ИДЗ 6

5 июня 2022 г. 11:12

Вариант 27

№1

$17x^2 - 18y^2 - 12xy + 22x - 48y = 51$

$B = \begin{pmatrix} 17 & -6 \\ -6 & -18 \end{pmatrix} \qquad \chi(t) = \begin{vmatrix} 17-t & -6 \\ -6 & -18-t \end{vmatrix} = t^2 + t - 306 - 36 = t^2 + t - 342 = (t+19)(t-18)$

Вид квадратичной формы $-19x_1^2 + 18y_1^2$:

$t = -19: \begin{pmatrix} 36 & -6 \\ -6 & 1 \end{pmatrix} \to \begin{pmatrix} -6 & 1 \\ 0 & 0 \end{pmatrix} \to \bar{x}_1 = \frac{1}{\sqrt{37}} \begin{pmatrix} -6 \\ 1 \end{pmatrix}$

$t = 18: \begin{pmatrix} -1 & -6 \\ -6 & -36 \end{pmatrix} \to \begin{pmatrix} 1 & 6 \\ 0 & 0 \end{pmatrix} \to \bar{x}_2 = \frac{1}{\sqrt{37}} \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

Переходим к базису $\bar{x}_1, \bar{x}_2$

$$\begin{cases} x = \frac{1}{\sqrt{37}}(x_1 + 6y_1) \\ y = \frac{1}{\sqrt{37}}(-6x_1 + y_1) \end{cases}$$

Подставим в исходное уравнение:

$-19x_1^2 + 18y_1^2 - 266 \frac{1}{\sqrt{37}} x_1 + 180 \frac{1}{\sqrt{37}} y_1 = 51$

Выделим полные квадраты:

$-19\left(x_1 + \frac{7}{\sqrt{37}}\right)^2 + 18\left(y_1 + \frac{5}{\sqrt{37}}\right)^2 = 38$

$-0{,}5\underbrace{\left(x_1 + \frac{7}{\sqrt{37}}\right)}_{u} + \frac{9}{19}\underbrace{\left(y_1 + \frac{5}{\sqrt{37}}\right)}_{v} = 1$

$-\frac{1}{2}u^2 + \frac{9}{19}v^2 = 1$ - гипербола

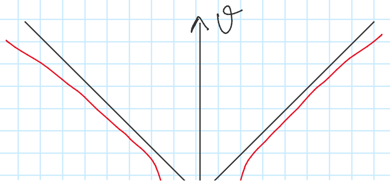
Координаты центра: $\Big(\underset{x_1}{-\frac{7}{\sqrt{37}}};\ \underset{y_1}{-\frac{5}{\sqrt{37}}}\Big) \to \Big(\underset{x}{1},\ \underset{y}{1}\Big)$

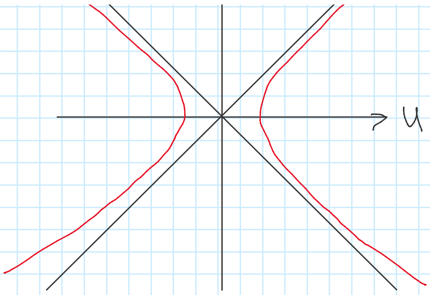
Координаты фокусов:

$c^2 = a^2 + b^2 \Rightarrow c = \frac{1}{3}\cdot\sqrt{37}$

$F_1 = \Big(\underset{u}{\frac{\sqrt{37}}{3}};\ \underset{v}{0}\Big) \to \Big(\underset{x_1}{\frac{16\sqrt{37}}{111}},\ \underset{y_1}{-\frac{5}{\sqrt{37}}}\Big) \to \Big(\underset{x}{-\frac{2}{3}},\ \underset{y}{-1}\Big)$

$F_2 = \Big(\underset{u}{-\frac{\sqrt{37}}{3}};\ \underset{v}{0}\Big) \to \Big(\underset{x_1}{-\frac{58\sqrt{37}}{11}},\ \underset{y_1}{-\frac{5}{\sqrt{37}}}\Big) \to \Big(\underset{x}{-\frac{4}{3}},\ \underset{y}{3}\Big)$





N 2

$$5x^2+2y^2+5z^2+6xy-10xz-6yz-x-2y-z=-4$$

$a_{11}=5$ $\quad a_{12}=3$ $\quad a_1=-0,5$ $\quad a_0=4$

$a_{22}=2$ $\quad a_{13}=-5$ $\quad a_2=-1$

$a_{33}=5$ $\quad a_{23}=-3$ $\quad a_3=-0,5$

$$T_1 = a_{11}+a_{22}+a_{33} = 12$$

$$T_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{13} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix} = 1+0+1=2$$

$$\delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = \begin{vmatrix} 5 & 3 & -5 \\ 3 & 2 & -3 \\ -5 & -3 & 5 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_1 \\ a_{12} & a_{22} & a_{23} & a_2 \\ a_{13} & a_{23} & a_{33} & a_3 \\ a_1 & a_2 & a_3 & a_0 \end{vmatrix} = \begin{vmatrix} 5 & 3 & -5 & -0,5 \\ 3 & 2 & -3 & -1 \\ -5 & -3 & 5 & -0,5 \\ -0,5 & -1 & -0,5 & 4 \end{vmatrix} = -1$$

ТК $\delta=0$ и $\Delta<0$ => эллиптический параболоид