Vediamo schemi per le onde

$$\partial_t u = v\,\partial_x u \qquad u_J^{n+1} = \frac{1}{2}\left(u^n_{J+1} + u^n_{J-1}\right) - \frac{v\Delta t}{2\Delta r}\left(u^n_{J+1} - u^n_{J-1}\right)$$

Problema: propagazione onde gravitazionali

$$\boxed{u_{tt} - u_{rr} =} \overset{=0}{V} u + S$$

come lo affrontiamo?

introduciamo $\underline{f} = \begin{pmatrix} u \\ u_t + u_r \end{pmatrix}$ $\quad \underline{g} = \begin{pmatrix} u \\ -u_t - u_r \end{pmatrix}$

$$f_t = \begin{pmatrix} u_t \\ u_{tt} + u_{rt} \end{pmatrix} \qquad g_r = \begin{pmatrix} u_r \\ -u_{tr} - u_{rr} \end{pmatrix}$$

$$f_t + g_r = \begin{pmatrix} u_t + u_r \\ u_{tt} - u_{rr} \end{pmatrix}$$

$u_{tt} - u_{rr} = Vu + S$ si può scrivere come

$$f_t + g_r = \begin{pmatrix} u_t + u_r \\ \underset{=0}{V u + S} \end{pmatrix} = h$$

$$f_t = -g_r + h$$

FTCS $\quad f_J^{n+1} = f_J^n + \Delta t\left(h_J^n - \frac{1}{2\Delta r}\left(g^n_{J+1} - g^n_{J-1}\right)\right)$

LAX $\quad f_J^{n+1} = \frac{1}{2}\left(f^n_{J+1} + f^n_{J-1}\right) + \Delta t\left(h_J^n - \frac{1}{2\Delta r}\left(g^n_{J+1} - g^n_{J-1}\right)\right)$

$$\begin{pmatrix} u_J^{n+1} \\ (u_t + u_r)_J^{n+1} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} u^n_{J+1} + u^n_{J-1} \\ (u_t + u_r)^n_{J+1} + (u_t + u_r)^n_{J-1} \end{pmatrix} +$$

$$+ \Delta t \begin{pmatrix} (u_t + u_r)_J^n \\ V_J u_J^n + S_J^n \end{pmatrix} - \frac{\Delta t}{2\Delta r}\begin{pmatrix} u^n_{J+1} - u^n_{J-1} \\ \boxed{(u_t + u_r)^n_{J+1} - (u_t + u_r)^n_{J-1}} \end{pmatrix}$$

$$f_t = g_r \qquad \frac{f_J^{n+1} - f_J^n}{\Delta t} = \frac{g^n_{J+1} - g^n_{J-1}}{2\Delta r}$$

$u_t + u_x$ $\qquad u = e^{-\frac{(x-vt)^2}{2}}$

$u_t = u\left(+(x-vt)v\right)$ $\qquad u_t + u_x = 0$

$u_x = u\left(-(x-vt)\right)$ $\qquad (1-2x)u$

$u_t = u\left(-(x+vt)v\right)$ $\qquad -2xu$ $\qquad \begin{cases} u_t = c v_x \\ v_t = c u_x \end{cases}$

$u_x = u\left(-(x+vt)\right)$

$u_{tt} = c v_{xt}$
$v_{tx} = c u_{xx}$ $= c^2 u_{xx}$

$u_{tt} - c^2 u_{xx} = 0 \Rightarrow u(x,t)$

---

caso con $c = c(x)$

$$u_{tt} - \partial_x c\, \partial_x c\, u = 0$$

come lo trattiamo?

