CS/SE 4/6FO3 - Solutions to [illegible] questions for sample questions for Test 2.

**(12)** M/M/1 system: expected revenue per hour

$$(\lambda)(1) - \lambda(\rho)(0.2) - 1$$
$$= 10 - (10)(10/12)(0.2) - 1 = 7.33$$

M/M/2 system: expected revenue per hour

$$(\lambda)(1) - \lambda(1-\pi_0-\pi_1)(0.2) - 2$$
$$= 10 - (10)(0.25)(0.2) - 2 = 7.50$$

Two servers are preferred.

**(14)** (a) Need

$$\underbrace{\frac{1}{\mu-\lambda}}_{E[T]} - \frac{1}{\mu} < 5$$

(includes time in process)

$$\Rightarrow \lambda < 0.139$$

(b) There is always some positive probability that a job waits more than 5 minutes (unless there are an infinite number of servers).

**(16)** $\lambda = 30$, $\mu = 35$ (units are hours)

$$E(N) = \frac{\rho}{1-\rho} = 6$$

New $\rho'$ is $\frac{\rho'}{1-\rho'} = 3 \Rightarrow \rho' = 0.75$

This is a decrease from $\rho = 0.86$. The fact that this is not a large decrease is due to the nonlinear nature of $E[N]$.

18. $M/M/1$: $\rho = 0.9$

$$\text{Cost rate} = 25 + (5)\left(\frac{\rho}{1-\rho}\right) = 70$$

$M/M/2$

$$\text{Cost rate} = 50 + (5)(E[N])$$
$$= 50 + (5)(1.1285) = 55.6$$

$M/M/2$ preferred.

20. $$\frac{1}{\mu - \lambda} = 3 \qquad \pi_0 \approx \frac{1}{6} = \left(1 - \frac{\lambda}{\mu}\right)$$

Solving gives $\mu = 1/2$ (and $\lambda = 5/12$)

22. Arrival rate of jobs that enter is $(1)(0.9) = 0.9$. Using Little's Law,

$$E[T] = (2.5)/0.9$$
$$= 2.78$$

24\. The high priority jobs form an M/M/1 queue with $\lambda_h = 5$ and $\mu_h = 20$ (they are not affected by the low priority jobs. As a result

$$E[N_h] = \frac{\rho_h}{1-\rho_h} = 1/3$$

The total number of jobs in the system is an M/M/1 queue with $\lambda = 10$ and $\mu = 20$. So, the expected total number of jobs in the system is

$$E[N] = \frac{\rho}{1-\rho} = 1$$

Therefore, the Expected number of low priority jobs is

$$E[N_\ell] = E[N] - E[N_h] = 2/3.$$

~~[illegible] ρ = 2/3, [illegible] the one server. For an M/M/2 system [illegible]~~

26 The system is unstable with one server. For two servers, using an M/M/2 queue

$$P\{wait\} = 1 - \pi_0 - \pi_1 = 0.64$$

For an M/M/3 queue

$$P\{wait\} = 1 - \pi_0 - \pi_1 - \pi_2 = 0.24$$

Three servers are required.

28. (a) $P\{N < 4\} = P\{N \le 3\} = 1 - P\{N > 3\}$
$$= 1 - \rho^4$$

So, $1 - (\lambda/\mu)^4 = 0.9 \Rightarrow (\lambda)^4 = 0.1$
$$\Rightarrow \lambda = 0.56$$

30. (a) Actual arrival rate to system, from Little's Law
$$\lambda' = \frac{E(N)}{E(T)} = 0.5$$
So, blocking probability is $\left(1 - \frac{\lambda'}{\lambda}\right) = \frac{1}{3}$.

(b) $\lambda = 5$, $\mu = 3$ (time in minutes). So, $c = 2$.

(c) No. Performance is worse.

