20/10/2020

SISTEMI QUANTISTICI A MOLTI CORPI - INTRODUZIONE

Eq. di Schrödinger $i\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$ $\quad (\hbar = 1)$

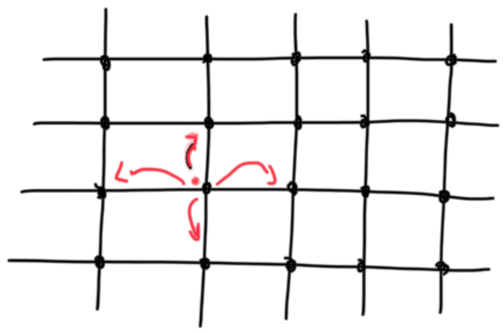
→ sistemi non relativistici
→ sistemi che evolvono in modo unitario (sistemi chiusi)

operatore Hamiltoniano $\hat{H}$ $\quad (\hat{H}^\dagger = \hat{H})$

→ sistemi finito-dimensionali (spazi di Hilbert finiti $\mathbb{C}^N$)

⇒ algebra lineare

→ sistemi su reticolo



tight-binding

$$\hat{H} = -t \sum_{\langle i,j\rangle} \left(\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i\right)$$

operatori di creazione/distruzione su uno dei siti reticolari

creazione i → $\hat{c}_i^\dagger$

distruzione i → $\hat{c}_i$

particelle fermioniche (es. elettroni)

$\{\hat{c}_i^\dagger, \hat{c}_j\} = \delta_{ij}$

$\{\hat{c}_i^\dagger, \hat{c}_j^\dagger\} = \{\hat{c}_i, \hat{c}_j\} = 0$

vuoto: $|\Omega\rangle$

$\left(c_2^\dagger c_1^\dagger|\Omega\rangle \neq c_1^\dagger c_2^\dagger|\Omega\rangle\right)$

regole di anticommutazione
$\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$

(II quantizzazione)

bosoni → $\hat{b}_i, \hat{b}_i^\dagger$ → $[\hat{b}_i, \hat{b}_j^\dagger] = \delta_{ij}$
$[\hat{b}_i^\dagger, \hat{b}_j^\dagger] = [\hat{b}_i, \hat{b}_j] = 0$

regole di commutazione
$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$

→ elettroni ⇒ devo aggiungere un indice per lo spin $\sigma = \uparrow, \downarrow$ $\quad (s = 1/2)$

$$\hat{H} = -t \sum_{\langle i,j\rangle} \sum_{\sigma=\uparrow,\downarrow} \left(\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + \hat{c}_{j,\sigma}^\dagger \hat{c}_{i,\sigma}\right) \quad \text{(tight binding)}$$

intensità con la quale gli elettroni si spostano
$t > 0$

i, j siti vicini sul reticolo (connessi direttamente)

uno il complesso coniugato dell'altro

$\left(\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma}\right)^\dagger = \hat{c}_{j,\sigma}^\dagger \hat{c}_{i,\sigma}$

$H = H^\dagger$

processo di hopping (elettroni saltano da un sito e quello adiacente)

$$\hat H = -2t \sum_{\vec k}\left\{\sum_{\vec\delta}\cos(\vec k\cdot\vec\delta)\right\}\hat c^\dagger_{\vec k}\,\hat c_{\vec k}$$

trasf. di Fourier

$$\hat c_{\vec k} = \frac{1}{\sqrt N}\sum_j e^{-i\vec k\cdot\vec r_j}\,\hat c_j$$

→ $\hat H$ con interazioni repulsive su ogni sito:

$$\hat H = -t\sum_{\langle i,j\rangle}\sum_\sigma\left(\hat c^\dagger_{i,\sigma}\hat c_{j,\sigma}+\hat c^\dagger_{j,\sigma}\hat c_{i,\sigma}\right) + U\sum_i \hat n_{i,\uparrow}\hat n_{i,\downarrow}$$

modello di Hubbard (1963)

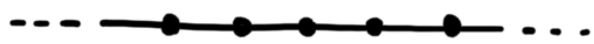
dove $\hat n_{i,\sigma} = \hat c^\dagger_{i,\sigma}\hat c_{i,\sigma}$ (numero di elettroni con spin σ, sul sito i)