$$u_t = \partial_x c v$$
$$c v_t = c\, \partial_x c u$$

NB ok $v_t = \partial_x c u$

$\partial_t u_t = u_{tt} = \partial_t \partial_x c v$

$\partial_x c v_t = \partial_x c \partial_x c u$

$\Downarrow$

$$u_{tt} = \partial_x c \partial_x c u$$

---

caso 2D

$$u_{tt} = c^2 (u_{xx} + u_{yy})$$

$u_t = \partial_x c v_x + \partial_y c w_y$

$v_t = \partial_x c u_{i,j}$

$w_t = \partial_y c u$

$u^{n+1}_{i,j} = u^n_{i,j} + \Delta t (\quad)$

$\| $

$\frac{1}{4}\left(u^n_{i-1,j} + u^n_{i+1,j} + u^n_{i,j-1} + u^n_{i,j+1}\right)$

$u_{tt} = \partial_x c^x \partial_x c^x u + \partial_y c^y \partial_y c^y u$

$u_t = \partial_x c^x v + \partial_y c^y w$

$c^x v_t = c^x \partial_x c^x u$

$c^x w_t = c^x \partial_y c^y u$

$$\begin{cases} u_t = c\, v_x \\ v_t = c\, u_x \end{cases}$$

~~FTCS~~

$u_{tt} - c^2 u_{xx} = 0$

LAX

$$\begin{cases} u_J^{n+1} = \cancel{u_J^n} + \dfrac{\Delta t}{2\Delta x} c \left(v_{J+1}^n - v_{J-1}^n \right) + \dfrac{1}{2}\left(u_{J+1}^n + u_{J-1}^n \right) \\ v_J^{n+1} = \cancel{v_J^n} + \dfrac{\Delta t}{2\Delta x} c \left(u_{J+1}^n - u_{J-1}^n \right) + \dfrac{1}{2}\left(v_{J+1}^n + v_{J-1}^n \right) \end{cases}$$

$$u(x,t)\Big|_{t=0} = e^{-\frac{(x+vt)^2}{2}}$$

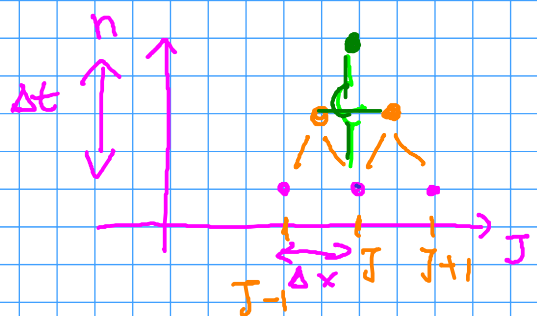
$u_t = (x+vt)\, v\, u(x,t)$

$u_x = (x+vt)\, u(x,t)$

$c = v \qquad v_x = u_x \qquad u_t = c\, v_x$

$u_t = c\, v_x + h_x$

$v_t = c\, u_x + h_y$



$$u_{J+\frac{1}{2}}^{n+1/2} = \underbrace{\frac{1}{2}\left(u_J^n + u_{J+1}^n \right)}_{} + \underbrace{\frac{\Delta t\, c}{2\Delta x}\left(v_{J+1}^n - v_J^n \right)}_{}$$

$$v_{J+\frac{1}{2}}^{n+1/2} = \frac{1}{2}\left(v_J^n + v_{J+1}^n \right) + \frac{\Delta t\, c}{2\Delta x}\left(u_{J+1}^n - u_J^n \right)$$

$$u_J^{n+1} = u_J^n + \frac{\Delta t\, c}{\Delta x}\left(v_{J+\frac{1}{2}}^{n+1/2} - v_{J-\frac{1}{2}}^{n+1/2} \right)$$

$$v_J^{n+1} = v_J^n + \frac{\Delta t\, c}{\Delta x}\left(u_{J+\frac{1}{2}}^{n+1/2} - u_{J-\frac{1}{2}}^{n+1/2} \right)$$

$$u_{J+\frac{1}{2}}^{n+1/2} = u_{J+\frac{1}{2}} + \frac{\Delta t}{2}\left\{ h_J + \underbrace{\frac{c}{\Delta x}\left(v_{J+1}^n - v_J^n \right)}_{} \right\}$$

$u_{J+\frac{1}{2}} \to \frac{1}{2}\left(u_{J+1}^n + u_J^n \right)$

$h_J = h(x_J)$

$h_{J+\frac{1}{2}} \begin{cases} h(x_{J+\frac{1}{2}}) \\ \frac{1}{2}(h_{J+1} + h_J) \end{cases}$

$$u_J^{n+1} = u_J^n + \frac{\Delta t\, c}{\Delta x}\left(v_{J+\frac{1}{2}}^{n+1/2} - v_{J-\frac{1}{2}}^{n+1/2} \right) + \Delta t\, h_J^{n+1/2}$$

$u_{tt} - u_{xx} = 0 \qquad f = \begin{pmatrix} u \\ u_t + u_x \end{pmatrix} \qquad g = \begin{pmatrix} u \\ -(u_t + u_x) \end{pmatrix}$

$$f_t + g_x = \begin{pmatrix} u_t + u_x \\ u_{tt} + \cancel{u_{xt}} - \cancel{u_{tx}} - u_{xx} \end{pmatrix} \qquad f_t + g_x = 0$$

$$u = e^{-\frac{(x-t)^2}{2}} \qquad u_t = +(x-t)\,u \qquad u_x = -(x-t)\,u$$

$$f = \begin{pmatrix} u \\ 0 \end{pmatrix} \qquad g = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$u = e^{-\frac{(x+t)^2}{2}} \qquad u_t = -(x+t)u \qquad u_x = -(x+t)u$$

$$f = \begin{pmatrix} u \\ -2(x+t)u \end{pmatrix} \qquad g = \begin{pmatrix} u \\ +2(x+t)u \end{pmatrix}$$