732A99/TDDE01 Machine Learning

Lecture 1c Block 2: Online Learning

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Contents

Literature

- Main sources
  - Murphy, K. P. *Machine Learning - A Probabilistic Perspective*. MIT Press, 2012. Sections 8.5 and 11.4.8.
  - Shalev-Shwartz, S. Online Learning and Online Convex Optimization. *Foundations and Trends in Machine Learning*, 4:107-194, 2011. Sections 1-1.2 and 3-3.1.
- Additional sources
  - Blum, A. On-Line Algorithms in Machine Learning. In *Online Algorithms: The State of the Art*. Springer, 1998.
  - Bordes, A. et al. Fast Kernel Classifiers with Online and Active Learning. *Journal of Machine Learning Research*, 6:1579-1619, 2005.

Online Learning

- Offline learning: Batch of data used to optimize a function such as

$$\sum_n L(t_n, y(\boldsymbol{x}_n, \boldsymbol{\theta}))$$

where $L(\cdot)$ is log loss, squared error, 0/1 loss, etc.

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for $n = 1, 2, \ldots$
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- Examples: Weather prediction, spam filtering, small devices.
- Goal: Minimize the cumulative loss suffered by updating the predictor.

Online Learning

- Assume that $|\Theta| < \infty$ and $t_n = y(\boldsymbol{x}_n, \boldsymbol{\theta}^*)$ for all n with $\boldsymbol{\theta}^* \in \Theta$. The latter is also known as realizability.

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- The algorithm above makes at most $\log_2 |\Theta|$ errors.

Online Learning

- If we drop the realizability assumption, then our goal may be restated as minimizing the regret

$$\sum_n L(t_n, y(\boldsymbol{x}_n, \boldsymbol{\theta}_n)) - \min_{\boldsymbol{\theta}^* \in \boldsymbol{\Theta}} \sum_n L(t_n, y(\boldsymbol{x}_n, \boldsymbol{\theta}^*))$$

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- These goals seem hopeless anyway, as the true answers may come from an adversary who waits for our prediction and then answers the opposite label (*regret* $= N - N/2$ at least, e.g. if the batch solution is majority voting).
- Solution: Mild change of game. The adversary has to decide the true label before we make a prediction. Moreover, we are allowed to randomize our predictions, to cope with non-realizability.

Online Learning

Weighted majority (input: $\eta \in (0,1)$)

$\boldsymbol{w}_1 = (1/M, \ldots, 1/M)$ where $M = |\boldsymbol{\Theta}|$

for $n = 1, 2, \ldots$

Choose $m \sim \boldsymbol{w}_n$ and predict $y(\boldsymbol{x}_n, \boldsymbol{\theta}_m)$

Compute cost $c_{nm} = |y(\boldsymbol{x}_n, \boldsymbol{\theta}_m) - t_n|$ for all m

Update $w_{n+1i} = \frac{w_{ni} exp(-\eta c_{ni})}{\sum_j w_{nj} exp(-\eta c_{nj})}$ for all i

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- The expected 0-1 loss on round n can be written as

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- Moreover, if $\eta = \sqrt{N \log M}$ then

$$\sum_n |p_n - t_n| \leq \min_{1 \leq m \leq M} \sum_n c_{nm} + 2\sqrt{N \log M}$$

which implies a sub-linear regret wrt N, i.e. $\frac{\text{regret}}{N} \to 0$ as $N \to \infty$.

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- Moreover, if $\eta = 1/2$ then

$$\sum_n |p_n - t_n| \leq 2 \min_{1 \leq m \leq M} \sum_n c_{nm} + 4 \log M$$

which reduces to $4 \log M$ for the realizable case (similar to Halving).

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- If we drop the assumption that $|\boldsymbol{\Theta}| < \infty$, minimizing the regret may be achieved by online gradient descent, which updates the parameters as

$$\boldsymbol{\theta}_{n+1} = proj_{\boldsymbol{\Theta}}(\boldsymbol{\theta}_n - \eta_n \nabla L(t_n, y(\boldsymbol{x}_n, \boldsymbol{\theta}_n)))$$

  where $proj_{\boldsymbol{\Theta}}(\boldsymbol{v}) = \arg\min_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} ||\boldsymbol{v} - \boldsymbol{\theta}||$, and it is needed only if $\boldsymbol{\Theta}$ is a subset of $\mathbb{R}^D$.

