Probabilistic data structures. Part 3. Frequency

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PROBABILISTIC DATA STRUCTURES

ALL YOU WANTED TO KNOW BUT WERE AFRAID TO ASK

PART 3: FREQUENCY

Andrii Gakhov
tech talk @ ferret

FREQUENCY

Agenda:

- Count-Min Sketch
- Majority Algorithm
- Misra-Gries Algorithm

THE PROBLEM

- To estimate number of times an element occurs in a set

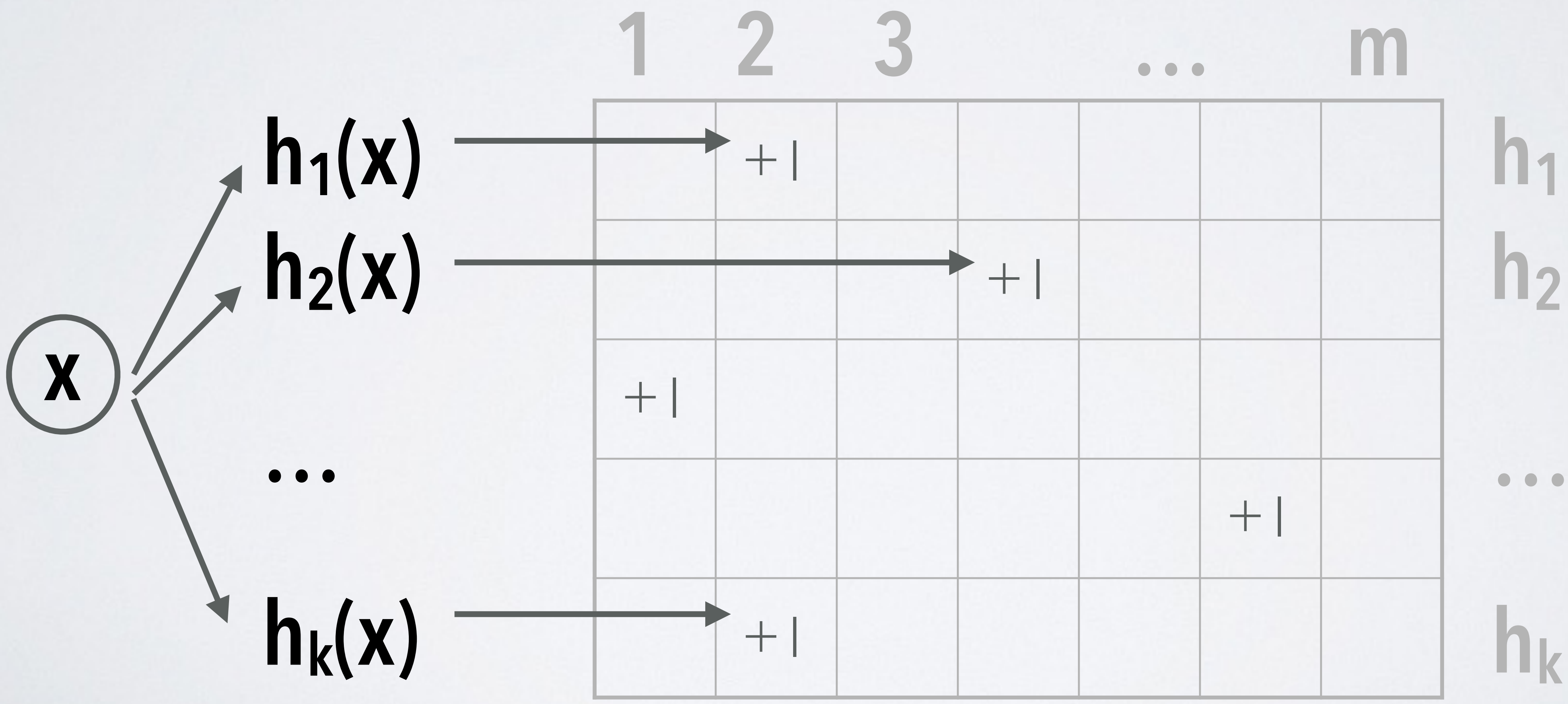
COUNT-MIN SKETCH

COUNT-MIN SKETCH

- proposed by G. Cormode and S. Muthukrishnan in 2003
- CM Sketch is a sublinear space data structure that supports:
  - **add** element to the structure
  - **count** number of times the element has been added (frequency)
- Count-Min Sketch is described by 2 parameters:
  - **m** - number of buckets
    (independent from **n**, but much smaller)
  - **k** - number of different hash functions, that map to 1…m
    (usually, k is much smaller than m)
- required fixed space: **m*k** counters and **k** hash functions

