This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike License. Your use of this material constitutes acceptance of that license and the conditions of use of materials on this site.

Copyright 2009, The Johns Hopkins University and John McGready. All rights reserved. Use of these materials permitted only in accordance with license rights granted. Materials provided "AS IS"; no representations or warranties provided. User assumes all responsibility for use, and all liability related thereto, and must independently review all materials for accuracy and efficacy. May contain materials owned by others. User is responsible for obtaining permissions for use from third parties as needed.

JOHNS HOPKINS
BLOOMBERG
SCHOOL of PUBLIC HEALTH

An Introduction to Hypothesis Testing: The Paired t-Test

John McGready
Johns Hopkins University

Lecture Topics

- Comparing two groups: the paired data situation
- Hypothesis testing: the null and alternative hypotheses
- Relationships between confidence intervals and hypothesis testing when comparing means
- p-values: definition, calculations, and more information

JOHNS HOPKINS
BLOOMBERG
SCHOOL of PUBLIC HEALTH

Section A

The Paired t-Test; the Confidence Interval Component

Comparison of Two Groups

- Are the population means different? (continuous data)

- Paired design
  - Before-after data
  - Twin data
  - Matched case-control

- Two independent sample design
  - Randomized trial
  - Smokers to non-smokers

Paired Design—Example: Before vs. After

- Why pairing?
  - Control extraneous noise
  - Each observation acts as a control
  - Good way to get preliminary data/estimates to be used to develop further research

Paired Design—Example: Before vs. After

- Ten non-pregnant, pre-menopausal women 16–49 years old who were beginning a regimen of oral contraceptive (OC) use had their blood pressures measured prior to starting OC use and three-months after consistent OC use[1]

- The goal of this small study was to see what, if any, changes in average blood pressure were associated with OC use in such women

- The data on the following slides shows the resulting pre- and post-OC use systolic BP measurements for the 10 women in the study

[1] Data taken from Rosner B Fundamentals of Biostatistics, 6th ed. (2005) Duxbury Press.

Blood Pressure and Oral Contraceptive Use

	BP Before OC	BP After OC	After-Before
1.	115	128	13
2.	112	115	3
3.	107	106	-1
4.	119	128	9
5.	115	122	7
6.	138	145	7
7.	126	132	6
8.	105	109	4
9.	104	102	-2
10.	115	117	2

$\bar{x}_{bef\ ore} = 115.6$ $\bar{x}_{af\ ter} = 120.4$ $\bar{x}_{dif\ f} = 4.8$

Blood Pressure and Oral Contraceptive Use

- The sample average of the differences is 4.8
  - Also note $\bar{x}_{diff} = \bar{x}_{after} - \bar{x}_{before}$

- The sample standard deviation (s) of the differences is s_{diff} = 4.6
  - Standard deviation of differences found by using the formula:

$$s_{diff} = \sqrt{\frac{\sum_{i=1}^{n} (x_{diff} - \bar{x}_{diff})^2}{n-1}}$$

  - Where:
    - Each x_{diff_i} represents an individual difference and
    - x_{diff_i} is the mean difference

Blood Pressure and Oral Contraceptive Use

	BP Before OC	BP After OC	After-Before
1.	115	128	13
2.	112	115	3
3.	107	106	-1
4.	119	128	9
5.	115	122	7
6.	138	145	7
7.	126	132	6
8.	105	109	4
9.	104	102	-2
10.	115	117	2

$\bar{x}_{bef\ ore} = 115.6$ $\bar{x}_{af\ ter} = 120.4$ $\bar{x}_{dif\ f} = 4.8$

Note

- In essence, what we have done is reduce the BP information on two samples (women prior to OC use, women after OC use) into one piece of information: information on the differences in BP between the samples

- This is standard protocol for comparing paired samples with a continuous outcome measure

Confidence Interval Approach

- Want to draw a conclusion about a population parameter
  - In a population of women who use oral contraceptives, is the average (expected) change in blood pressure (after-before) 0 or not?

- Sometimes the term *expected* is used for the population average

- μ is the expected (population) mean change in blood pressure

- CI approach allows us to create a range of possible values for μ using data from a single, imperfect (paired) sample

95% Confidence Interval

- 95% confidence interval for mean change in BP in population of women taking oral contraceptives, after starting OC use compared to before OC use

$$\bar{x}_{diff} \pm t_{.95,9} \times S\hat{E}(\bar{x}_{diff})$$

$$\bar{x}_{diff} \pm t_{.95,9} \times \frac{s_{diff}}{\sqrt{10}}$$

$$4.8 \pm 2.26 \times \left(\frac{4.6}{\sqrt{10}} \right)$$

$$1.5\ mmHG\ to\ 8.1\ \ mmHg$$

95% Confidence Interval

- 95% confidence interval for mean change in BP in population of women taking oral contraceptives, after starting OC use compared to before OC use using *cii* in Stata

```
. cii 10 4.8 4.6

    Variable |        Obs        Mean    Std. Err.       [95% Conf. Interval]
-------------+---------------------------------------------------------------
             |         10         4.8    1.454648        1.509358    8.090642
```

95% Confidence Interval

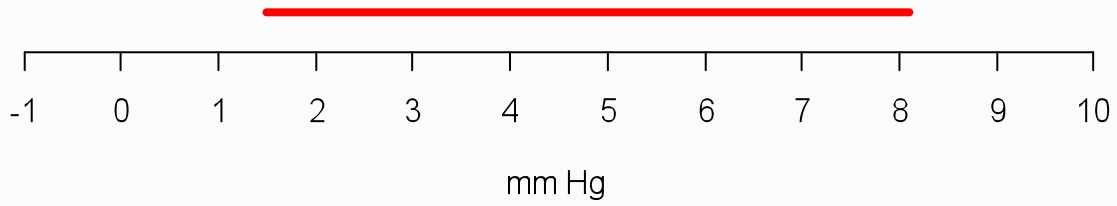
- 95% confidence interval for mean change in BP in population of women taking oral contraceptives, after starting OC use compared to before OC use using *cii* in Stata

```
. cii 10 4.8 4.6

    Variable |        Obs        Mean    Std. Err.       [95% Conf. Interval]
-------------+---------------------------------------------------------------
             |         10         4.8    1.454648        1.509358    8.090642
```

Notes

- The number 0 is **NOT** in confidence interval (1.5-8.1)



Notes

- The number 0 is **not** in confidence interval (1.5–8.1)
  - Because 0 is not in the interval, this suggests there is a non-zero change in BP over time
  - The phrase “statistically significant” change is used to indicate a non-zero mean change

Notes

- The BP change could be due to factors other than oral contraceptives
  - Changes in weather over pre- and –post period
  - Changes in personal stress
  - Other changes?

- A control group of comparable women who were not taking oral contraceptives would strengthen this study
  - This is an example of a *pilot study*—a small study done just to generate some evidence of a possible association
  - This can be followed up with a larger, more scientifically rigorous study