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- To ensure that stochastic gradient descent converges, we need to check that

$$\sum_{n=1}^{\infty} \eta_n = \infty \text{ and } \sum_{n=1}^{\infty} \eta_n^2 < \infty$$

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- Convergence vs adaptation or concept drift, e.g. $\eta_n = 1/2$.

Online EM Algorithm

EM algorithm

Set $\boldsymbol{\pi}$ and $\boldsymbol{\mu}$ to some initial values
Repeat until $\boldsymbol{\pi}$ and $\boldsymbol{\mu}$ do not change
  Compute $p(z_{nk}|\boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi})$ for all n /* E step */
  Set π_k to π_k^{ML}, and μ_{ki} to μ_{ki}^{ML} for all k and i /* M step */

Mixture of Bernoullis

$$\pi_k^{ML} = \frac{\sum_n p(z_{nk}|\boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi})}{N}$$

$$\mu_{ki}^{ML} = \frac{\sum_n x_{ni} p(z_{nk}|\boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi})}{\sum_n p(z_{nk}|\boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi})}$$

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$$\boldsymbol{\Sigma}_k^{ML} = \frac{\sum_n (\boldsymbol{x}_n - \boldsymbol{\mu}_k^{ML})(\boldsymbol{x}_n - \boldsymbol{\mu}_k^{ML})^T p(z_{nk}|\boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi})}{\sum_n p(z_{nk}|\boldsymbol{x}_n, \boldsymbol{\mu}, \boldsymbol{\pi})}$$

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- Online EM algorithm: Replace the E step with a stochastic E step such as

$$S_{1:n} = (1 - \eta_n) S_{1:n-1} + \eta_n \mathbb{E}_{\boldsymbol{Z}}[S_n | \boldsymbol{\mu}_n, \boldsymbol{\pi}_n]$$

where $\{\eta_n\}$ is the learning rate, e.g. $\eta_n = 1/n$. Similarly for the mixture of Gaussians.

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- Convergent under mild conditions. Adaptative too.

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$$y(\boldsymbol{x}) = \begin{cases} 0 & \text{if } \sum_n \mathbf{1}_{\{t_n=1\}} k\left(\frac{\boldsymbol{x}-\boldsymbol{x}_n}{h}\right) \leq \sum_n \mathbf{1}_{\{t_n=0\}} k\left(\frac{\boldsymbol{x}-\boldsymbol{x}_n}{h}\right) \\ 1 & \text{otherwise} \end{cases}$$

where $k : \mathbb{R}^D \to \mathbb{R}$ is a kernel function, which is usually non-negative and monotone decreasing along rays starting from the origin. The parameter h is called smoothing factor or width.

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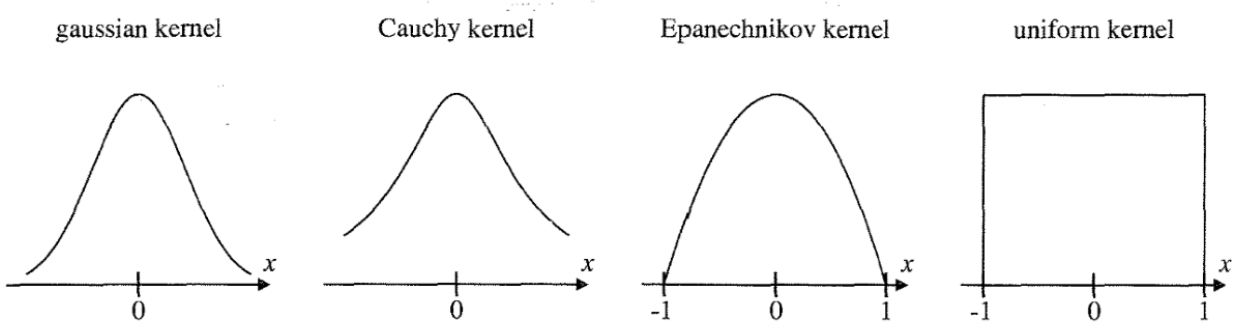

