

---

---

---

---

---



## Acquisition fn $\alpha(x)$

- Depends on past info (implicit)
- Sample location  $x$

$\alpha(x)$  in simple EI case is

$$\mathbb{E}[(f(x) - f_*)_+]$$

$\uparrow$   
random piece, 1D Gaussian  
Compute integral in "closed form"

Want to optimize via gradient descent

General:  $\alpha(x) = \mathbb{E}[g(x)]$

$$\nabla \alpha(x) = \mathbb{E}[\nabla g(x)]$$

(with enough reg.)

$$\hat{\alpha}(x) = \frac{1}{N} \sum_{i=1}^N g(x; \xi_i)$$

$\uparrow$   
random draws

$$\nabla \hat{\alpha}(x) = \frac{1}{N} \sum_{i=1}^N \nabla g(x; \xi_i)$$

## Rollout acquisition

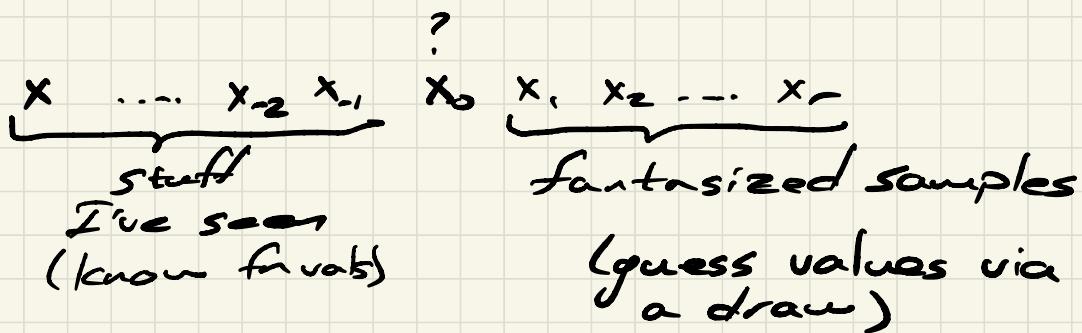
$$\alpha(x) = \mathbb{E}_{\substack{g \sim \text{posterior} \\ \text{of}}} \left[ \min_{\text{traj. } x_t} (g(x_t) - g(\hat{x})) \right]$$

where  $\hat{x}$  is the best known sample point (before this)

Trajectory (for  $g$ )

$$x_0 = x \quad (\text{starting point } \alpha(x))$$

$$x_{k+1} = \underset{x}{\operatorname{argmax}} \text{EI}(x \mid \text{prior data up to } k)$$



$f$  unknown fn, assumed drawn from GP

Have sampled

$$x_m, \dots, x_1 \text{ & got } y_m, \dots, y_1, y_j = f(x_j)$$

Posterior dist  $\mathcal{G}$ , gaussian proc conditioned on samples at  $x_m, \dots, x_1$

$$\alpha(x | g) = \max_{x_1, \dots, x_r} [g(x_k) - y_*]_+$$

$x_1, \dots, x_r$  depend on  $g$  hist defined by

$$x_{k+1} = \operatorname{argmax}_x EI(x \mid \text{samples through } x_k)$$

$$\hat{\alpha}(x) = \mathbb{E}_{g \sim \mathcal{G}} [\alpha(x | g)]$$

$$\hat{x} \hat{\alpha}(x) = \frac{1}{N} \sum_{i=1}^N \alpha(x | g^i), \quad g^i \sim \mathcal{G} \text{ are draws}$$

To est derivs,

$$\hat{\nabla} \hat{\alpha}(x) = \frac{1}{N} \sum_{i=1}^N \nabla \alpha(x | g^i)$$

Assume  $x_b$  is unique best pt. on trajectory for  $x_0 = x$  for  $g^i$

Then we want

$$\nabla [g(x_b) - y_*]$$

$$x_{k+1} = \underset{x}{\operatorname{argmax}} \text{EI}(x \mid x_{-m}, \dots, x_k, y_{-m}, \dots, y_k, g)$$

Stationarity:  $x_{k+1}$  solves

$$\nabla_x \text{EI}(x \mid x_{-m}, \dots, x_k, g) = 0$$

Solution to this is an implicitly defined function

$$x_{k+1}(x_{-m}, \dots, x_k, g)$$

Get:

$$\nabla^2 \text{EI}(x_{k+1} \mid \dots) \delta x_{k+1}$$

$$+ \sum_{j=-m}^k \frac{\partial}{\partial x_j} \text{EI}(x_k \mid x_{-m}, \dots, x_k, g) \frac{\delta x_k}{?} = 0$$

Algebra! And we have

$$\delta x_b = \frac{\partial x_b}{\partial x_0 = x}$$

$$\nabla \alpha(x \mid g) = g'(x_b) \frac{\partial x_b}{\partial x_0}$$