. If the shingles of the new page match shingles from the database, then it is likely that the new page bears a strong resemblance to an existing page.
- Because Web pages are surrounded by navigational and other information that changes from site to site it is useful to use nearest-neighbor solution that can employ LSH functions.

LSH: RETRIEVING IMAGES

- LSH can be used in image retrieval as an object recognition tool. We compute a detailed metric for many different orientations and configurations of an object we wish to recognize.
- Then, given a new image we simply check our database to see
 if a precomputed object's metrics are close to our query. This
 database contains millions of poses and LSH allows us to
 quickly check if the query object is known.

LSH: RETRIEVING MUSIC

- In music retrieval conventional hashes and robust features are typically used to find musical matches.
- The features can be fingerprints, i.e., representations of the audio signal that are robust to common types of abuse that are performed to audio before it reaches our ears.
 - Fingerprints can be computed, for instance, by noting the peaks in the spectrum (because they are robust to noise) and encoding their position in time and space. User then just has to query the database for the same fingerprint.
- However, to **find similar songs we cannot use fingerprints** because these are different when a song is remixed for a new audience or when a different artist performs the same song. Instead, we can use several seconds of the song—**a snippet**—as a shingle.
 - To determine if two songs are similar, we need to query the database and see if a large enough number of the query shingles are close to one song in the database.
- Although closeness depends on the feature vector, we know that long shingles provide specificity. This is particularly important because we can eliminate duplicates to improve search results and to link recommendation data between similar songs.

LSH: PYTHON

- https://github.com/ekzhu/datasketch
 datasketch gives you probabilistic data structures that can process vary large amount of data
- https://github.com/simonemainardi/LSHash
 LSHash is a fast Python implementation of locality sensitive hashing with persistence support
- https://github.com/go2starr/lshhdc
 LSHHDC: Locality-Sensitive Hashing based High
 Dimensional Clustering

LSH: READ MORE

- http://www.cs.princeton.edu/courses/archive/spr04/cos598B/ bib/IndykM-curse.pdf
- http://infolab.stanford.edu/~ullman/mmds/book.pdf
- http://www.cs.princeton.edu/courses/archive/spr04/cos598B/ bib/CharikarEstim.pdf
- http://www.matlabi.ir/wp-content/uploads/bank_papers/ g_paper/g15_Matlabi.ir_Locality-Sensitive%20Hashing%20for%20Finding%20Nearest%20Neighbors.pdf
- https://en.wikipedia.org/wiki/Locality-sensitive_hashing

MINHASH

MINHASH: RESEMBLANCE SIMILARITY

- While talking about relations of two documents A and B, we mostly interesting in such
 concept as "roughly the same", that could be mathematically formalized as their
 resemblance (or Jaccard similarity).
- Every document can be seen as a set of features and such issue could be reduced to a set intersection problem (find similarity between sets).
- The **resemblance r**(**A**,**B**) of two documents is a number between 0 and 1, such that it is close to 1 for the documents that are "roughly the same":

$$J(A,B) = r(A,B) = \frac{|A \cap B|}{|A \cup B|}$$

- For **duplicate detection problem** we can reformulate is as a task to find pairs of documents whose resemblance r(A,B) exceeds certain threshold **R**.
- For the clustering problem we can use d(A,B) = 1 r(A,B) as a metric to design algorithms intended to cluster a collection of documents into sets of closely resembling documents.

MINHASH

- MinHash (minwise hashing algorithm) has been proposed by Andrei Broder in 1997
- The minwise hashing family applies a random permutation $\pi \colon \Omega \to \Omega$, on the given set **S**, and stores **only the minimum values after the permutation mapping**.
- Formally MinHash is defined as:

$$h_{\pi}^{\min}(S) = \min(\pi(S)) = \min(\pi_1(S), \dots, \pi_k(S))$$

MINHASH: MATRIX REPRESENTATION

- Collection of sets could be visualised as characteristic matrix CM.
- The columns of the matrix correspond to the sets, and the rows correspond to elements of the universal set from which elements of the sets are drawn.
 - CM(r, c) = 1 if the element for row r is a member of the set for column c, otherwise CM(r,c) = 0
- **NOTE**: The characteristic matrix is <u>unlikely</u> to be the way the data is stored, but it is useful as a way to visualize the data.
 - For one reason not to store data as a matrix, these matrices are almost always sparse (they have many more 0's than 1's) in practice.

