Vediamo schemi per le onde

$$\partial_t u = v \partial_x u \qquad u^{n+1}_J = \frac{1}{2}(u^n_{J+1} + u^n_{J-1}) - \frac{v\Delta t}{2\Delta x}(u^n_{J+1} - u^n_{J-1})$$

Problema: propagazione onde gravitazionali

$$u_{tt} - u_{rr} = V u + S \qquad \text{come lo affrontiamo?}$$

introduciamo $\underline{f} = \begin{pmatrix} u \\ u_t + u_r \end{pmatrix}$ $\quad \underline{g} = \begin{pmatrix} u \\ -u_t - u_r \end{pmatrix}$

$$f_t = \begin{pmatrix} u_t \\ u_{tt} + u_{rt} \end{pmatrix} \qquad g_r = \begin{pmatrix} u_r \\ -u_{tr} - u_{rr} \end{pmatrix}$$

$$f_t + g_r = \begin{pmatrix} u_t + u_r \\ u_{tt} - u_{rr} \end{pmatrix}$$

$u_{tt} - u_{rr} = Vu + S$ si può scrivere come

$$f_t + g_r = \begin{pmatrix} u_t + u_r \\ V u + S \end{pmatrix} = h$$

$$f_t = -g_r + h$$

FTCS $\quad f^{n+1}_J = f^n_J + \Delta t \left(h^n_J - \frac{1}{2\Delta r}(g^n_{J+1} - g^n_{J-1}) \right)$

LAX $\quad f^{n+1}_J = \frac{1}{2}\left(f^n_{J+1} + f^n_{J-1}\right) + \Delta t \left(h^n_J - \frac{1}{2\Delta r}\left(g^n_{J+1} - g^n_{J-1}\right) \right)$

$$\begin{pmatrix} u^{n+1}_J \\ (u_t + u_r)^{n+1}_J \end{pmatrix} = \frac{1}{2} \begin{pmatrix} u^n_{J+1} + u^n_{J-1} \\ (u_t + u_r)^n_{J+1} + (u_t + u_r)^n_{J-1} \end{pmatrix} +$$

$$+ \Delta t \begin{pmatrix} (u_t + u_r)^n_J \\ V_J u^n_J + S^n_J \end{pmatrix} - \frac{\Delta t}{2\Delta r} \begin{pmatrix} u^n_{J+1} - u^n_{J-1} \\ (u_t + u_r)^n_{J+1} - (u_t + u_r)^n_{J-1} \end{pmatrix}$$

$$f_t = g_r \qquad \frac{f^{n+1}_J - f^n_J}{\Delta t} = \frac{g^n_{J+1} - g^n_{J-1}}{2\Delta r}$$