COMP SCI/SFWR ENG 4/6E03 – Test 1 Solutions

1.

$$\begin{aligned} E[D_{disk}] &= (2)(25\ ms) \\ &= 0.05\ s \\ U_{disk} &= E[D_{disk}]X \\ X &= 0.5/0.05 \\ &= 10 \text{ per } s \\ E[T] &= \frac{M}{X} - E[Z] \\ &= \frac{150}{10} - 10 \\ &= 5\ s \end{aligned}$$

2. (a) Let X be the number of hosts that need to be searched. Then, the first probability asked for is

$$P\{X = 1\} + P\{X = 2\} = 0.25 + (0.25)(0.75) = 7/16.$$

Due to independence, the second probability asked for is the same as the first one, or 7/16.

(b)

$$\begin{aligned} \int_0^k 3x^2 &= 1 \\ x^3|_0^k &= 1 \\ k &= 1 \end{aligned}$$

$$\begin{aligned} E[X] &= \int_0^1 3x^3 \\ &= \frac{3}{4}x^4|_0^1 \\ &= \frac{3}{4} \end{aligned}$$

3. (a)

$$P = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/4 & 3/4 & 0 \end{pmatrix}$$

(b) From the properties of DTMCs, the given probability is equivalent to

$$\begin{aligned} P\{X_5 = 0|X_4 = 0\}P\{X_6 = 0|X_5 = 0\}P\{X_7 = 2|X_6 = 0\} &= P(0,0)P(0,0)P(0,2) \\ &= (1/3)(1/3)(1/3) \\ &= 1/27 \end{aligned}$$

(c) One possibility for the equations for the steady-state probabilities is

$$\begin{aligned} 1/3\pi_0 + 3/4\pi_2 &= \pi_1 \\ 1/3\pi_0 + 1/2\pi_1 &= \pi_2 \\ \pi_0 + \pi_1 + \pi_2 &= 1 \end{aligned}$$

This yields $\pi_1 = 14/41$.