32. For $M|M|1$: $\pi_0 = \left(1 - \frac{54}{60}\right) = 0.1$

So, earn revenue at rate $(54)(1)(0.1) + (54)(0.5)(0.9)$
$$= 29.70 \text{ dollars / hour.}$$

For $M|M|1|5$: $\pi_0 = \dfrac{1 - \lambda/\mu}{1 - (\lambda/\mu)^6} = 0.213$
$$\pi_5 = \pi_0 \left(\frac{\lambda}{\mu}\right)^5 = 0.126$$

New arrival rate. $(54)(1 - 0.126) = 47.20$

Earn revenue at rate

$$(47.20)(1)(0.213) + (47.20)(0.5)(0.787)$$
$$= 28.63$$

M/M/1 preferred (option (a))

34. Two M/M/1's: $E[T_1] = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{30-20} = \frac{1}{10}$ hours

$= 6$ minutes

$E(T_2) = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{30-15} = \frac{1}{15}$ hours

$= 4$ minutes

M/M/2: $\lambda = \lambda_1 + \lambda_2$ $\quad E[T] = 0.0505$ hours

$= 3.03$ minutes

<u>All</u> jobs are better off with M/M/2, even the ones that are at the lower utilized queue.

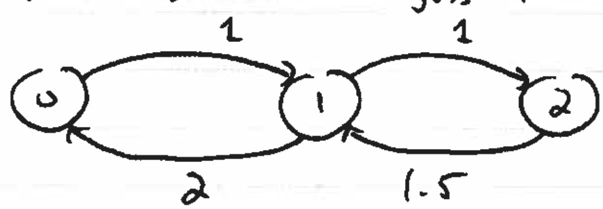
36. Cost (per minute) $= \mu + (10)(\rho)(\lambda)$

$$= \mu + \frac{40}{\mu}$$

Minimized by setting $\frac{d}{d\mu}\left(\mu + \frac{40}{\mu}\right) = 0 \Rightarrow \mu = \sqrt{40}$

38. (a) T
(b) F
(c) T

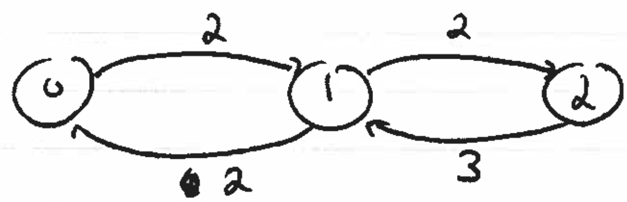
40. Let the state be # of jobs in system




$$\pi_0 = \frac{1}{1 + \frac{1}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{1.5}\right)} = 0.545$$

$$\Rightarrow \quad \pi_1 = \left(\frac{1}{2}\right)\pi_0 = 0.272$$

42. (a) Let the state be # of jobs in system

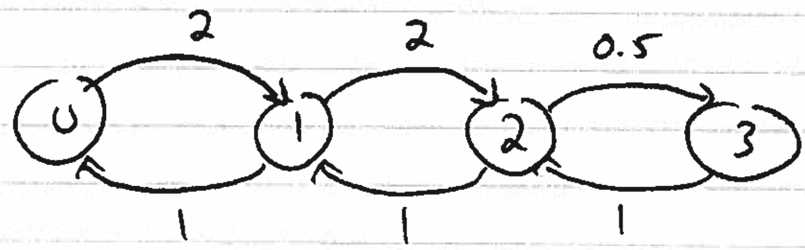



(time in minutes

Probability server idle is $\pi_0 = \dfrac{1}{1 + \left(\frac{2}{2}\right) + \left(\frac{2}{2}\right)\left(\frac{2}{3}\right)} = 0.375$

(b) By the memoryless property, thirty seconds from the current time.

44.

