CS/SE 4E03 – Solutions to Sample Questions for Test 1.

1. (a) 1 minute from current time.

(b) $E[X] = (\frac{1}{3})(0) + (\frac{1}{3})(1) + (\frac{1}{3})(2) = 1$

$E[X^2] = (\frac{1}{3})(0)^2 + (\frac{1}{3})(1)^2 + (\frac{1}{3})(2)^2 = \frac{5}{3}$

$\Rightarrow Var(X) = E[X^2] - (E[X])^2 = 2/3$

3. (a) $P\{X \le 2000\} = 1 - e^{-2000/1000}$

$= 0.86$

(b) $P\{X \le 2000\} = \frac{2000 - 500}{5000 - 500} = \frac{1}{3}$

(c) $E[X] = \int_0^1 3x^3\,dx = \frac{3}{4}$

$E[X^2] = \int_0^1 3x^4\,dx = \frac{3}{5}$

$\Rightarrow Var(X) = \frac{3}{5} - (\frac{3}{4})^2 = 0.038$

5. (a) $D_{disk} = (25)(25) = 625$ ms

$X = \frac{\rho_{disk}}{D_{disk}} = 4.8 \times 10^{-4}$ /ms

$= 0.48$ / s

(b) $E[T] = \frac{E[N]}{X} = \frac{23}{3}$ minutes.

7. $V_{CPU} = 25$
$V_A = 20$
$V_B = 4$

$\Rightarrow D_{CPU} = 750$ ms, $D_A = 500$ ms, $D_B = 160$ ms

(a) CPU

(b) $\frac{1}{D_{CPU}} = \frac{1000}{750}$ (per second) $= \frac{4}{3}$ per second

(c) $X \leq \min\left\{ \frac{1}{D_{CPU}}, \frac{N}{D + E[Z]} \right\}$

Here, $D_{CPU} = 0.75$ s, $D = 1.41$ s and $E[Z] = 5$ s.

(d) $\frac{D_B}{D_{CPU}} = \frac{160}{750} = 0.21$

9. (a) $X = \frac{\rho_i}{E[D_i]} = \frac{0.48}{0.63} = 0.76$ per second

(b) $E[T] = \frac{M}{X} - E[Z]$

$= \frac{230}{0.76} - 300$

$= 2.63$

11. (a) $D_{disk} = 45$ msec $\Rightarrow$ disk is bottleneck
$D_{CPU} = 15$ msec

(b) ~~Need to know~~ Not enough information to determine this. Even with think time, can get at best lower bound

$$E[T] \geq \max(D, MD_{max} - E[Z])$$

(c) (i) and (ii): either (i) or (ii) in isolation has a new disk demand greater than 15 msec.

13. (a) If D_B is between 2 and 4, and thus $D_C = 6 - D_B$ is also between 2 and 4. For this range, D_A is always the bottleneck.

(b) $D_B = 6$ or $D_C = 6$.

(c) Let $D_B = D_C = 3$. Then maximum number of users supported is ~~dec~~ M^* solving:

$$D = M^* D_{max} - E[Z]$$

$$10 = (M^*)(4) - 40$$

$$\Rightarrow M^* = \left\lfloor \frac{50}{4} \right\rfloor = 13$$

15. (a) $D_A = 100$
$D_B = 120$
$D_C = 150$

(iii) reduces D_c the most.

(b) $E(N_i) = \lambda_i E[T_i]$. Here $\lambda_i = XV_i$, so $\lambda_A = 5X$

So, $2 = 5X(50)$
$\Rightarrow X = \frac{2000}{250}$ per second
$= 8$ per second.

(Note that this is more than the maximum possible throughput, so that data is not achievable.)

