Quantum Time-Dilation in Qubit Hypersurfaces

Dario Cafasso
University of Pisa

May 30, 2024

The Problem of Time

Evolution without evolution

Time-Dilated Schrödinger equation

Qubit Hypersurfaces

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- Evolution without evolution
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The Problem of Time

[Höhn et al., 2021, Castro-Ruiz et al., 2020, Isham, 1992]

In **General Relativity**: covariance under coordinate transformations $\implies$
time plays no fundamental role.

In practice: time is measured using **physical clocks**,
obeying the laws of our most fundamental theories.

In **Quantum Mechanics**: superposition of energy eigenstates $\implies$
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This conflict is known as the **Problem of Time**!
Different approaches...

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[Page and Wootters, 1983]

> *"If a quantum system is **truly closed**, there's **no place** in the theory for an **external (time)** parameter."*

$$\text{Given} \quad |\Psi\rangle \in \mathcal{H} \implies \quad i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Psi\rangle = \hat{\mathcal{H}}\,|\Psi\rangle = 0$$

The clock C is part of the Universe

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_C + \hat{\mathcal{H}}_S$$

Evolution: correlations between C and S.

$$\text{History state} \mapsto |\Psi\rangle = \int \mathrm{d}t\,|t\rangle_C\,|\psi(t)\rangle_S$$

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- **Ideal clocks** are non-physical

$$\hat{T}\,|t\rangle_C = t\,|t\rangle_C \quad \left[\hat{T}\,,\hat{\mathcal{H}}_C\right] = i\hbar$$

since $\hat{\mathcal{H}}_C$ is unbounded from below. [Isham, 1992]

- **Interactions** introduce temporal non-locality

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\,|\psi(t)\rangle_S = \int \mathrm{d}s\,K(t,s)\,|\psi(s)\rangle_S$$

in the modified Schrödinger equation. [Smith and Ahmadi, 2019]

- **Measurements** make the clock stuck

$$\mathcal{A}(t_f\,|\,t_i) = \delta(t_f - t_i)$$

giving rise to Kuchar's criticisms. [Hausmann et al., 2023]

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[Smith, 2022]

About **measurements**, Page argued:

> *"I simply wish to argue that all of the testable predictions of quantum mechanics appear to arise from one-time conditional probabilities. [...] We can never directly test what happened yesterday, but we can check the consequences that a hypothetical scenario for yesterday has on the situation today." [Page, 1989]*

About **the past**, in the words of Wheeler:

> *"It is wrong to think of the past as 'already existing' in all detail. The 'past' is theory. The past has no existence except as it is recorded in the present." [Wheeler, 1978]*

The Problem of Time
State of the art and Open Problems

[Hausmann et al., 2023]

Approach:	Purified measurement [Giovannetti et al., 2015]	Twirled observable [Hoehn et al., 2021]
Measurement:	Enters the constraint, i.e. the history state	Commutes with the constraint
Time ordering:	Internal	External
Non-ideal clock case:	Discrete-time or non-unitary evolution	Measurements are delocalized

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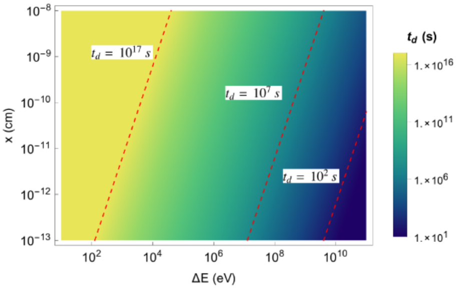
Assuming the relativistic **mass-energy equivalence** $\Longrightarrow$ time-local dynamics, but undefined metric structure.



The clocks get entangled and decoherence time t_d becomes accessible at *GeV* energy scales. [Castro-Ruiz et al., 2017]

Conventional time-dilation can be recovered via a semi-classical coupling with the gravitational field. [Favalli and Smerzi, 2022]

Here we consider the following **Questions**:

- (0.) Is it necessary to introduce a **built-in notion of time**?
- (1.) A **laboratory quantum system** as an evolving universe?
- What changes with a **gravitational-like interaction**?
- What if we attach a **qubit clock** to every point in space?

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Evolution without evolution

The Quantum Clock
The rules for our description

A **finite-dimensional** quantum system C with Hilbert space $\mathcal{H}_C$.

