COMP SCI/SFWR ENG 4/6E03 — Assignment 6 Solutions

1. (a) Let the states be labelled by state i represents i players present, $i = 0, 1, 2, 3$. Then the balance equations are (rate out on the left, rate out on the right, and time units are minutes):

$$\begin{aligned} 0.5\pi_0 &= 0.05\pi_3 \\ 0.5\pi_1 &= 0.5\pi_0 \\ 0.5\pi_2 &= 0.5\pi_1 \\ 0.05\pi_3 &= 0.5\pi_2 \end{aligned}$$

With the sum of the the probabilities being one, the solution is $\pi_0 = \pi_1 = \pi_2 = 1/13$, $\pi_3 = 10/13$. So, the required probability is $\pi_3 = 10/13$.

(b) Note that state 1 now needs to be split into two, according to whether a game is being played or not. Leave state 1 to be one player in the system and no game being played, and state 4 be one player waiting and a game being played (four players in the system). Then the balance equations are

$$\begin{aligned} 0.5\pi_0 &= 0.05\pi_3 \\ 0.5\pi_1 &= 0.5\pi_0 + 0.05\pi_4 \\ 0.5\pi_2 &= 0.5\pi_1 \\ 0.55\pi_3 &= 0.5\pi_2 \\ 0.05\pi_4 &= 0.5\pi_3 \end{aligned}$$

Solving (also using the fact that the probabilities sum to one) yields $\pi_0 = 0.0075$, $\pi_1 = \pi_2 = 0.0827$, $\pi_3 = 0.0752$, and $\pi_4 = 0.7519$. The required probability is $\pi_3 + \pi_4 = 0.8271$. Note that this is higher than the probability in (a), which should be expected.

2. From the data, we have estimates of $\rho = 5/6$ and $E[T] = 3$. Now,

$$\begin{aligned} E[T] &= \frac{1/\mu}{1-\rho} \\ \mu &= 2 \end{aligned}$$

3. We need to analyze an M/M/c system and increase c until the probability of queueing is less than 0.5. This means evaluating the Erlang C formula. For $c = 1$, the system is unstable. For $c = 2$, we get the probability of queueing is 0.6429. For $c = 3$, the probability of queueing is 0.2368, so choose three servers.

4. This is an M/M/∞ system, so the expected number of jobs in the system is $\lambda/\mu = 40/2 = 20$.