An introduction to numerical optimization

ROOT mathematical environment

L. Bianchini (INFN Sezione di Pisa)



ROOT (C++): an example

```
double Quadratic(const double *xx){
                                       \Rightarrow f_0 + g^T x + \frac{1}{2} x^T B x
TVectorD x(NDIM, xx);
return f0 + q*x + 0.5*(x*(B*x));
ROOT::Math::Minimizer* minimum = ROOT::Math::Factory::CreateMinimizer(minName, algoName);
minimum->SetMaxFunctionCalls(10000);
minimum->SetTolerance(0.001);
ROOT::Math::Functor f( &Quadratic, NDIM);
minimum->SetFunction(f);
double step[NDIM] = {};
for(unsigned int i=0; i<NDIM; ++i) step[i] +=0.01;
double start[NDIM] = \{\};
for(unsigned int i=0; i<NDIM; ++i){
  minimum->SetVariable(i, Form("x%d",i), start[i], step[i]);
minimum->Minimize();
const double *xs = minimum->X();
double *cov[NDIM*NDIM];
minimum->GetCovMat(cov);
```

```
("Minuit2","migrad")
("Minuit2","simplex"),
("GSLMultiMin","conjugatefr")
("GSLMultiMin","conjugatepr")
("GSLMultiMin","bfgs2")
("GSLMultiMin","steepestdescent")
("GSLMultiFit","") // Levemberg-Marquardt NLLQ
("GSLSimAn","")
```

GSL (C)

with gradient

```
gsl_multimin_fdfminimizer_conjugate_fr
                                                 Conjugate-gradients
gsl_multimin_fdfminimizer_conjugate_pr
gsl_multimin_fdfminimizer_vector_bfgs2
                                               Quasi-Newton (BFGS)
gsl multimin fdfminimizer vector bfgs
gsl_multimin_fdfminimizer_steepest_descent
                                               Steepest descent
```

-Nelder-Mead Simplex

scipy.optimize (Python)

estimation. Or, objects implementing **HessianUpdateStrategy** interface can be used to approximate the Hessian. Available quasi-Newton methods implementing this interface are:

- BFGS:
- SR1.

Type of solver. Should be one of

- 'Nelder-Mead' (see here)
- 'Powell' (see here)
- 'CG' (see here)
- 'BFGS' (see here)
- 'Newton-CG' (see here)
- 'L-BFGS-B' (see here)
- 'TNC' (see here)
- 'COBYLA' (see here)
- 'SLSQP' (see here)
- 'trust-constr'(see here)
- 'dogleg' (see here)
- 'trust-ncg' (see here)
- 'trust-exact' (see here)
- 'trust-krylov' (see here)

Gradient-free with linear search

Quasi-Newton with linear search

Approximate Newton with linear search & bound constraints

Constrained fit

Quasi-Newton (dogleg) with trust-region

Quasi-Newton with exact trust-region solution

Approximate Newton with trust-region

scipy.optimize: an example

```
import math
import numpy as np
stddev = 1.0
theta points = np.array([-1.0, 2.0, 0.0, 0.0, 0.0])
n theta = theta points.size
x_{points} = np.array([0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0], dtype=np.float32)
n x = x points.size
npA = np.array([math.pow(x points[i], j)/math.factorial(j) for i in range(n x) for j in range(n theta)]).reshape((n x,n theta))
npV = np.diag( [(stddev*stddev)]*n x )
npVinv = np.linalq.inv(npV)
npB = np.matmul(npA.T, npVinv)
def npfun(x, args):
  npy = args
                                                                     => (y - Ax)^{T} V^{-1} (y - Ax)
  npdelta = npy - np.matmul(npA,x)
  nploss = np.matmul( np.matmul( npdelta.T, npVinv), npdelta )
  return np.sum(nploss)/(n x-n theta)
rnd data = np.random.multivariate normal( np.matmul(npA, theta points), npV )
res = minimize(fun=npfun, x0=theta points, args=ynp, method='BFGS')
print('Loss- BFSG :', res.fun )
```

tf.keras.optimizers (Python)

class Adadelta: Optimizer that implements the Adadelta algorithm.

class Adagrad: Optimizer that implements the Adagrad algorithm.

class Adam: Optimizer that implements the Adam algorithm.

class Adamax: Optimizer that implements the Adamax algorithm.

class Ftrl: Optimizer that implements the FTRL algorithm.

class Nadam: Optimizer that implements the NAdam algorithm.

class Optimizer : Updated base class for optimizers.

class RMSprop: Optimizer that implements the RMSprop algorithm.

class SGD: Stochastic gradient descent and momentum optimizer.

Stochastic Gradient Descent with momentum

tf.keras.optimizers (Python)

```
Algorithm 1: Adam, our proposed algorithm for stochastic optimization. See section 2 for details,
class Adadelta:
                              and for a slightly more efficient (but less clear) order of computation. g_t^2 indicates the elementwise
                              square g_t \odot g_t. Good default settings for the tested machine learning problems are \alpha = 0.001,
class Adagrad: (\beta_1 = 0.9, \beta_2 = 0.999 and \epsilon = 10^{-8}. All operations on vectors are element-wise. With \beta_1^t and \beta_2^t
                              we denote \beta_1 and \beta_2 to the power t.
                              Require: \alpha: Stepsize
class Adam: Optin
                              Require: \beta_1, \beta_2 \in [0, 1): Exponential decay rates for the moment estimates
                              Require: f(\theta): Stochastic objective function with parameters \theta
class Adamax : Of Require: \theta_0: Initial parameter vector
                                 m_0 \leftarrow 0 (Initialize 1<sup>st</sup> moment vector)
                                 v_0 \leftarrow 0 (Initialize 2<sup>nd</sup> moment vector)
class Ftrl: Optin
                                 t \leftarrow 0 (Initialize timestep)
                                 while \theta_t not converged do
class Nadam: Opt
                                   t \leftarrow t + 1
                                    g_t \leftarrow \nabla_{\theta} f_t(\theta_{t-1}) (Get gradients w.r.t. stochastic objective at timestep t)
                                   m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t (Update biased first moment estimate) v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2 (Update biased second raw moment estimate)
class Optimizer
                                    \widehat{m}_t \leftarrow m_t/(1-\beta_1^t) (Compute bias-corrected first moment estimate)
class RMSprop: (
                                   \hat{v}_t \leftarrow v_t/(1-\beta_2^t) (Compute bias-corrected second raw moment estimate)
                                    \theta_t \leftarrow \theta_{t-1} - \alpha \cdot \widehat{m}_t / (\sqrt{\widehat{v}_t} + \epsilon) (Update parameters)
class SGD: Stoch
                                 end while
                                 return \theta_t (Resulting parameters)
```

Minuit + Tensorflow (PyROOT)

```
Ref: <u>TensorFlowAnalysis</u>
```

```
def MinuitFit(sess, model, data_sample, integ_sample, call_limit = 50000) :
  tfpars = tf.trainable_variables() # Create TF variables
  float tfpars = [ p for p in tfpars if p.floating() ]
  nll = UnbinnedLogLikelihood(model, data sample, integ sample)
  gradient = tf.gradients(nll, float tfpars)
  def fcn(npar, gin, f, par, istatus) :
   for i,p in enumerate(float_tfpars) : p.update(sess, par[i])
   f[0] = sess.run(nll)
                           # Calculate log likelihood
                        # If gradient calculation is needed
   if istatus == 2 :
     dnll = sess.run(gradient) # Calculate analytic gradient _
     for i in range(len(float tfpars)) : qin[i] = dnll[i] # Pass gradient to MINUIT
    fcn.n += 1
   if fcn.n % 10 == 0 : print(fcn.n, istatus, f[0], sess.run(tfpars))
  fcn.n = 0
  minuit = TVirtualFitter.Fitter(0, len(float tfpars))
  minuit.Clear()
  minuit.SetFCN(fcn)
  arglist = array.array('d', 10*[0])
  for n,p in enumerate(float_tfpars) :
   minuit.SetParameter(n, p.par name, p.init value, p.step size, p.lower limit, p.upper limit)
  minuit.ExecuteCommand("SET GRA", arglist, 0)
  arglist[0] = call_limit
  minuit.ExecuteCommand("MIGRAD", arglist, 1)
```