Given a non-degenerate Hermitian operator $\hat{\mathcal{H}}_C \in \mathcal{B}(\mathcal{H}_C)$, we have

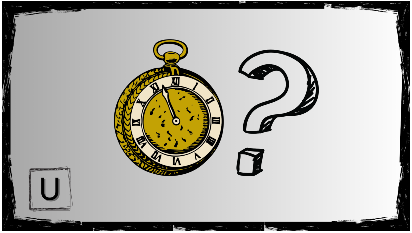
$$\hat{V}_C(t) := e^{-\frac{i}{\hbar}\hat{\mathcal{H}}_C t} \implies \hat{\mathcal{H}}_C \hat{V}_C(t) = i\hbar \frac{\mathrm{d}}{\mathrm{d}t} \hat{V}_C(t)$$

Given a reference state $|e\rangle_C$ we introduce a *time state*

$$|t\rangle_C := V_C(t)\,|e\rangle_C$$

and a resolution of the Identity

$$\mathbb{1}_C = \int \mathrm{d}\mu(t)\,|t\rangle\langle t|_C$$



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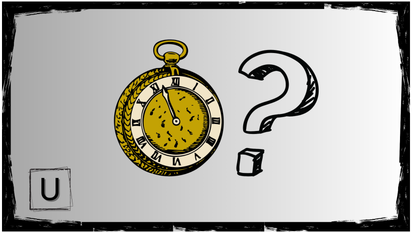

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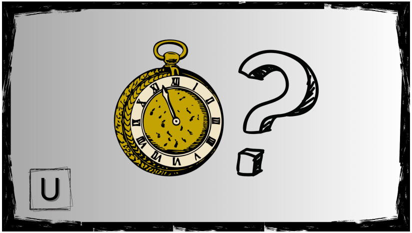

The eigenstates of $\hat{\mathcal{H}}_C$ as

$$\hat{\mathcal{H}}_C \left| w_k \right\rangle_C = \hbar w_k \left| w_k \right\rangle_C \quad \text{with} \quad w_k = w_0 + r_k \, \frac{2\pi}{T} \, ,$$

in which

$$\frac{r_k}{r_1} := \frac{A_k}{B_k} = \frac{w_k - w_0}{w_1 - w_0} \qquad \text{and} \qquad T = \frac{2\pi \, r_1}{w_1 - w_0} \, ,$$

where $A_k, B_k \in \mathbb{N}$ are coprimes and r_1 is the LCM of B_k.

If $r_k = 0, 1, 2, ..., d_C - 1$ and $\Delta t = T/d_C \implies$
two possible procedures for $d_C \to +\infty$:

- Continuous limit, i.e. $\Delta t \to 0$
- Thermodynamic limit, i.e. $T \to +\infty$

Performed in a sequence $\implies$ **ideal clock model**

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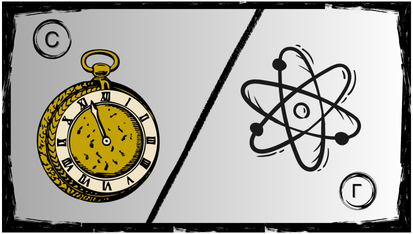
The Quantum Universe

The system to describe

Bipartite quantum system $U = C + S$ with $\mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_S$.

Given $\hat{\mathcal{H}} \in \mathcal{B}(\mathcal{H})$, the state $|\Psi\rangle \in \mathcal{H}$ such that $\hat{\mathcal{H}}\,|\Psi\rangle = 0$ is stationary.



Schmidt decomposition with rank r

$$|\Psi\rangle = \sum_{n=0}^{r-1} \sqrt{\lambda_n}\, |\phi_n\rangle_C\, |\xi_n\rangle_S$$

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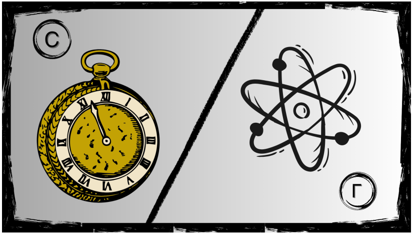

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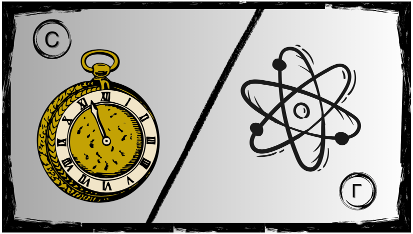