$U>0$ esprime un processo di repulsione su ogni sito del reticolo

$\hat N = \sum_i \sum_{\sigma=\uparrow,\downarrow}\hat n_{i,\sigma}$ operatore numero

$= \hat N_\uparrow + \hat N_\downarrow$ dove $\hat N_\sigma = \sum_i \hat n_{i,\sigma}$

$\Rightarrow [\hat H,\hat N] = [\hat H,\hat N_\sigma] = 0$



Modello di Hubbard in 1D

↳ Bethe ansatz (sol. analitica non banale)

Nagaosa "Quantum field theory in strongly correlated systems"

- mapping con il modello di Heisenberg (spin)

$\boxed{U \gg t}$ (limite fortemente interagente)

→ sistema ad half filling (n° di elettroni = n° di siti)

$\hat H = \hat H_0 + \delta\hat H$

$\hat H_0$ ↓ interazioni (U)

$\delta\hat H$ ↘ hopping (t)

2 siti e 2 particelle

o — o
1   2

↓

stati possibili

$\{|\uparrow,\uparrow\rangle;\ |\uparrow,\downarrow\rangle;\ |\downarrow,\uparrow\rangle;\ |\downarrow,\downarrow\rangle$
$\cancel{|\uparrow\downarrow,0\rangle};\ \cancel{|0,\uparrow\downarrow\rangle}\}$

hanno un peso sfavorevole in energia U (→ li trascuro) se sono interessato al ground state

considero solo gli stati a singola occupazione

(spin ½) + (spin ½)

tripletto:
$$\begin{cases}|1,1\rangle = |\uparrow,\uparrow\rangle\\ |1,0\rangle = \frac{1}{\sqrt2}\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)\\ |1,-1\rangle = |\downarrow,\downarrow\rangle\end{cases}$$

↓ $S=1$ ↘ S_z

singoletto $|0,0\rangle = \frac{1}{\sqrt2}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)$

↓ $S=0$ ↘ $S_z=0$

→ teoria delle perturbazioni in $\delta\hat{H}$

I ordine: $\langle 1,1|\delta\hat{H}|1,1\rangle = 0$ $\qquad$ $\langle 1,0|\delta\hat{H}|1,0\rangle = 0$

scambio

stato intermedio

II ordine: $$\delta E_n^{(2)} = \sum_{m\neq n} \frac{\langle n|\delta\hat{H}|m\rangle\langle m|\delta\hat{H}|n\rangle}{E_n^{(0)} - E_m^{(0)}}$$

$\delta\hat{H}|1,1\rangle = \delta\hat{H}|1,-1\rangle = 0$ → non danno contributo al II ordine

$\delta\hat{H}|1,0\rangle$ e $\delta\hat{H}|0,0\rangle \neq 0$

$$\delta\hat{H}|\uparrow,\downarrow\rangle = -t\left(\cancel{\hat{c}^\dagger_{1,\uparrow}\hat{c}_{2,\uparrow}} + \hat{c}^\dagger_{2,\uparrow}\hat{c}_{1,\uparrow} + \hat{c}^\dagger_{1,\downarrow}\hat{c}_{2,\downarrow} + \cancel{\hat{c}^\dagger_{2,\downarrow}\hat{c}_{1,\downarrow}}\right)\left[\hat{c}^\dagger_{1,\uparrow}\hat{c}^\dagger_{2,\downarrow}|\Omega\rangle\right] =$$

ho scelto un ordinamento

$= -\hat{c}^\dagger_{2,\downarrow}\hat{c}^\dagger_{1,\uparrow}|\Omega\rangle$

$$= -t\left(\hat{c}^\dagger_{2,\uparrow}\hat{c}_{1,\uparrow} + \hat{c}^\dagger_{1,\downarrow}\hat{c}_{2,\downarrow}\right)\hat{c}^\dagger_{1,\uparrow}\hat{c}^\dagger_{2,\downarrow}|\Omega\rangle$$

$$= -t\left(\hat{c}^\dagger_{2,\uparrow}\hat{c}^\dagger_{2,\downarrow} + \hat{c}^\dagger_{1,\downarrow}\hat{c}^\dagger_{1,\uparrow}\right)|\Omega\rangle$$