17. $D_{CPU} = 5$
$D_A = \frac{(80)(50)}{1000} = 4$ (times in seconds)
$D_B = \frac{(100)(30)}{1000} = 3$
$E[T] = 18$
$X_A = 15.70$
$M = 17$

$X = \frac{X_A}{V_A} = \frac{15.70}{80} = 0.196$

$\Rightarrow D_{CPU} = 0.98$, $D_A = 0.78$, $D_B = 0.59$ $(\rho_i = XE[D_i])$

19. (a) $\binom{5}{3}(0.9)^3(0.1)^2$

(b) $P\{\text{win in } 3\} + P\{\text{win in } 4\} + P\{\text{win in } 5\}$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$(0.9)^3 \qquad \underbrace{\binom{3}{2}(0.9)^2(0.1)}_{\text{win two of three}}\underbrace{(0.9)}_{\text{game 4}} \qquad \binom{4}{2}\underbrace{(0.9)^2(0.1)^2}_{\text{win two of four}}\underbrace{(0.9)}_{\text{game 5}}$$

(c) $(0.9)^3$

21. This question is poorly worded.

23. There is also a problem with this question.

$\int_1^c \frac{1}{x^2}\,dx = 1$ is not consistent with

$$E(X) = \int_1^c \frac{1}{x}\,dx = 2.$$

25.
$$\begin{aligned} V_{cpu} &= 20 \\ V_A &= 11 \\ V_B &= 8 \end{aligned} \quad \Rightarrow \quad \begin{aligned} D_{cpu} &= 1 \\ D_A &= 0.88 \\ D_B &= 0.32 \end{aligned}$$

(a) CPU is bottleneck

(b) $E(T) \geq \underbrace{(30)}_{M}\underbrace{(1)}_{D_{max}} - \underbrace{20}_{E(Z)} = 10$

Need $D_{cpu} = 28/30$, implies speedup of $30/28$.

27. (c) As no t-tables will be given, the confidence interval calculation is not on the exam. You would need to increase the number of samples by a factor of 4.

(b)
$$\int_0^1 kx(1-x) = 1$$
$$\Rightarrow k = 6$$
$$F(x) = \begin{cases} 3x^2 - 2x^3 & 0 \le x < 1 \\ 1 & x \ge 1 \\ 0 & x < 0 \end{cases}$$

$F^{-1}(a) =$ this is quite complicated, moreso than is reasonable for an exam.

29. (a) $(3)(0.6)(0.3)(0.1) = 0.054$

(b) $1 - P\{\text{always stands}\}$
$= 1 - (0.6)^{10}$

31. This can happen: the true value is expected to be in the confidence interval 95 percent of the time, so this single observation is not inconsistent.

33. Not on exam (no t-tables given)

35. Not on exam (no t-tables given)

37. (a) $\int_0^4 c e^{-2x}\,dx = 1 \Rightarrow c = \dfrac{2}{1-e^{-8}}$

(b) $P\{X>2 \mid X>1\} = \dfrac{P\{X>2\}}{P\{X>1\}}$

$$= \frac{\int_2^4 \cancel{c} e^{-2x}\,dx}{\int_1^4 \cancel{c} e^{-2x}\,dx} = 0.1332$$

(c) No, as exponential distribution is only memoryless distribution

39. $F(x) = x^2/4, \quad 0 \le x \le 2$

$F^{-1}(u) = 2\sqrt{u}$ ⇐ use this function.

41. (a) $\int_1^c \frac{4}{15} x^3 = 1 \Rightarrow c = 2$

(b) $F(x) = \dfrac{x^4}{15} - \dfrac{1}{15} \qquad 1 \le x \le 2$

$F^{-1}(u) = (15u+1)^{1/4}$ ⇐ use this function

43. $D_1 = 50$ ⇐ bottleneck

$D_2 = 35$

$D_3 = \dfrac{0.5}{\left(\frac{1}{50}\right)} = 25$

45. $G^{-1}(u) = \sqrt{u}$ $\Leftarrow$ use this function

47. 0: defective
1: not defective

(a) $$P = \begin{bmatrix} 0.3 & 0.7 \\ 0.1 & 0.9 \end{bmatrix}$$

(b) $P^2(0,0) = 0.16$

49. $$P = \begin{bmatrix} .4 & .4 & .2 \\ .2 & .4 & .4 \\ .3 & 0 & .7 \end{bmatrix}$$

(a) $P^1(1,1) \cdot P^1(1,1) = .16$

(b) $P^2(1,1) = .30$. Value is different as the intermediate state can be <u>anything</u>, not just state 1 as in (a).