FIGURE 10.3. *Various kernels on $\mathcal{R}$.*

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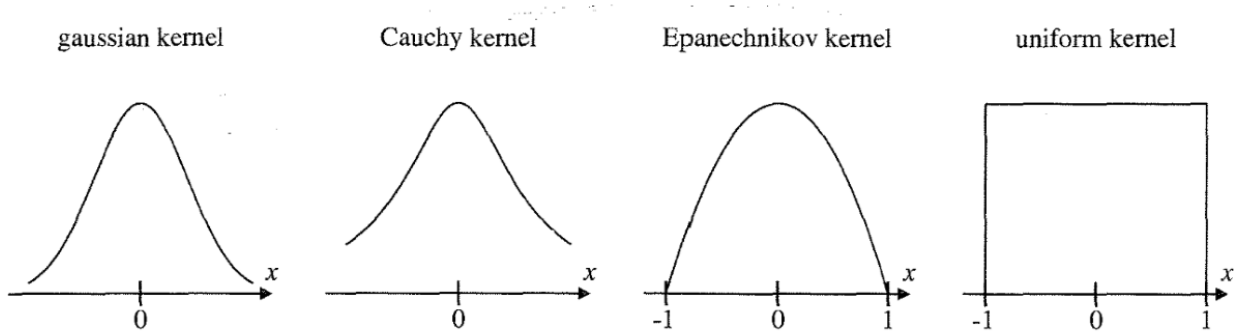

FIGURE 10.3. *Various kernels on $\mathcal{R}$.*

- Gaussian kernel: $k(u) = exp(-||u||^2)$ where $||\cdot||$ is the Euclidean norm.
- Cauchy kernel: $k(u) = 1/(1 + ||u||^{D+1})$
- Epanechnikov kernel: $k(u) = (1 - ||u||^2)\mathbf{1}_{\{||u||\le 1\}}$