(perché $\hat{c}_{1,\uparrow}\hat{c}_{1,\uparrow} = 1$)

$$\delta\hat{H}|\downarrow,\uparrow\rangle = t\left(\hat{c}^\dagger_{1,\uparrow}\hat{c}^\dagger_{1,\downarrow} + \hat{c}^\dagger_{2,\uparrow}\hat{c}^\dagger_{2,\downarrow}\right)|\Omega\rangle$$

$$\delta\hat{H}|1,0\rangle = \delta\hat{H}\left(\frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\right)\right) = 0$$

$$\delta\hat{H}|0,0\rangle = \delta\hat{H}\left(\frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)\right) = -\frac{2t}{\sqrt{2}}\left(|\uparrow\downarrow,0\rangle + |0,\uparrow\downarrow\rangle\right)$$

il singoletto è favorito rispetto al tripletto al II ordine in $\delta\hat{H}$

$$\delta E^{(2)}_{|0,0\rangle} = \frac{2t\cdot 2t}{0-U} = -\frac{4t^2}{U}$$

⇒ il sistema •—• tende a mettersi in un singoletto

$$\hat{\vec{S}}_1\cdot\hat{\vec{S}}_2 = \frac{1}{2}\left((\hat{\vec{S}}_1+\hat{\vec{S}}_2)^2 - \hat{\vec{S}}_1^2 - \hat{\vec{S}}_2^2\right)$$

$$\Rightarrow \hat{\vec{S}}_1\cdot\hat{\vec{S}}_2 = \begin{cases} \frac{1}{2}\Big[\underbrace{(0)^2}_{s(s+1)} - \frac{3}{4} - \frac{3}{4}\Big] = -\frac{3}{4} & \text{singoletto } (s=1) \\ \frac{1}{2}\Big[\underbrace{(2)^2}_{s(s+1)} - \frac{3}{4} - \frac{3}{4}\Big] = +\frac{1}{4} & \text{tripletto } (s=0) \end{cases}$$

$\hat{\vec{S}}_1 = (\hat{S}_1^x, \hat{S}_1^y, \hat{S}_1^z) = \frac{1}{2}(\hat{\sigma}_1^x, \hat{\sigma}_1^y, \hat{\sigma}_1^z)$ dove $\hat{\sigma}_1^\alpha$ sono matrici di Pauli per lo spin 1

$\hat{D}$ ... $(1/4 - (-3/4)) = 1$ singoletto

$$P_{\text{singoletto}}(1,2) = \frac{1}{4} - S_1 \cdot S_2 = \begin{cases} \dots \\ 1/4 - (+1/4) = 0 & \text{tripletto} \end{cases}$$

$$\Rightarrow \hat{H}_{eff} \sim -J \sum_{\langle i,j \rangle} \hat{P}_{\text{singoletto}}(i,j) \sim J \sum_{\langle i,j \rangle} \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j$$

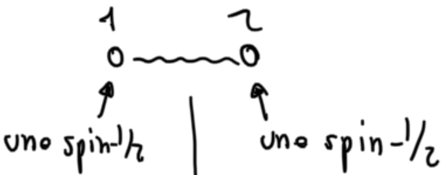
con $\underbrace{J = \frac{4t^2}{U}}$

$$\Downarrow$$

$$\hat{H}_{eff} \sim J \sum_{\langle i,j \rangle} \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j$$

modello di Heisenberg per spin-1/2

(→ si risolve analiticamente Bethe ansatz)



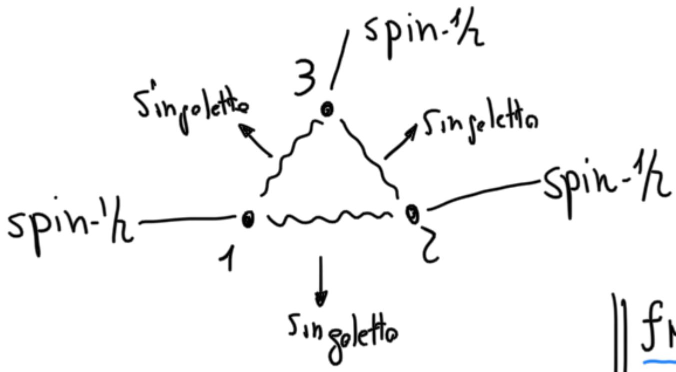
nello stato fondamentale il sistema tende a disporsi in un singoletto

$|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle$

[Coppie di Bell

→ presenta entanglement (E) massimo per una coppia di spin-1/2

(viola massimamente le disuguaglianze di Bell)



NON è possibile!