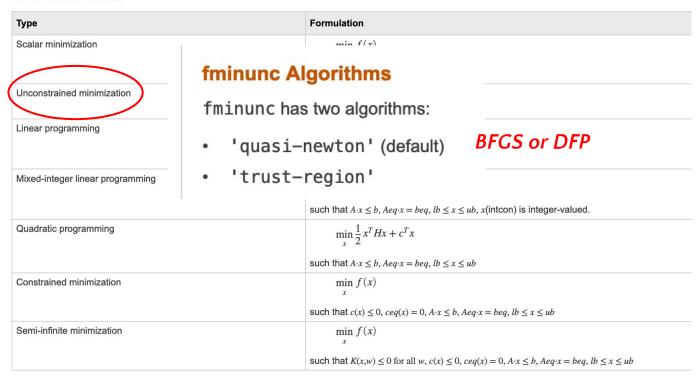
Objective function is a Tensorflow op => benefit from Tensorflow speed-ups

Exploit native *automatic differentiation* of tensorflow

Implementation of objective function is transparent to Minuit

Matlab

Minimization Problems





Mathematica

NMinimize

```
NMinimize [f, x] minimizes f numerically with respect to x.

NMinimize [f, \{x, y, ...\}] minimizes f numerically with respect to x, y, ...

NMinimize [f, cons\}, \{x, y, ...\}] minimizes f numerically subject to the constraints cons.

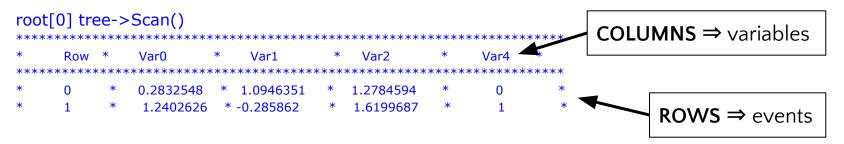
NMinimize [..., x \in reg] constrains x to be in the region reg.
```



```
Optimization`NMinimizeDump`$Methods
```

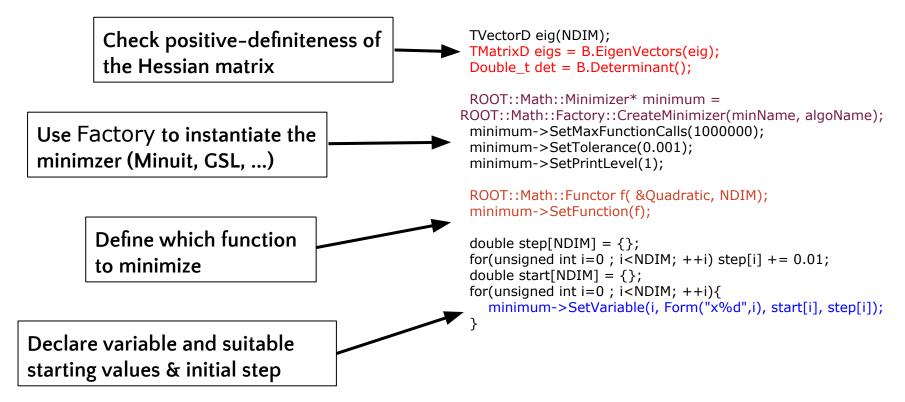
Interesting features (from a HEP perspective)

- Efficient storage and read-out of multidimensional, arbitrary-type data (⇒ TTree)
 - O Can store (arrays of, pointers to) basic types, C structures, C++ classes



- Mathematical environment (⇒ ROOT::Math::)
- Linear algebra (⇒ TLorentzVector, TMatrixD, TDecomp*, SMatrix)
- N-dimensional minimization and integration (⇒ Minuit, GSL)
- Monte Carlo studies (⇒ TRandom)
- MVA's for regression/classification (⇒ TMVA)
- Unfolding (⇒ TUnfold)

```
#include "Math/Minimizer.h"
                                                               for(unsigned int i=0; i<NDIM; ++i){
#include "Math/Factory.h"
                                                                double r = ran.Rndm();
#include "Math/Functor.h"
                                                                q[i] = qmin*r + qmax*(1-r);
#include "TRandom3.h"
#include "Math/IntegratorMultiDim.h"
const int NDIM = 4;
                                                         Use an ill-conditioned Hessian matrix
                                                         to make the task more challenging
TMatrixD B = THilbertMatrixD(NDIM,NDIM);
double qrad[NDIM] = \{\};
TVectorD g(NDIM, grad);
const double f0 = 1.0:
                                                          The objective function we wish to
double Quadratic(const double *xx)
                                                           minimize.
 TVectorD x(NDIM, xx);
                                                           XX[i] is i-th variable
return f0 + q*x + 0.5*(x*(B*x));
void minimizer(const char * minName = "Minuit", const
char *algoName = "Migrad"){
                                                Create non-zero gradient to
 TRandom3 ran(1234);
 double amin = -1.:
                                                make the solution non-trivial
 double amax = +1.:
```



```
minimum->Minimize();
                                              Trigger the minimizer
const double *xs = minimum->X();
std::cout << std::endl;</pre>
std::cout << "NUMERICAL: f(";
                                                                     Retrieve the information
for(unsigned int i=0; i<NDIM; ++i) std::cout << xs[i] << ",";
std::cout << "): " << minimum->MinValue() << std::endl;</pre>
                                                                     about the minimum
TVectorD x0(NDIM, start);
TMatrixD Binv(B);
TVectorD xs ana = x0 - Binv.InvertFast()*(B*x0 + g);
                                                                          Alternatively, find the
std::cout << "ANALYTICAL: f(";
                                                                          solution by one Newton step
for(unsigned int i=0; i<NDIM; ++i) std::cout << xs ana[i] << ",";
std::cout << "): " << Quadratic(xs ana.GetMatrixArray()) << std::endl;</pre>
                                               Evaluate the objective
                                               function at the analytical
                                               solution
```

```
double xL[NDIM] = \{\};
double xU[NDIM] = {};
                                                      Integration range
for(unsigned int i=0; i<NDIM; ++i) xL[i] = -1.0;
for(unsigned int i=0; i<NDIM; ++i) xU[i] = +1.0;
double val = 0.:
ROOT::Math::IntegratorMultiDim ig1(ROOT::Math::IntegrationMultiDim::kVEGAS);
ig1.SetFunction(f);
val = ig1.Integral(xL,xU);
return;
                                                                            Several MC algorithms
                                                                            available from GSL library
                                                                            (kVEGAS, kADAPTIVE,
           Integrate the function
                                                                            kPLAIN, ...)
            in the box [-1,1]
```

Setup the environment

All the material can be also found here: https://github.com/bianchini/SNS_DAS

- 1. Open a terminal
- 2. Create a working directory
 - > mkdir ROOT Tutorial
 - > cd ROOT_Tutorial/
- 3. Download the material
 - > wget https://github.com/bianchini/SNS DAS/archive/master.zip
 - > unzip master.zip; cd SNS_DAS-master/
 - > sudo apt-get install libgsl23 libgsl-dev

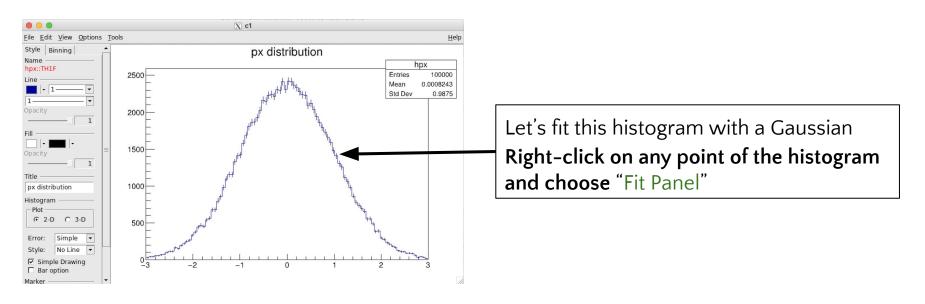
Basic ROOT commands

- Open ROOT without displaying initial logo
 - > root -l
- Open ROOT in batch mode (no graphic windows)
 - > root -b
- List content of the ROOT current directory root[0] .ls
- Exit ROOT root[0] .q
- The ROOT prompt provides auto-completion (TAB), forward-backward search (\uparrow , \downarrow) and regex search in the history (CTRL-R)