Kernel Classification

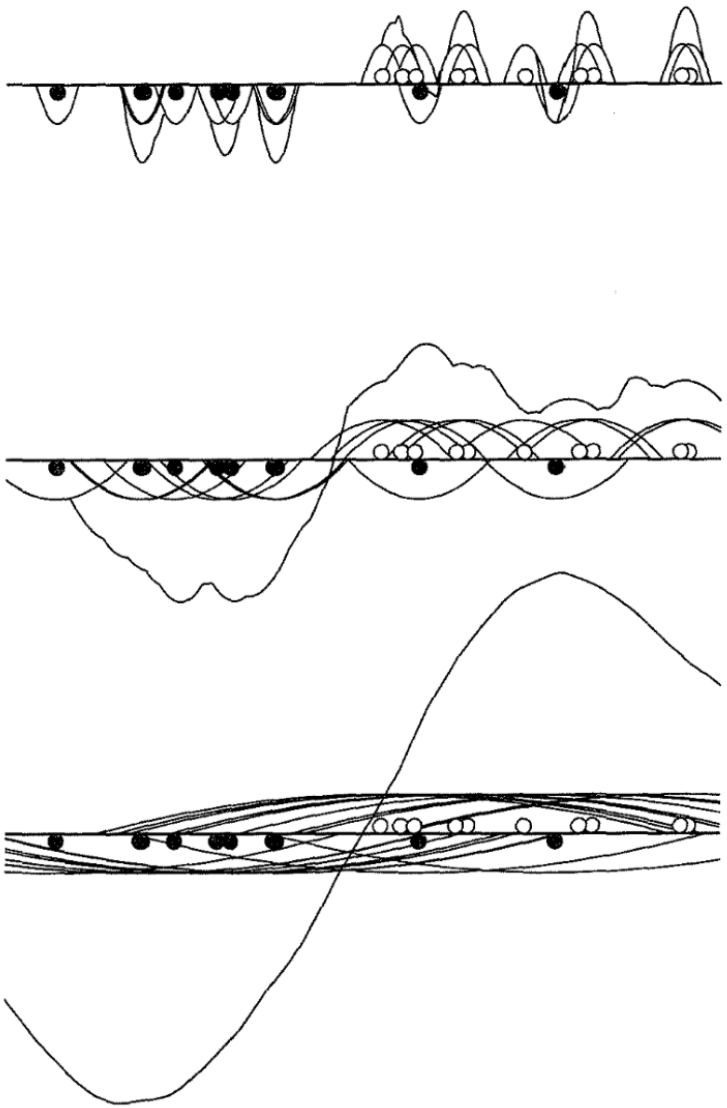

FIGURE 10.2. *Kernel rule on the real line. The figure shows* $\sum_{i=1}^{n}(2Y_i-1)K((x-X_i)/h)$ *for* $n = 20$, $K(u) = (1-u^2)I_{\{|u|\le 1\}}$ *(the Epanechnikov kernel), and three smoothing factors* h. *One definitely undersmooths and one oversmooths. We took* $p = 1/2$, *and the class-conditional densities are* $f_0(x) = 2(1-x)$ *and* $f_1(x) = 2x$ *on* $[0, 1]$.

Online Kernel Classification

- Kernel classifier:

$$y(\boldsymbol{x}) = \begin{cases} 0 & \text{if } \sum_n \mathbf{1}_{\{t_n=1\}} k\left(\frac{\boldsymbol{x}-\boldsymbol{x}_n}{h}\right) \leq \sum_n \mathbf{1}_{\{t_n=0\}} k\left(\frac{\boldsymbol{x}-\boldsymbol{x}_n}{h}\right) \\ 1 & \text{otherwise} \end{cases}$$

- The kernel classifier implies no learning and, thus, it can be applied to online learning directly. Deploying it may however become computationally demanding (specially with scarce resources), since it sums over all the training points.

Support Vector Machines for Classification

- Consider binary classification with input space $\mathbb{R}^D$.

Support Vector Machines for Classification

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- Consider a training set $\{(\boldsymbol{x}_n, t_n)\}$ where $t_n \in \{-1, +1\}$.

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$$y(\boldsymbol{x}) = \boldsymbol{w}^T \phi(\boldsymbol{x}) + b$$

so that a new point $\boldsymbol{x}$ is classified according to the sign of $y(\boldsymbol{x})$.

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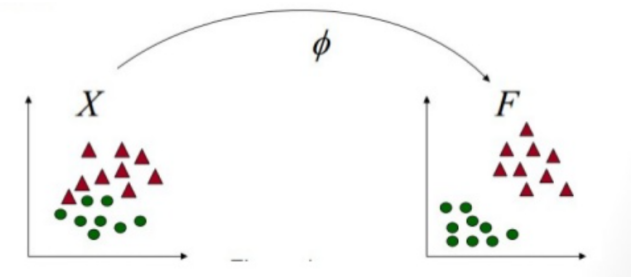
  so that a new point $\boldsymbol{x}$ is classified according to the sign of $y(\boldsymbol{x})$.
- Assume that the training set is linearly separable in the feature space (but not necessarily in the input space), i.e. $t_n y(\boldsymbol{x}_n) > 0$ for all n.



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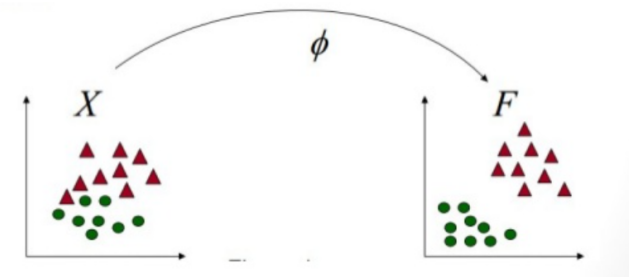
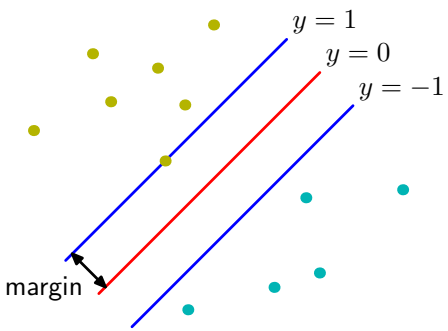
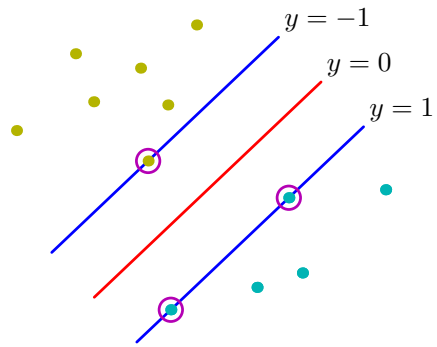


