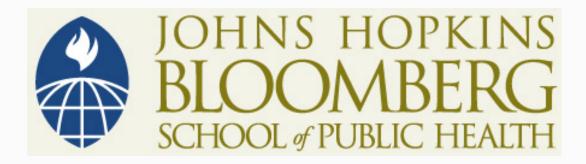
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#### Section H

Small Sample Considerations for Confidence Intervals for Population Proportions

## The Central Limit Theorem (CLT)

- The Central Limit Theorem (CLT) is a powerful mathematical tool that gives several useful statistics including:
  - The sampling distribution of sample proportions based on all samples of same size *n* is *approximately* normal
  - Mother/infant transmission example, placebo group:
  - CLT 95% CI:(can be done by hand)
  - Exact 95% CI:(requires computer, always correct)

```
(.158,.282) \approx (.16,.28)

\rightarrow 16\% \text{ to } 28\%
```

```
From Ocii 183 400 command
[95% Conf. Interval]
-----
.160984 .2855248
```

# Notes on 95% Confidence Interval for Proportion

- The CLT based formula for a 95% CI is only approximate; it works very well if you have enough data in your sample
- The approximation works better the bigger  $n \times \hat{p} \times (1 \hat{p})$
- "Large sample" for binary outcomes is not only a function of total sample size *n*, but the split between "yes" and "no" outcomes

## Mother/Infant Transmission: AZT Group

Mother/infant transmission example, AZT group:

- (n = 180, 
$$\hat{p} = \frac{13}{180} = .07$$
)

- CLT 95% CI: (can be done by hand)
- Exact 95% CI: (requires computer, always correct)

$$(.032,.108) \approx (.03,.11)$$
  
 $\rightarrow 3\% \text{ to } 11\%$ 

From Òcii 180 13Ó command

[95% Conf. Interval]

----.0390137 .1203358

#### Mother/Infant Transmission Cls

In the placebo sample

$$n \times \hat{p}_{plac} \times (1 - \hat{p}_{plac}) =$$

$$183 * .22 * .78 \approx 31$$

In the AZT sample

$$n \times \hat{p}_{AZT} (1 - \hat{p}_{AZT}) = 180 * .07 * .93 \approx 12$$

## Notes on 95% Confidence Interval for Proportion

- You do not use the t-correction for small sample sizes like we did for sample means
  - We use exact binomial calculations
- Interpretation of 95% CIs exactly the same with either method
  - In real life, using computer will always give valid result
  - CLT only breaks down with "small" sample sizes
  - In testing situations you will not be required to do exact CIs!

# Really Small Sample Example for Illustration

 Random sample of 16 patients on drug A: two of sixteen patients experience drug failure in first month

- CLT 95% CI: 
$$\hat{p} \pm 2 \times S\hat{E}(\hat{p}) \rightarrow \frac{2}{16} \pm 2 \times \sqrt{\frac{(2/16) \times (1 - 2/16)}{16}} \rightarrow (-0.05, 0.28)$$

Exact 95% CI: (0.02, 0.38)

Binomial Exact					
[95% Conf. Inter	. Err.	Std.	Mean	Obs	Variable
					+
.0155136 .3834	26797	.082	.125	16	