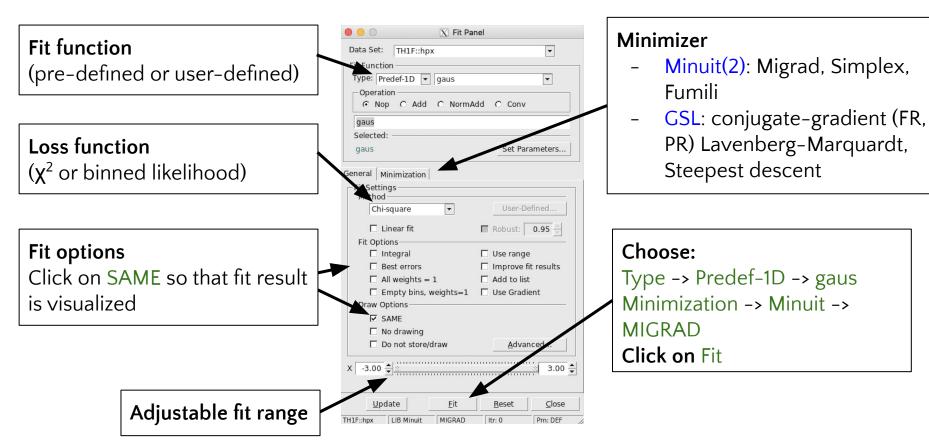
17

Fit Panel

- Let's run the macro tree.C
 - > root -l -e 'gROOT->ProcessLine(".L tree.C"); gROOT->ProcessLine("read_tree()")'
- A default TCanvas is created and displayed
 - On the top, select "View" -> "Editor", then click on any point of the histogram



A simple 1D fit to a binned data set

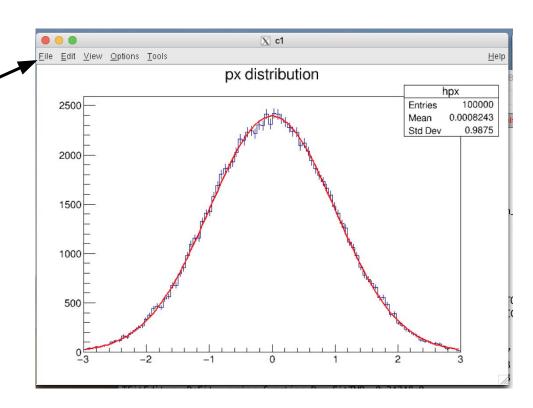


Saving graphic data into a .C file

You can customize the plot as you wish. Then: "File" -> "Save As..." to save it in graphic form.

Select the ".C" format, and save it as example.C; Now do:

root[1] .q
> root -l example.C



RooFit: a ROOT package for statistical analysis

ROOT has some limitations when dealing with complicated fit

- Standard ROOT function framework insufficient to handle complicated functions
- P.d.f's are normalized densities, not just functions
- Computation performances important when NDIM>>1, unbinned data, many events, ...

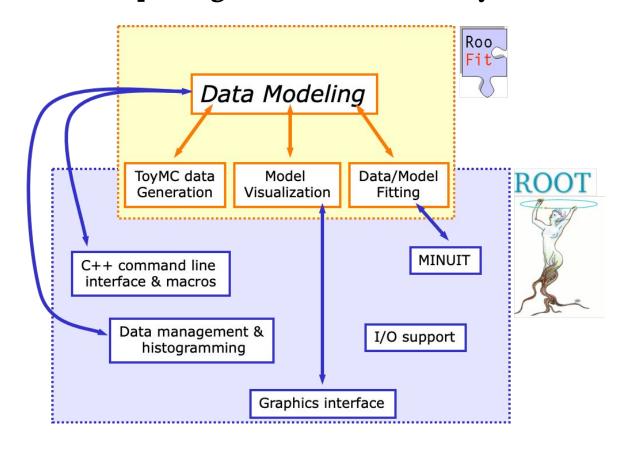
You can write a specific fit using ROOT interface to Minuit (⇒ minimizer.C)

can requires a lot of coding and ad hoc optimization

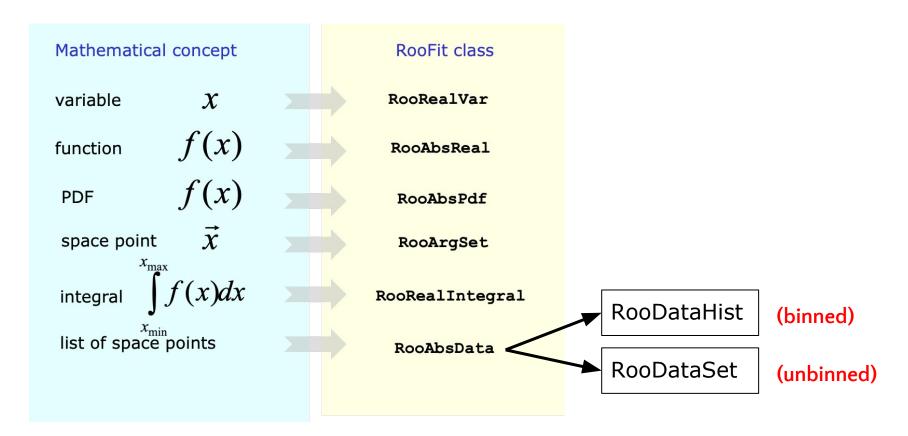
⇒ RooFit

- Interface to Minuit transparent to the user
- Many pre-compiled models
- Takes care of p.d.f. normalization, variable marginalization, conditioning, simultaneous fit.
 Using optimizations whenever possible
- o Profile-likelihood scans, likelihood contours, GoF, MC studies, ...

RooFit: a ROOT package for statistical analysis



RooFit: a ROOT package for statistical analysis



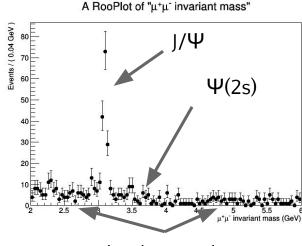
Cross section measurement

- J/Ψ: a particle with mass of 3.096 GeV. It can be produced in pp collisions
 - J/Ψ -> μμ is a neat narrow peak on top of a smooth background
 - A second excited state (⇒ Ψ(2s)) exists at the slightly larger mass of 3.686 GeV
- The experiment produces a certain number of binary collisions ($\Rightarrow \bot$), and has an efficiency ($\Rightarrow \varepsilon$) of collecting $\Psi(2s) \rightarrow \mu\mu$ events.

We wish to determine:

$$\sigma = N/(L \epsilon)$$
, N = number of measured $\Psi(2s)$

 The data is a RooDataSet (⇒ DataSet_lowstat.root) containing the μμ mass of 500 events.



background

```
import ROOT
```

Gaussian", 1.5, 0.5, 5.)

```
fInput = ROOT.TFile("DataSet_lowstat.root")
dataset = fInput.Get("data")

mass = ROOT.RooRealVar("mass", "mumu mass", 2.0, 6.0, "GeV")

meanJpsi = ROOT.RooRealVar("meanJpsi", "The mean of the Jpsi
Gaussian", 3.1, 2.8, 3.2)
sigmaJpsi = ROOT.RooRealVar("sigmaJpsi", "The width of the Jpsi
Gaussian", 0.3, 0.0001, 1.)
alphaJpsi = ROOT.RooRealVar("alphaJpsi", "The alpha of the Jpsi
Gaussian", 1.5, -5., 5.)
nJpsi = ROOT.RooRealVar("nJpsi", "The alpha of the Jpsi
```

```
CBJpsi = ROOT.RooCBShape("CBJpsi","The Jpsi Crystall Ball",mass,meanJpsi,sigmaJpsi,alphaJpsi,nJpsi)
```

```
meanpsi2S = ROOT.RooRealVar("meanpsi2S","The mean of the psi(2S) Gaussian",3.7,3.65,3.75)
gausspsi2S = ROOT.RooGaussian("gausspsi2S","The psi(2S)
Gaussian",mass,meanpsi2S,sigmaJpsi)

a1 = ROOT.RooRealVar("a1","The a1 of background",-0.7,-2.,2.)
a2 = ROOT.RooRealVar("a2","The a2 of background",0.3,-2.,2.)
a3 = ROOT.RooRealVar("a3","The a3 of background",-0.03,-2.,2.)
backgroundPDF = ROOT.RooChebvchev("backgroundPDF","The
```

background PDF", mass, ROOT. RooArgList(a1,a2,a3))

