Table III. *Finite-difference expressions for ordinary differential equations*

		Formulae [Notation: $y_j = y(jh)$, $y'_j = y'(jh)$, etc.]	The next non-vanishing term of the Taylor expansion
Formulae for y'	symmetric	$y'_0 = \frac{1}{2h}(-y_{-1} + y_1) +$	$-\frac{1}{6}h^2 y_0''' - \cdots$
		$y'_0 = \frac{1}{12h}(y_{-2} - 8y_{-1} + 8y_1 - y_2) +$	$+\frac{1}{30}h^4 y_0^{\mathrm{V}} + \cdots$
		$y'_0 = \frac{1}{60h}(-y_{-3} + 9y_{-2} - 45y_{-1} + 45y_1 - 9y_2 + y_3) +$	$-\frac{1}{140}h^6 y_0^{\mathrm{VII}} + \cdots$
		$y'_{-1} + 4y'_0 + y'_1 + \frac{3}{h}(y_{-1} - y_1) = 0 +$	$+\frac{1}{30}h^4 y_0^{\mathrm{V}} + \cdots$
		$y'_{-1} + 3y'_0 + y'_1 + \frac{1}{12h}(y_{-2} + 28y_{-1} - 28y_1 - y_2) = 0 +$	$-\frac{1}{420}h^6 y_0^{\mathrm{VII}} - \cdots$
		$y'_{-2} + 16y'_{-1} + 36y'_0 + 16y'_1 + y'_2 + \frac{5}{6h}(5y_{-2} + 32y_{-1} - 32y_1 - 5y_2) = 0 +$	$+\frac{1}{630}h^8 y_0^{\mathrm{IX}} + \cdots$
		$7y'_{-2} + 32y'_{-1} + 12y'_0 + 32y'_1 + 7y'_2 + \frac{45}{2h}(y_{-2} - y_2) = 0 +$	$+\frac{4}{21}h^6 y_0^{\mathrm{VII}} + \cdots$
		$y'_{-2} + 4y'_{-1} + 4y'_1 + y'_2 + \frac{1}{6h}(19y_{-2} - 8y_{-1} + 8y_1 - 19y_2) = 0 +$	$+\frac{1}{35}h^6 y_0^{\mathrm{VII}} + \cdots$
	unsymmetric	$y'_0 = \frac{1}{h}(-y_0 + y_1) +$	$-\frac{1}{2}h y_0'' - \cdots$
		$y'_0 = \frac{1}{2h}(-3y_0 + 4y_1 - y_2) +$	$+\frac{1}{3}h^2 y_0''' + \cdots$
		$y'_0 = \frac{1}{12h}(-3y_{-1} - 10y_0 + 18y_1 - 6y_2 + y_3) +$	$-\frac{1}{20}h^4 y_0^{\mathrm{V}} + \cdots$
		$y'_0 = \frac{1}{60h}(2y_{-2} - 24y_{-1} - 35y_0 + 80y_1 - 30y_2 + 8y_3 - y_4) +$	$+\frac{1}{105}h^6 y_0^{\mathrm{VII}} + \cdots$
		$y'_0 + y'_1 + \frac{2}{h}(y_0 - y_1) = 0 +$	$+\frac{1}{6}h^2 y_0''' + \cdots$
		$y'_{-1} + 9y'_0 + 9y'_1 + y'_2 + \frac{1}{3h}(11y_{-1} + 27y_0 - 27y_1 - 11y_2) = 0 +$	$+\frac{1}{140}h^6 y_0^{\mathrm{VII}} + \cdots$
Formulae for y''	symmetric	$y''_0 = \frac{1}{h^2}(y_{-1} - 2y_0 + y_1) +$	$-\frac{1}{12}h^2 y_0^{\mathrm{IV}} + \cdots$
		$y''_0 = \frac{1}{12h^2}(-y_{-2} + 16y_{-1} - 30y_0 + 16y_1 - y_2) +$	$+\frac{1}{90}h^4 y_0^{\mathrm{VI}} + \cdots$
		$y''_0 = \frac{1}{180h^2}(2y_{-3} - 27y_{-2} + 270y_{-1} - 490y_0 + 270y_1 - 27y_2 + 2y_3) +$	$-\frac{1}{560}h^6 y_0^{\mathrm{VIII}} + \cdots$
		$y''_{-1} + 10y''_0 + y''_1 - \frac{12}{h^2}(y_{-1} - 2y_0 + y_1) = 0 +$	$+\frac{1}{20}h^4 y_0^{\mathrm{VI}} + \cdots$
		$2y''_{-1} + 11y''_0 + 2y''_1 - \frac{3}{4h^2}(y_{-2} + 16y_{-1} - 34y_0 + 16y_1 + y_2) = 0 +$	$-\frac{23}{5040}h^6 y_0^{\mathrm{VIII}} + \cdots$

Table III (continued)