- Aim for the separating hyperplane that maximizes the margin (i.e. the smallest perpendicular distance from any point to the hyperplane) so as to minimize the generalization error.





Support Vector Machines for Classification

- The maximum margin separating hyperplane is given by

$$\arg\min_{\boldsymbol{w},b} \frac{1}{2}\|\boldsymbol{w}\|^2$$

subject to $t_n(\boldsymbol{w}^T\phi(\boldsymbol{x}_n) + b) \geq 1$ for all n, which is equivalent to

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$$\arg\min_{\boldsymbol{w},b} \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_n a_n\Big(t_n(\boldsymbol{w}^T\phi(\boldsymbol{x}_n)+b)-1\Big)$$

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where $a_n \geq 0$ are Lagrange multipliers, whose solution is

  - $\boldsymbol{w} = \sum_n a_n t_n \phi(\boldsymbol{x}_n)$, and
  - $a_n > 0$ if and only if $\boldsymbol{x}_n$ lies on the margin boundaries. Such points are called support vectors, or $\mathcal{S}$ for short.

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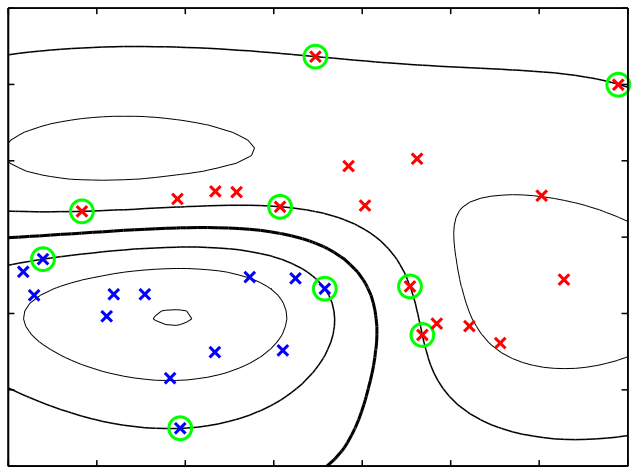

Online Support Vector Machines for Classification

- Support vector machines:

$$y(\boldsymbol{x}) = \sum_{m \in \mathcal{S}} a_m t_m k(\boldsymbol{x}, \boldsymbol{x}_m) + b$$

- Training a support vector machine needs to be adapted for online learning, i.e. retraining is not an option as the training data may grow very fast. Deploying it is feasible due to its sparseness, even with scarce resources.

Online Support Vector Machines for Classification

	Online SVM
1	$\mathcal{S} = \varnothing$
2	$b = 0$
3	Receive a random example $(\boldsymbol{x}_i, t_i)$
4	Compute $y(\boldsymbol{x}_i) = \sum_{m \in \mathcal{S}} a_m t_m k(\boldsymbol{x}_i, \boldsymbol{x}_m) + b$
5	If $t_i y(\boldsymbol{x}_i) \leq 0$ then
6	$\mathcal{S} = \mathcal{S} \cup \{i\}$
7	$a_i = 1$
8	Go to step 3

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- It converges to a separating hyperplane in the separable case.

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- It converges to a separating hyperplane in the separable case.
- It does not attempt to maximize the margin.

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- It converges to a separating hyperplane in the separable case.
- It does not attempt to maximize the margin.
- It only adds support vectors.
- Therefore, it may overfit the training data (and increase computational cost for deploying it).