MINHASH: MATRIX REPRESENTATION

- Consider the following sets documents (represented as bag of words):
 - S1 = {python, is, a, programming, language}
 - S2 = {java, is, a, programming, language}
 - S3 = {a, programming, language}
 - S4 = {python, is, a, snake}
- We can assume that our universal set limits to words from these 4 documents. The characteristic matrix of these sets is

	row	SI	S2	S 3	S4
a	0				
is	I			0	
java	2	0		0	0
language	3				0
programming	4	1	F = []		0
python	5		0	0	
snake	6	0	0	0	

MINHASH: INTUITION

- The *minhash* value of any column of the characteristic matrix is the **number of the first row, in the permuted order, in which the column has a 1**.
- To minhash a set represented by such columns, we need to pick a permutation of the rows.

MINHASH: ONE STEP EXAMPLE

Consider the characteristic matrix:

row	SI	S2	S 3	S4	permuted row
1		0	1	0	2
2		0	0	0	5
3	0	0		1	6
4		0	0		
5	0	0			4
6	0			0	3

Record the first "1" in each column:

$$m(S1) = 2$$
, $m(S2) = 3$, $m(S3) = 2$, $m(S4) = 6$

• Estimate the Jaccard similarity: J(Si,Sj) = 1 if m(Si) = m(Sj), or 0 otherwise

$$J(S1,S3) = 1, J(S1,S4) = 0$$

MINHASH: INTUITION

- It is not feasible to permute a large characteristic matrix explicitly.
 - Even picking a random permutation of millions of rows is timeconsuming, and the necessary sorting of the rows would take even more time.
- Fortunately, it is possible to **simulate the effect of a random permutation** by a **random hash function** that maps row numbers to as many buckets as there are rows.
 - So, instead of picking \mathbf{n} random permutations of rows, we pick \mathbf{n} randomly chosen hash functions $\mathbf{h_1}, \mathbf{h_2}, \ldots, \mathbf{h_n}$ on the rows

MINHASH: PROPERTIES

- Connection between minhash and resemblance (Jaccard) similarity of the sets that are minhashed:
 - The probability that the minhash function for a random permutation of rows produces the same value for two sets equals the Jaccard similarity of those sets
- Minhash(π) of a set is the number of the row (element) with first non-zero in the permuted order π .
- MinHash is an LSH for resemblance (Jaccard) similarity, which defined over binary vectors.

MINHASH: ALGORITHM

- Pick k randomly chosen hash functions h₁, h₂, ..., h_k
- From the column representing set S, construct the *minhash* signature for S the vector $[h_1(S), h_2(S), \ldots, h_k(S)]$.
- Build the characteristic matrix CM for the set S.
- Construct a signature matrix SIG, where SIG(i, c) corresponds to the ith hash function and column c. Initially, set SIG(i, c) = ∞ for each i and c.
- On each row **r**:
 - Compute $h_1(r)$, $h_2(r)$, ..., $h_k(r)$
 - For each column **c**:
 - If CM(r,c) = 1, then set $SIG(i,c) = min\{SIG(i,c), h_i(r)\}$ for each i = 1...n
- Estimate the resemblance (Jaccard) similarities of the underlying sets from the final signature matrix

MINHASH: EXAMPLE

- Consider the same set of 4 documents and universal set that consists of 7 words
- Consider 2 hash functions:
 - $h1(x) = x + 1 \mod 7$
 - $h2(x) = 3x + 1 \mod 7$

	row	SI	S2	S 3	S4	hl(row)	h2 (row)
a	0						
is				0		2	4
java	2	0		0	0	3	0
language	3				0	4	3
programming	4				0	5	6
python	5		0	0		6	2
snake	6	0	0	0		0	5