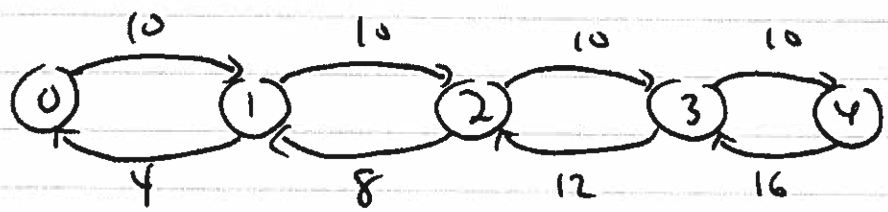



$$\pi_0 = \frac{1}{1 + \left(\frac{2}{1}\right) + \left(\frac{2}{1}\right)\left(\frac{2}{1}\right) + \left(\frac{2}{1}\right)\left(\frac{2}{1}\right)\left(\frac{0.5}{1}\right)}$$

$$= 1/9$$

$\Rightarrow$ Required probability is $\pi_0 + \pi_1 = 1/9 + (1/9)(2)$
$= 3/9$

46.




$$\pi_0 = \frac{1}{1 + \left(\frac{10}{4}\right) + \left(\frac{10}{4}\right)\left(\frac{10}{8}\right) + \left(\frac{10}{4}\right)\left(\frac{10}{8}\right)\left(\frac{10}{12}\right) + \left(\frac{10}{4}\right)\left(\frac{10}{8}\right)\left(\frac{10}{12}\right)\left(\frac{10}{16}\right)}$$

$$= 0.0921$$

utilization of server $1 - \pi_0 = 0.9079$

Blocking probability $\pi_4 = \pi_0 \left(\frac{10}{4}\right)\left(\frac{10}{8}\right)\left(\frac{10}{12}\right)\left(\frac{10}{16}\right)$

$$= 0.1499$$

48. Total cost rate $= 7.50 + (0.50)\frac{\rho}{1-\rho}$

Here, $\rho = \frac{21}{24} = 7/8$

So, total cost rate is 11.00

Proposed new $\mu = \dfrac{1}{\frac{7}{24} - \frac{1}{12}} = \dfrac{24}{5}$. So, new

$\rho = \dfrac{15}{24}$, which gives cost due to waiting of

$(0.50)(1.667) = 0.833$, as compared to $3.500$. So,
could allow an increase of up to $3.500 - 0.833$.

50. Expected time to download first file: $\dfrac{1}{1+1+\frac{1}{2}}$

$= 0.4$ minutes

If first file downloaded is $f_1$ or $f_2$, expected time
to download second file: $\dfrac{1}{1+\frac{1}{2}} = 0.67$ min

$P\{f_1 \text{ or } f_2 \text{ downloaded before } f_3\}$: $\dfrac{1+1}{1+1+\frac{1}{2}} = 0.8$

If first file downloaded is $f_3$, expected time
to download second file: $\dfrac{1}{1+1} = 0.5$ minute

$P\{f_3 \text{ downloaded before } f_1 \text{ or } f_2\}$: $\dfrac{1/2}{1+1+1/2} = 0.2$

Therefore, total expected time is

$$0.4 + (0.67)(0.8) + (0.5)(0.2) = 1.04 \text{ minute}$$

M/M/1 52. $E(T) = 10 = \dfrac{1}{\mu - \lambda}$ $\quad \rho = \dfrac{\lambda}{\mu} = 0.8$

$$\Rightarrow \lambda = 2/5 \quad , \quad \mu = 1/2$$

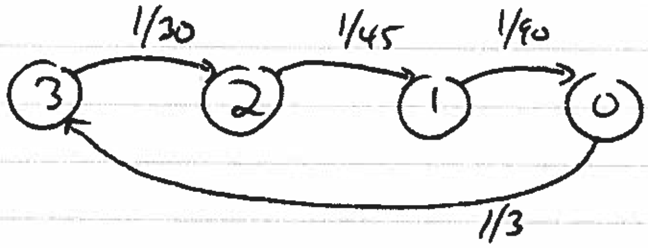
~~Let the state be the # of working~~ machines