		Formulae [Notation: $y_j = y(jh)$, $y'_j = y'(jh)$, etc.]	The next non-vanishing term of the Taylor expansion
Formulae for y''	symmetric	$23y''_{-2} + 688y''_{-1} + 2358y''_0 + 688y''_1 + 23y''_2 - \frac{15}{h^2}(31y_{-2} + 128y_{-1} - 318y_0 + 128y_1 + 31y_2) = 0 +$	$+\frac{79}{1260}h^8 y_0^{\mathrm{X}} + \cdots$
		$y''_{-1} - 8y''_0 + y''_1 + \frac{9}{h}(y'_{-1} - y'_1) + \frac{24}{h^2}(y_{-1} - 2y_0 + y_1) = 0 +$	$+\frac{1}{2520}h^6 y_0^{\mathrm{VIII}} + \cdots$
		$y''_{-1} - y''_1 + \frac{1}{h}(7y'_{-1} + 16y'_0 + 7y'_1) + \frac{15}{h^2}(y_{-1} - y_1) = 0 +$	$-\frac{1}{315}h^5 y_0^{\mathrm{VII}} + \cdots$
	unsymmetric	$y''_0 = \frac{1}{h^2}(2y_0 - 5y_1 + 4y_2 - y_3) +$	$+\frac{11}{12}h^2 y_0^{\mathrm{IV}} + \cdots$
		$y''_0 = \frac{1}{12h^2}(11y_{-1} - 20y_0 + 6y_1 + 4y_2 - y_3) +$	$+\frac{1}{12}h^3 y_0^{\mathrm{V}} + \cdots$
		$y''_0 = \frac{1}{180h^2}(-13y_{-2} + 228y_{-1} - 420y_0 + 200y_1 + 15y_2 - 12y_3 + 2y_4) +$	$-\frac{1}{90}h^5 y_0^{\mathrm{VII}} + \cdots$
Formulae for y'''	symmetric	$y'''_0 = \frac{1}{2h^3}(-y_{-2} + 2y_{-1} - 2y_1 + y_2) +$	$-\frac{1}{4}h^2 y_0^{\mathrm{V}} + \cdots$
		$y'''_0 = \frac{1}{8h^3}(y_{-3} - 8y_{-2} + 13y_{-1} - 13y_1 + 8y_2 - y_3) +$	$+\frac{7}{120}h^4 y_0^{\mathrm{VII}} + \cdots$
		$y'''_{-1} + 2y'''_0 + y'''_1 + \frac{2}{h^3}(y_{-2} - 2y_{-1} + 2y_1 - y_2) = 0 +$	$-\frac{1}{60}h^4 y_0^{\mathrm{VII}} + \cdots$
		$y'''_{-2} + 56y'''_{-1} + 126y'''_0 + 56y'''_1 + y'''_2 + \frac{120}{h^3}(y_{-2} - 2y_{-1} + 2y_1 - y_2) = 0 +$	$-\frac{1}{252}h^6 y_0^{\mathrm{IX}} + \cdots$
	unsymmetric	$y'''_0 = \frac{1}{2h^3}(-3y_{-1} + 10y_0 - 12y_1 + 6y_2 - y_3) +$	$+\frac{1}{4}h^2 y_0^{\mathrm{V}} + \cdots$
		$y'''_0 = \frac{1}{8h^3}(-y_{-2} - 8y_{-1} + 35y_0 - 48y_1 + 29y_2 - 8y_3 + y_4) +$	$-\frac{1}{15}h^4 y_0^{\mathrm{VII}} + \cdots$
Formulae for y^{IV}	symmetric	$y_0^{\mathrm{IV}} = \frac{1}{h^4}(y_{-2} - 4y_{-1} + 6y_0 - 4y_1 + y_2) +$	$-\frac{1}{6}h^2 y_0^{\mathrm{VI}} + \cdots$
		$y_0^{\mathrm{IV}} = \frac{1}{6h^4}(-y_{-3} + 12y_{-2} - 39y_{-1} + 56y_0 - 39y_1 + 12y_2 - y_3) +$	$+\frac{7}{240}h^4 y_0^{\mathrm{VIII}} + \cdots$
		$y_{-1}^{\mathrm{IV}} + 4y_0^{\mathrm{IV}} + y_1^{\mathrm{IV}} - \frac{6}{h^4}(y_{-2} - 4y_{-1} + 6y_0 - 4y_1 + y_2) = 0 +$	$+\frac{1}{120}h^4 y_0^{\mathrm{VIII}} + \cdots$
		$y_{-2}^{\mathrm{IV}} - 124y_{-1}^{\mathrm{IV}} - 474y_0^{\mathrm{IV}} - 124y_1^{\mathrm{IV}} + y_2^{\mathrm{IV}} + \frac{720}{h^4}(y_{-2} - 4y_{-1} + 6y_0 - 4y_1 + y_2) = 0 +$	$+\frac{5}{21}h^6 y_0^{\mathrm{X}} + \cdots$