$(N > 2)$

|| frustrazione quantistica

se si hanno

$\Rightarrow$ 3 spin (1/2)

$$E^2_{1|2} + E^2_{1|3} \leq E^2_{1|2,3}$$

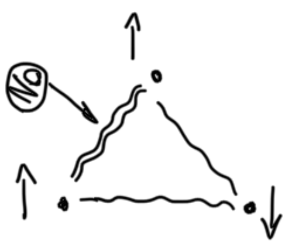
monogamia dell'entanglement

(Coffman, Kundu, Wooters, Phys. Rev. A 61, 052306 (2000))

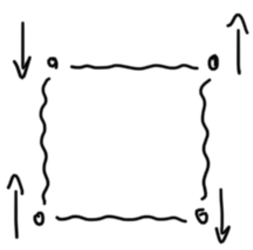
- la frustrazione quantistica è da distinguersi da quella geometrica

es: spin classici, sistema antiferromagnetico

$$H = -J \sum_{\langle i,j \rangle} S_i S_j \quad (\text{Ising classico})$$



reticolo triangolare
↓
frustrazione



reticolo quadrato
↓
No frustrazione

21/10/2020

Spazio di Hilbert $\mathcal{H}$
per sistemi di spin-1/2 su reticolo

$\dim \mathcal{H} = 2^N$

1 sito: $\mathcal{H}^{(1)} = \text{span}\{|\uparrow\rangle, |\downarrow\rangle\}$ (1 spin-1/2 $\equiv$ qubit) — quantum bit

$\dim \mathcal{H}^{(1)} = 2$

2 siti: $\mathcal{H}^{(2)} = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} = \text{span}\{|\uparrow,\uparrow\rangle, |\uparrow,\downarrow\rangle, |\downarrow,\uparrow\rangle, |\downarrow,\downarrow\rangle\}$

$\dim \mathcal{H}^{(2)} = 4$

3 siti: $\mathcal{H}^{(3)} = \text{span}\{|\uparrow,\uparrow,\uparrow\rangle, |\uparrow,\uparrow,\downarrow\rangle, |\uparrow\downarrow\uparrow\rangle, |\downarrow\uparrow\uparrow\rangle, |\downarrow\downarrow\uparrow\rangle, |\downarrow\uparrow\downarrow\rangle, |\uparrow\downarrow\downarrow\rangle, |\downarrow\downarrow\downarrow\rangle\}$

$\dim \mathcal{H}^{(3)} = 8$

$N=10$ spin-1/2 $\quad \dim \mathcal{H}^{(10)} = 2^{10} = 1024 \quad \dim \hat{H} = 2^{10} \times 2^{10}$

trovare lo spettro e autovettori di una matrice 1024×1024

$N=20 \rightarrow \dim \mathcal{H}^{(20)} = 2^{20} \sim 10^6 \qquad \hat{H} = 10^6 \times 10^6$!

($N=14$ circa upper bound per i computer attuali)

In questo modulo:

- diagonalizzazione esatta - forza bruta $\quad \approx N \leq 14$
- diag. Lanczos - bene per lo stato fondamentale $\quad \approx N \leq 28$
  - ↳ matrici sparse (tanti elementi sono zeri)

$$\hat{H} = J \sum_{i=1}^{N} \hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1} \qquad (1D)$$

$\hat{H}$ agisce su $\mathcal{H}^{(N)}$ (dimensione 2^N)

$|\psi\rangle \in \mathcal{H}^{(N)}$

$|\psi\rangle \in \mathcal{H}^{(1)} \otimes \mathcal{H}^{(1)} \otimes \ldots \otimes \mathcal{H}^{(1)}$