MINHASH: EXAMPLE

Initially the signature matrix will have all ∞:

	SI	S2	S3	S4
hI	00	00	00	∞
h2	00	00	00	∞

_		_	_	_	_	
r	SI	S2	S3	S4	hl	h2
0	I	- 1	1	- [1	1
I	I		0	I	2	4
2	0	I	0	0	3	0
3	I	-	- 1	0	4	3
4	I	I	1	0	5	6
5	I	0	0	1	6	2
6	0	0	0	I	0	5

• First, consider row 0, all values h1(0) and h2(0) are both 1. So, for all sets with 1 in the row 0 we set value h1(0)=h2(0)=1 in the signature matrix:

	SI	S2	S 3	S4
hI	L		I	
h2				

Next, consider row 1, its values h1(1) = 2 and h2(1) = 4. In this row only S1, S2 and S4 have 1's, so we set the appropriate cells with the *minimum* between existing values and 2 for h1 or 4 for h2:

	SI	S2	S 3	S4
hl			I	
h2				

MINHASH: EXAMPLE

Continue with other rows and after row 5 the signature matrix is:

	SI	S2	S 3	S4
hl			T	
h2		0		

		00	00			1.0
r	SI	52	S3	S4	hl	h2
0	I	- 1	- 1	-1	1	1
I	I	-	0	I	2	4
2	0	Ι	0	0	3	0
3	Н	- [- 1	0	4	3
4	I	I	- 1	0	5	6
5	I	0	0	1	6	2
6	0	0	0		0	5

• Finally, consider row 6, its values h1(6) = 0 and h2(6) = 5. In this row only S4 has 1's, so we set the appropriate cells with the *minimum* between existing values and 0 for h1 or 5 for h2. The final signature matrix is:

	SI	S2	S 3	S4
hl				0
h2		0		

- Document S1 is similar to S3 (identical columns), so similarity is 1 (the true value $J \approx 0.75$)
- Documents S2 and S4 have no rows in common, so similarity is 0 (the true value $J \approx 0.28$)

MINHASH: PYTHON

- https://github.com/ekzhu/datasketch
 datasketch gives you probabilistic data structures that
 can process vary large amount of data
- https://github.com/anthonygarvan/MinHash
 MinHash is an effective pure python implementation of Minhash

MINHASH: READ MORE

- http://infolab.stanford.edu/~ullman/mmds/book.pdf
- http://www.cs.cmu.edu/~guyb/realworld/slidesS13/ minhash.pdf
- https://en.wikipedia.org/wiki/MinHash
- http://www2007.org/papers/paper570.pdf
- https://www.cs.utah.edu/~jeffp/teaching/cs5955/L4-Jaccard+Shingle.pdf

B-BIT MINHASH

B-BIT MINHASH

- A modification of minwise hashing, called **b**-bit minwise hashing, was proposed by Ping Li and Arnd Christian König in 2010.
- The method of **b**-bit minwise hashing provides a simple solution by storing only the lowest **b** bits of each hashed data, reducing the dimensionality of the (expanded) hashed data matrix to just **2**^b•**k**.
- **b**-bit minwise hashing is simple and requires only minimal modification to the original minwise hashing algorithm.

B-BIT MINHASH

- Intuitively, using fewer bits per sample will increase the estimation variance, compared to the original MinHash, at the same "sample size" k.
 Thus, we have to increase k to maintain the same accuracy.
- Interestingly, the theoretical results demonstrate that, when resemblance is not too small (e.g. R ≥ 0.5 as a common threshold), we do not have to increase k much.
 - This means that, compared to the earlier approach, the **b**-bit minwise hashing can be used to **improve** estimation accuracy and significantly reduce storage requirement at the same time.
- For example, for b=1 and R=0.5, the estimation **variance will increase** at most by factor of 3.
 - So, in order not to lose accuracy, we have to increase the sample size k by factor 3.
 - If we originally stored each hashed value using 64 bits, the improvement by using b=1 will be 64/3=21.3

