Locally mediated entanglement through gravity from first principles

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Observing entanglement generation mediated by a local field certifies that the field cannot be classical. This information–theoretic argument is at the heart of the race to observe gravity–mediated entanglement in a 'table-top' experiment. Previous derivations of the effect assume the locality of interactions, while using an instantaneous interaction to derive the effect. We correct this by giving a first principles derivation of mediated entanglement using linearised gravity. The framework is Lorentz covariant—thus local—and yields Lorentz and gauge invariant expressions for the relevant quantum observables. For completeness we also cover the electromagnetic case. An experimental consequence of our analysis is the possibility to observe *retarded* mediated entanglement, which avoids the need of taking relativistic locality as an assumption. This is a difficult experiment for gravity, but could be feasible for electromagnetism. Our results confirm that the entanglement is dynamically mediated by the gravitational field.

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It is often assumed that quantum gravitational effects only show up at high–energy or short length scale regimes, out of reach of current technology. Recent proposals for low–energy table–top experiments could be game changers [1–7]. Rapid technological progress in quantum manipulation of solid-state matter at larger microscopic mass scales [8, 9], and in gravitational measurements at smaller mesoscopic mass scales [10], have raised expectations that probing gravitational phenomena of quantum source masses may be within reach [11]. In particular, it might be possible to detect entanglement between two masses generated by their gravitational interaction, or GIE (Gravity Induced Entanglement)[2, 3].

Verifying GIE would spectacularly support what is expected from most tentative quantum gravity theories: spacetime has quantum properties. It would also falsify—or put limits on—the alternatives that have been considered in the absence of empirical evidence for quantum gravity: for example, that gravity is a classical field obeying semiclassical Einstein equations [12–14], or that quantum mechanics breaks down at a scale before measurable quantum gravity effects appear [15–17]. Specifically, a general quantum information argument has been invoked to argue that GIE would rule out the possibility that the gravitational field is a local, classical field [2, 3, 18–21]. The argument is based on the fact that local operations and classical communication (LOCC) cannot produce entanglement according to quantum theory [22], as well as to more general approaches [19, 21]. Therefore, observing GIE certifies that gravity cannot be described by classical physics.

However, this interpretation has been debated. While some claims have been made that the experiment is irrelevant to quantum gravity [23–25], other claims state quite oppositely that it tests an effect of 'virtual gravitons' [18, 26–28].

Here we argue that these claims respectively either understate or overstate the implications of the experiment. The confusion stems from the fact that the effect has been computed within the approximation of an instantaneous interaction—in other words, since the experiment is in the Newtonian regime, gravity can be described without the need of a dynamical field. This is sufficient for estimating the final state of the masses, but obfuscates the core of the argument: locality of interactions. A firm grasp of the role of locality is critical to deduce that entanglement is mediated by a field that cannot be classical.

To clarify the role of locality, it is better to have a manifestly local first–principles derivation of the effect. A tool to do so is the path–integral formalism, which keeps the symmetries explicit. Starting from two established paradigms of physics—general relativity and quantum field theory—we show here that the quantum phases responsible for gravity mediated entanglement production are on–shell actions (cf. Eq. (5)), which we compute below (cf. Eq. (9)). This provides an explicitly Lorentz invariant, hence local, and gauge invariant description of GIE. This is our main result.

The framework is also applicable to arbitrary non–stationary relativistic particle trajectories, and to an arbitrary number of particles with different masses.[1] Crucially, it shows that two quantum masses that start off in a product state begin to get entangled due to their gravitational interaction only *after a light crossing time has elapsed*.

This confirms that GIE requires the gravitational field to be in a quantum superposition, and therefore amounts to witnessing a quantum superposition of spacetimes [32]. In fact, the effect dynamically arises due to a superposition of macroscopically distinct field configurations: information travels in a quantum superposition of wavefronts in the field.

Finally, we consider an experiment where both entanglement and effects of the locality of the interaction can be observed, thus, incompatible with an instantaneous interaction description. For gravity, this experiment is currently out of reach. We show that the analogous experiment is possible in electromagnetism, which would inform the outcome of the gravitational experiment.

Locally Mediated Entanglement from the Path Integral of the Quantum Field

Consider the experimental setup in [2] that comprises two[2] masses m_a ($a = 1, 2$), each with an embedded spin–1/2 degree of freedom. At time t^i, the particles are at initial positions x_a^i and are then put in a spin–dependent planar motion $x_a^{s_a}(t)$, by being passed through inhomogeneous and possibly time varying magnetic fields B_z oriented along the axis z, perpendicular to the plane of motion. We denote $|\sigma\rangle = \otimes_a |s_a\rangle$ the spin configurations, where $s_a \in \{\uparrow, \downarrow\}$.