Get the data (same I/O than ROOT!)

import ROOT

Gaussian", 1.5, 0.5, 5.)

fInput = ROOT.TFile("DataSet_lowstat.root")
dataset = fInput.Get("data")

mass = ROOT.RooRealVar("mass", "mumu mass", 2.0, 6.0, "GeV")

meanJpsi = ROOT.RooRealVar("meanJpsi","The mean of the Jpsi Gaussian",3.1,2.8,3.2) sigmaJpsi = ROOT.RooRealVar("sigmaJpsi" "The width of the Jpsi Gaussian",0.3,0.0001,1.) alphaJpsi = ROOT.RooRealVar("alphaJpsi","The alpha of the Jpsi Gaussian",1.5,-5.,5.) nJpsi = ROOT.RooRealVar("nJpsi","The alpha of the Jpsi

CBJpsi = ROOT.RooCBShape("CBJpsi","The Jpsi Crystall Ball",mass,meanJpsi,sigmaJpsi,alphaJpsi,nJpsi)

meanpsi2S = ROOT.RooRealVar("meanpsi2S","The mean of the psi(2S) Gaussian",3.7,3.65,3.75)
gausspsi2S = ROOT.RooGaussian("gausspsi2S","The psi(2S)
Gaussian",mass,meanpsi2S,sigmaJpsi)

a1 = ROOT.RooRealVar("a1","The a1 of background",-0.7,-2.,2.)
a2 = ROOT.RooRealVar("a2","The a2 of background",0.3,-2.,2.)
a3 = ROOT.RooRealVar("a3","The a3 of

backgroundPDF = ROOT.RooChebychev("backgroundPDF","The

background PDF", mass, ROOT. RooArgList(a1,a2,a3))

The name of the observable saved into

data is mass

(⇒ data imported by name)

background",-0.03,-2.,2.)

import ROOT

fInput = ROOT.TFile("DataSet_lowstat.root")
dataset = fInput.Get("data")

mass = ROOT.RooRealVar("mass", "mumu mass", 2.0, 6.0, "GeV")

meanJpsi = ROOT.RooRealVar("meanJpsi","The mean of the Jpsi Gaussian",3.1,2.8,3.2)

sigmaJpsi = ROOT.RooRealVar("sigmaJpsi","The width of the Jpsi Gaussian",0.3,0.0001,1.)

alphaJpsi = ROOT.RooRealVar("alphaJpsi","The alpha of the Jpsi Gaussian",1.5,-5.,5.)

nJpsi = ROOT.RooRealVar("nJpsi","The alpha of the Jpsi Gaussian",1.5,0.5,5.)

CBJpsi = ROOT.RooCBShape("CBJpsi","The Jpsi Crystall Ball",mass,meanJpsi,sigmaJpsi,alphaJpsi,nJpsi)

An empirical p.d.f. for the J/ Ψ (\Rightarrow Crystal Ball function)

A simpler (\Rightarrow Gaussian) function for $\Psi(2s)$

meanpsi2S = ROOT.RooPealVar("meanpsi2S","The mean of the psi(2S) Gaussian", 3.7, 3.65, 3.75)
gausspsi2S = ROOT.RooGaussian("gausspsi2S", "The psi(2S)
Gaussian", mass, meanpsi2S, sigmaJpsi)

a1 = ROOT.RooRealVar("a1","The a1 of background",-0.7,-2.,2.)
a2 = ROOT.RooRealVar("a2","The a2 of background",0.3,-2.,2.)
a3 = ROOT.RooRealVar("a3","The a3 of background",-0.03,-2.,2.)
backgroundPDF = ROOT.RooChebychev("backgroundPDF","The background PDF",mass,ROOT.RooArgList(a1,a2,a3))

An empirical (⇒ polynomial) function for the background

```
NJpsi = ROOT.RooRealVar("NJpsi", "The Jpsi signal
events",1500.,0.1,10000.)
Nbkg = ROOT.RooRealVar("Nbkg", "The bkg
events",5000.,0.1,50000.)
eff_psi = ROOT.RooRealVar("eff_psi","The psi
efficiency", 0.75, 0.00001, 1.)
lumi psi = ROOT.RooRealVar("lumi_psi","The CMS
luminosity", 0.64, 0.00001, 50., "pb-1")
cross_psi = ROOT.RooRealVar("cross_psi","The psi
xsec",3.,0.,40.,"pb")
Npsi =
ROOT.RooFormulaVar("Npsi","@0*@1*@2",ROOT.RooArgList(eff
psi,lumi psi,cross psi))
eff psi.setConstant(1)
lumi psi.setConstant(1)
totPDF = ROOT.RooAddPdf("totPDF", "The total
PDF",ROOT.RooArgList(CBJpsi,gausspsi2S,backgroundPDF),ROOT
.RooArgList(NJpsi,Npsi,Nbkg))
totPDF.fitTo(dataset, ROOT.RooFit.Extended(1))
```

```
xframe = mass.frame()
dataset.plotOn(xframe)
totPDF.plotOn(xframe)
totPDF.plotOn(xframe,
ROOT.RooFit.Components("backgroundPDF"),
ROOT.RooFit.LineStyle(ROOT.kDashed),
ROOT.RooFit.LineColor(ROOT.kRed))
c1 = ROOT.TCanvas()
xframe.Draw()
c1.SaveAs("exercise 0.png")
fOutput =
ROOT.TFile("Workspace mumufit.root","RECREATE")
fInput.cd()
ws = ROOT.RooWorkspace("ws")
getattr(ws,'import')(totPDF)
getattr(ws,'import')(dataset)
ws.writeToFile("Workspace mumufit.root")
del ws
fOutput.Write()
fOutput.Close()
```

28

Total yields (for extended ML fit)

```
NJpsi < ROOT.RookealVar("NJpsi", "The Jpsi signal
                                                                   xframe = mass.frame()
events",1500.,0.1,10000.)
                                                                   dataset.plotOn(xframe)
Nbkg = ROOT.RooRealVar("Nbkg", "The bb
                                                                   totPDF.plotOn(xframe)
events",5000.,0.1,50000.)
                                                                   totPDF.plotOn(xframe,
                                                                   ROOT.RooFit.Components("backgroundPDF"),
eff_psi = ROOT.RooRealVar("eff_psi","The psi
                                                                   ROOT.RooFit.LineStyle(ROOT.kDashed),
efficiency", 0.75, 0.00001
                                                                   ROOT.RooFit.LineColor(ROOT.kRed))
lumi_psi = ROOT.RooRealVar("lumi_psi","The CMS
luminosity", 0.64, 0.0001, 50., "pb-1")
                                                                   c1 = ROOT.TCanvas()
cross_psi = POOT.RooRealVar("cross_psi","The psi
                                                                   xframe.Draw()
xsec",3.0.,40.,"pb")
                                                                   c1.SaveAs("exercise 0.png")
Npsi =
ROOT.RooFormulaVar("Npsi","@0*@1*@2",ROOT.RooArgList(eff
                                                                   fOutput =
psi,lumi psi,cross psi))
                                                                   ROOT.TFile("Workspace mumufit.root", "RECREATE")
eff psi.setConstant(1)
                                                                   fInput.cd()
                                     Total p.d.f. is S+B
lumi psi.setConstant(1)
                                                                   ws = ROOT.RooWorkspace("ws")
                                                                   getattr(ws,'import')(totPDF)
totPDF = ROOT.RooAddPdf("totPDF","The total
                                                                   getattr(ws,'import')(dataset)
PDF",ROOT.RooArgList(CBJpsi,gausspsi2S,backgroundPDF),ROOT
                                                                   ws.writeToFile("Workspace mumufit.root")
.RooArgList(NJpsi,Npsi,Nbkg))
                                                                   del ws
totPDF.fitTo(dataset, ROOT.RooFit.Extended(1))
                                                                   fOutput.Write()
                                                                   fOutput.Close()
                                The fit is done here!
```