Online Support Vector Machines for Classification

Budget online SVM (input: β and M)

1 $\mathcal{S} = \varnothing$
2 $b = 0$
3 Receive a random example $(\boldsymbol{x}_i, t_i)$
4 Compute $y(\boldsymbol{x}_i) = \sum_{m \in \mathcal{S}} a_m t_m k(\boldsymbol{x}_i, \boldsymbol{x}_m) + b$
5 If $t_i y(\boldsymbol{x}_i) \leq \beta$ then
6 $\quad \mathcal{S} = \mathcal{S} \cup \{i\}$
7 $\quad a_i = 1$
8 $\quad$ If $|\mathcal{S}| > M$ then $\mathcal{S} = \mathcal{S} \setminus \{\arg\max_{m \in \mathcal{S}} t_m (y(\boldsymbol{x}_m) - a_m t_m k(\boldsymbol{x}_m, \boldsymbol{x}_m))\}$
9 Go to step 3

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- **Quiz**: Why does the removal criterion in step 8 make sense ?

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- **Quiz**: Why does the removal criterion in step 8 make sense ?
- It tolerates mild noisy data but does not maximize the margin.
- It may still overfit the training data (and increase computational cost for deploying it) due to its myopic nature.

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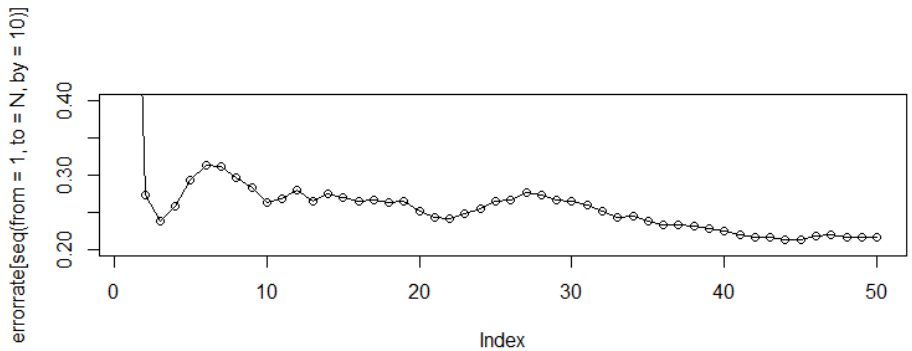
9. Go to step 3

- **Quiz**: Why does the removal criterion in step 8 make sense ?
- It tolerates mild noisy data but does not maximize the margin.
- It may still overfit the training data (and increase computational cost for deploying it) due to its myopic nature.
- More sophisticated addition and removal criteria are needed, e.g. LASVM which handles noisy data (slack variables) and converges to the batch solution.

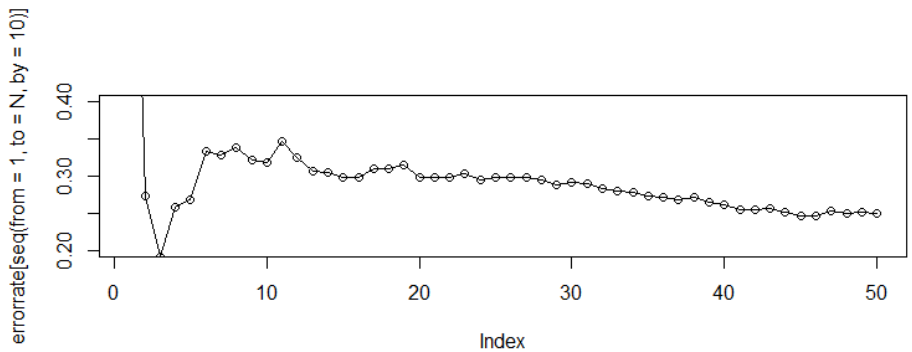
Online Support Vector Machines for Classification

- Online classification of 500 instances from the spambase data, using only the first 48 attributes.
- Gaussian kernel with $h = 1$.

