$$u_J^{n+1} = u_J^n - \frac{v\Delta t}{2\Delta x}\left(u_{J+1}^n - u_{J-1}^n\right)$$

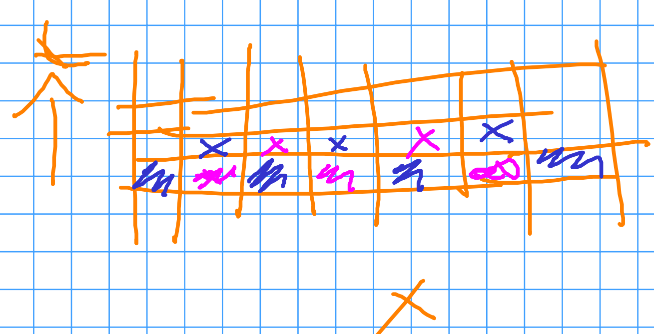
$$u_J^n = \xi^n e^{ik\Delta x J}$$

$$\cancel{\xi^{n}}\,\xi\,\cancel{e^{ikJ\Delta x}} = \cancel{\xi^n}\,\cancel{e^{ikJ\Delta x}} - \frac{v\Delta t}{2\Delta x}\left(\cancel{\xi^n} e^{ik(J+1)\Delta x} - \cancel{\xi^n} e^{ik(J-1)\Delta x}\right)$$

$$\xi = 1 - \frac{v\Delta t}{2\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

$$= 1 - \frac{v\Delta t}{\Delta x} i \sin k\Delta x$$

$$|\xi|^2 = 1 + \left(\frac{v\Delta t}{\Delta x}\sin k\Delta x\right)^2$$



$$u_J^{n+1} = \frac{u_{J+1}^n + u_{J-1}^n}{2} - \frac{v\Delta t}{2\Delta x}\left(u_{J+1}^n - u_{J-1}^n\right)$$

$$u_J^n = \xi^n e^{ikJ\Delta x}$$

$$\xi = \frac{e^{ik\Delta x} + e^{-ik\Delta x}}{2} - \frac{v\Delta t}{2\Delta x}\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)$$

$$\cos k\Delta x - \frac{i v\Delta t}{\Delta x}\sin k\Delta x$$

$$|\xi|^2 = \cos^2(k\Delta x) + \frac{v^2\Delta t^2}{\Delta x^2}\sin^2(k\Delta x)$$

$$\frac{v^2\Delta t^2}{\Delta x^2} < 1 \qquad |\xi|^2 < 1$$

$$1 - k^2\Delta x^2 + \frac{v^2\Delta t^2}{\Delta x^2} k^2\Delta x^2$$

$$1 - \underbrace{\left(1 - \frac{v^2\Delta t^2}{\Delta x^2}\right)} k^2\Delta x^2$$

$$\partial_t u = \partial_x F(u) \qquad F(u) = vu$$

$$u_J^{n+1} = \frac{u^n_{J+1} + u^n_{J-1}}{2} + \frac{\Delta t}{2\Delta x}\left(F(u^n_{J+1}) - F(u^n_{J-1})\right)$$

$$u^n_J$$

$$u_J^{n+1} = u_J^{n-1} - \frac{v\Delta t}{\Delta x}\left(u^n_{J+1} - u^n_{J-1}\right)$$

$$u^n_J = \xi^n e^{ik\Delta x J}$$

$$\xi^2 = 1 + \frac{2iv\Delta t}{\Delta x}\xi\sin(k\Delta x) = 0$$

$$\xi = \frac{-\frac{iv\Delta t}{\cancel{\Delta x}}\sin \pm \sqrt{-\frac{v^2\Delta t^2}{\Delta x^2}s^2 + 1}}{}$$

$$|\xi|^2 = (\quad)^2 + \quad 1 \quad = 1$$

$$\frac{v^2\Delta t^2}{\Delta x^2} < 1$$

$\sin^2 k\Delta x$

$k\Delta x \ll 1$

$$\sqrt{1 - v^2\Delta t^2 k^2}$$

$$\frac{v^2\Delta t^2}{\Delta x^2} > 1$$

$k\Delta x$ piccoli

caso limite $\sin(k\Delta x) = 1$

$$\frac{v^2 \Delta t^2}{\Delta x^2} > 1$$

$$-\frac{i v \Delta t}{\Delta x} \pm i \sqrt{\frac{v^2 \Delta t^2}{\Delta x^2} - 1}$$