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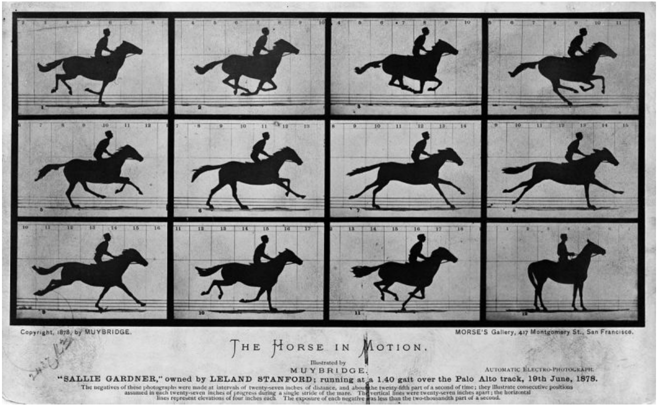
Schmidt + Identity $\implies$ *history state*

$$|\Psi\rangle = \int \mathrm{d}\mu(t)\; a(t)\, |t\rangle_C\, |\psi(t)\rangle_S$$

where

$$|\psi(t)\rangle_S = \frac{1}{a(t)} \langle t|_C\, |\Psi\rangle \quad \text{with} \quad a(t) = \|\langle t|_C\, |\Psi\rangle\|$$

is called a *conditional Schrödinger state*.



Copyright, 1878, by MUYBRIDGE.
MORSE'S Gallery, 417 Montgomery St., San Francisco.
THE HORSE IN MOTION.
Illustrated by
MUYBRIDGE.
AUTOMATIC ELECTRO-PHOTOGRAPH.
"SALLIE GARDNER," owned by LELAND STANFORD; running at a 1.40 gait over the Palo Alto track, 19th June, 1878.
The negatives of these photographs were made at intervals of twenty-seven inches of distance, and about the twenty-fifth part of a second of time; they illustrate consecutive positions assumed in each twenty-seven inches of progress during a single stride of the mare. The vertical lines were twenty-seven inches apart; the horizontal lines represent elevations of four inches each. The exposure of each negative was less than the two-thousandth part of a second.

Considering $\hat{\mathcal{H}} = \hat{\mathcal{H}}_C + \hat{\mathcal{H}}_S$ we have

$$\langle t|_C \hat{\mathcal{H}} |\Psi\rangle = \left(-i\hbar\frac{\mathrm{d}}{\mathrm{d}t} + \hat{\mathcal{H}}_S\right) \langle t|_C |\Psi\rangle = 0$$

or equivalently

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi(t)\rangle_S = -\frac{i}{\hbar}\hat{\mathcal{H}}_S |\psi(t)\rangle_S \;\text{ with }\; a(t) = const$$

Evolution without evolution!

Time-Dilated Schrödinger equation

Multiple Quantum Clocks
Non-interacting case

Clocks structure $J = A, B, ..., M, N$

associated with $\mathcal{H}_C = \otimes_J \mathcal{H}_J$

$$\hat{V}_J(\tau) = e^{-\frac{i}{\hbar}\hat{\mathcal{H}}_J \tau}$$

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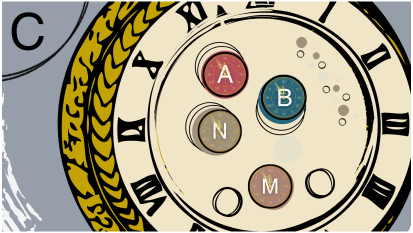

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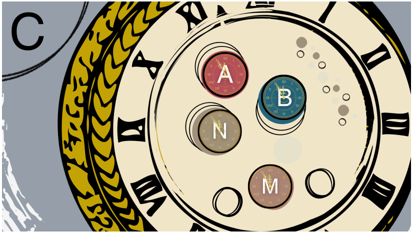

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Conditioning the constraint on A's time, we obtain

$$\langle\tau|_A\, \hat{\mathcal{H}}\, |\Psi\rangle = \left(-i\hbar\frac{\mathrm{d}}{\mathrm{d}\tau} + \sum_{J\neq A} \hat{\mathcal{H}}_J + \hat{\mathcal{H}}_S\right) \langle\tau|_A\, |\Psi\rangle = 0$$

They describe the same dynamics. **What about interactions?**

Multiple Quantum Clocks
History state and dynamical laws

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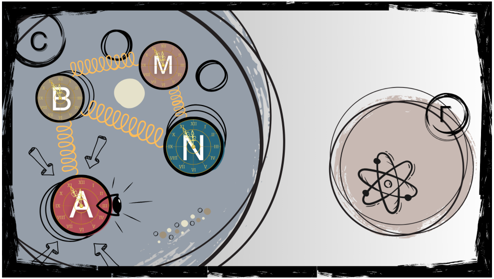
Introducing an interaction between the local clocks $J, K = A, B, ..., M, N$

$$\hat{\mathcal{H}}_C = \sum_J \hat{\mathcal{H}}_J + \hat{\mathcal{H}}_{int} \quad \text{with} \quad \hat{\mathcal{H}}_{int} = -\sum_{J<K} g_{JK}\, \hat{\mathcal{H}}_J \otimes \hat{\mathcal{H}}_K$$