$\in \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2$ (N volte)

$$\hat{\vec{S}}_i = \frac{1}{2}\begin{pmatrix} \hat{\sigma}_i^x \\ \hat{\sigma}_i^y \\ \hat{\sigma}_i^z \end{pmatrix}$$

$\hat{\sigma}_i^x, \hat{\sigma}_i^y, \hat{\sigma}_i^z$ matrici di Pauli agiscono su stati $\in \mathcal{H}^{(1)}$ ($\mathbb{C}^2$)

$$\hat{\sigma}_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; ; \; \hat{\sigma}^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; ; \; \hat{\sigma}_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (\dim 2\times 2)$$

$\hat{\sigma}_i^x$ se ho N siti intendo: $\hat{\sigma}_i^x = \underbrace{\hat{\mathbb{1}}^{(1)}}_{\text{sul 1° spin}} \otimes \underbrace{\hat{\mathbb{1}}^{(1)}}_{\text{sul 2° spin}} \otimes \ldots \otimes \underbrace{\hat{\sigma}^x}_{\text{sull'i-esimo spin}} \otimes \underbrace{\mathbb{1}^{(1)}}_{\text{sul (i+1)-esimo}} \otimes \ldots \otimes \underbrace{\mathbb{1}^{(1)}}_{\text{sul N° spin}}$

prodotto tensore

$$\hat{A} \otimes \hat{B} = \begin{pmatrix} a_{11} B & a_{12} B \\ \ldots & \ldots \end{pmatrix}$$

$$\hat{A}\otimes\hat{B} = \begin{pmatrix} a_{21}\cdot B & a_{22}\cdot B \end{pmatrix}$$

$\hat{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ agisce sul 1° spin 1/2

$\hat{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ " 2° spin 1/2

$$\hat{A}\otimes\hat{B} = \left(\begin{array}{cc|cc} a_{11}b_{11} & a_{11}b_{12} & a_{12}b_{11} & a_{12}b_{12} \\ a_{11}b_{21} & a_{11}b_{22} & a_{12}b_{21} & a_{12}b_{22} \\ \hline a_{21}b_{11} & a_{21}b_{12} & a_{22}b_{11} & a_{22}b_{12} \\ a_{21}b_{21} & a_{21}b_{22} & a_{22}b_{21} & a_{22}b_{22} \end{array}\right)$$

$$\hat{H} = J\sum_i \hat{\vec{S}}_i\cdot\hat{\vec{S}}_{i+1} = J\sum_i\left(\hat{S}_i^x\hat{S}_{i+1}^x + \hat{S}_i^y\hat{S}_{i+1}^y + \hat{S}_i^z\hat{S}_{i+1}^z\right)$$

$$\equiv \hat{\mathbb{1}}\otimes\hat{\mathbb{1}}\otimes\ldots\otimes\hat{S}_i\otimes\hat{S}_{i+1}\otimes\hat{\mathbb{1}}\otimes\ldots\otimes\hat{\mathbb{1}} \qquad \hat{\mathbb{1}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

→ $\hat{H}$ è una matrice piena di zeri → (sparsa)

(ipotizzando di avere interazioni a 2 corpi)

MAPPING FERMIONI ⟷ SPIN

modello t-V: $\hat{H} = -t\sum_j\left(\hat{c}_j^\dagger\hat{c}_{j+1} + \hat{c}_{j+1}^\dagger\hat{c}_j\right) + V\sum_j\hat{n}_j\hat{n}_{j+1}$

(1D)

fermioni "spinless" $\{\hat{c}_i^\dagger, \hat{c}_j^\dagger\} = \{\hat{c}_i, \hat{c}_j\} = 0$; $\{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij}$

→ mapping esatto su modello di spin-1/2

Pauli: $[\hat{\sigma}_i^\alpha, \hat{\sigma}_j^\beta] = 2i\,\varepsilon_{\alpha\beta\gamma}\hat{\sigma}^\gamma\delta_{ij}$ regole di commutazione

$\hat{c}_i^{(\dagger)} \leftrightarrow \hat{\sigma}_i^\alpha$?

