COMP SCI/SFWR ENG 4/6E03 - Sample Questions for Test 1

1. Short Answer Questions.

   (a) The time between user requests is exponentially distributed with mean 1 minute. One minute has passed since the last request. What is the expected time to the next request?

   (b) Calculate the variance of the following random variable: $P(X = 0) = 1/3$, $P(X = 1) = 1/3$ and $P(X = 2) = 1/3$.

2. The time (measured in years) required to complete a software project has a density of the form

$$f(x) = \begin{cases} kx(1-x), & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$$

   What is the probability that a project is finished in less than four months?

3. (a) The lifetime of a component is exponentially distributed with mean 1000 hours. What is the probability a particular component has a lifetime that does not exceed 2000 hours? (b) Repeat part (a) if the lifetime is uniformly distributed between 500 and 5000 hours. (c) A random variable X has density $f(x) = 3x^2$ for x between 0 and 1, and 0 otherwise. Calculate the variance of X.

4. A discrete random variable takes on the values 1, 2 or 3 with $P(X = 1) = 0.5$, $P(X = 2) = a$, $P(X = 3) = b$. The mean and variance of X are 1.40 and 1.04, respectively. Find a and b.

5. (a) A particular disk is busy 30 percent of the time. Each transaction requires 25 disk accesses and each access takes 25 milliseconds. In transactions per second, what is the throughput of the disk?

   (b) A system has throughput of 3 jobs per minute, and the average number in the system is seen to be 23. What is the average time in system (from arrival to departure) of a job?

6. (a) A system resource executes an average of 10 requests per minute, with an average service time of 4 seconds per request. Another resource in the same system executes an average of 5 requests per minute, with an average service time of 11 seconds per request. Which resource is the bottleneck?

   (b) A transaction requires 2 visits on average to Resource A and 1 visit on average to Resource B. Each visit to Resource A takes 20 seconds, each visit to Resource B takes 30 seconds. Which Resource is the bottleneck?

   (c) I have three resources with utilizations 0.8, 0.3 and 0.9. Given only this information, which resource would it be most useful to upgrade?

7. Consider a system with N users, a CPU and two disks. A request from the users first visits the CPU. After processing at the CPU, with probability 0.8 the request visits disk A, 0.16 it visits disk B and 0.04 it is completed and returns to the user. After processing at either disk, the request returns to the CPU. The mean think time at the users is 5 seconds, the mean processing times (per visit) at the CPU, disk A, and disk B are 30, 25 and 40 milliseconds, respectively.
(a) What is the bottleneck device?
(b) What is the maximum possible system throughput (measured at the users)?
(c) Plot an upper bound on the system throughput as a function of N.
(d) What is the maximum possible disk B utilization?

8. Consider a system with N users, a CPU and two disks. A request from the users first visits the CPU. After processing at the CPU, with probability 0.48 the request visits disk A, 0.48 it visits disk B and 0.04 it is completed and returns to the user. After processing at either disk, the request returns to the CPU. The mean think time at the users is 3 seconds, the mean processing times (per visit) at the CPU, disk A, and disk B are 30, 25 and 40 milliseconds, respectively.
(a) What is the minimum expected response time?
(b) You are asked to give a recommendation for the number of users this system can support. Provide such a recommendation and justify it.
(c) If you double the processing rate at the bottleneck device, what improvement (in percentage terms) would you get for the maximum possible system throughput?

9. A server that is used for compilation jobs is monitored and reveals the following performance data (compilations are the only activity performed by the server):

   - Average number of active user logins: 230
   - Average time to generate a new compilation request: 300 seconds per user
   - Average server utilization: 48 percent
   - Average CPU processing demand: 0.63 seconds per compilation

   (a) What is the system throughput (in compiles per second)?
   (b) What is the average compilation time (from compilation submission by a user to completion)?