Commutation relation:

$$\left[\hat{\mathcal{H}}_J\,, \hat{\mathcal{H}}_{int}\right] = 0$$

Global-local mapping:

$$|t\rangle_C = e^{-\frac{i}{\hbar}\hat{\mathcal{H}}_{int}\, t} \otimes_J^N |t\rangle_J$$

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Gravitational-like interaction

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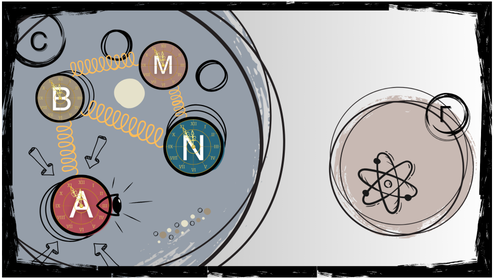

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Introducing an interaction between the local clocks $J, K = A, B, ..., M, N$

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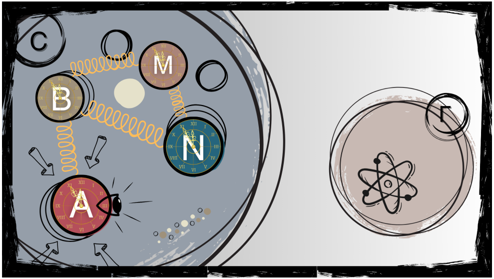

Commutation relation:

$$\left[\,\hat{\mathcal{H}}_J\,,\,\hat{\mathcal{H}}_{int}\,\right] = 0$$

Global-local mapping:

$$|t\rangle_C = e^{-\frac{i}{\hbar}\hat{\mathcal{H}}_{int}t} \otimes_J^N |t\rangle_J$$

Time-Dilated Schrödinger equation

The redshift operator

From $\langle\tau|_A \hat{\mathcal{H}} |\Psi\rangle = 0$, when $g_{AJ} = 0$ for every sub-clock J

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}\tau} |\psi(\tau)\rangle_{U|A} = \hat{\mathcal{H}}^{(A)} |\psi(\tau)\rangle_{U|A}$$

where $\hat{\mathcal{H}}^{(A)} := \hat{\mathcal{H}}_S + \sum_{J\neq A} \hat{\mathcal{H}}_J - \frac{1}{2} \sum_{J,K\neq A} g_{JK} \hat{\mathcal{H}}_J \hat{\mathcal{H}}_K$.

where $\hat{R}$ is called *redshift operator*

$$\hat{R}(A) = \mathbb{1} - \sum_{J\neq A} g_{AJ} \hat{\mathcal{H}}_J$$

From $\langle\tau|_A \hat{\mathcal{H}} |\Psi\rangle = 0$, when $g_{AJ} \neq 0$ for some sub-clock J

$$i\hbar \hat{R}(A) \frac{\mathrm{d}}{\mathrm{d}\tau} |\psi(\tau)\rangle_{U|A} = \hat{\mathcal{H}}^{(A)} |\psi(\tau)\rangle_{U|A}$$

where $\hat{\mathcal{H}}^{(A)} := \hat{\mathcal{H}}_S + \sum_{J\neq A} \hat{\mathcal{H}}_J - \frac{1}{2}\sum_{J,K\neq A} g_{JK}\, \hat{\mathcal{H}}_J \hat{\mathcal{H}}_K$.

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Time-Dilated Schrödinger equation
Gravitational time dilation

Relative distances r_{JK}, gravitational constant G and speed of light c

$$g_{JK} = \frac{G}{r_{JK}\, c^4} \quad \text{and} \quad \hat{\Phi}_J(r_{JK}) := -\frac{G}{r_{JK}\, c^2}\hat{\mathcal{H}}_J \,.$$

From A's perspective, we have

$$i\hbar\left(\mathbb{1} + \frac{1}{c^2}\hat{\Phi}(A)\right)\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \\ = \hat{\mathcal{H}}_S + \sum_{J\neq A}\hat{\mathcal{H}}_J + \sum_{J,K\neq A}\frac{1}{2c^2}\hat{\Phi}_J(r_{JK})\hat{\mathcal{H}}_K \left|\psi(\tau)\right\rangle_{U|A}$$

where $\hat{\Phi}(A) := \sum_{J\neq A}\hat{\Phi}_J(r_{AJ})$.
Well-suited for a semi-classical description, but ...

Relative distances r_{JK}, gravitational constant G and speed of light c

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Well-suited for a semi-classical description, but ...