→ **Trasformazione di Jordan-Wigner** (J.-W.)

su 1 sito $\{|0\rangle, |1\rangle\}$ $\hat{c}^\dagger|0\rangle = |1\rangle$ $\hat{c}|1\rangle = |0\rangle$

$\hat{c}^\dagger|1\rangle = 0$ $\hat{c}|0\rangle = 0$

$\hat{c}^\dagger \sim \hat{\sigma}^+$ $\hat{\sigma}^+ = \frac{1}{2}(\hat{\sigma}^x + i\hat{\sigma}^y)$

$\{|\uparrow\rangle, |\downarrow\rangle\}$ $\hat{\sigma}^+|\downarrow\rangle = |\uparrow\rangle$

$\hat{\sigma}^+|\uparrow\rangle = 0$

$\hat{c} \sim \hat{\sigma}^-$ $\hat{\sigma}^- = \frac{1}{2}(\hat{\sigma}^x - i\hat{\sigma}^y)$ $\hat{\sigma}^-|\uparrow\rangle = |\downarrow\rangle$

$\hat{\sigma}^-|\downarrow\rangle = 0$

$(i\neq j)$ $\hat{c}_i^\dagger\hat{c}_j \overset{?}{\rightarrow} \hat{\sigma}_i^+\hat{\sigma}_j^-$ problema legato alle regole di commutazione

$\quad\;\; \| \qquad\qquad \|$

$-\hat{c}_j\hat{c}_i^\dagger \qquad +\hat{\sigma}_j^-\hat{\sigma}_i^+$

J-W: $\boxed{\hat{c}_i = \left(\prod\hat{\sigma}_k^z\right)\hat{\sigma}_i^-}$ è utile ... $k<j$ ⇒ ordinamento imposto dei siti

"stringa" di operatori

è solo qui → 1D → 1 2 3 4 5 ....

2D → ordinamento va scelto (J-W poco utile...)

$$\hat{c}_j^+ = \Big(\prod_{k<j} \hat{\sigma}_k^z\Big)\hat{\sigma}_j^+ = \hat{\sigma}_j^+ \Big(\prod_{k<j}\hat{\sigma}_k^z\Big)$$

se $\{\hat{\sigma}_j^\alpha\}$ seguono le regole di comm. degli spin ½ (Pauli)

⇒ si dimostra che $\{\hat{c}_j^{(+)}\}$ anticommutano

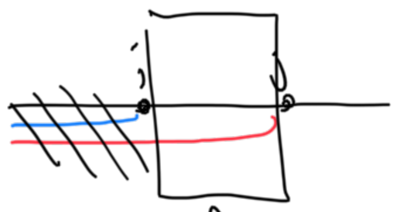
$$[\hat{\sigma}_i^+, \hat{\sigma}_j^-] = \delta_{ij}\hat{\sigma}_j^z$$

$$[\hat{\sigma}_i^z, \hat{\sigma}_j^\pm] = \pm 2\delta_{ij}\hat{\sigma}_j^\pm$$

$(i<j)$

$$c_i^+ c_j = \Big(\prod_{k<i}\hat{\sigma}_k^z\Big)\hat{\sigma}_i^+ \cdot \Big(\prod_{\ell<j}\hat{\sigma}_\ell^z\Big)\hat{\sigma}_j^- = \qquad \begin{pmatrix}\text{uso} \\ [\hat{\sigma}^z]^2 = \mathbb{1}\end{pmatrix}$$

$$= \hat{\sigma}_i^+ \Big(\prod_{i\le k<j}\hat{\sigma}_k^z\Big)\hat{\sigma}_j^-$$



$(i<j)$

$$\hat{c}_j\hat{c}_i^+ = \Big(\prod_{k<j}\hat{\sigma}_k^z\Big)\hat{\sigma}_j^- \Big(\prod_{\ell<i}\hat{\sigma}_\ell^z\Big)\hat{\sigma}_i^+$$

$$= \Big(\prod_{i\le k<j}\hat{\sigma}_k^z\Big)\hat{\sigma}_j^-\hat{\sigma}_i^+ = \Big(\prod_{i\le k<j}\hat{\sigma}_k^z\Big)\hat{\sigma}_i^+\hat{\sigma}_j^- = -\hat{\sigma}_i^+\Big(\prod \ldots\Big)\hat{\sigma}_j^-$$

