COMP SCI/SFWR ENG 4/6E03 — Assignment 5

1. A system consists of three components, labelled A, B, and C. The time for each component to fail follows an exponential distribution with means 100 days, 50 days, and 100 days, respectively.

   (a) What is the probability that component A fails between 50 and 150 days?

   (b) What is the probability that component A fails first?

   (c) What is the expected time until the first failure (of any component)?

   (d) What is the probability that at least two of the three components are still working after 100 days?

   (e) Suppose that all of the components are working after 100 days. What is the expected time until the first failure (of any component).

2. Suppose that $N(t)$ is a Poisson process with rate 3. Let T_n denote the time of the nth arrival. Compute the following quantities:

   (a) $P\{N(2) = 4\}$

   (b) $P\{N(3) = 4|N(1) = 1\}$

   (c) $P\{N(1) = 1|N(3) = 4\}$

   (d) $E[T_{12}]$

   (e) $E[T_{12}|N(2) = 5]$

   (f) $E[N(5)|N(2) = 5]$

3. The time that a server fails is exponentially distributed with mean one year. The lifetime of a component is defined to be the time at which a server is 60 percent certain to still be working.

   (a) Calculate the lifetime of a server.

   (b) If you buy 10 servers, what is the likelihood that at least one server will fail before the lifetime calculated in (a)?

   (c) Suppose that a server has reached its lifetime. How long would you expect it to continue working?

4. Messages are transmitted according to a Poisson process with rate $\lambda = 10$ per second. With probability 0.1 a transmitted message is lost, otherwise it is received (assume zero transmission time).

   (a) What is the expected time until a message is lost?

(b) During a period of 10 seconds, 12 messages are lost. How many messages would you expect to be received?

(c) During a period of one second, nine messages are transmitted. What is the probability that exactly one of these was lost?

(d) Suppose that we know the last message was lost 0.2 seconds ago and the last message was received 0.05 seconds ago? What is the probability that the next message is lost?