Relative distances r_{JK}, gravitational constant G and speed of light c

$$g_{JK} = \frac{G}{r_{JK}c^4} \quad \text{and} \quad \hat{\Phi}_J(r_{JK}) := -\frac{G}{r_{JK}c^2}\hat{\mathcal{H}}_J\,.$$

From A's perspective, we have

$$i\hbar\left(1+\frac{1}{c^2}\Phi(A)\right)\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} =$$

$$= M_E c^2 + \hat{\mathcal{H}}_S + \sum_{J\neq A,E}\hat{\mathcal{H}}_J + \sum_{J\neq A,E}\frac{1}{c^2}\Phi(J)\hat{\mathcal{H}}_K\left|\psi(\tau)\right\rangle_{U|A}$$

where $\Phi(J) := -GM_E/r_{JE}$. Then $\mathrm{d}\tau = \sqrt{g_{00}}\,\mathrm{d}t < \mathrm{d}t$!
Well-suited for a semi-classical description, but ...

Expressing the redshift operator as

$$\hat{R}(A) = \mathbb{1} - \hat{\Phi}(A)\,,$$

if the spectral radius $\rho(\hat{\Phi}(A)) < 1$, we get a geometric series:

$$\hat{R}^{-1}(A) = \sum_{n=0}^{+\infty} \hat{\Phi}^n(A)\,.$$

Multiplying by $\hat{R}^{-1}(A)$ from the left, we obtain

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}\tau} \left|\psi(\tau)\right\rangle_{U|A} = \sum_{n=0}^{+\infty} \hat{\Phi}^n(A) \hat{\mathcal{H}}^{(A)} \left|\psi(\tau)\right\rangle_{U|A}$$

Time-Dilation induced Interaction Transfer (TiDIT) mechanism

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Time-Dilation induced Interaction Transfer (TiDIT) mechanism

Quantum time dilation $\xrightarrow{TiDIT}$ new interaction terms!

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \hat{\mathcal{H}}_{eff}^{(A)}\left|\psi(\tau)\right\rangle_{U|A}$$

At -order in g_{AJ}

$$\hat{\mathcal{H}}_{eff}^{(A)} := \hat{\mathcal{H}}_S + \sum_{J\neq A} g_{AJ}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_S + \sum_{J,K\neq A} g_{AJ}g_{AK}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K\hat{\mathcal{H}}_S +$$

$$+\sum_{J\neq A}\hat{\mathcal{H}}_J - \frac{1}{2}\sum_{J,K\neq A}(g_{JK})\,\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K$$

and so on...

Quantum time dilation $\xrightarrow{TiDIT}$ new interaction terms!

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \hat{\mathcal{H}}^{(A)}_{eff}\left|\psi(\tau)\right\rangle_{U|A}$$

At zeroth-order in g_{AJ}

$$\hat{\mathcal{H}}^{(A)}_{eff} := \hat{\mathcal{H}}_S + \sum_{J\neq A} g_{AJ}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_S + \sum_{J,K\neq A} g_{AJ}g_{AK}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K\hat{\mathcal{H}}_S +$$

$$+\sum_{J\neq A}\hat{\mathcal{H}}_J - \frac{1}{2}\sum_{J,K\neq A}\left(g_{JK}\right)\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K$$

and so on...

Quantum time dilation $\xrightarrow{\textit{TiDIT}}$ new interaction terms!

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}\tau} \left|\psi(\tau)\right\rangle_{U|A} = \hat{\mathcal{H}}^{(A)}_{\textit{eff}} \left|\psi(\tau)\right\rangle_{U|A}$$

At first-order in g_{AJ}

$$\hat{\mathcal{H}}^{(A)}_{\textit{eff}} := \hat{\mathcal{H}}_S + \sum_{J\neq A} g_{AJ}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_S + \sum_{J,K\neq A} g_{AJ}g_{AK}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K\hat{\mathcal{H}}_S + \\ + \sum_{J\neq A}\hat{\mathcal{H}}_J - \frac{1}{2}\sum_{J,K\neq A}\left(g_{JK} - 2g_{AK}\right)\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K$$

and so on...

Quantum time dilation $\xrightarrow{\textit{TiDIT}}$ new interaction terms!

