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Entanglement of quantum clocks via gravity

Page and Wootters formalism

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Contents

Introduction to Quantum Gravity

When we talk about **Quantum Gravity**, rather than a theory, we refer to a **family of problems** about the relation between our fundamental theories:

Quantum Mechanics and General Relativity

These problems lead physicists to pursue different approaches toward the "complete" picture, but all these paths originated from the same question:

If spacetime is a dynamical field and dynamical fields are quantized, then should we expect a kind of quantum spacetime?[1]
there's something wrong in our foundations?

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there's something wrong in our foundations?

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In the present picture, our fundamental theories are able to describe with astonishing precision a huge amount of natural phenomena, but, in the words of Einstein[2]:

> *"One is struck [by the fact] that the theory [of special relativity]... introduces two kinds of physical things, i.e., measuring rods and clocks, all other things, e.g., the electromagnetic field, the material point, etc. This, in a certain sense, is inconsistent..."*

and so, as pointed out by C. Rovelli[3]:

> *"The search for a quantum theory of gravity raises once more old questions such as: What is space? What is time? What is the meaning of "moving"? Is motion to be defined with respect to objects or with respect to space? And also: What is causality? What is the role of the observer in physics?"*

[2] Cfr. ref. [2] for the sources.
[3] Cfr. section 1.1.3 of ref. [1].

Our work is based on a different way of conceiving time, not more as a classical parameter, but defining it from an operative point of view as the observable measured by the clock of an observer.

As pointed out in ref. [2]:

> *"For the sake of consistency, it is natural to assume that the clocks, being physical, behave according to the principles of our most fundamental physical theories."*

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How can we take care of the physical, observer dependent, nature of time?

Timeless Quantum Mechanics

PaW framework
Quantum Time

Here we follow the approach introduced by Page and Wootters[4], and then modified by Giovannetti, Lloyd and Maccone[5] to define the concept of "Quantum Time" as the one measured via the "Time Operator".

We can describe a clock as a quantum system C, such that

$$|t\rangle_C \in \mathcal{H}_C \qquad \hat{T}\,|t\rangle_C = t\,|t\rangle_C \qquad \hat{H}_C = \hbar\hat{\Omega} \tag{1}$$

where $\hat{\Omega}$ is the conjugate variable to $\hat{T}$, i.e. $[\hat{T}, \hat{\Omega}] = i\mathbb{1}_C$. We also have

$$_C\langle t'|t\rangle_C = \delta(t'-t) \quad and \quad _C\langle t'|\,\hat{\Omega}\,|t\rangle_C = \delta(t'-t)\partial_t \tag{2}$$

[4] Cfr. ref. [3] for the original approach to this framework.
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In this framework, we assume that the time measured by **the clock** C is used by an observer to make measures about the system S.
It follows that C **has to be regarded as a part of the quantum system** $S+C$, subject to a global constraint $\hat{H}$ in the form of a **Wheeler-DeWitt Equation**, such that

$$|\Psi\rangle \in \mathcal{H} := \mathcal{H}_C \otimes \mathcal{H}_S \qquad \hat{H}\,|\Psi\rangle = 0 \;\; with \;\; \hat{H} = \hat{H}_C \otimes \mathbb{1}_S + \mathbb{1}_C \otimes \hat{H}_S \tag{3}$$

This allows an observer-dependent and timeless description of QM!

Note that we are assuming C to be a perfect clock, i.e. isomorphic to a particle on a line, and $\hat{H}_S$ to be time independent. For generalization to the time dependent case, cfr. ref. [4].

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PaW framework
Schrödinger Equation

Indeed, we recover the **Schrödinger Equation** from equation 3 conditioning on time via projection on the generalized eigenvectors of the $\hat{T}$ operator. Using equations 2, we find:

$$_C \langle t | \hat{H} | \Psi \rangle = 0 \iff \partial_t |\psi(t)\rangle_S = \hat{H}_S |\psi(t)\rangle_S \tag{4}$$

where $|\psi(t)\rangle_S := {}_C \langle t | \Psi \rangle$ is the **Schrödinger State** we know from QM.[6]

A general solution of equation 3 is called **history state** and can be written as

$$|\Psi\rangle = \int dt\, |t\rangle_C \otimes |\psi(t)\rangle_S \tag{5}$$

It incorporates all of the information of the quantum system.