$M = 500, \beta = 0$



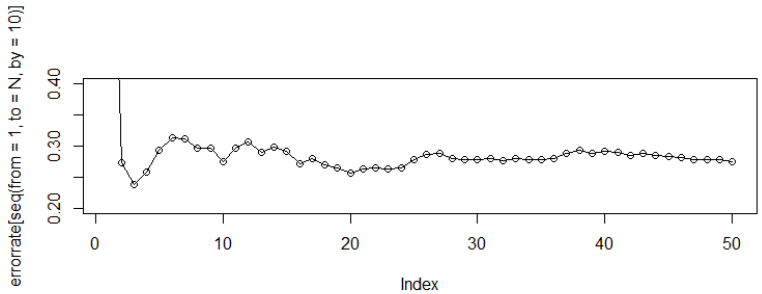
$M = 500, \beta = 0.05$



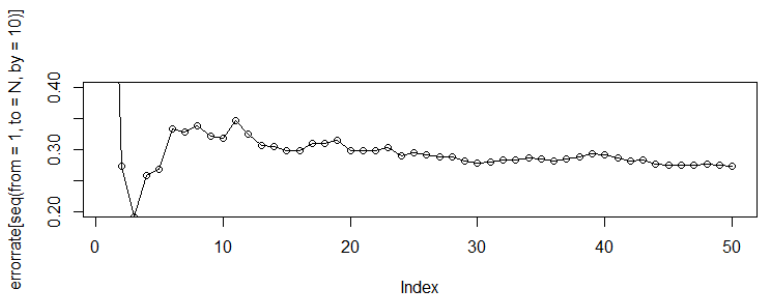
- $M = 500, \beta = 0$: Final error rate of 0.212 with 106 support vectors.
- $M = 500, \beta = 0.05$: Final error rate of 0.248 with 46 support vectors.

Online Support Vector Machines for Classification

$M = 20, \beta = 0$



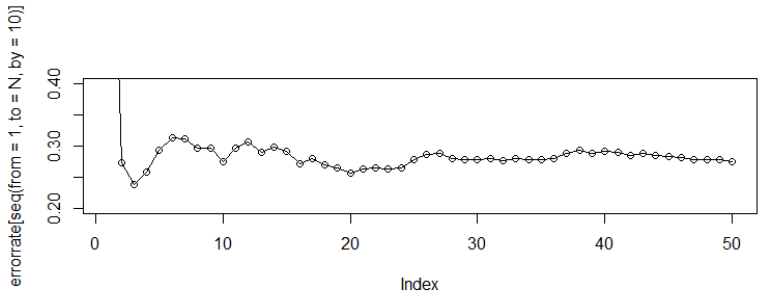
$M = 20, \beta = 0.05$



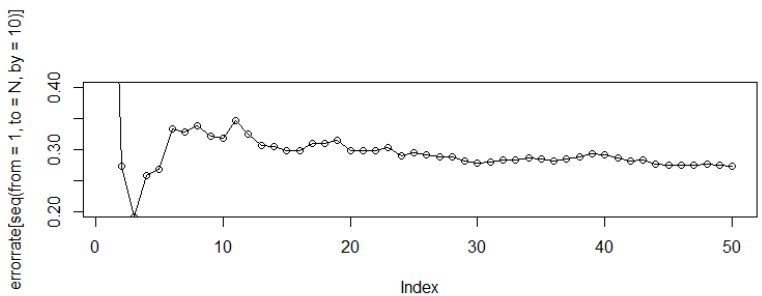
- $M = 20, \beta = 0$: Final error rate of 0.274 with 20 support vectors.
- $M = 20, \beta = 0.05$: Final error rate of 0.278 with 20 support vectors.

Online Support Vector Machines for Classification

$M = 20, \beta = 0$



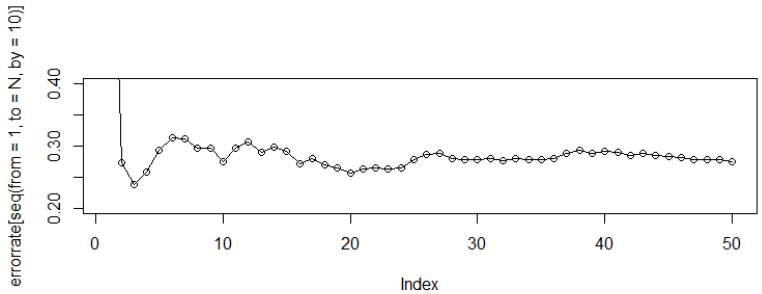
$M = 20, \beta = 0.05$



- $M = 20, \beta = 0$: Final error rate of 0.274 with 20 support vectors.
- $M = 20, \beta = 0.05$: Final error rate of 0.278 with 20 support vectors.
- Plots 1 and 2 show that correcting for non-serious errors is beneficial if we do not have an upper limit to the number of support vectors.