10. Consider a system with three nodes: a user node, a CPU node, and an I/O node. Users have think times with mean 40 seconds. A request first visits the CPU. After processing at the CPU, it goes to the I/O with probability 0.9, otherwise the request is completed and returned to the user. After processing at the I/O, requests are always returned to the CPU. The *total* demand per request for the CPU is 4.2 seconds, while the mean processing times at the I/O are 0.6 seconds per visit.
(a) Determine the bottleneck device and the maximum possible throughput for the system (measured at the user node).

(b) How many users can this system support?
(c) You have a budget of 20 dollars. You can decrease the mean demand at the CPU by 0.1 seconds times the amount spent in dollars (fractions are allowed). Similarly, you can decrease the mean time per visit at the I/O by 0.04 seconds times the amount spent in dollars. If the goal is to maximize the number of users that the system can support, how would you allocate the 20 dollars?

11. A database server receives requests from 50 clients. Each request to the DB server requires that five reads be made on average from the server's single disk. The average read time per record is 9 msec. Each database request requires on average 15 msec of CPU time to be processed.
(a) What is the bottleneck in the DB server?
(b) With the 50 clients, what is the throughput of the DB server?
(c) Consider the following three proposed design changes: (i) the average number of disk reads per access can be reduced from 5 to 2.5; (ii) the disk is replaced by one 60 percent faster; (iii) the CPU speed is doubled. Budgetary constraints allow for the implementation of two of these three. If the goal is to maximize the system capacity (maximum possible throughput), which two would you choose? Justify your answer.

12. A computer system has one CPU and two disks. After monitoring the system for two hours, the following observations were made. The utilization of the CPU was 43 percent and the utilization of the first disk was 66 percent. Each transaction to the system makes 5 I/O requests to the first disk and 6 to the second disk. The average processing time (per request) at the two disks are both 20 msec.
    (a) What was the throughput of the system (in transactions per second)?
    (b) What was the utilization of the second disk?
    (c) What is the total processing demand (in milliseconds per transaction) at the CPU?

13. You have three devices, A, B and C. Device A has average demand $D_A = 4$ and devices B and C are such that $D_B + D_C = 6$.
(a) Give all possible values of D_B and D_C that yield the highest possible maximum throughput. As part of your answer, give the highest possible maximum throughput.
(b) Give all possible values of D_B and D_C that yield the lowest possible maximum throughput. As part of your answer, give the lowest possible maximum throughput.
(c) For a specific choice of values of D_B and D_C from (a), compute the maximum number of users that the system can support if the average user think time is $Z = 40$.

14. Consider the following database server model. Each request to the server requires on average five records to be read from the server's single disk. The average read time per record is 9 msec. Each database request requires 15 msec of CPU time.
(a) What is the maximum number of users that this system can support if the average user think time is 100 seconds?
(b) You are given three upgrade options:

(i) More indexes can be built into the database to reduce the average number of reads per access from 5 to 2.5.
(ii) The disk can be replaced by a disk 60 percent faster, so the average processing time would drop to 5.63 msec.
(iii) The CPU speed can be doubled, i.e. the processing time per request is reduced to 7.5 msec.

If the goal is to maximize the maximum possible system throughput, which option would you choose? (You can only choose one option.)

15. A server system consists of three resources. Each user request has resource demands as follows: the processing times at resources A, B and C are 20, 30 and 15 msec, respectively. The average number of visits to A, B and C are 5, 4 and 10, respectively.
(a) You are given the following three upgrade options:

(i) Reduce the average number of visits to resource C to 7.

(ii) Reduce the average processing time at resource B to 25 msec.

(iii) Reduce the average processing time at resource C to 10 msec.

If the design goal is to maximize the number of users that can be supported, which option would you choose?
(b) If the average number of jobs at resource A is 2 and the average waiting time (from arrival to departure at resource A) is 50 msec, what is the throughput of the system (in user requests completed per second)?

16. Consider a system consisting of a single CPU, one disk, and some terminals (clients). The average think times for each of the terminals is 60 seconds. Suppose that we are presented with three configurations:

System	D_{CPU}	D_{disk}
A	4.6	4.0
B	5.1	1.9
C	3.1	1.9

(a) For System A, the utilization of the CPU is measured to be 0.85. What is the utilization of the disk?
(b) For System B, what is the maximum possible throughput of the disk?
(c) If there is a requirement that the system be able to support at least 14 users, which systems satisfy this requirement?