16 Dic. 2019, Pisa

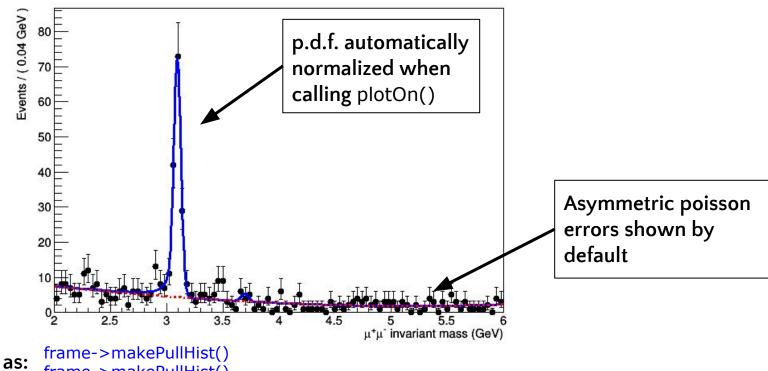
```
NJpsi = ROOT.RooRealVar("NJpsi", "The Jpsi signal
events",1500.,0.1,10000.)
Nbkg = ROOT.RooRealVar("Nbkg", "The bkg
events",5000.,0.1,50000.)
eff_psi = ROOT.RooRealVar("eff_psi","The psi
efficiency", 0.75, 0.00001, 1.)
lumi_psi = ROOT.RooRealVar("lumi_psi","The CMS
luminosity", 0.64, 0.00001, 50., "pb-1")
cross_psi = ROOT.RooRealVar("cross_psi","The psi
xsec",3.,0.,40.,"pb")
Npsi =
ROOT.RooFormulaVar("Npsi","@0*@1*@2",ROOT.RooArgList(eff
psi,lumi psi,cross psi))
eff psi.setConstant(1)
lumi psi.setConstant(1)
totPDF = ROOT.RooAddPdf("totPDF", "The total
PDF",ROOT.RooArgList(CBJpsi,gausspsi2S,backgroundPDF),ROOT
.RooArgList(NJpsi,Npsi,Nbkg))
totPDF.fitTo(dataset, ROOT.RooFit.Extended(1))
 A fix for bad memory handling...
```

```
Draw the data & fit result
xframe = mass.frame()
dataset.plotOn(xframe)
totPDF.plotOn(xframe)
totPDF.plotOn(xframe,
ROOT.RooFit.Components("backgroundPDF"),
ROOT.RooFit.LineStyle(ROOT.kDashed),
ROOT.RooFit.LineColor(ROOT.kRed))
c1 = ROOT.TCanvas()
xframe.Draw()
c1.SaveAs("exercise 0.png")
fOutput =
ROOT.TFile("Workspace mumufit.root", "RECREATE")
fInput.cd()
ws = ROOT.RooWorkspace("ws")
getattr(ws,'import')(totPDF)
getattr(ws,'import')(dataset)
ws.writeToFile("Workspace mumufit.root"
del ws
                Save a comprehensive summary
fOutput.Write()
```

into a RooWorkspace

fOutput.Close()

A RooPlot of "\u03c4+\u03c4-\u00e4 invariant mass"



Pulls/residuals readily obtained as:

frame->makePullHist()

So, what is the measured cross section of $\Psi(2s)$ -> $\mu\mu$?

```
> root Workspace_mumufit.root
root[1] w = (RooWorkspace*) gDirectory->Get("ws")
root[2] w->Print()
root[3] w->var("cross_psi")->Print()
```

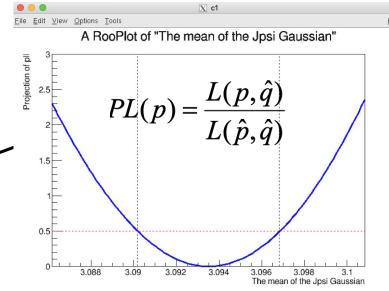
```
RooRealVar::cross_psi = 9.65042 +/- 7.94308 L(0 - 40) // [pb]
```

Fitting with RooFit (⇒ profile.C)

Suppose we want to look at the profile log-likelihood for one POI (say, J/Ψ mass)

- an <u>intensive task</u> (one minimization per value of the POI)
- RooFit has a built-in method for doing it

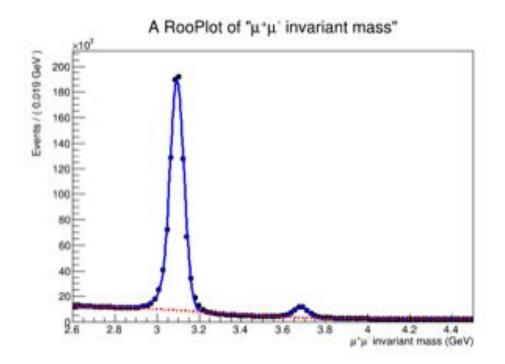
```
{
    TFile* f = TFile::Open("Workspace_mumufit.root");
    RooWorkspace* w = (RooWorkspace*)f->Get("ws");
    w->Print();
    double xL = w->var("meanJpsi")->getVal() - 2.2*
w->var("meanJpsi")->getError();
    double xU = w->var("meanJpsi")->getVal() + 2.2*
w->var("meanJpsi")->getError();
    w->var("meanJpsi")->setRange(xL, xU);
    RooNLLVar nll("nll","nll",*(w->pdf("totPDF")),*(w->data("data")));
    RooProfileLL pll("pll","pll", nll,*w->var("meanJpsi"));
    RooPlot* frame = w->var("meanJpsi")->frame(xL, xU);
    pll.plotOn(frame);
    frame->Draw();
}
```



Hands-on

- Download a large sample of CMS open data (L=11.6/fb)
 - Pre-processed for you into a flat TTree with one branch only
 wget -O DataSet highstat.root cern.ch/arizzi/out.root
- Modify roofit.py to run on the new input
 - The input file contains a TTree, not a RooDataSet. *Hint:* RooDataSet reference guide
 - Remember that data is imported by name. Hint: Events->Print()
 - Is it necessary to perform an unbinned fit? *Hint: RooDataHist reference guide*
 - Are the RooRealVar ranges feasible? *Hint: what is the total number of events?*
 - Is the fitting range adequate? *Hint: take a quick look at the data before making a fit*
 - o Can you improve the background/signal modeling? *Hint: add more d.o.f.*

Hands-on



Backup

ROOT is...

A scientific software toolkit for data analysis

- User code (⇒ macros) is standard C++ using ROOT classes (⇒ T***)
 - TFile ⇒ basic I/O
 - TH1F ⇒ 1D-histogram with floating-point precision
 - TF1 ⇒ a generic real-valued 1D-function
- Class methods start with capital letters (⇒ histo.Draw();)
- The code can be either interpreted or compiled
- Any ROOT class can be used through automatic Python binding (⇒ PyROOT)

```
import ROOT
f = ROOT.TFile.Open(...)
h = ROOT.TH1F()
```

Project developed & maintained by CERN

- Online manual, tutorials, forum: https://root.cern.ch/
- Open-source: can either compile the source code, or use binaries



I/O & operations in ROOT

Input/output

- Data stored in binary files (\Rightarrow .root). Many compressions possible (ZLIB, LZMA, LZ4, ZSTD)
- Can save graphic objects into most used formats (.eps, .png, .pdf, .jpeg, .C)

Operations

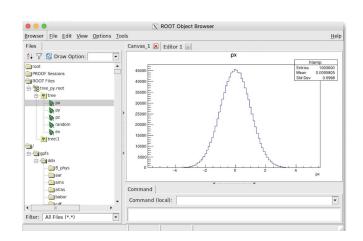
Commands issued into the ROOT prompt:

```
> root
root [0] TFile* f = TFile::Open("my_file.root")
```