[6] Some definitions may vary because of the introduction of a normalization factor into the time generalized eigenvectors. Note that here they're improper states. Cfr. ref. [4].

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At this point:

- How to describe measures and probabilities in this framework?
- Which criticisms have been raised about the clock model?
- Which is the interpretation of the emerging picture?

In ref. [4], the **correct expression** for the n-times **propagator** has been recovered. For the derivation, they start adopting the **von Neumann formulation of the measurement apparatus** and then they cast the outcome in the time dependent PaW framework.

It is a "purification" of the procedure, **introducing a memory system** M for each measure. This allows us to **write the probability** of n-outcomes in n-times **accordingly to the Bayes rule** for conditional probabilities.

As an example, the conditional probability of getting b at time t given a at time $t > t_0$ reads

$$P(b,t|a,t_0) = \frac{P(b,a,t)}{P(a,t_0)} = \frac{\left\| \left({}_C\langle t| \otimes {}_{M_b}\langle b| \otimes {}_{M_a}\langle a| \right) |\Psi\rangle \right\|^2}{\left\| \left({}_C\langle t_0| \otimes {}_{M_a}\langle a| \right) |\Psi\rangle \right\|^2} \tag{6}$$

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Timeless QM

Conditional probabilities

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Here we describe the **von Neumann formulation** of the measurement apparatus.[7]

We consider the system S as the sum of the **subsystem of interest** Q and a **memory system** M. Then, we describe the **process of measuring** as an **instantaneous transformation** which induces the unitary mapping

$$|\psi(t_I)\rangle_Q \otimes |r\rangle_M \longmapsto \sum_a \hat{K}_a |\psi(t_I)\rangle_Q \otimes |a\rangle_M \tag{7}$$

in which $\{\hat{K}_a\}$ are called **Kraus operators** and fulfill the normalization condition $\sum_a \hat{K}_a^\dagger \hat{K}_a = \mathbb{1}$. The transformation can be expressed as an **interaction term** between Q and M in the Hamiltonian of S as

$$\hat{H}_S(t) = \hat{H}_Q + \hat{f}(t) \qquad \hat{f}(t) = \delta(t - t_I)\hat{h}_{QM} \tag{8}$$

[7] More details in section "Measurements" of ref. [4].
In eq. 7, the memory state "r" stands for "ready" and "a" stands for "after".
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Let's define the unitary operator $\hat{V}_{QM} := e^{-\frac{i}{\hbar}\hat{h}_{QM}}$.[8] It is responsible of the mapping in equation 7 and its action can be expressed as

$$\hat{V}_{QM}\left|\psi(t)\right\rangle_Q \otimes \left|r\right\rangle_M = \sum_a \left\langle a\right|_M \hat{V}_{QM}\left|r\right\rangle_M \left|\psi(t)\right\rangle_Q \otimes \left|a\right\rangle_M = \sum_a \hat{K}_a\left|\psi(t)\right\rangle_Q \otimes \left|a\right\rangle_M = \sum_a \sqrt{P(a,t)}\left|\phi_a\right\rangle_Q \otimes \left|a\right\rangle_M \quad (9)$$

in which, according to equation 6, $\left|\phi_a\right\rangle_Q$ is the state of Q after the collapse

$$\left|\phi_a\right\rangle_Q := \frac{1}{\sqrt{P(a,t)}}\hat{K}_a\left|\psi(t)\right\rangle_Q \qquad P(a,t) := \left\|\hat{K}_a\left|\psi(t)\right\rangle_Q\right\|^2$$

[8] Its relation with the experimental apparatus is far away simple and is the needing for the introduction of the Kraus operators.

