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# Title

## Subtitle

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# Introduction to Quantum Gravity

So, which are the underlying structures in our fundamental theories?

- In General Relativity, space-time has a differential manifold structure determined by the energy distribution of the fields lying on it, via the Einstein Equation.
- In Quantum Mechanics, the evolution of a system in relation to a laboratory time, i.e. with time as a classical parameter, is governed by its Hamiltonian via the Schrödinger Equation.

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In both the cases, **we ignore the fact that** clocks measuring time are physical systems and, for precision measures, **they're quantum systems**!

How can we take in account that?

# Page and Wootters framework

As firstly described by Page and Wootters [1], and then further formalized by Giovannetti and Maccone [2], we may think at the clock as part of the quantum system, with its own Hamiltonian, but subject to a global constraint in the form of a Wheeler-DeWitt Equation as

$$\hat{C}\,|\Psi\rangle = 0 \qquad \textit{with} \quad \hat{C} = \hat{H}_T \otimes \mathbb{1}_S + \mathbb{1}_T \otimes \hat{H}_S \tag{1}$$

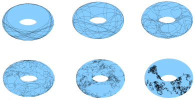
# Example section

# EM shower simulator

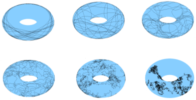
## Physics and reasons for our work

EM showers develops through bremsstrahlung and pair production processes. Main physical paramaters:

- $X_0$ = radiation lenght;
- $\lambda_\gamma$ = photon absorption lenght;
- $R_M = \frac{E_S}{E_c} X_0$;



Kinetic Brownian motion on the flat torus for $\sigma = 10^{-2}, 10^{-1}, 1, 2, 4, 10$.



Kinetic Brownian motion on the flat torus for $\sigma = 10^{-2}, 10^{-1}, 1, 2, 4, 10$.

Once installed, it can be used typing:

**from bash**

```
$ simulate-EM-shower -f 10. 1
```

**from Python**

```
>>> import em_shower_simulator as em
>>> em.simulate([10., 1.], verbose=0)
```

# SDE and Fokker-Planck Equation

## Forward Fokker-Planck



Equation ?? is more correctly expressed as a stochastic differential equation. Switching to several dimensions, the motion of the particle is described by

$$dq_i = -f_i(q)dt + g_{ij}(q)d\omega_j \tag{2}$$

The SDE above is associated to the Fokker-Planck Equation

$$\partial_t P(q,t|q_0,t_0) = \hat{\mathcal{L}} P(q,t|q_0,t_0) \tag{3}$$

in which $\hat{\mathcal{L}}$ is the differential operator defined as

$$\hat{\mathcal{L}} \equiv \sum_i \partial_{q_i} f_i(q) + \frac{1}{2} \sum_{i,j} \partial_{q_i}\partial_{q_j}[g(q)g^T(q)]_{ij} \tag{4}$$

The general idea is that, once the solution to the equation 3 is known, the statistical properties of the process are completely defined.

Cfr. section 4.3 of [3]

# Conclusions

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What we achieved:

- something

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What we learned:

- something else

# Conclusions



What we achieved:

- something

What we learned:

- something else

- **In the end!**

# Acknowledgments and References

# References I



[1] Don N. Page and William K. Wootters.
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*Phys. Rev. D*, 92:045033, Aug 2015.

[3] Esteban Castro-Ruiz, Flaminia Giacomini, A. Belenchia, and Časlav Brukner.
Quantum clocks and the temporal localisability of events in the presence of gravitating quantum systems.
*Nature Commun.*, 11(1):2672, 2020.

Thank you for your attention!