COMP SCI/SFWR ENG 4/6E03 — Assignment 7 Solutions

1. (a) The probability of an arriving job waiting in an M/M/1 queue is equal to ρ. Here, $\rho = 400/N$, so we need

$$\begin{aligned} 400/N &< 0.2 \\ N &> 2000. \end{aligned}$$

   (b) Use the Erlang-C formula - you need to be careful using this, as there can be numerical difficulties in evaluating the terms – in particular, taking the factorial of numbers on the order of 400 can result in numerical difficulties. I used an online calculator at: `http://www.gerkoole.com/CCO/downloads/tools.xls`, with a Time-to-Answer of 0 seconds. This yielded 422 servers required.

   (c) Using the Capacity Provisioning Theorem, we need to first solve

$$\frac{c\Phi(c)}{\phi(c)} = \frac{1 - P_Q}{P_Q}.$$

   This needs to be solved numerically, but it yields $c = 1.06$. So, the required number of servers is

$$\begin{aligned} N &= \lceil R + c\sqrt{R} \rceil \\ &= \lceil 400 + (1.06)(20) \rceil \\ &= 422. \end{aligned}$$

   Note that this agrees exactly with (b). If you used the $R + \sqrt{R}$ heuristic (which is reasonable in this case), the choice of 420 would also be reasonable (although it does not quite meet the requirement).

2. (a) First, solve the traffic equations:

$$\begin{aligned} \lambda_1 &= r_1 \\ \lambda_2 &= 0.5r_1 + 0.4\lambda_1 + 0.25\lambda_3 \\ \lambda_3 &= 0.5r_1 + 0.6\lambda_1 \end{aligned}$$

   This gives $\lambda_1 = r_1$, $\lambda_2 = 1.175r_1$, $\lambda_3 = 1.1r_1$. So, we need $1.175r_1 < 1$, or $r_1 < 0.851$.

   (b) Here, $\lambda_1 = 0.5$, so the required probability is

$$\sum_{n=0}^{2} \rho_1^n (1 - \rho_1) = 0.875.$$

   (c) Here, $\rho_3 = 0.55$, so

$$E[N_3] = \frac{0.55}{1 - 0.55} = 1.22.$$

(d) The visit ratios for a job arriving to node 1 can be found by solving

$$\begin{aligned} V_1 &= 1 \\ V_2 &= 0.4V_1 + 0.25V_3 \\ V_3 &= 0.6V_1 \end{aligned}$$

So, $V_1 = 1$, $V_2 = 0.55$, $V_3 = 0.6$. This gives, using $E[T_i] = \frac{1/\mu_i}{1-\rho_i}$:

$$E[T] = V_1 E[T_1] + V_2 E[T_2] + V_3 E[T_3] = 4.68.$$

3. (a) First, solve the traffic equations:

$$\begin{aligned} \lambda_1 &= 0.2\lambda_1 + \lambda_2 + \lambda_3 \\ \lambda_2 &= 0.6\lambda_1 \\ \lambda_3 &= 0.2\lambda_1 \end{aligned}$$

Letting $\lambda_1 = 1$ yields $\lambda_2 = 0.6$, $\lambda_3 = 0.2$. The steady-state distribution is then

$$\pi_{(n_1,n_2,n_3)} = C(0.1)^{n_1}(0.12)^{n_2}(0.2)^{n_3}$$

Solving for C:

$$C((0.1)^2 + (0.12)^2 + (0.2)^2 + (0.1)(0.12) + (0.1)(0.2) + (0.12)(0.2)) = 1$$

yields $C = 8.31$. The bottleneck is node 3 (highest utilization), so the required probability is

$$\pi_{(0,0,2)} = (8.31)(0.2)^2 = 0.33.$$

(b)

$$1 \cdot (\pi_{(1,1,0)} + \pi_{(1,0,1)}) + 2 \cdot (\pi_{(2,0,0)}) = 0.43.$$