COMP SCI/SFWR ENG 4/6E03 — Assignment 5

1. (a)

$$\int_{50}^{150} 0.01e^{-0.01x}dx = 0.3834$$

(b)

$$\frac{\lambda_A}{\lambda_A + \lambda_B + \lambda_C} = \frac{0.01}{0.04} = 0.25$$

(c)

$$\frac{1}{\lambda_A + \lambda_B + \lambda_C} = \frac{1}{0.04} = 25$$

(d) The probability of A and C working are both $e^{-1} = 0.3679$. The probability that B is working is $e^{-2} = 0.1353$. So, the required probability is

$$(0.3679)^2(0.1353) + (0.3679)^2(1 - 0.1353) + 2(0.3679)(0.1353)(1 - 0.3679) = 0.1983$$

(e) By the memoryless property, this answer is the same as part (c), so 25.

2. (a)

$$\frac{6^4}{4!}e^{-6} = 0.1339$$

(b) This is

$$P\{N(2) = 3\} = \frac{6^3}{3!}e^{-6} = 0.0892$$

(c) The arrival time of each of the four arrivals is uniformly distributed between 0 and 3. So, the required probability is

$$\binom{4}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3 = 0.3951$$

(d) $12/3 = 4$

(e) $2 + E[T_7] = 4.33$

(f) $5 + (3)(3) = 14$

3. (a)

$$
\begin{aligned}
P\{X > L\} &= 0.6 \\
e^{-L} &= 0.6 \\
L &= 0.5108 \text{ years}
\end{aligned}
$$

   (b)

$$1 - P\{\text{no servers failed}\} = 1 - (0.6)^{10} = 0.994$$

   (c) 1 year

4. (a) Lost messages follow a Poisson process with rate 1 per second, so the expected time is 1 second.

   (b) The Poisson processes of received and lost messages are independent, so one would expect 90 messages to be received.

   (c)

$$\binom{9}{1}(0.1)^1(0.9)^8 = 0.3874$$

   (d) The information provided does not affect the calculation, so the probability is 0.1.