B-BIT MINHASH: READ MORE

- https://www.microsoft.com/en-us/research/publication/bbit-minwise-hashing/
- https://www.microsoft.com/en-us/research/publication/ theory-and-applications-of-b-bit-minwise-hashing/

SIMHASH

SIMHASH

- SimHash (a sign normal random projection) algorithm has been proposed by Moses S. Charikar in 2002.
- SimHash originate form the concept of sign random projections (SRP):
 - In short, for a given vector x SRP utilizes a random vector w with each component generated from i.i.d normal, i.e. $w_i \sim N(0,1)$, and only **stores the sign of the projected data**.
- Currently, SimHash is the only known LSH for cosine similarity

$$sim(A,B) = \frac{A \cdot B}{||A|| \cdot ||B||} = \frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\sum_{i=1}^{n} a_i^2} \sqrt{\sum_{i=1}^{n} b_i^2}}$$

SIMHASH

- In fact, it is a **dimensionality reduction technique** that maps high-dimensional vectors to \mathbf{f} -bit fingerprints, where \mathbf{f} is small (e.g. 32 or 64).
- SimHash algorithm consider a fingerprint as a "hash" of its properties and assumes that similar documents have similar hash values.
 - This assumption requires the hash functions to be LSH and, as we already know, it isn't trivial as it could be seen for the first time.
 - If we consider 2 documents that are different just in a *single byte* and apply such popular hash functions as SHA-1 or MD5, they will definitely get two *completely different* hash-values, that aren't close. This is why such approach requires a special rule how to define hash functions that could have such required property.
- An attractive feature of the SimHash hash functions is that the output is
 a single bit (the sign)

SIMHASH: FINGERPRINTS

- to generate an f-bit fingerprint, maintain an f-dimensional vector V (all dimensions are 0 at the beginning).
- a feature is hashed into an f-bit hash value (using a hash function) and can be seen as a binary vector of f bits
- these f bits (unique to the feature) increment/decrement the f components of the vector by the weight of that feature as follows:
 - if the i^{th} bit of the hash value is 1 = > the i^{th} component of V is incremented by the weight of that feature
 - if the i^{th} bit of the hash value is 0 = > the i^{th} component of V is decremented by the weight of that feature.
- When all features have been processed, components of V are positive or negative.
- The signs of components determine the corresponding bits of the final fingerprint.

SIMHASH: FINGERPRINTS EXAMPLE

- Consider a document after POS tagging: {("ferret", NOUN), ("go", VERB)}
 - decide to have weights for features based on their POS tags: {"NOUN": 1.0, "VERB": 0.9}
- h(x) some hash function, and we will calculate **16-bit** (f=16) fingerprint **F**
- Maintain V = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0) for the document
 - h("hello") = 010101010101111
 - h(``hello'')[1] == 0 => V[1] = V[1] 1.0 = -1.0
 - h("hello")[2] == 1 => V[2] = V[2] + 1.0 = 1.0
 - •
- V = (-1.0, 1.0, -1.0, 1.0, -1.0, 1.0, -1.0, 1.0, -1.0, 1.0, -1.0, 1.0, -1.0, 1.0, 1.0, 1.0)
 - h(``go'') = 0111011100100111
 - h(``go'')[1] == 0 => V[1] = V[1] 0.9 = -1.9
 - h(``go'')[2] == 1 => V[2] = V[2] + 0.9 = 1.9
 - •
- V = (-1.9, 1.9, -0.1, 1.9, -1.9, 1.9, -0.1, 1.9, -1.9, 0.1, -0.1, 0.1, -1.9, 1.9, 1.9, 1.9)
 - sign(V) = (-, +, -, +, -, +, -, +, -, +, -, +, +, +, +)
- fingerprint F = 010101010101111

SIMHASH: FINGERPRINTS

After converting document into simhash fingerprints, we face the following design problem:

 Given a f-bit fingerprint of a document, how quickly discover other fingerprints that differ in at most k bit-positions?