55. (a) $\dfrac{1/20}{1/20 + 1/30} = \dfrac{3}{5}$

(b) Let $X \sim \text{Exp}(1/30)$

$\therefore$ Answer is $P\{X > 20\} = e^{-20/30} = 0.5134$

57. (a) Let the state be the # of working sensors

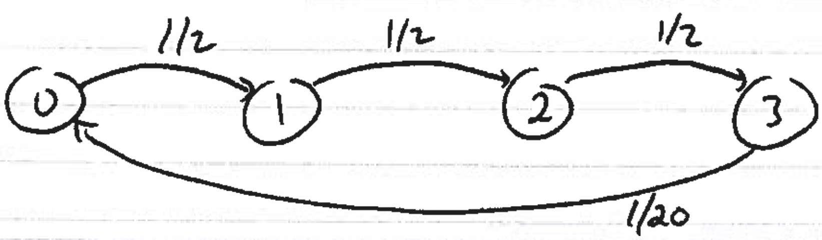



$$\frac{1}{30}\pi_3 = \frac{1}{3}\pi_0$$
$$\frac{1}{45}\pi_2 = \frac{1}{30}\pi_3$$
$$\frac{1}{90}\pi_1 = \frac{1}{45}\pi_2$$
$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$$

$\Rightarrow$ $\pi_3 = 0.1786$

(b) $(\pi_1)(1/90) = 0.003$ visits per day.

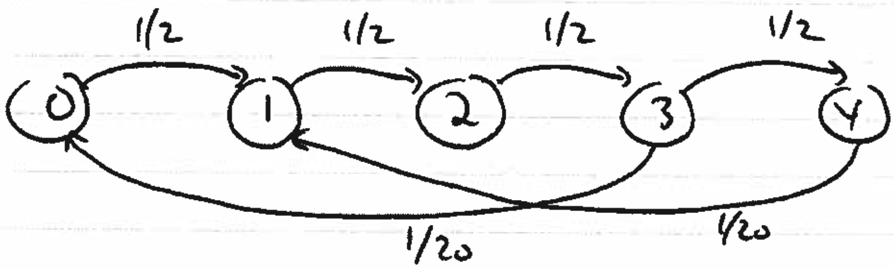
59. (a) Let the state be the # of players in the system



$$\tfrac{1}{2}\pi_0 = \tfrac{1}{20}\pi_3$$
$$\tfrac{1}{2}\pi_1 = \tfrac{1}{2}\pi_0$$
$$\tfrac{1}{2}\pi_2 = \tfrac{1}{2}\pi_1$$
$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$$

$\Rightarrow \pi_0 = \pi_1 = \pi_2 = 1/13, \quad \pi_3 = 10/13$

(b)




(63) Note, as stated M/M/2 is unstable: $\dfrac{\lambda}{2\mu} = 1.2$! Change question to $\mu = 1.2$, rather than $\dfrac{1}{\mu} = 1.2$.

For M/M/2, $E(T) = 2.73$

For M/M/1, need $\dfrac{1}{\mu - 1} = 2.73 \Rightarrow \mu = 1.37$.

(65) $(e^{-1})(e^{-1}) = 0.135$

67.

$$\lambda_{CPU} = 0.05\,\lambda_{CPU} + \lambda_{I/O}$$
$$\lambda_{I/O} = 0.95\,\lambda_{CPU}$$

Let $\lambda_{CPU} = 1$, so $\lambda_{I/O} = 0.95$

Time in seconds

$$\pi(n_1, n_2) = (0.02)^{n_1}(0.038)^{n_2}$$

$$C = \frac{1}{(0.02)^2 + (0.038)^2 + (0.02)(0.038)}$$
$$= 384.0$$

Utilization of CPU = $\pi(2,0) + \pi(1,1) = 0.45$

69.

$$\lambda_1 = 0.9\,\lambda_2 + 0.9\,\lambda_3 + \lambda_4$$
$$\lambda_2 = 0.5\,\lambda_1$$
$$\lambda_3 = 0.5\,\lambda_1$$
$$\lambda_4 = 0.1\,\lambda_2 + 0.1\,\lambda_3$$