○ Code interpreted on-the-fly (⇒ Cling):

```
> root
root[0] .L my_macro.C
root[1] my_function()
root[2] .x snippet.C
```

Code compiled, linked, and executed (⇒ <u>ACLiC</u>)
 root
 root[0] .L my_macro.C++
 root[1] my_function()

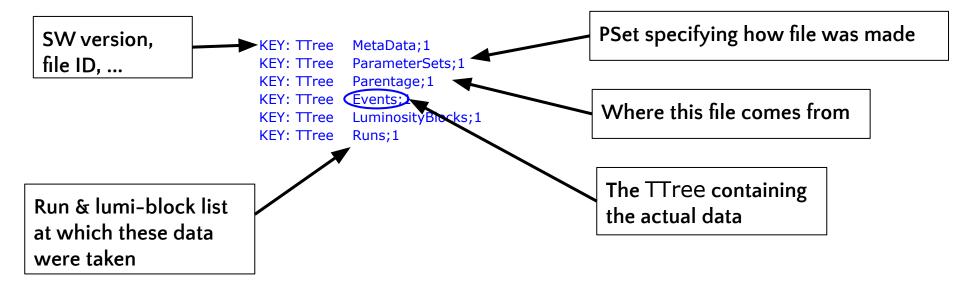


ROOT comes with its own **GUI** for inspecting files, visualize, and customize plots

A concrete example: ROOT-based data for CMS

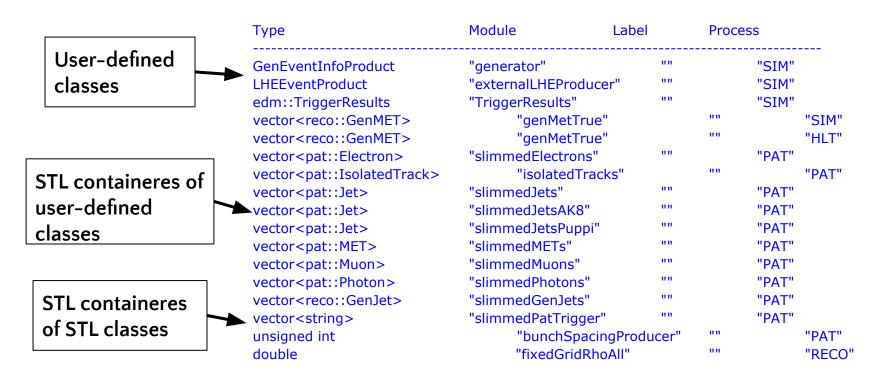
Extract of a typical file content for simulated data (-50 kB/event)

34705FB5-CE8C-E911-B0E4-00144F45BD0E.root (1 runs, 14 lumis, 13258 events, 565781558 bytes)



A concrete example: ROOT-based data for CMS

Extract of a typical Event content for simulated data



```
#include "TFile.h"
#include "TTree.h"
#include "TH2.h"
#include "TRandom.h"
void make tree(Int t nevents=100000)
 TFile* file = new TFile("tree_C.root", "RECREATE", "");
 TTree* tree = new TTree("tree","A simple Tree");
 Float_t px, py, pz;
 Double t random;
 Int tev;
 tree->Branch("px",&px,"px/F");
 tree->Branch("py",&py,"py/F");
 tree->Branch("pz",&pz,"pz/F");
 tree->Branch("random",&random,"random/D");
 tree->Branch("ev",&ev,"ev/I");
```

```
for (Int_t i = 0 ; i<nevents ; i++) {
   gRandom->Rannor(px,py);
   pz = px*px + py*py;
   random = gRandom->Rndm();
   ev = i;
   tree->Fill();
}
tree->Write();
```

```
#include "TFile.h"
                                          No need to set full path. ROOT has its own collection of
#include "TTree.h"
                                          predefined include paths (\Rightarrow root[0] .include)
#include "TH2.h"
#include "TRandom.h"
void make tree(Int t nevents=100000)
                                                               for (Int_t i = 0; i<nevents; i++) {
                                                                gRandom->Rannor(px,py);
                                                                 pz = px*px + py*py;
 TFile* file = new TFile("tree_C.root", "RECREATE", "");
                                                                random = gRandom->Rndm();
 TTree* tree = new TTree("tree","A simple Tree");
                                                                ev = i:
                                                                tree->Fill();
 Float_t px, py, pz;
 Double t random;
 Int tev;
                                                               tree->Write();
 tree->Branch("px",&px,"px/F");
 tree->Branch("py",&py,"py/F");
 tree->Branch("pz",&pz,"pz/F");
 tree->Branch("random",&random,"random/D");
 tree->Branch("ev",&ev,"ev/I");
```

```
#include "TFile.h"
#include "TTree.h"
#include "TH2.h"
#include "TRandom.h"
void make tree(Int t nevents=100000)
 TFile* file = new TFile("tree_C.root", "RECREATE", "");
 TTree* tree = new TTree("tree","A simple Tree");
 Float_t px, py, pz;
 Double t random;
 Int_t ev;
 tree->Branch("px",&px,"px/F");
 tree->Branch("py",&py,"py/F");
 tree->Branch("pz",&pz,"pz/F");
 tree->Branch("random",&random,"random/D");
 tree->Branch("ev",&ev,"ev/I");
```

A classical C++ function. No need to write main()

N.B. ROOT defines new types for int, float, ...

```
E.g.: int \rightarrow Int_t
```

```
for (Int_t i = 0 ; i<nevents ; i++) {
   gRandom->Rannor(px,py);
   pz = px*px + py*py;
   random = gRandom->Rndm();
   ev = i;
   tree->Fill();
}
tree->Write();
```



No need to call explicit destructors here.

- ⇒ object will be destroyed once out-of-scope.
- ⇒ it allows to work interactively with the TTree within the ROOT prompt

```
#include "TFile.h"
#include "TTree.h"
#include "TH2.h"
#include "TRandom.h"
void make tree(Int t nevents=100000)
 TFile* file = new TFile("tree_C.root", "RECREATE");
 TTree* tree = new TTree("tree","A simple Tree");
 Float_t px, py, pz;
 Double t random;
 Int tev;
 tree->Branch("px",&px,"px/F");
 tree->Branch("py",&py,"py/F");
 tree->Branch("pz",&pz,"pz/F");
 tree->Branch("random",&random,"random/D");
 tree->Branch("ev",&ev,"ev/I");
```

ROOT file created from scratch (⇒ "RECREATE")

A new Tree object is instantiated

```
for (Int_t i = 0 ; i<nevents ; i++) {
   gRandom->Rannor(px,py);
   pz = px*px + py*py;
   random = gRandom->Rndm();
   ev = i;
   tree->Fill();
}
tree->Write();
```

Objects that we want to be persistently save (e.g. tree) will be written into file

```
#include "TFile.h"
 #include "TTree.h"
 #include "TH2.h"
 #include "TRandom.h"
 void make tree(Int t nevents=100000)
                                                                for (Int t i = 0; i<nevents; i++) {
                                                                 gRandom->Rannor(px,py);
                                                                 pz = px*px + py*py;
  TFile* file = new TFile("tree_C.root", "RECREATE", "");
                                                                 random = gRandom->Rndm();
  TTree* tree = new TTree("tree","A simple Tree");
                                                                 ev = i;
                                                                 tree->Fill();
  Float_t px, py, pz;
                             Auxiliary variables
  Double trandom;
                             to store the output
  Int tev;
                                                                tree->Write();
  tree->Branch("px",&px,"px/F");
  tree->Branch("py",&py,"py/F");
  tree->Branch("pz",&pz,"pz/F");
  tree->Branch("random",&random,"random/D");
  tree->Branch("ev",&ev,"ev/I");
                                                         Leaf type gets specified
                                                         here (e.g. "/F", "/I", "/D")
Define the TBranches
for this TTree
```

```
#include "TFile.h"
#include "TTree.h"
#include "TH2.h"
#include "TRandom.h"
void make tree(Int t nevents=100000)
 TFile* file = new TFile("tree_C.root", "RECREATE", "");
 TTree* tree = new TTree("tree","A simple Tree");
 Float_t px, py, pz;
 Double t random;
 Int tev;
 tree->Branch("px",&px,"px/F");
 tree->Branch("py",&py,"py/F");
 tree->Branch("pz",&pz,"pz/F");
 tree->Branch("random",&random,"random/D");
 tree->Branch("ev",&ev,"ev/I");
```

```
for (Int_t i = 0; i < nevents; i++) {
    gRandom->Rannor(px,py);
    pz = px*px + py*py;
    random = gRandom->Rndm();
    ev = i;
    tree->Fill();
}
tree->Write();
```

Start the "event" loop.