COUNT–MIN SKETCH: ALGORITHM

- Count-Min Sketch is simply an *matrix of counters* (initially all 0), where each row corresponds to a hash function $\mathbf{h_i}$, **i=1…k**
- To **add** element into the sketch - calculate all **k** hash functions increment counters in positions **[i, $\mathbf{h_i}$(element)], i=1…k**



COUNT-MIN SKETCH: ALGORITHM

- Because of **soft collisions**, we have **k** estimations of the true frequency of the element, but because we never decrement counter it can only ***overestimate*, not underestimate**.
- To **get frequency** of the element we calculate all **k** hash functions and return the minimal value of the counters in positions **[i, h_i(element)], i=1…k**.
- **Time** needed to add element or return its frequency is a fixed constant **O(k)**, assuming that every hash function can be evaluated in a constant time.

COUNT-MIN SKETCH: EXAMPLE

- Consider Count-Min Sketch with 16 columns (m=16)
- Consider 2 hash functions (k=2):

  h_1 is **MurmurHash3** and h_2 is **Fowler-Noll-Vo**

  (to calculate the appropriate index, we divide result by mod 16)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

- Add element to the structure: "berlin"

  h_1("berlin") = 4, h_2("berlin") = 12

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

COUNT-MIN SKETCH: EXAMPLE

- Add element "*berlin*" 5 more times (so, 6 in total):

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0

- Add element "bernau": h_1("bernau") = 4, h_2("bernau") = 4

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	7	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	0	0	0	0	0	0	0	6	0	0	0

- Add element "paris": h_1("paris") = 11, h_2("paris") = 4

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	7	0	0	0	0	0	0	1	0	0	0	0
2	0	0	0	0	2	0	0	0	0	0	0	0	6	0	0	0

COUNT-MIN SKETCH: EXAMPLE

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	7	0	0	0	0	0	0	1	0	0	0	0
2	0	0	0	0	2	0	0	0	0	0	0	0	6	0	0	0

- Get frequency for element: “london”:

  h_1(“london”) = 7, h_2(“london”) = 4

  freq(“london”) = min (0, 2) = 0

- Get frequency for element: “berlin”:

  h_1(“berlin”) = 4, h_2(“berlin”) = 12

  freq(“berlin”) = min (7, 6) = 6

- Get frequency for element: “warsaw”:

  h_1(“warsaw”) = 4, h_2(“warsaw”) = 12

  freq(“warsaw”) = min (7, 6) = 6 **!!! due to hash collision**

COUNT-MIN SKETCH: PROPERTIES

- Count-Min Sketch only returns **overestimates** of true frequency counts, never underestimates
- To achieve a target error probability of **δ**, we need **k ≥ ln 1/δ**.
  - for **δ** around **1%**, k = 5 is good enough
- Count-Min Sketch is essentially the same data structure as the Counting Bloom filters.

  The difference is followed from the usage:
  - Count-Min Sketch has a sublinear number of cells, related to the desired approximation quality of the sketch
  - Counting Bloom filter is sized to match the number of elements in the set

COUNT–MIN SKETCH: APPLICATIONS

- **AT&T** has used Count-Min Sketch in network switches to perform analyses on network traffic using limited memory
- At **Google**, a precursor of the count-min sketch (called the "count sketch") has been implemented on top of their MapReduce parallel processing infrastructure
- Implemented as a part of Algebird library from **Twitter**

COUNT-MIN SKETCH: PYTHON

- https://github.com/rafacarrascosa/countminsketch
  CountMinSketch is a minimalistic Count-min Sketch in pure Python
- https://github.com/farsightsec/fsisketch
  FSI Sketch a disk-backed implementation of the Count-Min Sketch algorithm

