COMP SCI/SFWR ENG 4/6E03 — Assignment 7

1. A system of N servers has an arrival rate of 400 requests per second. The performance requirement is that the probability of an arriving job waiting is at most 0.2. The processing times are exponentially distributed with mean 1 second.

   (a) How many servers would be required using N M/M/1 queues in parallel, where an arrival is equally likely to join any of the N queues?

   (b) How many servers would be required using an M/M/N model (use the exact model, not the square root rule)?

   (c) How many servers does the square root rule suggest?

2. Consider an open queueing network with three nodes. External arrivals occur according to Poisson processes with rate r_i to node i. We have that $r_2 = r_3 = r_1/2$. Suppose that the non-zero routing probabilities are $P_{12} = 0.4$, $P_{13} = 0.6$, and $P_{32} = 0.25$. Processing times at each node (each node has a single server) are exponentially distributed with rates $\mu_1 = \mu_2 = \mu_3 = 1$.

   (a) How large can r_1 be while maintaining stability?

   (b) For the remainder of the question, let $r_1 = 0.5$. What is the steady-state probability that there are less than three jobs at node 1?

   (c) What is the mean number of jobs (in steady-state) at node 3?

   (d) What is the expected time that a job arriving to node 1 spends in the network? (Hint: your solution should at some point involve the visit ratios to each of the nodes for a job arriving to node 1.)

3. Consider the following simple model of a computer system. Node 1 is the CPU, node 2 is a disk, and node 3 is an I/O device. Assume that $P_{11} = 0.2$, $P_{12} = 0.6$, $P_{13} = 0.2$, and $P_{21} = P_{31} = 1$. Two jobs circulate. The processing rates are $\mu_1 = 10$, $\mu_2 = 5$, and $\mu_3 = 1$.

   (a) Calculate the steady-state probability that both jobs are at the bottleneck node.

   (b) Calculate the expected number of jobs at node 1.