SIMHASH: DATA STRUCTURE

- SimHash data structure consists of t tables: $T_1, T_2, ..., T_t$. With each table T_i is associated 2 quantities:
 - an integer p_i (number of top bit-positions used in comparisons)
 - a permutation π_i over the f bit-positions
- Table T_i is constructed by applying permutation π_i to each existing fingerprint F and the resulting set of permuted f-bit fingerprints is sorted.
 - To optimize such structure to store it in the main-memory, it is possible to compress
 each such table (that can decrease it size by half approximately).
- Example:

$$D_F = \{0110, 0010, 0010, 1001, 1110\}$$

$$\pi_i = \{1 \rightarrow 2, 2 \rightarrow 4, 3 \rightarrow 3, 4 \rightarrow 1\}$$
• $\pi_i (0110) = 0011, \pi_i (0010) = 0010, \pi_i (1001) = 1100, \pi_i (1110) = 0111$

$$T_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

SIMHASH: ALGORITHM

- Actually, the algorithm follows the general approach we saw for LSH
- Given a fingerprint **F** and an integer **k** we probe tables from the data structure in parallel:
 - For each table T_i we identify all permuted fingerprints in T_i whose top p_i bit-positions match the top p_i bit-positions of $\pi_i(\mathbf{F})$.
 - After that, for each such permuted fingerprints, we check if it differs from the $\pi_i(\mathbf{F})$ in at most k bit-positions (in fact, comparing the Hamming distance between such fingerprints).
- Identification of the first fingerprint in table T_i whose top p_i bit-positions match the top p_i bit-positions of $\pi_i(\mathbf{F})$ (Step 1) can be done in $O(p_i)$ steps by binary search.
- If we assume that each fingerprint were truly a random bit sequence, it is possible to shrinks the run time to $O(log p_i)$ steps in expectation (interpolation search).

SIMHASH: PARAMETERS

- For a repository of **8B web-pages**, f = 64 (64-bit simhash fingerprints) and k = 3 are reasonable. (Gurmeet Singh Manku et al., 2007)
- For the given f and k, to find a reasonable combination of t (number of tables) and p_i (number of top bit-positions used in comparisons) we have to note that they are <u>not independent</u> from each other:
 - Increasing the number of tables t increases p_i and hence reduces the query time. (large values for various p_i let to avoid checking too many fingerprints in Step 2)
 - Decreasing the number of tables t reduces storage requirements, but reduces p_i and hence increases the query time (small set of permutations let to avoid space blowup)

SIMHASH: PARAMETERS

- Consider f = 64 (64-bit fingerprints) and k = 3, so documents whose fingerprints differ at most 3 bit-positions will be similar for us (duplicates).
- Assume we have $8B = 2^{34}$ existing fingerprints and decide to use t = 10 tables:
 - For instance, $t = 10 = {5 \choose 2}$, so we can use **5** blocks and choose **2** out of them in 10 ways.
 - Split 64 bits into 5 blocks having 13, 13, 13, 13 and 12 bits respectively.
 - For each such choice, permutation π corresponds to making the bits lying in the chosen blocks the leading bits.
 - p_i is the total number of bits in the chosen blocks, thus $p_i = 25$ or 26.
 - On average, a probe retrieves at most $2^{34-25} = 512$ (permuted) fingerprints.
 - if we seek all (permuted) fingerprints which match the top p_i bits of a given (permuted) fingerprint, we expect 2^{d-pi} fingerprints as matches
- The total number of probes for 64-bit fingerprints and k = 3 is $\binom{64}{3} = 41664$ probes

SIMHASH: PYTHON

- https://github.com/leonsunliang/simhash. A Python implementation of **Simhash**.
- http://bibliographie-trac.ub.rub.de/browser/simhash.py
 Implementation of Charikar's simhashes in Python

SIMHASH: READ MORE

- http://www.cs.princeton.edu/courses/archive/spr04/ cos598B/bib/CharikarEstim.pdf
- http://jmlr.org/proceedings/papers/v33/ shrivastava14.pdf
- http://www.wwwconference.org/www2007/papers/ paper215.pdf
- http://ferd.ca/simhashing-hopefully-made-simple.html

THANK YOU

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