COMP SCI/SFWR ENG 4/6E03 — Assignment 1 Solutions

1. (a) Let X be the number of bugs in a randomly selected program. Then

$$\begin{aligned}
P\{X=0\} &= (1/2)(1/2)+(1/2)(1/5) &= 0.35 \\
P\{X=1\} &= (1/2)(1/2)+(1/2)(1/5) &= 0.35 \\
P\{X=2\} &= (1/2)(1/5) &= 0.10 \\
P\{X=3\} &= (1/2)(1/5) &= 0.10 \\
P\{X=4\} &= (1/2)(1/5) &= 0.10
\end{aligned}$$

So,

$$E[X] = (1)(0.35)+(2)(0.10)+(3)(0.10)+(4)(0.10) = 1.25.$$

(b)

$$\begin{aligned}
P\{A|X=1\} &= \frac{P\{A\cap\{X=1\}\}}{P\{X=1\}} \\
&= \frac{0.25}{0.35} \\
&= \frac{5}{7}
\end{aligned}$$

2. (a) $(1-0.1)(1-0.2)(1-0.3)(1-0.4)(1-0.5) = 0.1512$

(b) The probability that an error is found by at least two tests is one minus the probability that an error is found by either zero tests or exactly one test. Given (a), we just need to calculate the latter:

$$\begin{aligned}
&(0.1)(0.8)(0.7)(0.6)(0.5)+(0.9)(0.2)(0.7)(0.6)(0.5)+(0.9)(0.8)(0.3)(0.6)(0.5) \\
&+(0.9)(0.8)(0.7)(0.4)(0.5)+(0.9)(0.8)(0.7)(0.6)(0.5) = 0.3714
\end{aligned}$$

So, the required probability is $1-0.1512-0.3714 = 0.4774$.

3. Let A be the event that a component is defective. Let B be the event that a component tests defective. We use Bayes' rule:

$$P\{A|B\} = P\{B|A\}\frac{P\{A\}}{P\{B\}}$$

We know that $P\{B|A\}=1$, $P\{A\}=0.006$, and

$$\begin{aligned}
P\{B\} &= P\{B|A\}P\{A\}+P\{B|A^c\}P\{A^c\} \\
&= (1)(0.006)+(0.02)(0.994) \\
&= 0.0259
\end{aligned}$$

So, $P\{A|B\} = 0.2318$.

4. (a) The probability that any draw is a heart is 0.25. So, the required probability is $1 - 0.25 - (0.25)(0.75) = 0.5625$.

   (b) Using the properties of the geometric distribution, the expected number of draws is 4 (the probability of success is 0.25).

5. (a)

$$\begin{aligned} P\{X > L\} &= 0.9 \\ e^{-L} &= 0.9 \\ L &= 0.1054 \text{ years} \end{aligned}$$

   (b) $1 - (0.9)^5 = 0.4095$

   (c) Using the memoryless property, the expected time (after L) is one year.