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Title

Subtitle

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Introduction to Quantum Gravity



So, which are the underlying structures in our fundamental theories?

- In General Relativity, space-time has a differential manifold structure determined by the energy distribution of the fields lying on it, via the Einstein Equation,
- In Quantum Mechanics, the evolution of a system in relation to a laboratory time, i.e. with time as a classical parameter, is governed by its Hamiltonian via the Schrödinger Equation.



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In both the cases, **we ignore the fact that** clocks measuring time are physical systems and, for precision measures, **they're quantum systems!**

How can we take in account that?

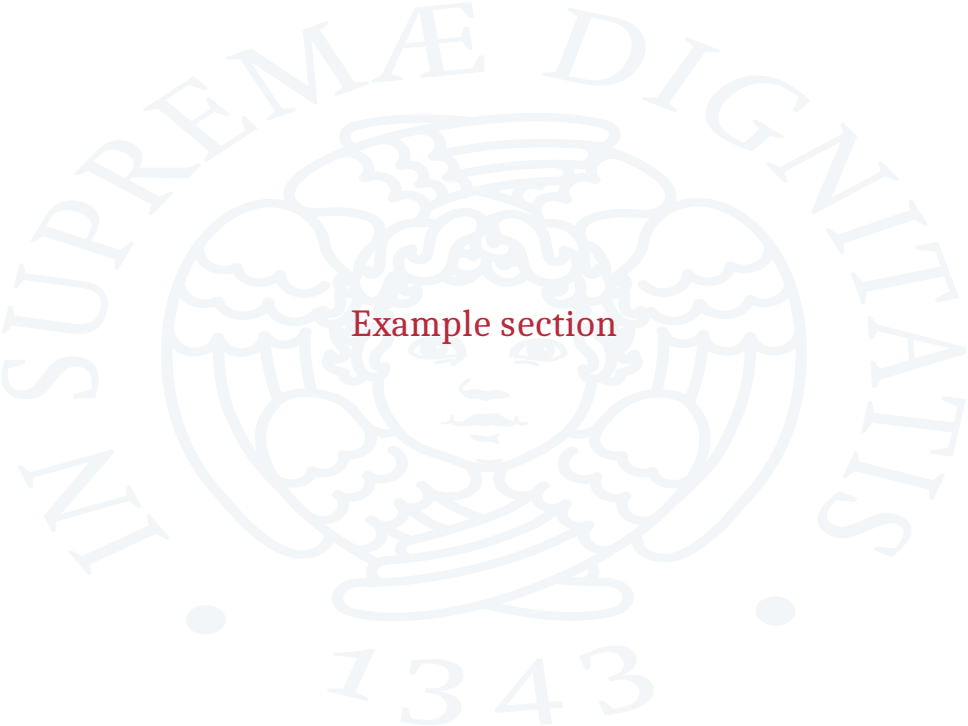


Page and Wootters framework



As firstly described by Page and Wootters [1], and then further formalized by Giovannetti and Maccone [2], we may think at the clock as part of the quantum system, with its own Hamiltonian, but subject to a global constraint in the form of a Wheeler-DeWitt Equation as

$$\hat{C}|\Psi\rangle = 0 \quad \text{with} \quad \hat{C} = \hat{H}_T \otimes \mathbb{1}_S + \mathbb{1}_T \otimes \hat{H}_S \quad (1)$$



Example section

EM showers develop through bremsstrahlung and pair production processes. Main physical parameters:

- ▶ X_0 = radiation length;
- ▶ λ_γ = photon absorption length;
- ▶ $R_M = \frac{E_s}{E_c} X_0$;



Once installed, it can be used typing:

from bash

```
$ simulate-EM-shower -f 10.  
1
```

from Python

```
>>> import em_shower_simulator as em  
>>> em.simulate([10., 1.],  
verbose=0)
```



Equation ?? is more correctly expressed as a stochastic differential equation. Switching to several dimensions, the motion of the particle is described by

$$dq_i = -f_i(q)dt + g_{ij}(q)d\omega_j \quad (2)$$

The SDE above is associated to the Fokker-Planck Equation

$$\partial_t P(q, t|q_0, t_0) = \hat{\mathcal{L}}P(q, t|q_0, t_0) \quad (3)$$

in which $\hat{\mathcal{L}}$ is the differential operator defined as

$$\hat{\mathcal{L}} \equiv \sum_i \partial_{q_i} f_i(q) + \frac{1}{2} \sum_{i,j} \partial_{q_i} \partial_{q_j} [g(q)g^T(q)]_{ij} \quad (4)$$

The general idea is that, once the solution to the equation 3 is known, the statistical properties of the process are completely defined.



Conclusion

What we achieved:

- ▶ something

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What we learned:

- ▶ something else

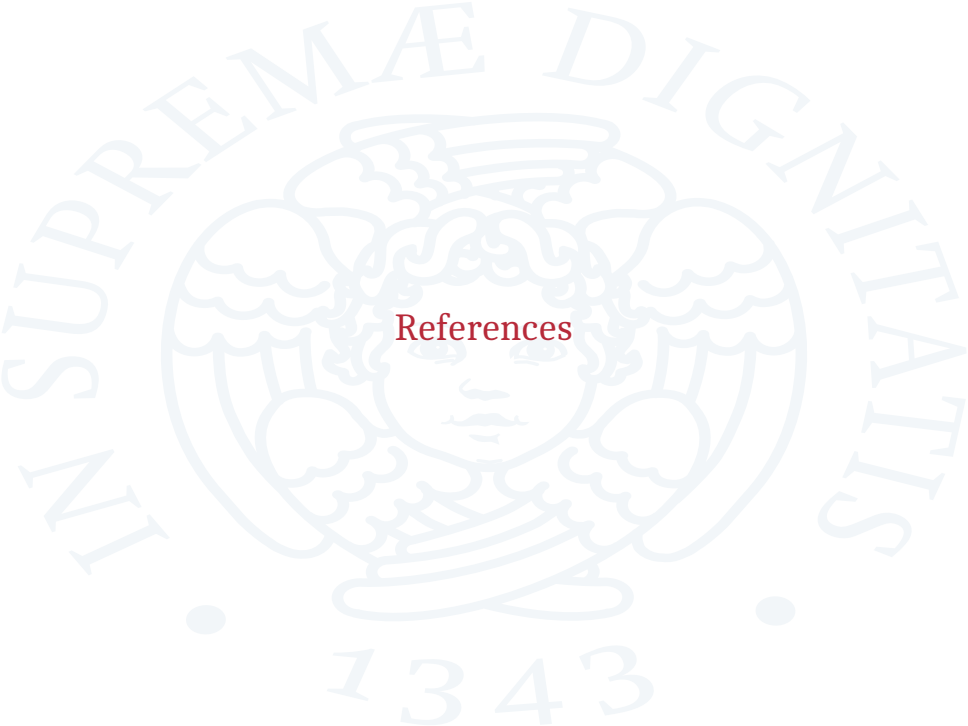
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What we learned:

- ▶ something else

▶ **In the end!**



References

- [1] Don N. Page and William K. Wootters.
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Quantum time.
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- [3] Esteban Castro-Ruiz, Flaminia Giacomini, A. Belenchia, and Časlav Brukner.
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Nature Commun., 11(1):2672, 2020.



Thank you for your attention!