At each iterate i, the Fill() command fills one row of the TTree using current data found at the branch address

Inspecting a ROOT file (⇒ tree_C.root)

• Let's run the macro tree.C

```
> root -l
root[0] .L tree.C
root[1] make_tree(100000)
root[2] .q
Equivalent to:
```

```
Browser File Edit View Options Tools

Files

Canvas,1 So Editor 1 So

Prof. Sessions

Soft Prof. Sessions

Soft Prof. Sof
```

```
> root -l -b -e 'gROOT->ProcessLine(".L tree.C"); gROOT->ProcessLine("make_tree(100000)")'
```

We could also have compiled the code (⇒ tree_C.so) and run it. Quit ROOT, then:

```
> root -l -b -e 'gROOT->ProcessLine(".L tree.C++"); gROOT->ProcessLine("make_tree(100000)")'
```

• Let's inspect the file content (what is the size of the output file?)

```
> root -l tree_C.root
root[0] .ls
root[1] tree->Print()
root[2] new TBrowser()
```

Inspecting a ROOT file (⇒ tree_C.root)

• From the ROOT prompt, you can draw the content of any TBranch

```
root[0] tree->Draw("px")
root[1] tree->Draw("px", "py>0 && random<0.5")
root[2] tree->Draw("px:py", "py>0 && random<0.5", "colz")
root[3] tree->Draw("px:py:pz", "py>0 && random<0.5", "lego")</pre>
```

• The output of Draw() can be put into a binned histogram (e.g. for future work):

```
root[4] tree->Draw("px>>h(100,-3,3)")
root[5] h->GetMean()
```

• Actually, you can draw any function of the input branches:

```
root[6] tree->Draw("px*TMath::Sin(py)/TMath::Log(pz+1)")
When functions are too complicated to fit into a line:
```

- 1. Write the function in a separate macro (\Rightarrow func.C)
- 2. Load the function macro within ROOT (\Rightarrow root[1] .L func.C)
- 3. Now, you can now use **func(...)** within **Draw()**. E.g.: root[7] tree->Draw("px*TMath::Sin(py)/TMath::Log(pz+1):func(px,py,pz)","","COLZ")

```
#include "TFile.h"
#include "TTree.h"
#include "TH2.h"
#include "TRandom.h"
void read tree()
 TFile *infile = new TFile("tree C.root", "READ");
 TTree *tree = (TTree*)infile->Get("tree");
 Float t px, py, pz;
 Double t random;
 Int tev;
 tree->SetBranchAddress("px",&px);
 tree->SetBranchAddress("py",&py);
 tree->SetBranchAddress("pz",&pz);
 tree->SetBranchAddress("random",&random);
 tree->SetBranchAddress("ev",&ev);
```

```
TH1F *hpx = new TH1F("hpx","px distribution",100,-3,3);
TH2F *hpxpy = new TH2F("hpxpy","py vs px",30,-3,3,30,-3,3);

Long64_t nentries = tree->GetEntries();
for (Long64_t i = 0; i < nentries; i++) {
    tree->GetEntry(i);
    hpx->Fill(px);
    hpxpy->Fill(px,py);
}

hpx->Draw("HISTE");
TFile* outfile = new TFile("histos.root", "RECREATE");
hpx->Write();
outfile->Close();
}
```

```
This is analogous to make_tree() with the replacements:
#include "TFile.h"
                           "RECREATE" ⇒ "READ"
#include "TTree.h"
                           tree->Branch(...) \Rightarrow tree->SetBranchAddress(...)
#include "TH2.h"
#include "TRandom.h"
                                                                TH1F *hpx = new TH1F("hpx","px distribution",100,-3,3);
void read tree()
                                                                TH2F *hpxpy = new TH2F("hpxpy","py vs px",30,-3,3,30,-3,3);
                                                               Long64 t nentries = tree->GetEntries();
 TFile *infile = new TFile("tree C.root", "READ");
                                                                for (Long64 t i = 0; i < nentries; i++) {
 TTree *tree = (TTree*)infile->Get("tree");
                                                                 tree->GetEntrv(i);
 Float t px, py, pz;
                                                                 hpx->Fill(px);
 Double t random;
                                                                 hpxpy->Fill(px,py);
 Int tev;
 tree->SetBranchAddress("px",&px);
 tree->SetBranchAddress("py",&py);
                                                                hpx->Draw("HISTE");
 tree->SetBranchAddress("pz",&pz);
                                                                TFile* outfile = new TFile("histos.root", "RECREATE");
 tree->SetBranchAddress("random",&random);
                                                               hpx->Write();
 tree->SetBranchAddress("ev",&ev);
                                                                outfile->Close();
```

```
We define 1D and 2D histograms to
#include "TFile.h"
#include "TTree.h"
                           be filled with the TTree content.
#include "TH2.h"
#include "TRandom.h"
void read_tree()
 TFile *infile = new TFile("tree_C.root", "READ");
 TTree *tree = (TTree*)infile->Get("tree");
 Float t px, py, pz;
 Double t random;
 Int tev;
 tree->SetBranchAddress("px",&px);
 tree->SetBranchAddress("py",&py);
 tree->SetBranchAddress("pz",&pz);
 tree->SetBranchAddress("random",&random);
 tree->SetBranchAddress("ev",&ev);
            Draw a bar histogram ("HIST")
```

with Poisson error ("E")

```
TH1F *hpx = new TH1F("hpx","px distribution",100,-3,3);
TH2F *hpxpy = new TH2F("hpxpy","py vs px",30,-3,3,30,-3,3);
Long64 t nentries = tree->GetEntries();
for (Long64 t i = 0; i < nentries; i++) {
 tree->GetEntrv(i);
 hpx->Fill(px);
 hpxpy->Fill(px,py);
hpx->Draw("HISTE");
TFile* outfile = new TFile("histos.root", "RECREATE");
hpx->Write();
outfile->Close():
                     Histogram persistently written
```

into file for future use

```
#include "TFile.h"
#include "TTree.h"
#include "TH2.h"
#include "TRandom.h"
                                                                TH1F *hpx = new TH1F("hpx","px distribution",100,-3,3);
void read tree()
                                                                TH2F *hpxpy = new TH2F("hpxpy","py vs px",30,-3,3,30,-3,3);
                                                                Long64 t nentries = tree->GetEntries();
 TFile *infile = new TFile("tree C.root", "READ");
                                                                for (Long64 t i = 0; i < nentries; i++) {
 TTree *tree = (TTree*)infile->Get("tree");
                                                                 tree->GetEntrv(i):
 Float t px, py, pz;
                                                                 hpx->Fill(px);
 Double t random;
                                                                 hpxpy->Fill(px,pv);
 Int tev;
 tree->SetBranchAddress("px",&px);
 tree->SetBranchAddress("py",&py);
                                                                hpx->Draw("HISTE"):
 tree->SetBranchAddress("pz",&pz);
                                                                TFile* outfile = new TFile("histos.root", "RECREATE");
 tree->SetBranchAddress("random",&random
                                                                hpx->Write();
 tree->SetBranchAddress("ev",&ev);
                                                                outfile->Close();
                                                               }
 Start the "event" loop.
 At each iterate i, the GetEntry(i) command
```