Online Support Vector Machines for Classification

$M = 20, \beta = 0$



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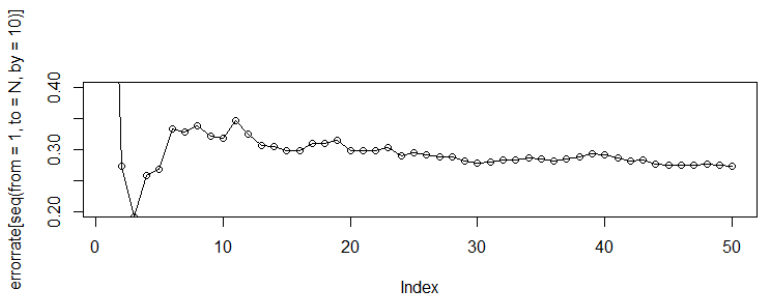
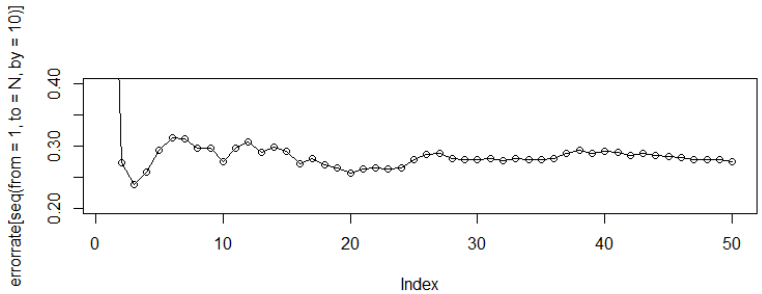


- $M = 20, \beta = 0$: Final error rate of 0.274 with 20 support vectors.
- $M = 20, \beta = 0.05$: Final error rate of 0.278 with 20 support vectors.
- Plots 1 and 2 show that correcting for non-serious errors is beneficial if we do not have an upper limit to the number of support vectors.
- If we have such a limit, then plots 3 and 4 show that it may be better to use $\beta > 0$ so as to correct only for serious errors. Note that setting 4 is preferable since it results in a smoothly monotone decreasing error rate.

Online Support Vector Machines for Classification

$M = 20, \beta = 0$



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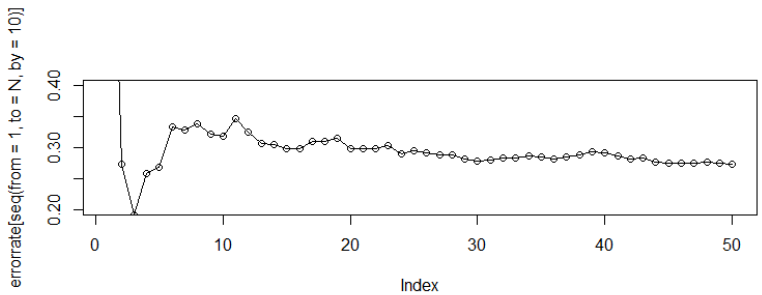

- $M = 20, \beta = 0$: Final error rate of 0.274 with 20 support vectors.
- $M = 20, \beta = 0.05$: Final error rate of 0.278 with 20 support vectors.
- Plots 1 and 2 show that correcting for non-serious errors is beneficial if we do not have an upper limit to the number of support vectors.
- If we have such a limit, then plots 3 and 4 show that it may be better to use $\beta > 0$ so as to correct only for serious errors. Note that setting 4 is preferable since it results in a smoothly monotone decreasing error rate.
- Setting 3 is the slowest since it is the one that removes more vectors.

Summary

- New concepts: Online learning, adversarial environment, regret.
- New algorithms:
  - Realizable and finite class of models: Consistent and halving.
  - Finite class of models: Weighted majority.
  - Online EM, online SVM, and budget online SVM.