$$\frac{v^2 \Delta t^2}{\Delta x^2} + \frac{v^2 \Delta t^2}{\Delta x^2} - 1$$

$$i \frac{\psi_j^{n+1} - \psi_j^n}{\Delta t} = - \frac{\psi_{j+1}^n - 2\psi_j^n + \psi_{j-1}^n}{\Delta x^2}$$

$$\dot{x} = f(x)$$

$$\frac{x_{n+1} - x_n}{h} = f(x_{n+1}) \quad (x_n)$$

$$\frac{x_{n+1} - x_n}{h} = x_n$$

$$\dot{x} = \lambda x$$

$$x_{n+1} - x_n = h\lambda x_{n+1} \quad x_n \rightarrow x_{n+1} = \frac{x_n}{1 - \lambda h} \rightarrow 0 \quad \lambda < 0$$

λh

$\lambda h \sim 0.1$

$$x_{n+1} = (1 + \lambda h) x_n$$

$\lambda h <$

$|\lambda h| < 1$

$$x_{n+1} = (1+\Delta) x_n$$

$$(1 + \lambda h)$$

$$\lambda = -2$$

$$\frac{x_n}{1+2h} \rightarrow 0$$

$$(1-2h)^n$$

$$h < \frac{1}{2}$$

$$\frac{1}{1 - \lambda h}$$

$$\psi = \xi^n e^{ik\Delta x j}$$

$$i\frac{(\xi - 1)}{\Delta t} = + \frac{i \Delta t}{\Delta x^2}\left(e^{ik\Delta x} - 2 + e^{-ik\Delta x}\right)$$

$$\xi = 1 + i\frac{\Delta t}{(\Delta x)^2}\left(e^{ik\Delta x} - 2 + e^{-ik\Delta x}\right)\xi$$

$$\sin^2(k\Delta x$$

$$\left(e^{i\frac{k\Delta x}{2}} - e^{-i\frac{k\Delta x}{2}}\right)^2$$

$$2i\sin\frac{k\Delta x}{2}$$

$$1 \pm 4\frac{i\Delta t}{(\Delta x)^2}\sin^2\frac{k\Delta x}{2}$$

$$\xi\left(1 \pm 4\frac{i\Delta t}{(\Delta x)^2}\sin^2\frac{k\Delta x}{2}\right) = 1$$

$$\xi = \frac{1}{(\quad)}$$

$$\left(1+\frac{iA}{2}\right)\psi^{n+1} = \left(1-\frac{iA}{2}\right)\psi^n$$

$$e^{\frac{iA}{2}}\psi^{n+1} = e^{-\frac{iA}{2}}\psi^n$$

$$\psi^{n+1} = e^{-iA}\psi^n$$

$$\psi^n = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix}^n$$

$$\psi^{n+1} = \frac{1-\frac{iA}{2}}{1+\frac{iA}{2}}\psi^n$$

$$1-\frac{iA}{2} = \begin{pmatrix}\ddots & \ddots & \ddots\end{pmatrix}$$

$$\underbrace{\begin{pmatrix}\ddots & \ddots & \ddots\end{pmatrix}}_{1+\frac{iA}{2}}\psi^{n+1} = \overbrace{\begin{pmatrix}\ddots & \ddots & \ddots\end{pmatrix}}^{1-\frac{iA}{2}}\psi^n$$

$$\begin{pmatrix}\ddots & \ddots\end{pmatrix}\psi^{n+1} = \begin{pmatrix} b \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & 0\\ a_{21} & a_{22} & a_{23}\\ 0 & a_{32} & a_{33}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}b_1\\ b_2\\ b_3\end{pmatrix}$$

$$\Big|\, b \,\Big|\, x \;=\; \Big|\, b \,\Big|$$

$$\begin{pmatrix} x & x & o & o \\ x & x & x & o \\ o & x & x & x \\ o & o & x & x \end{pmatrix} \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$$

$$\psi^{n+1} = \frac{1}{1+\frac{iA}{2}} \left(1 - \frac{iA}{2}\right) \psi^{n}$$