COMP SCI/SFWR ENG 4/6E03 — Assignment 2

1. Consider an experiment of mating mice. We are interested in the evolution of a particular gene that has two types, G and g. A mouse has a pair of genes, either GG (dominant), Gg (hybrid – gG is the same as Gg), or gg (recessive). When two mice are mated, the offspring inherits one of the genes from each of its parents with equal probability. For example, a dominant parent mated with a hybrid parent yields a dominant offspring with probability 1/2 and a hybrid offspring with probability 1/2. We start with a mouse of one of the three types and mate it with a hybrid. The offspring produced is mated with a hybrid, and the process is repeated through a number of generations, always mating with a hybrid.

   (a) Give the transition matrix for the DTMC that describes the evolution of the offspring.

   (b) Assume that the initial mouse is a hybrid. What is the probability that the third generation of offspring is dominant?

   (c) What is the steady-state probability that an offspring is recessive?

2. (This question is a version of 10.1 from the text.) Suppose that a web server has three web pages, labelled 1, 2, and 3. The probabilities that a user moves from one page to another are:

$$\begin{array}{lll} P(1 \to 1) = 0 & P(1 \to 2) = x & P(1 \to 3) = 1 - x \\ P(2 \to 1) = y & P(2 \to 2) = 0 & P(2 \to 3) = 1 - y \\ P(3 \to 1) = 0 & P(3 \to 2) = 1 & P(3 \to 3) = 0 \end{array}$$

   (For example, when a user is currently at page 1, they request page 2 next with probability x and page 3 with probability $(1 - x)$.) Assume that $0 < x < y < 1/2$.

   Suppose that the web server's cache has enough memory to store two pages. Whenever a request is for a page that is not in the cache, the browser will store that page in the cache, replacing the page least likely to be requested next. For example, if the cache contained pages 2 and 3, and page 1 was requested, the cache would be updated to contain pages 1 and 3 (since $x < 1 - x$).

   (a) Find the proportion of time (requests) that the cache contains pages 1 and 2. (**Hint:** be careful about your choice of state.)

   (b) Find the probability of a cache miss (a request is not available in the cache).

3. A server can be either up or down. It can go down due to either a hardware or software failure. Suppose that a server that is up today will be down tomorrow due to a hardware failure with probability 1/6, or will be down tomorrow due to a software failure with probability 1/4 (otherwise it remains up). A server that is down today due to a hardware failure is up tomorrow with probability 1. A server that is down today due to a software failure is up tomorrow with probability 3/4.

(a) If the server is initially up, what is the probability that it remains up for the first five days?

(b) If the server is initially up, what is the probability that it is down on the fifth day?

(c) What fraction of time is the server up (in steady-state)?