17. In a timesharing system, accounting log data produced the following profile for user programs

- each program requires 5 seconds of CPU time, makes 80 I/O requests to disk A and 100 I/O requests to disk B

- average think-time of a user was 18 seconds
- from the device specifications, it was determined that disk A takes 50 milliseconds to satisfy an I/O request and disk B takes 30 milliseconds per request
- with 17 users, disk A throughput was observed to be 15.70 I/O requests per second

Find the system throughput and the utilizations of the CPU, disk A and disk B.

18. Jobs arrive to a system at average rate 15 per hour. On observing the system, the average number occupying it is 10 jobs. What is the mean response time for a job?

19. Suppose that a chess player has a 90 percent chance of winning any game in a competition.
    (a) What is the probability that she wins exactly 3 games, if she plays a total of 5 games?
    (b) Suppose that a match ends when one player wins 3 games over another player. What is the probability that she wins a match?
    (c) What is the probability that she wins a match in exactly three games?

20. Consider a system with three nodes: a user node, a CPU node, and an I/O node. Users have think times with mean 40 seconds. A request first visits the CPU. After processing at the CPU, it goes to the I/O with probability 0.9, otherwise the request is completed and returned to the user. After processing at the I/O, requests are always returned to the CPU. The mean total demand (over all visits) per request for the CPU is 4.2 seconds, while the mean processing times at the I/O are 0.6 seconds per visit.
    (a) Determine the bottleneck device and the maximum possible throughput for the system (measured at the user node).
    (b) How many users can this system support?
    (c) You have a budget of 20 dollars. You can decrease the mean demand at the CPU by 0.1 seconds times the amount spent in dollars (fractions are allowed). Similarly, you can decrease the mean time per visit at the I/O by 0.04 seconds times the amount spent in dollars. If the goal is to maximize the number of users that the system can support, how would you allocate the 20 dollars?

21. (a) Database transactions perform an average of 4.5 disk operations on a database server with a single disk. The database server was monitored during one hour and during this period, 7,200 transactions were executed. What is the throughput of the disk (in transactions per second)? If each disk operation takes 20 msec on average, what is the disk utilization?
    (b) What is the average processing time (per operation) for the disk in (a)?
    (c) A corporate portal provides Web services to the company's employees. On average, 500 employees are online requesting Web services from the portal. An analysis of the portal's log reveals that 6,480 requests are processed per hour on average. The average response time was measured as 5 seconds. What is the average think time of an employee?

22. A database server receives requests from 50 clients, each client having think time 2 seconds. The server has one CPU and two disks. Each request requires an average of six accesses to the first disk and four accesses to the second disk. The average time per access to each disk is 10 msec and 12 msec, respectively. Each request requires an average of 30 msec of CPU time.

    (a) What is the bottleneck device?

    (b) What is the average response time of the server?

    (c) You are allowed to make two of the following three changes: (i) the average time per access for the first disk can be decreased to 7 msec; (ii) the average time per access for the second disk can be decreased to 11 msec; (iii) the CPU speed can be doubled, resulting in average CPU time of 15 msec. Which two would you choose, if the goal is to maximize the maximum possible throughput? Justify your answer.

23. A random variable X has density

$$f(x) = \begin{cases} x^{-2} & 1 \leq x < c \\ 0 & \text{otherwise} \end{cases}$$

    where $c > 0$. If $E[X] = 2$, what is c? For your choice of c, calculate $Var(X)$.

24. Suppose you have N identical servers. At a given point of time each server has probability $2/3$ of working, independent of all of the other servers. What is the minimum value of N to guarantee that the probability of at least 2 servers working is at least $2/3$? Give both your value of N and the probability that at least 2 servers are working.