Let $\lambda_1 = 1 \Rightarrow \lambda_2 = 0.5,\ \lambda_3 = 0.5,\ \lambda_4 = 0.1$

$$\pi(n_1, n_2, n_3, n_4) = C\left(\frac{1}{5}\right)^{n_1}\left(\frac{1}{20}\right)^{n_2}\left(\frac{1}{20}\right)^{n_3}\left(\frac{1}{10}\right)^{n_4}$$

Solve for $C$ using states $(2,0,0,0)$, $(0,2,0,0)$, $(0,0,2,0)$, $(0,0,0,2)$, $(1,1,0,0)$, $(1,0,1,0)$, $(1,0,0,1)$, $(0,1,1,0)$, $(0,1,0,1)$, $(0,0,1,1)$: $C = 9.30$

Throughput (at node 1):

$$5\left(\pi_{(2,0,0,0)} + \pi_{(1,1,0,0)} + \pi_{(1,0,1,0)} + \pi_{(1,0,0,1)}\right)$$
$$= 3.72$$

Node 1 is the bottleneck. The bottleneck ~~class~~ is not a function of the number of jobs in the system.

(7) 1. Time units minutes

$$\lambda_1 = 2 + 0.5\lambda_2 + 0.5\lambda_3$$
$$\lambda_2 = 0.4\lambda_1 + 0.5\lambda_3$$
$$\lambda_3 = 0.5\lambda_1 + 0.5\lambda_2$$

$\Rightarrow \lambda_1 = 20, \lambda_2 = 17.3, \lambda_3 = 18.7$

System unstable! $\lambda_1 > \mu_1, \lambda_2 > \mu_2, \lambda_3 > \mu_2.$

Change $r_1$ to 1 $\Rightarrow \lambda_1 = 10, \lambda_2 = 8.7, \lambda_3 = 9.3$

(a) Total number in system: $\frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} = 5.3$

$(\rho_1 = 10/20, \ \rho_2 = 8.7/12, \ \rho_3 = 9.3/15)$

By Little's Law, $E[T] = 5.3/1 = 5.3$ minutes

(b) Highest $\rho_i$, which is at node 2.

73. Choose a cyclic network, $\lambda_1 = \lambda_2 = 1$. Processing rate at each server is $\mu$

$$\pi(n_1, n_2) = C\left(\frac{1}{\mu}\right)^{n_1}\left(\frac{1}{\mu}\right)^{n_2}$$

$$\Rightarrow \quad C = \frac{1}{\left(\frac{1}{\mu}\right)^2 + \left(\frac{1}{\mu}\right)^2 + \left(\frac{1}{\mu}\right)^2} = \frac{\mu^2}{3}$$

Throughput is $\mu \cdot \frac{\mu^2}{3}\left(\frac{2}{\mu^2}\right)$. So, $\frac{2\mu}{3} = 1$ and $\mu = 3/2$.

75.

$$\lambda_1 = \gamma + \lambda_2 + \lambda_3$$
$$\lambda_2 = 0.4\lambda_1$$
$$\lambda_3 = 0.3\lambda_1$$

$$\Rightarrow \quad \lambda_1 = 10\gamma/3, \quad \lambda_2 = 4\gamma/3, \quad \lambda_3 = \gamma$$

Time unit seconds

(a) $\frac{\lambda_3}{\mu_3} = \frac{\gamma}{2} = 0.7 \Rightarrow \gamma = 1.4$

(b) $\rho_1 = \frac{1.4}{3}$, $\rho_2 = \frac{2.8}{3}$, $\rho_3 = \frac{1.4}{2}$

Bottleneck is node 2, change $\rho_2$ to $\frac{1.4}{3}$. Then total # in system is $\frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} = 4.08$

77.

$$\lambda_1 = \lambda_4 + \lambda_2 + 0.6\lambda_3$$
$$\lambda_2 = 0.5\lambda_1$$
$$\lambda_3 = 0.5\lambda_1$$
$$\lambda_4 = 4$$