COUNT–MIN SKETCH: READ MORE

- http://dimacs.rutgers.edu/~graham/pubs/papers/cm-latin.pdf
- http://dimacs.rutgers.edu/~graham/pubs/papers/cmencyc.pdf
- http://dimacs.rutgers.edu/~graham/pubs/papers/cmsoft.pdf
- http://theory.stanford.edu/~tim/s15/l/l2.pdf
- http://www.cs.dartmouth.edu/~ac/Teach/CS49-Fall11/Notes/lecnotes.pdf

MAJORITY PROBLEM

- To find a single element that occurs strictly more than n / 2 times

n - number of elements in a set

MAJORITY PROBLEM

- Mostly a toy problem, but it let us better understand the frequency problems for data streams
- **Majority problem** has been formulated as a research problem by J Strother Moore in the *Journal of Algorithms* in 1981.
- It's possible that such element does not exists
- It could be at most one majority element in the set
- If such element exists, it will be equal to the median. So, the **Naïve linear-time solution** - compute the median of the set
  - Problem: it requires multiple pass through the stream

MAJORITY PROBLEM

- The **Boyer-Moore Majority Vote Algorithm** has been invented by Bob Boyer and J Strother Moore in 1980 to solve the Majority Problem in a single pass through the data stream.
  - The similar solution was independently proposed by Michael J. Fischer and Steven L. Salzberg in 1982. This is the most popular algorithm for undergraduate classes due to its simplicity.
- An important **pre-requirement** is that the **majority element actually exists**, without it the output of the algorithm will be an **arbitrary** element of the data stream.
- Algorithm requires only 1 left-to-right pass!
- The Data structure for the Majority Algorithm just a pair of an integer **counter** and monitored element **current**

MAJORITY ALGORITHM: ALGORITHM

- Initialise **counter** with 1 and **current** with the first element from the left
- Going from left to right:
  - If **counter** equals to 0, then take the current element as the **current** and set **counter** to 1
  - if **counter** isn't 0, then increase **counter** by 1 if the element equals to **current** or decrease **counter** by 1 otherwise
- The last **current** is the majority element (if **counter** bigger that 0)

Intuition: each element that contains a non-majority-value can only "cancel out" one copy of the majority value. Since more than n/2 of the elements contain the majority value, there is *guaranteed to be a copy of it left standing* at the end of the algorithm.

MAJORITY ALGORITHM: EXAMPLE

- Consider the following set of elements: {3,2,3,2,2,3,3,3}
- Iterate from left to right and update counter:

3	2	3	2	2	3	3	3

counter: **1** current: **3**

3	2	3	2	2	3	3	3

counter: **1** current: **3**

3	2	3	2	2	3	3	3

counter: **1** current: **2**

3	2	3	2	2	3	3	3

counter: **1** current: **3**

3	2	3	2	2	3	3	3

counter: **0** current: 3

3	2	3	2	2	3	3	3

counter: **0** current: 3

3	2	3	2	2	3	3	3

counter: **0** current: 2

3	2	3	2	2	3	3	3

counter: **2** current: 3

MAJORITY: READ MORE

- https://courses.cs.washington.edu/courses/cse522/14sp/lectures/lect04.pdf
- http://theory.stanford.edu/~tim/s16/l/l2.pdf

HEAVY HITTERS PROBLEM

- To find elements that occurs more than **n / k** times (n >> k)

* also known as **k-frequency-estimation**

HEAVY HITTERS PROBLEM

- There are **not more** than ***k*** such values
- The ***majority problem*** is a particular case of the ***heavy hitters*** problem
  - **k** $\approx 2 - \delta$ for a small value $\delta > 0$
  - with the additional promise that a majority element exists
- **Theorem:** There is no algorithm that solves the heavy hitters problems in one pass while using a sublinear amount of auxiliary space
- ε-*approximate heavy hitters* (ε-HH) problem:
  - every value that occurs at least **n/k** times is in the list
  - every value in the list occurs at least **n/k – εn** times in the set
- ε-HH can be solved with Count-Min Sketch as well, but we consider Misra-Gries / Frequent algorithm here …