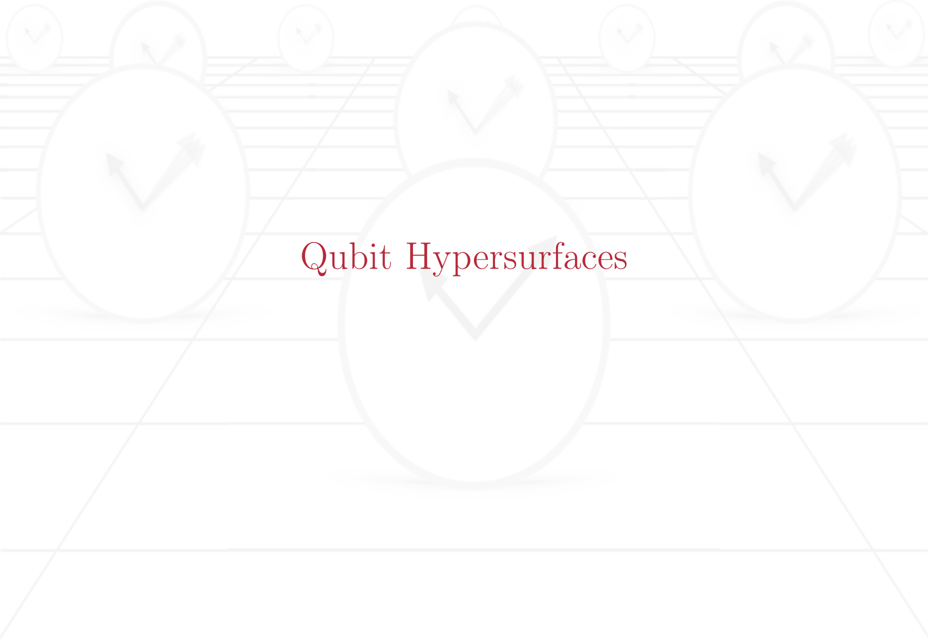
$$i\hbar \frac{\mathrm{d}}{\mathrm{d}\tau} |\psi(\tau)\rangle_{U|A} = \hat{\mathcal{H}}^{(A)}_{\mathit{eff}} |\psi(\tau)\rangle_{U|A}$$

At second-order in g_{AJ}

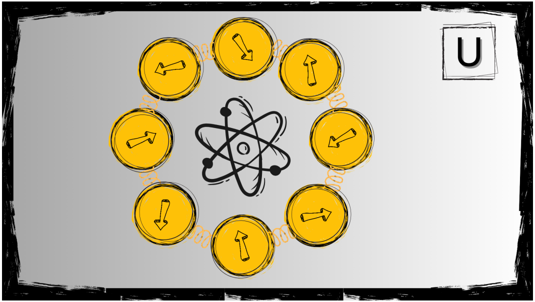
$$\hat{\mathcal{H}}^{(A)}_{\mathit{eff}} := \hat{\mathcal{H}}_S + \sum_{J\neq A} g_{AJ}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_S + \sum_{J,K\neq A} g_{AJ}g_{AK}\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K\hat{\mathcal{H}}_S + \\ + \sum_{J\neq A}\hat{\mathcal{H}}_J - \frac{1}{2}\sum_{J,K\neq A}(g_{JK} - 2g_{AK})\left(\mathbb{1} + \sum_{M\neq A} g_{AM}\hat{\mathcal{H}}_M\right)\hat{\mathcal{H}}_J\hat{\mathcal{H}}_K$$

and so on...

Qubit Hypersurfaces



$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_S + \sum_J h_J \hat{\sigma}_J^x + \sum_{\langle J,K \rangle} g_{JK} \hat{\sigma}_J^x \hat{\sigma}_K^x$$



For a qubit clock A we have

$$\hat{\mathcal{H}}_A = \hbar w\, \hat{\sigma}^x_A \quad \Longrightarrow \quad \hat{V}_A(\tau) = e^{-iw\tau\hat{\sigma}^x_A}\,.$$

Time states are obtained as

$$|e\rangle_A = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \quad \Longrightarrow \quad |\tau\rangle_A = \hat{V}_A(\tau)\,|e\rangle_A\,.$$

Orthogonal with $\tau_0 = 0$ and $\tau_1 = \pi/2w$

$$|\tau_0\rangle_A = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \quad \text{and} \quad |\tau_1\rangle_A = \frac{-i}{\sqrt{2}}(|+\rangle - |-\rangle)$$

The resolutions of the Identity

$$\mathbb{1}_A = \sum_{n=0,1} |\tau_n\rangle\langle\tau_n|_A = \frac{2w}{\pi}\int_0^{\frac{\pi}{w}} \mathrm{d}\tau\ |\tau\rangle\langle\tau|_A\,.$$

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Qubit Hypersurfaces
The rules for the qubit clock

For a qubit clock A we have

$$\hat{\mathcal{H}}_A = \hbar w \, \hat{\sigma}_A^x \quad \Longrightarrow \quad \hat{V}_A(\tau) = e^{-iw\tau\hat{\sigma}_A^x} \, .$$