25. A system operates as follows. There are 30 users, each with average think time 20 seconds. User requests first go to a CPU, with average processing time 0.05 seconds. From the CPU, with probability 0.05 a request returns to the user, with probability 0.55 it goes to Disk A, and with probability 0.40 it goes to Disk B. After visiting either disk, requests always return to the CPU. The average processing times at Disk A and Disk B are 0.08 seconds and 0.04 seconds, respectively.

    (a) What is the bottleneck device?

    (b) Is an 8 second average response time feasible? If not, what minimum amount of CPU speedup is required?

26. Suppose a continuous random variable X has density function $f(x) = k/x^2$ for $x > 2$ and 0 otherwise. What is $Var(X)$? (As part of your solution, you should solve for k.)

27. (a) Response times from six independent runs of a simulation are (in msec) 102.5, 101.7, 103.1, 100.9, 100.5 and 102.2. What is the 95 percent confidence interval for the mean response time? How many more simulation runs would you recommend in order to reduce the width of the confidence interval by a factor of two?

(b) Using a random number generator that outputs samples from a $U[0, 1]$ distribution, how would you generate a sample for the random variable in question 2?

28. A user's behaviour in interacting with a database system is as follows (we assume that the user is always connected). There are three actions: query, modify/delete, add. A query action is followed by another query action with probability 0.8, by a modify/delete action with probability 0.18 and by an add operation with probability 0.02. A modify/delete action is followed by a modify/delete action with probability 0.05, otherwise it is followed by a query action. Finally, an add action is always followed by a query action.

    (a) If the first action is a query, what is the probability that the third action is not a query?

    (b) In steady-state, what is the probability that one sees two query actions in a row?

29. A simple agent in a computer game operates as follows. At each time-step, the agent stands with probability 0.6, walks with probability 0.3, or runs with probability 0.1. Its action at each time-step is independent of its action at all other time-steps.

    (a) What is the probability that the agent stands, walks and runs, *in any order*, in 3 consecutive time-steps?

    (b) What is the probability over 10 time steps the agent will walk or run at least once?

30. You have the following data: 9.50, 2.31, 6.07, 4.86, 8.91, 7.62, 4.56, 0.19, 8.21, 4.45. The confidence interval constructed for the mean was $[3.94, 7.40]$. What confidence level was used?

31. You are asked to construct a 95 percent confidence interval for a mean and construct it to be $(.22, 1.35)$. You then get another set of data, which you believe is from the same distribution, that gives an average of 1.40. Is this consistent with the confidence interval that you constructed?

32. Calculate a 95 percent confidence interval for the following set of data: 10.95, 10.23, 10.61, 10.49, 10.89, 10.76, 10.46, 10.02, 10.82, 10.44.

33. Ten independent simulation runs give values for the average number in system of 3.36, 3.65, 4.11, 3.31, 3.44, 3.50, 3.50, 3.90, 3.57, 3.50. With what level of confidence can you say that the true mean number in system is less than 4.12?

34. An experimenter feels that for her experiment, the sample standard deviation will be no larger than 100. A 95 percent confidence interval is desired to be constructed for the mean which has a width of at most 5.0. What sample size would you recommend?

35. Ten data points are collected, measuring the execution time of a computer job (in seconds): 9.5, 2.3, 6.1, 4.9, 8.9, 7.6, 4.5, 0.2, 8.2, 4.4. With what level of confidence can we say that the mean execution time is less than 7.5 seconds?

36. A web server system produces the following measurements.

    - Each web server request requires an average of 1.2 seconds of CPU time and generates an average of 40 disk accesses to Disk A and 20 disk accesses to Disk B.
    - On average, an access to either Disk A or Disk B requires 10 msec.
    - Disk A throughput is measured at 30 disk accesses per second.
    - The average response time of Disk A is measured to be 25 msec.

    (a) What is the utilization of the CPU?

    (b) What is the average number of accesses waiting for Disk A?

37. A random variable X has density

$$f(x) = \begin{cases} ce^{-2x} & 0 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

    (a) What is c?

    (b) Calculate $P(X > 2|X > 1)$.

    (c) Is this distribution memoryless? Justify your answer.