MISRA-GRIES SUMMARY

MISRA–GRIES / FREQUENT ALGORITHM

- A generalisation of the **Majority Algorithm** to track multiple frequent items, known as **Frequent Algorithm**, was proposed by Erik D. Demaine, et al. in 2002.
    - After some time it was discovered that the Frequent algorithm actually the same as the algorithm that was published by Jayadev Misra and David Gries in 1982, known now as **Misra-Gries Algorithm**
- The trick is to run the **Majority** algorithm, but with ***many*** *counters instead of 1*
- The *time cost* of the algorithm is dominated by the **O(1)** dictionary operations per update, and the cost of decrementing counts

MISRA–GRIES: ALGORITHM

The **Data structure** for Misra-Gries algorithm consists of 2 arrays:

- an array of **k-1** frequency counters **C** (all 0) and **k-1** locations **X*** (empty set)

For every element x_i *in the set:*

- If x_i is already in **X*** at some index **j**:
  - *increase* its corresponding frequency counter by 1: **C[j] = C[j] + 1**
- If x_i not in **X***:
  - If size of **X*** less than **k** (so, there is free space in the top):
    - *append* x_i to **X*** at index **j**
    - *set* the corresponding frequency counter **C[j] =1**
  - else If **X*** is already full:
    - *decrement* all frequency counters by 1: **C[j] = C[j] - 1, j=1..k-1**
    - *remove* such elements from **X*** whose counters are 0.

Top k-1 frequent elements : **X*[j]** with frequency estimations **C[j], j=1..k-1**

MISRA-GRIES:EXAMPLE

- Consider the following set of elements: {3,2,1,2,2,3,1,3}, n= 8
- We need to find top **2** elements in the set that appears at least **n/3** times.

C:	0	0
X*:		

- Step 1: {**3**,2,1,2,2,3,1,3}
  Element {3} isn't in **X*** already and **X*** isn't full, so we add it at position 0 and set C[0] = 1

C:	1	0
X*:	3	

- Step 2: {3,**2**,1,2,2,3,1,3}
  Element {2} isn't in **X*** already and **X*** isn't full, so we add it at position 1 and set C[1] = 1

C:	1	1
X*:	3	2

MISRA-GRIES:EXAMPLE

- Step 3: {3,2,**1**,2,2,3,1,3}
  Element {1} isn't in **X*** already, but **X*** is full, so decrement all counters and remove elements that have counters less then 1:

C:	0	0
X*:		

- Step 4: {3,2,1,**2**,2,3,1,3}
  Element {2} isn't in **X*** already and **X*** isn't full, so we add it at position 0 and set C[0] = 1

C:	1	0
X*:	2	

- Step 5: {3,2,1,2,**2**,3,1,3}
  Element {2} is in **X*** at position 0, so we increase its counter C[0] = C[0] + 1 = 2

C:	2	0
X*:	2	

MISRA-GRIES:EXAMPLE

- Step 6: {3,2,1,2,2,**3**,1,3}
  Element {3} isn't in **X*** already and **X*** isn't full, so we append {3} at position 1 and set its counter to 1: C[1] = 1

C:	2	1
X*:	2	3

- Step 7: {3,2,1,2,2,3,**1**,3}
  Element {1} isn't in **X*** already, but **X*** is full, so decrement all counters and remove elements that have 0 as the counters values:

C:	1	0
X*:	2	

- Step 8: {3,2,1,2,2,3,1,**3**}
  Element {3} isn't in **X*** already and **X*** isn't full, so we append {3} at position 1 and set its counter to 1: C[1] = 1

C:	2	1
X*:	**2**	**3**

Top 2 elements

MISRA-GRIES: PROPERTIES

- The algorithm identifies at most **k-1** candidates without any probabilistic approach.
- It's still open question how to process such updates quickly, or particularly, how to decrement and release several counters simultaneously.
    - For instance, it was proposed to use doubly linked list of counters and store only the differences between same-value counter's groups and use. With such data structure each counter no longer needs to store a value, but rather its group and its monitored element.

MISRA-GRIES: READ MORE

- https://courses.cs.washington.edu/courses/cse522/14sp/lectures/lect04.pdf
- http://dimacs.rutgers.edu/~graham/pubs/papers/encalgs-mg.pdf
- http://drops.dagstuhl.de/opus/volltexte/2016/5773/pdf/3.pdf
- http://theory.stanford.edu/~tim/s16/l/l2.pdf

THANK YOU

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- linkedin.com/in/gakhov
- www.datacrucis.com