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Given $|\Psi\rangle \in \mathcal{H}$ such that $\hat{\mathcal{H}}\,|\Psi\rangle = 0$, we have

$$
\begin{aligned}
|\Psi\rangle &= \frac{2w}{\pi}\int_0^{\frac{\pi}{w}} \mathrm{d}\tau\, a(\tau)\; |\tau\rangle_A\, |\psi(\tau)\rangle_{U|A} \\
&= a_0\,|\tau_0\rangle_A\,|\psi(\tau_0)\rangle_{U|A} + a_1\,|\tau_1\rangle_A\,|\psi(\tau_1)\rangle_{U|A}
\end{aligned}
$$

where

$$
a(\tau) = \sqrt{a_0^2\cos^2(w\tau) + a_1^2\sin^2(w\tau)}
$$

Notice:

$$
\hat{\mathcal{H}} = \hat{\mathcal{H}}_A + \hat{\mathcal{H}}_{U|A} \implies a(\tau) = a_1 = a_0 = 1/\sqrt{2}
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$$

A two-qubit clock model

$$\hat{\mathcal{H}} = \hbar w\left(\hat{\sigma}_A^x + \alpha\hat{\sigma}_B^x\right) + \hat{\mathcal{H}}_S$$

The Schrödinger equation reads

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \left(\alpha\hbar w\hat{\sigma}_B^x + \hat{\mathcal{H}}_S\right)\left|\psi(\tau)\right\rangle_{U|A}$$

where

$$\hat{R} = \mathbb{1} - g\hat{\sigma}_B^x$$

A two-qubit clock model

$$\hat{\mathcal{H}} = \hbar w\left(\hat{\sigma}_A^x + \alpha\hat{\sigma}_B^x - g\hat{\sigma}_A^x\hat{\sigma}_B^x\right) + \hat{\mathcal{H}}_S$$

The Schrödinger equation reads

$$i\hbar\hat{R}\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \left(\alpha\hbar w\hat{\sigma}_B^x + \hat{\mathcal{H}}_S\right)\left|\psi(\tau)\right\rangle_{U|A}$$

where

$$\hat{R} = \mathbb{1} - g\hat{\sigma}_B^x$$

The redshift operator takes the form

$$\hat{R} = \mathbb{1} - g\hat{\sigma}^x_B \implies \lambda = \begin{cases} 1+g & \text{on } \left|-\right\rangle_B \\ 1-g & \text{on } \left|+\right\rangle_B \end{cases}$$

When $g \neq 1$, i.e. $\hat{R}$ is invertible, we observe two new effects...

$$i\hbar(1-g^2)\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} =$$

$$= \left(\alpha\hbar w g + \alpha\hbar w \hat{\sigma}^x_B \quad \hat{\mathcal{H}}_S + g\hat{\sigma}^x_B\hat{\mathcal{H}}_S\right)\left|\psi(\tau)\right\rangle_{U|A}$$

Qubit Hypersurfaces
The TiDIT mechanism

The redshift operator takes the form

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When $g \neq 1$, i.e. $\hat{R}$ is invertible, we have $\mathrm{d}\tau = \left(1 - g^2\right)\mathrm{d}t$

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Time-Dilation!

The redshift operator takes the form

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Interaction Transfer!

The redshift operator takes the form

$$\hat{R} = \mathbb{1} - g\hat{\sigma}^x_B \implies \lambda = \begin{cases} 1+g & \text{on } \left|-\right\rangle_B \\ 1-g & \text{on } \left|+\right\rangle_B \end{cases}$$

When $g \neq 1$, i.e. $\hat{R}$ is invertible, we have $\mathrm{d}\tau = \left(1-g^2\right)\mathrm{d}t$

$$i\hbar\left(1-g^2\right)\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \\ = \left(\alpha\hbar w g + \alpha\hbar w\hat{\sigma}^x_B + \hat{\mathcal{H}}_S + g\hat{\sigma}^x_B\,\hat{\mathcal{H}}_S\right)\left|\psi(\tau)\right\rangle_{U|A}$$

Time-Dilation induced Interaction Transfer (TiDIT) mechanism

Qubit Hypersurfaces
The TiDIT mechanism

Quantum time dilation $\xrightarrow{\textit{TiDIT}}$ new interaction terms!

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}\tau} \left|\psi(\tau)\right\rangle_{U|A} = \hat{\mathcal{H}}_{eff}^{(A)} \left|\psi(\tau)\right\rangle_{U|A}$$

$$\hat{\mathcal{H}}_{eff}^{(A)} = \frac{1}{1-g^2}\left(\alpha\hbar w g + \alpha\hbar w \hat{\sigma}_B^x + \hat{\mathcal{H}}_S + g\hat{\sigma}_B^x \hat{\mathcal{H}}_S\right)$$

When $g = 1$ the history state becomes

$$|\Psi\rangle = \int \mathrm{d}\mu(\tau)\, a_-(\tau) \left|\tau\right\rangle_A \left|-\right\rangle_B \left|\phi_-(\tau)\right\rangle_S + \\ + \left(\int \mathrm{d}\mu(\tau)\, a_+(\tau) \left|\tau\right\rangle_A\right) \left|+\right\rangle_B \left|-\alpha\hbar w\right\rangle_S$$

Quantum time dilation $\xrightarrow{TiDIT}$ new interaction terms!