38. In a file-sharing system, peers are chosen at random, and there is a probability .25 that a peer has a requested file. The outcome when choosing a peer is independent of the outcomes of choosing other peers. Peers are chosen until the file is found.

    (a) What is the probability that the file is obtained from the third peer selected?

    (b) What is the expected number of peers selected before the file is obtained?

    (c) Once the file is found on a peer, the time to download it is exponentially distributed with mean one minute. What is the probability that the download takes longer than two minutes?

39. You wish to generate a sample from a distribution with density

$$f(x) = \begin{cases} x/2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

    Using only a sample from a $U[0, 1]$ distribution, how would you do so?

40. True or False.

    (a) The uniform distribution has the memoryless property.

    (b) If A and B are independent events, $P(B|A) = P(B)$.

    (c) The bottleneck in a system is the device with the slowest processing rate.

41. A continuous random variable has density

$$f(x) = \begin{cases} \frac{4}{15}x^3 & 1 \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is c?

   (b) Given a sample from a $U[0,1]$ distribution, how would one generate a sample for a random variable with density f?

42. True or False.

   (a) The variance of $X \sim Exp(1)$ is less than or equal to the variance of $Y \sim U[0,2]$.

   (b) If A and B are independent events, $P\{B|A\} = P\{B\}$.

   (c) If $X \sim Exp(10)$, then $P\{X > 15|X > 5\} = P\{X > 15\}$.

43. Suppose that we have the following data on three resources in a system:

   - Resource 1 has visit ratio of 5 and average processing time of 10 msec
   - Resource 2 has average demand of 35 msec.
   - Resource 3 has utilization of 0.5 and throughput of 1/50 msec.

   Which resource is the bottleneck? Justify your answer.

44. Consider a system with 50 users, and a server consisting of a CPU and three disks. The average response time perceived by a user is 10 seconds. The throughput of the system is 1.5 user requests per second. It is also known that each user request requires an average of 0.52 seconds of CPU time.

   (a) What is the average think time of a user?

   (b) What is the CPU utilization?

   (c) On average, how many requests is the server processing?

45. Suppose that a CPU's processing times follow the distribution

$$G(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

   Given a sample from a $U[0,1]$ distribution, how can one generate a sample from the distribution G?

46. Suppose that we have three requests that we must assign to one of three servers. Each request is assigned such that it is equally likely to go to each of the servers. The three requests are assigned independently.

(a) What is the probability that the first server receives no requests?

(b) Given that the second server receives no requests, what is the probability that the first server receives no requests?

47. Parts coming down an assembly line are inspected in order. If a part is defective, then with probability .3, the next one is defective. If a part is not defective, then with probability .1 the next part is defective. Let X_n be the condition (defective/not defective) of the nth part.

    (a) Give the matrix P which describes the Markov chain X.

    (b) We observe the 8th part is defective. What is the probability that the 10th part is also defective?

48. An analyst is looking at the trends in a certain market and wants to start with a simple model: he wants to check simply if is up or down on a given week. He finds that if one week it is up, it is down the next with probability .2 and if one week it is down, the next it is up with probability .3.
    (a) What is the expected number of weeks in the year that the trend is up?
    (b) Given that the trend is up one week, what is the probability that is is up two weeks later?

49. An electron can be in one of 3 energy states (1,2,3). The state is observed every second and modelled as a Markov chain where X_n is the energy state at the nth second. If in state 1, the next second it will be in state 2 with probability .4 and in state 3 with probability .2. If in state 2, the next second it will be in state 3 with probability .4 and in state 1 with probability .2. Finally, if in state 3, it is in state 1 the next second with probability .3 and in state 3 otherwise.
    (a) If at time 0 the electron is in state 1, what is the probability that it will be in state 1 at time 1 second *and* 2 seconds.
    (b) Explain in words the interpretation of $P^2(1,1)$. Why is this value different from your answer to (a)?