will read data row i for all the branches

```
import ROOT
import numpy as np
def make tree(nevents):
  outfile = ROOT.TFile("tree_py.root", "RECREATE")
  tree = ROOT.TTree("tree", "A simple Tree")
  px = np.empty((1), dtype="float32")
  py = np.empty((1), dtype="float32")
  pz = np.empty((1), dtype="float32")
  random = np.empty((1), dtype="float64")
  ev = np.empty((1), dtype="int32")
  tree.Branch("px", px, "px/F")
  tree.Branch("py", py, "py/F")
  tree.Branch("pz", pz, "pz/F")
  tree.Branch("random", random, "random/D")
  tree.Branch("ev", ev, "ev/I")
  for i in range(nevents):
     px[0] = np.random.normal()
     py[0] = np.random.normal()
     pz[0] = px[0]**2 + py[0]**2
     random[0] = np.random.random_sample()
     ev[0] = i
     tree.Fill()
  outfile.Write()
  return (outfile), tree
```

```
if __name__ == '__main__':
    _, tree = make_tree(100000)
    array, labels = tree.AsMatrix(return_labels=True)
```

```
import ROOT
import numpy as np
def make_tree(nevents):
  outfile = ROOT.TFile("tree_xy.root", "RECREATE")
  tree = ROOT.TTree("tree", "A simple Tree")
  px = np.empty((1), dtype="float32")
  py = np.empty((1), dtype="float32")
  pz = np.empty((1), dtype="float32")
  random = np.empty((1), dtype="float64"
  ev = np.empty((1), dtype="int32")
  tree.Branch("px", px, "px/F")
  tree.Branch("py", py, "py/F")
  tree.Branch("pz", pz, "pz/F")
  tree.Branch("random", random, "random/D")
  tree.Branch("ev", ev, "ev/I")
  for i in range(nevents):
     px[0] = np.random.normal()
     py[0] = np.random.normal()
     pz[0] = px[0]**2 + py[0]**2
     random[0] = np.random.random_sample()
     ev[0] = i
     tree.Fill()
  outfile.Write()
  return (outfile), tree
```

```
if __name__ == '__main__':
    _, tree = make_tree(100000)
    array, labels = tree.AsMatrix(return_labels=True)
```

That's it! From now on, any ROOT class Txxx accessible using ROOT.Txxx

numpy or array only needed for writing branches

```
import ROOT
import numpy as np
def make tree(nevents):
   outfile = ROOT.TFile("tree_py.root", "RECREATE")
  tree = ROOT.TTree("tree", "A simple Tree")
   px = np.empty((1), dtype="float32")
  py = np.empty((1), dtype="float32")
  pz = np.emptv((1), dtype="float32")
  random = np.empty((1), dtype="float64")
   ev = np.empty((1), dtype="int32")
  tree.Branch("px", px, "px/F")
  tree.Branch("py", py, "py/F")
  tree.Branch("pz", pz, "pz/F")
  tree.Branch("random", random, "random/D")
  tree.Branch("ev", ev, "ev/I")
  for i in range(nevents):
     px[0] = np.random.normal()
     pv[0] = np.random.normal()
     pz[0] = px[0]**2 + py[0]**2
     random[0] = np.random.random sample()
     ev[0] = i
     tree.Fill()
  outfile.Write()
  return (outfile), tree
```

```
if __name__ == '__main__':
    _, tree = make_tree(100000)
    array, labels = tree.AsMatrix(return_labels=True)
```

Same as before, with all the advantages of Python

```
import ROOT
import numpy as np
def make tree(nevents):
  outfile = ROOT.TFile("tree_py.root", "RECREATE")
  tree = ROOT.TTree("tree", "A simple Tree")
  px = np.empty((1), dtype="float32")
  py = np.empty((1), dtype="float32")
  pz = np.empty((1), dtype="float32")
  random = np.empty((1), dtype="float64")
   ev = np.empty((1), dtype="int32")
  tree.Branch("px", px, "px/F")
  tree.Branch("py", py, "py/F")
  tree.Branch("pz", pz, "pz/F")
  tree.Branch("random", random, "random/D")
  tree.Branch("ev", ev, "ev/I")
  for i in range(nevents):
     px[0] = np.random.normal()
     py[0] = np.random.normal()
     pz[0] = px[0]**2 + py[0]**2
     random[0] = np.random.random_sample()
     ev[0] = i
     tree.Fill()
  outfile.Write()
  return (outfile), tree
```

```
if __name__ == '__main__':
    _, tree = make_tree(100000)
    array, labels = tree.AsMatrix(return_labels=True)
```

A nice feature: automatic conversion

TTree -> numpy.array(nevents,nbranches) (working in multicore mode, if supported)

```
import ROOT
import numpy as np
def make tree(nevents):
  outfile = ROOT.TFile("tree_py.root", "RECREATE")
  tree = ROOT.TTree("tree", "A simple Tree")
  px = np.empty((1), dtype="float32")
  py = np.empty((1), dtype="float32")
  pz = np.empty((1), dtype="float32")
  random = np.empty((1), dtype="float64")
  ev = np.empty((1), dtype="int32")
  tree.Branch("px", px, "px/F")
  tree.Branch("py", py, "py/F")
  tree.Branch("pz", pz, "pz/F")
  tree.Branch("random", random, "random/D")
  tree.Branch("ev", ev, "ev/I")
  for i in range(nevents):
     px[0] = np.random.normal()
     py[0] = np.random.normal()
     pz[0] = px[0]**2 + py[0]**2
     random[0] = np.random.random_sample()
     ev[0] = i
     tree.Fill()
  outfile.Write()
  return (outfile), tree
```

```
if __name__ == '__main__':
    _, tree = make_tree(100000)
    array, labels = tree.AsMatrix(return_labels=True)
```

<u>Caveats</u>:

- I. PyROOT can be much slower than compiled C++ (see e.g. the two examples just discussed)
- Handling of memory sometimes problematic.
 Extra care needed to avoid memory leaks.

Exercise 4: a tree with variable-length branches (⇒ tree_array.py)

```
import ROOT
import numpy as np
def make tree(nevents):
  outfile = ROOT.TFile("tree_py.root", "RECREATE")
  tree = ROOT.TTree("tree", "A simple Tree")
  px = np.empty((1), dtype="float32")
  py = np.empty((1), dtype="float32")
  pz = np.empty((1), dtype="float32")
  random = np.empty((1), dtype="float64")
  ev = np.empty((1), dtype="int32")
  nHits = np.empty((1), dtype="int32")
  hits = np.empty((100), dtype="float32")
  tree.Branch("px", px, "px/F")
  tree.Branch("py", py, "py/F")
  tree.Branch("pz", pz, "pz/F")
  tree.Branch("random", random, "random/D")
  tree.Branch("ev", ev, "ev/I")
  tree.Branch("nHits", nHits, "nHits/I")
  tree.Branch("hits", hits, "hits[nHits]/F")
```

```
for i in range(nevents):
    px[0] = np.random.normal()
    py[0] = np.random.normal()
    pz[0] = px[0]**2 + py[0]**2
    random[0] = np.random.random_sample()
    ev[0] = i
    nHits[0] = 2 if i%2==0 else 4
    for h in range(nHits[0]):
        hits[h] = np.random.randint(0,20)
        tree.Fill()
outfile.Write()
return (outfile), tree
```

N.B.: convertion into **numpy** array will fail due to array-like branch

More into the future: RDataFrame (⇒ test_RDF.C)

A tool for declarative analysis

- "Users say what, ROOT chooses how"
- Analysis as a computational graph of connected nodes: data -> operations -> results
- Computation of the graph can be parallelized in a transparent way

```
data
                                                                           input
 ROOT::EnableImplicitMT();
                                                                                                    transformation
                                                                           x, y
 ROOT::RDataFrame df("tree", "tree_py.root", {"px", "py"});
                                                                                                    result
                                                                                     filter
 auto df2 = df.Filter("px > 0").Define("pT2", "px*px + py*py");
                                                                                     x > 0
 auto rHist = df2.Histo1D("pT2");
 rHist->Draw();
                                                                                              define
                                                                                                          ROOT file
 df2.Snapshot("newtree", "out.root");
                                                                                             r2 = x^2 + v
                                                                                                           x, y, r2
Lat's run it:
                                                                                                      histo
                                                                                                        r2
> root test RDF.C
```

59