The evolution of the system is the described by the unitary operator

$$\hat{U}_S(t, t_0) = \begin{cases} \hat{U}_Q(t, t_0) & \forall t < t_I \\ \hat{U}_Q(t, t_I)\hat{V}_{QM}\hat{U}_Q(t_I, t_0) & \forall t > t_I \end{cases} \tag{10}$$

Putting together equations 7, 9 and 10, the history state in 5 becomes

$$|\Psi\rangle = \int_{-\infty}^{t_I} \mathrm{d}t\, |t\rangle_C \otimes |\psi(t)\rangle_Q \otimes |r\rangle_M + \int_{t_I}^{+\infty} \mathrm{d}t\, |t\rangle_C \otimes \hat{U}_Q(t, t_I) \sum_a \hat{K}_a\, |\psi(t_I)\rangle_Q \otimes |a\rangle_M \tag{11}$$

Instantaneous transformations are experimentally impossible and a generalization to short-time interaction is necessary for a realistic description. So, **the present description is not free of ambiguities!**

Another important criticism, is about the clock's Hamiltonian:

The **clock model** we are using in this work **is unrealistic** and has to be intended as an approximation of a n-level system. More realistic choices of the clock, with lower bounded Hamiltonian, led to closed time-like curves. Other choices are a straightforward modification of the above theory and some of them can be found in references [24,29] of ref. [4].

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Finally, as a criticism, some physicists argued that this framework describes a **stationary universe**. The most interesting observation pointed out by the authors of ref. [4] is that we can define two points of view:

- the **external observer**, who sees the whole universe as a static system whose state is an eigenstate of its global Hamiltonian, i.e. equation 3.
- the **internal observer**, who can anyway observe a dynamics, i.e. time dependence, and Born-rule induced wave-function collapses.

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- the **internal observer**, who can anyway observe a dynamics, i.e. time dependence, and Born-rule induced wave-function collapses.

This is a kind of generalization of the **relativity principle**, which lead us to the introduction of the concept of **Time Reference Frame**.

What is a "Time Reference Frame"?

According to ref. [5],

- We define a "Time Reference Frame" as a quantum temporal reference frame associated to a quantum clock.
- Temporal localization of events, i.e. quantum measurements on the system, is defined by the time reference frame of an observer.
- In the presence of more than one clock, each of them describes the evolution of the rest system according to its "proper time".

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Time Reference Frames
Definition and basic concepts

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- Temporal localization of events, i.e. quantum measurements on the system, is defined by the time reference frame of an observer.
- In the presence of more than one clock, each of them describes the evolution of the rest system according to its "proper time".

Suppose to have a system made up of two clocks $I = A, B$ and a subsystem of interest S with a trivial Hamiltonian $\hat{H}_S = 0$. No interaction is considered between A, B, S.

Accordingly to equations 3 and 8, we introduce in S an experimental apparatus with two memory systems, each associated to one clock

$$\hat{H} = \hat{H}_A \otimes \mathbb{1}_{\bar{A}} + \hat{H}_B \otimes \mathbb{1}_{\bar{B}} + \hat{f}_A(\hat{T}_A) + \hat{f}_B(\hat{T}_B) \tag{12}$$

in which, defining R_I as the rest of $I + M_I + Q$, we have

$$\hat{f}_I(\hat{T}_I) = |t_I\rangle\langle t_I| \otimes \hat{h}_{QM_I} \otimes \mathbb{1}_{\bar{R}_I}$$

Expressing equation 5 in terms of $I = A, B$ reference frames, we have

$$|\Psi\rangle = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes |\psi_I(t_I)\rangle_{\bar{I}} = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes \hat{U}(t_I) \, |\psi_I(0)\rangle_{\bar{I}}$$

Using the relation

$$|\psi_A(t_A)\rangle_{\bar{A}} = \hat{U}_{\bar{A}}(t_A) \hat{S}_{AB} \hat{U}^{\dagger}_{\bar{B}}(t_B) \, |\psi_B(t_B)\rangle_{\bar{B}}$$

in which

$$\hat{S}_{AB} = \int \mathrm{d}t_B \, |t_B\rangle_B \otimes \langle t_A = 0|_A \, \hat{U}_{\bar{B}}(t_B) e^{i\hat{T}_A \hat{H}_S}$$

We can express them in terms of each others as

$$|\Psi\rangle = \int \mathrm{d}t_A \, |t_A\rangle_A \otimes e^{-it_A \hat{H}_B} \mathcal{T}\left\{ e^{-i\int_0^{t_A} \mathrm{d}s (\hat{f}_A(s) + \hat{f}_B(s + \hat{T}_B))} \right\} |\psi_A(0)\rangle_{\bar{A}}$$

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Interacting quantum clocks

Interacting quantum clocks

Ref. PNAS [2].

Perspectives

Perspective

Future research directions are:

- classical limit

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Thank you for your attention!