$\lambda_1 = 20, \quad \lambda_2 = 10, \quad \lambda_3 = 10, \quad \lambda_4 = 4$

(a) $\rho_1 = 0.8, \quad \rho_2 = 0.3, \quad \rho_3 = 0.6, \quad \rho_4 = 0.2$

$$(1-\rho_1)\rho_1^3 (1-\rho_2)\rho_2^2 (1-\rho_3)\rho_3^4 (1-\rho_4)\rho_4$$

$$= 5.35 \times 10^{-5}$$

(b) Expected # in system $\sum_{i=1}^{4} \frac{\rho_i}{1-\rho_i} = 6.18$, so

by Little's Law $E(T) = \frac{6.18}{4} = 1.54$ seconds.

79.

$$\lambda_1 = 2 + \lambda_2 + \lambda_3/2$$
$$\lambda_2 = \lambda_1/3$$
$$\lambda_3 = 2\lambda_1/3$$

Time in minutes

$\Rightarrow \lambda_1 = 4, \quad \lambda_2 = 4/3, \quad \lambda_3 = 8/3.$

$\Rightarrow \rho_1 = 4/5, \quad \rho_2 = 8/9, \quad \rho_3 = 2/3$

(a) $[(1-\rho_1)\rho_1(1-\rho_2)(1-\rho_3) + (1-\rho_1)\rho_2(1-\rho_2)(1-\rho_3) + (1-\rho_1)(1-\rho_2)(1-$

$= 0.04$

(3) The bottleneck, node 2.

(81)

$$\lambda_1 = 0.4\lambda_3$$
$$\lambda_2 = 0.6\lambda_3$$
$$\lambda_3 = \lambda_1 + \lambda_2$$

Let $\lambda_3 = 1$, $\lambda_1 = 0.4$, $\lambda_2 = 0.6$

$$\pi_{(n_1,n_2,n_3)} = C\left(\frac{1}{0.7}\right)^{n_1}(4)^{n_2}\left(\frac{0.6}{1.5}\right)^{n_3}$$

Solve to get $C = 0.0383$

(a) Bottleneck is node 2.

(b) $1.5\left(\pi_{(0,0,2)} + \pi_{(0,1,1)} + \pi_{(1,0,1)}\right) = 0.1341$

89. (a) $\lambda_1 = \lambda_2 + \lambda_3$

$\lambda_2 = 0.4\lambda_1$

$\lambda_3 = 0.6\lambda_1$

Let $\lambda_1 = 1$, then $\lambda_2 = 0.4$, $\lambda_3 = 0.6$

$\frac{\lambda_1}{\mu_1} = \frac{1}{\mu}$, $\lambda_2 = 0.4$, $\lambda_3 = 0.6$

$\therefore$ Node 1 is the bottleneck for $\mu < 10/6$.

(b) Let $\mu = 1$. Then

$$\pi_{(n_1, n_2, n_3)} = C\,(1)^{n_1}(0.4)^{n_2}(0.6)^{n_3}$$

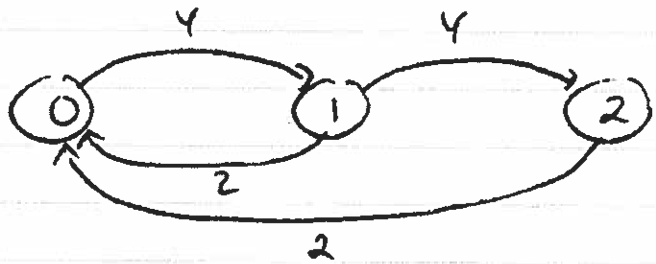
$$\therefore \quad C = \frac{1}{(1)^2 + (0.4)^2 + (0.6)^2 + (1)(0.4) + (1)(0.6) + (0.4)(0.6)}$$

$$= 0.3623$$

$$P\{\text{node 2 idle}\} = C\left(\pi_{(2,0,0)} + \pi_{(0,0,2)} + \pi_{(1,0,1)}\right)$$