→ $\hat{c}_j^+\hat{c}_{j+1} = +\hat{\sigma}_j^+\hat{\sigma}_{j+1}^-$

$\hat{c}_{j+1}^+\hat{c}_j = +\hat{\sigma}_{j+1}^+\hat{\sigma}_j^-$

$\hat{n}_j\hat{n}_{j+1} = \frac{1}{4}\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + \frac{1}{4}(\hat{\sigma}_j^z + \hat{\sigma}_{j+1}^z) + \text{cost.}$ ⟷ $\hat{c}_j^+\hat{c}_j \equiv \hat{n}_j = \frac{1}{2}(-\hat{\sigma}_j^z + \mathbb{1})$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1\end{pmatrix}$$

$$\Rightarrow t\text{-}V \iff \hat{H} = -\frac{t}{2}\sum_j \underbrace{\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y\right)}_{=2(\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_i^-\hat{\sigma}_{i+1}^+)} + \frac{V}{4}\sum_j \hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + \frac{V}{2}\sum_j\hat{\sigma}_j^z + \ldots$$

$\hat{c}_j^+\hat{c}_{j+1} + \hat{c}_{j+1}^+\hat{c}_j$

$\hat{n}_j\hat{n}_{j+1}$

costanti

spin-1/2

Hamiltoniana di Heisenberg. XXZ

anisotropia

$$\left(-\frac{t}{2}, -\frac{t}{2}, \frac{V}{4}\right)$$

in generale:

$J^x = t(1+\gamma) \quad ; \quad J^y = t(1-\gamma)$

$$\hat{H}_{spin} = \frac{1}{2}\sum_j\left(J^x\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x + J^y\hat{\sigma}_j^y\hat{\sigma}_{j+1}^y\right) + \frac{1}{4}V\sum_j\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + \frac{1}{2}V\sum_j\hat{\sigma}_j^z + \frac{V}{4}N$$

↕ J.W.

$$\hat{H}_f = -t\sum\Big[\left(\hat{c}_j^+\hat{c}_{j+1} + \gamma\hat{c}_j^+\hat{c}_{j+1}^+\right) + \left(\hat{c}_{j+1}^+\hat{c}_j + \gamma\hat{c}_{j+1}\hat{c}_j\right)\Big] + V\sum\hat{n}_j\hat{n}_{j+1}$$

$$\hat{H}_{fermi} = -t\sum_j \Big[\big(\hat{c}_j^\dagger \hat{c}_{j+1} + \underbrace{\gamma\, \hat{c}_j^\dagger \hat{c}_{j+1}^\dagger}\big) + \big(\hat{c}_{j+1}^\dagger \hat{c}_j + \underbrace{\gamma\, \hat{c}_{j+1} \hat{c}_j}\big)\Big] + V\sum_j \hat{n}_j \hat{n}_{j+1}$$

$\rightarrow$ pairing superconduttivo (non conserva il n° di fermioni)

- Cons. fermioni: $\hat{N}_f = \sum_j \hat{n}_j$

$\Updownarrow$

- Cons. magnetizzazione lungo $\hat{z}$: $\hat{S}^z = \sum_j \hat{\sigma}_j^z$ (si conserva se il sistema è isotropo sul piano XY)

$\Rightarrow$ proviamo a studiare questo modello più semplice:

$$\hat{H}_{Ising} = -t\sum_j \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x - h\sum_j \hat{\sigma}_j^z \qquad \text{Ising } \underline{1D}$$

accoppiamento tra gli spin (x)

campo magnetico (z) trasversale

se campo $\perp$ accoppiamento $\Rightarrow$ sol. analitica (sistema integrabile)

se non è $\perp$ $\Rightarrow$ $\underline{\text{NO}}$ $\Rightarrow$ solo numerica (sistema NON integrabile)

per es: $\hat{H}_2 = \hat{H}_{Ising} - h_x \sum_j \hat{\sigma}_j^x$

campo magnetico longitudinale (x)