50. The number of people in a waiting room is observed every hour and modelled as a Markov chain. If there are no people in the room, the next hour there is one with probability .5, two with probability .2 and zero otherwise. If there is one person in the room, the next hour there are none with probability .5, two with probability .1 and one otherwise. If there are two people in the room, the next hour there are none with probability .2, one with probability .6 and two otherwise. In steady-state, calculate
    (a) The average number of people waiting.
    (b) The variance of the number of people waiting.

51. The price of a commodity fluctuates between two values, call them high and low. The price is observed each week. If one week it is high, the next week it is low with probability .4. If one week it is low, the next week it is high with probability .1. Let $\{X_n\}$ be the

Markov chain which gives the price at week n.
(a) Suppose the initial distribution is $\Pr\{X_0 = \text{high}\} = .2$, $\Pr\{X_0 = \text{low}\} = .8$). Calculate the distribution at the first week (give $\Pr\{X_1 = \text{high}\}$ and $\Pr\{X_1 = \text{low}\}$).
(b) If the high price is \$110 and the low price is \$90, what is the expected cost at the first week (using the μ of (a))?

52. A worker can do three types of jobs $(1, 2, 3)$. If on a given day she is doing job 1, the next day she is doing job 1 with probability .3. If on a given day she is doing job 2, the next day she is doing job 2 with probability .6. If on a given day she is doing job 3, the next day she is doing job 3 with probability .9. If she changes jobs the following day, it is equally likely that she changes to each of the other two options.
(a) What proportion of days (in steady-state) is she doing each job?
(b) Suppose on day 1 she is doing job 1. What is the probability that the next 2 days she does **both** jobs 2 and 3 (in any order)?

53. A particle can be in one of two states: energized or deenergized. There are two particles that act independently of each other. At time n, if a particle is energized, it is energized at time $(n+1)$ with probability .8. At time n, if a particle is deenergized, it is energized at time $(n+1)$ with probability .1.
(a) Let X_n be the number of particles energized at time n. Give its transition matrix P.
(b) Compute $\Pr\{X_{52} = 2 | X_{51} = 0, X_{50} = 0\}$.

54. A machining center has two machines. The number of machines operating on day n is a Markov chain with state space $\{0, 1, 2\}$ and transition matrix

$$P = \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} .1 & .6 & .3 \\ .1 & .6 & .3 \\ 0 & .1 & .9 \end{bmatrix} \end{array}$$

The profit gained per day is \$100 per machine operating.
(a) In steady-state, find the expected profit per day.
(b) For a flat fee of \$10 per day, one can change $P(0,1) = P(1,1) = 0.5$ and $P(0,2) = P(1,2) = .4$, with the same profit as in (a). Is the change worthwhile?

55. A user submits requests to a database server. There are three kinds of requests supported: query, add/modify and delete. If the previous request was a query, then the next request is an add/modify request with probability 0.2, a delete request with probability 0.1 and otherwise it is another query request. An add/modify request is followed by another add/modify request with probability 0.1, otherwise it is followed by a query request. A delete request is followed by another delete request with probability 0.1, otherwise it is followed by a query request.

   (a) If the first request is a query, what is the probability that the third request is a query?
   (b) In steady-state, what is the proportion of requests that are queries?

(c) A query requires 100 msec of processing, while add/modify and delete requests require 200 msec of processing. What is the expected processing time per request (in steady-state)?

56. Consider a multiprocessor system with two processors and two memory modules. At each time point (time is discrete), requests are made in the following manner.

    (a) If both processors are accessing different memory modules, at the next time point each processor makes independent requests for the memory modules: with probability 0.4, a request is made for memory module 1, otherwise the request is made for memory module 2.

    (b) If both processors are accessing the same module, only one processor makes a new access request at the next time step (the other maintains its request for the same memory module). The request probabilities are the same as above.

    This can be modelled as a DTMC with three states. Using such a DTMC, calculate the steady-state probability that the processors are accessing different memory modules.

57. A computer program's execution is modelled as a DTMC. The state is the resource being used at time n. Suppose that there are three resources: CPU (state 1), Disk A (state 2) and Disk B (state 3). The probability transition matrix is

$$P = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0 & 0.9 \end{bmatrix}$$

    (a) What is the probability that given the program is initially using the CPU, it is using the CPU two time units later?