$$= 0.7101$$

91\. Let the state be the # of players in the system




$$4\pi_0 = 2\pi_1 + 2\pi_2$$
$$6\pi_1 = 4\pi_0$$
$$\pi_0 + \pi_1 + \pi_2 = 1$$

$$\Rightarrow \pi_0 = 0.33$$

93\. For $M/M/2$, $P\{\text{wait}\} = 1 - \pi_0 - \pi_1$
$= 0.6428$

For $M/M/1$, $1.5/\mu = 0.6428 \Rightarrow \mu = 2.33$

95\. $M/M/\infty$. $E(N) = \lambda/\mu = 40$

97\. (a) $\lambda_1 = 4$
$\lambda_2 = \lambda_1 + 0.5\lambda_2$
$\lambda_3 = 0.25\lambda_2$

$$\Rightarrow \lambda_1 = 4, \ \lambda_2 = 8, \ \lambda_3 = 2$$

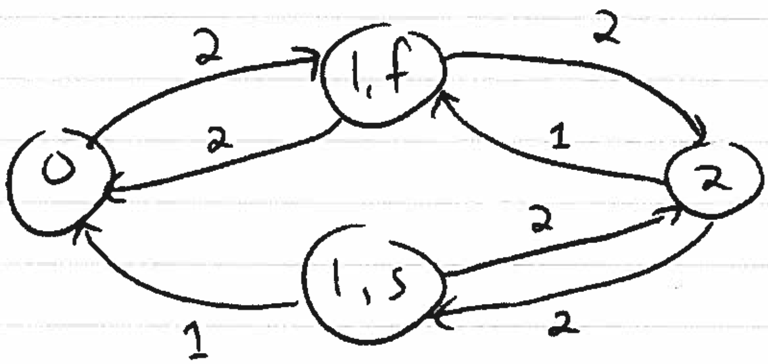
$\rho_1 = 4/6$, $\rho_2 = 8/16$, $\rho_3 = 2/2.5$

$$E(N) = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} = 7$$

By Little's Law, $E(T) = 7/4$

(5) Bottleneck is node 3, new $E(N)$ is $11/3$.

(98) Let the states be: 0 (system empty), 1,f (fast server only is busy), 1,s (slow server only is busy), and 2 (both servers busy).




$$2\pi_0 = 2\pi_{1,f} + \pi_{1,s}$$
$$4\pi_{1,f} = 2\pi_0 + \pi_2$$
$$3\pi_{1,s} = 2\pi_2$$
$$\pi_0 + \pi_{1,f} + \pi_{1,s} + \pi_2 = 1$$

$$\pi_0 = 0.3182$$
$$\pi_{1,f} = 0.2273$$
$$\pi_{1,s} = 0.1818$$
$$\pi_2 = 0.2727$$

$\rightarrow$ Probability slow server is busy is .4545

(99) Need $\dfrac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3} = 0.2$

$$\Rightarrow \frac{1}{\lambda_3} = 4.8$$

(100)

$$\lambda_1 = 10$$
$$\lambda_2 = 0.5\lambda_1 + 0.7\lambda_2$$
$$\lambda_3 = 0.5\lambda_1 + 0.3\lambda_2$$

$$\Rightarrow \quad \lambda_1 = 10.0$$
$$\lambda_2 = 16.7$$
$$\lambda_3 = 10.0$$

(a) $\dfrac{\rho_2}{1-\rho_2} = \dfrac{16.7/30}{1-16.7/30} = 1.26$

(b) $\rho_1 = 10/12, \quad \rho_2 = 16.7/30, \quad \rho_3 = 10/15$

smallest, so reduce cost

(104) (a) Probability of more than two in system: $\rho^3 = 0.05$.
This yields $\mu = 27.14$

(b) M/M/∞: $E[T] = \dfrac{1}{\mu} = 0.0368$

M/M/1: $E[T] = \dfrac{1}{\mu-\lambda} = 0.0583$

∴ Not a good approximation.