51. 0: low
1: high

$$P = \begin{bmatrix} .9 & .1 \\ .4 & .6 \end{bmatrix}$$

(a) $P\{X_1 = \text{low}\} = P\{X_1 = \text{low} \mid X_0 = \text{low}\}\ P\{X_0 = \text{low}\}$
$+ P\{X_1 = \text{low} \mid X_0 = \text{high}\}\ P\{X_0 = \text{high}\}$

$= (0.9)(0.8) + (0.4)(0.2)$
$= 0.80$

$\Rightarrow P\{X_1 = \text{high}\} = 0.20.$

(b) $(110)(0.2) + (90)(0.8) = 94.$

53. (a) $P = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0.18 & 0.74 & 0.08 \\ 0.04 & 0.32 & 0.64 \end{bmatrix}$

(b) $P\{X_{52} = 2 \mid X_{51} = 0, X_{50} = 0\}$
$= P\{X_{52} = 2 \mid X_{51} = 0\}$
$= 0.01$

55. Let states be: 0 - query
1 - add/modify
2 - delete

Then $P = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.9 & 0.1 & 0 \\ 0.9 & 0 & 0.1 \end{bmatrix}$

(a) $P^2(0,0) = 0.76$

(b) Solve $\pi P = \pi$, $\pi_0 + \pi_1 + \pi_2 = 1$

$\pi_0 = 0.75$
$\pi_1 = 0.17$
$\pi_2 = 0.08$

57. (a) $P^2(1,1) = 0.66$

(b) Solve $\pi P = \pi$, $\pi_1 + \pi_2 + \pi_3 = 1$

$\pi_1 = 1/3$
$\pi_2 = 1/3$
$\pi_3 = 1/3$

59. (a) $P^2(0,0) = 0.5$

(b) $X_0 = 0$ does not enter into steady-state calculations

Solve $\pi P = \pi$, $\pi_0 + \pi_1 + \pi_2 = 1$

$\pi_0 = 0.5$
$\pi_1 = 0.25$
$\pi_2 = 0.25$

61. (a) Let the state be the # of failed components

$$P = \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0.63 & 0.34 & 0.03 \\ 0.49 & 0.42 & 0.09 \end{bmatrix}$$

(b) Solve $\pi P = \pi$, $\pi_0 + \pi_1 + \pi_2 = 1$

$\pi_0 = 0.7656$
$\pi_1 = 0.2188$
$\pi_2 = 0.0156$

63. States: 0: both servers available
1: one server offline (no repairs), other available
2: one server offline (one day of repair), " "
3: both servers offline.

(a) $$P = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 0.9 & 0.1 \\ 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) $0.9\pi_0 + 0.9\pi_2 = \pi_0$
$0.1\pi_0 + 0.1\pi_2 = \pi_1$
$0.9\pi_1 + \pi_3 = \pi_2$
$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$

π_3 is the required probability

65. (a) States are

	LRU	MRU
0:	1	2
1:	1	3
2:	2	3
3:	2	1
4:	3	1
5:	3	2

$$P = \begin{array}{c|c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0.1 & 0 & 0.1 & 0.8 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0.8 & 0.1 \\ 0 & 0 & 0.1 & 0 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0 & 0.8 & 0 & 0 \\ 0.1 & 0.1 & 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0.1 & 0.8 & 0 & 0.1 \end{bmatrix} \end{array}$$

(b) $(0.8)(\pi_2 + \pi_5) + (0.1)(\pi_1 + \pi_4) + (0.1)(\pi_0 + \pi_3)$