    (b) What is the steady-state probability that Disk A is being used?

58. A user's behaviour in interacting with a database system is as follows (we assume that the user is always connected). There are three actions: query, modify/delete, add. A query action is followed by another query action with probability 0.8, by a modify/delete action with probability 0.18 and by an add operation with probability 0.02. A modify/delete action is followed by a modify/delete action with probability 0.05, otherwise it is followed by a query action. Finally, an add action is always followed by a query action.

    (a) If the first action is a query, what is the probability that the third action is not a query?

    (b) In steady-state, what is the probability that one sees two query actions in a row?

59. Consider a DTMC, $\{X_n\}$ with transition matrix

$$P = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \end{pmatrix}$$

The states are $\{0, 1, 2\}$.

(a) Calculate $P(X_3 = 0|X_1 = 0)$.

(b) Suppose that $X_0 = 0$. As $n \to \infty$, what is the probability that $X_n = 2$?

60. Consider a small system of three web pages, labelled A, B and C. Page A is always the first page visited. After visiting A, pages B and C are equally likely to be visited next. After page B is visited, page A is always visited next. After page C is visited, pages A and B are equally likely to be visited next.

    (a) What is the probability that A is the fourth page visited?

    (b) In steady-state, what is the probability that A is visited?

61. Consider a system with two components. We observe the state of the system every hour. A component operating at hour n has probability 0.1 of being failed at hour $n + 1$. A component that is failed at hour n has a probability of operating at hour $n + 1$ of 0.7. The components operate independently of each other.

    (a) Model this system as a DTMC. Define the states and give the transition matrix.

    (b) What is the limiting probability that both components are failed at the same hour?

62. A board is divided into two. There are four markers on the board. Time is discrete. At each time point, one marker is chosen at random and moved to the opposite side of the board. What is the limiting probability that the numbers of markers on both sides of the board are equal?

63. Suppose that a system has two identical servers. Only one server operates at a time. At the end of each day, the operating server is inspected. With probability 0.1 it is taken offline to be repaired and the other server becomes operational (if it is available). Repairs take *exactly* two days and both servers can be concurrently repaired.

    (a) Model this system as a DTMC and give the P matrix.

    (b) Write down the equations that you would use to solve for the limiting distribution (do not solve them). In terms of your limiting distribution, what is the probability that you see two consecutive days where a server is taken offline to be repaired?

64. Consider the following game. A dart will hit the random point Y on the interval $(0, 1)$ according to the density $f_Y(t) = 2t$. You must guess the value of Y (your guess must be a constant, not random). You will lose \$2 times the magnitude of your error if Y is to the left of your guess and will lose \$1 times the magnitude of your error if Y is to the right of your guess. What is the best guess to minimize your expected loss?

65. Consider a system of three web pages. There is a cache with room for two web pages. The Least Recently Used (LRU) algorithm is employed to determine the cache contents - the two most recently used web pages are placed in the cache. Suppose that the probabilities

that the web pages are accessed are 0.8 for the first page, 0.1 for the second page, and 0.1 for the third page.

(a) Determine an appropriate state space and a corresponding P matrix for a DTMC that can be used to calculate the probability of a cache miss (a request does not find its web page in the cache).

(b) In terms of the steady-state probabilities, what is the likelihood of a cache miss? (There is no need to calculate the steady-state probabilities.)

66. A database server receives requests from 100 clients. The average think time of a client is 10 seconds. The database server has two disks and one CPU. In the current configuration, the average number of disk accesses per client request is 20 for the first disk, and 30 for the second disk. For both disks, the average time per access is 10 msec. For each client request, the total amount of CPU time required has an average of 240 msec.

    (a) Is this number of clients well supported by this system? Justify your answer.

    (b) You are presented with two design options, both of equal cost. The first would balance the loads on the disks (keeping the total load over both disks unchanged), the second would decrease the demand on the CPU to 200 msec. Which option would you choose? Justify your choice.