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}\tau} |\psi(\tau)\rangle_{U|A} = \hat{\mathcal{H}}^{(A)}_{eff} |\psi(\tau)\rangle_{U|A}$$

$$\hat{\mathcal{H}}^{(A)}_{eff} = \frac{1}{1-g^2}\left(\alpha\hbar w g + \alpha\hbar w \hat{\sigma}^x_B + \hat{\mathcal{H}}_S + g\hat{\sigma}^x_B \hat{\mathcal{H}}_S\right)$$

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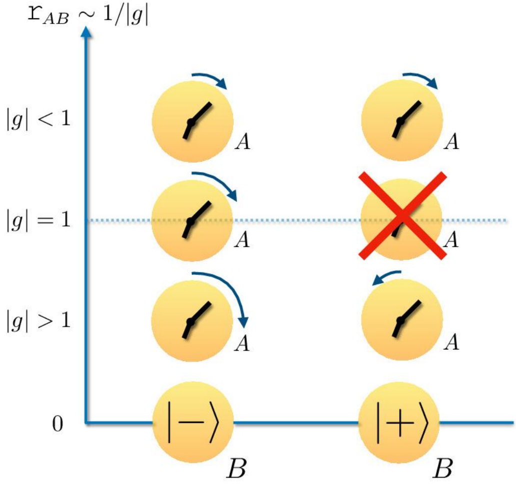
Effective Hamiltonian diverges

$\Downarrow$

The dynamical description is frozen "at the horizon"



Conclusion

Take home messages:

(1.) A history can be associated with a laboratory quantum system.

Gravitational-like interaction introduces a redshift operator.

The spin-spin coupling gives quantum time dilation and TiDiT.

Perspectives:

Generalization to more complex interaction terms.

Generalization to dynamical spatial degrees of freedom.

Explore the implications of the TiDiT mechanism in gravity.

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Thank you for your (classical) time!

References

References I

[Castro-Ruiz et al., 2020] Castro-Ruiz, E., Giacomini, F., Belenchia, A., and Brukner, v. (2020).
Quantum clocks and the temporal localisability of events in the presence of gravitating quantum systems.
Nature Commun.

[Castro-Ruiz et al., 2017] Castro-Ruiz, E., Giacomini, F., and Časlav Brukner (2017).
Entanglement of quantum clocks through gravity.
Proceedings of the National Academy of Sciences.

[Favalli and Smerzi, 2022] Favalli, T. and Smerzi, A. (2022).
A model of quantum spacetime.
AVS Quantum Science.

References II

[Giovannetti et al., 2015] Giovannetti, V., Lloyd, S., and Maccone, L. (2015).
Quantum time.
Phys. Rev. D.

[Hausmann et al., 2023] Hausmann, L., Schmidhuber, A., and Castro-Ruiz, E. (2023).
Measurement events relative to temporal quantum reference frames.

[Hoehn et al., 2021] Hoehn, P. A., Smith, A. R. H., and Lock, M. P. E. (2021).
Equivalence of Approaches to Relational Quantum Dynamics in Relativistic Settings.
Front. in Phys.

[Höhn et al., 2021] Höhn, P. A., Smith, A. R., and Lock, M. P. (2021).
Trinity of relational quantum dynamics.
Physical Review D, 104(6).

[Isham, 1992] Isham, C. J. (1992).
Canonical quantum gravity and the problem of time.

[Page, 1989] Page, D. N. (1989).
Time as an Inaccessible Observable.
NSF-ITP-89-18.

[Page and Wootters, 1983] Page, D. N. and Wootters, W. K. (1983).
Evolution without evolution.
Phys. Rev. D.

[Smith, 2022] Smith, A. (2022).
The page-wootters formalism: Where are we now?

[Smith and Ahmadi, 2019] Smith, A. R. H. and Ahmadi, M. (2019).
Quantizing time: Interacting clocks and systems.
Quantum.

References IV

[Wheeler, 1978] Wheeler, J. A. (1978).
The "past" and the "delayed-choice" double-slit experiment.
In Marlow, A. R., editor, *Mathematical Foundations of Quantum Theory*. Academic Press.