The spacetime curvature is assumed to be small, so the linear approximation of general relativity holds. We denote the gravitational perturbation sourced by the particles as $\mathcal{F}$.[3] Preparing each of the particles in a spin superposition state, the magnetic field B_z drives the particles into a path–superposition by coupling to the spins s_a. The field $\mathcal{F}$ couples to the masses m_a of the moving particles. After recombining the interferometer paths at time t_2, see Fig. 1, a spin measurement is performed on each particle at time t^f. The spins can become entangled due to the gravitational interaction between the masses m_a. The coupling of B_z with $\mathcal{F}$, the backreaction of s_a on B_z, and the backreaction of $\mathcal{F}$ on the particle trajectories $x_a^{s_a}(t)$, are taken to be negligible.

The partition function of the joint system is

$$\int \mathcal{D}\mathcal{F}'\mathcal{D}x' \, \exp\left(\frac{iS}{\hbar}\right) \tag{1}$$

where $S = [x'_a(t), \mathcal{F}'(x,t); m_a, B_z, \sigma]$ and $\mathcal{D}x' = \prod_a \mathcal{D}x'_a$. The integration is over field configurations $\mathcal{F}'(x,t)$ and paths of the particles $x'_a(t)$. The quantities m_a, B_z and σ are not affected by the dynamics in (1) .

The path that each particle takes is determined by the spin, which does not change along the path. Thus, the joint evolution is of the form

$$U_{i\to f} = \sum_\sigma |\sigma\rangle\langle\sigma| \otimes U^\sigma_{i\to f}, \tag{2}$$

with $U^\sigma_{i\to f}$ defined by folding (1) with initial and final states $|\psi^{i,f}\rangle = |\mathcal{F}^{i,f}[x_a^{i,f}]\rangle \otimes |x_a^{i,f}\rangle$, where the paths and field states are assumed pure and separable at $t^{i,f}$. The boundary conditions are taken the same for all spin configurations σ. The time t^f is far enough in the future for the field to have time to relax in the vicinity of the spin measurement. It is sufficient to take the boundary conditions as given by the static Newtonian field $\mathcal{F}^{i,f}[x_a^{i,f}]$ of masses sitting at the initial and final particle positions $x_a^{i,f}$.

The task is to calculate $U^\sigma_{i\to f}$ up to normalisation. The field integration can be heuristically performed by a stationary phase approximation, keeping the contribution of the sole field configurations $\mathcal{F}[x_a(t)]$ that solve the classical field equations sourced by particles of mass m_a with classical paths $x_a(t)$ and boundary conditions $|\psi^{i,f}\rangle$. Then,

$$U^\sigma_{i\to f} \propto \int_i^f \mathcal{D}x' \, \exp\left(\frac{iS\left[x'_a, \mathcal{F}[x'_a]\right]}{\hbar}\right) |\psi^f\rangle\langle\psi^i| \, . \tag{3}$$

This approximation allows us to sidestep the rigorous definition of the path–integral [33, 34] and neglects loop corrections. The latter are not relevant for the phenomenon, as justified a posteriori by the result of this calculation.

Between times t^i and t^f, for each spin configuration σ there is a classical path $x_a^{s_a}$ determined by the magnetic field B_z coupled to the spin s_a of each particle. These paths can be taken as orthogonal states, and the remaining integral approximated by a second stationary phase approximation, keeping only the contribution on these paths

$$U^\sigma_{i\to f} \propto \exp\left(\frac{iS^\sigma\left[x_a^{s_a}, \mathcal{F}[x_a^{s_a}]\right]}{\hbar}\right) |\psi^f\rangle\langle\psi^i| \, , \tag{4}$$

Here, for a given spin configuration σ, S^σ is the *on–shell action for the joint system of spins, paths and field.*

[1] For, example, it could be applied to the protocol in [29] and re-analysed in [30, 31].
[2] The formulas are the same for an arbitrary number of particles.
[3] For electromagnetism, $\mathcal{F}$ is the four–potential.

The action S splits as $S = S_0 + S_{\mathcal{F}}$. S_0 does not depend on $\mathcal{F}$, it contains the matter kinetic terms and the coupling of B_z with the spins s_a. S_0 can be calculated, or measured, separately. For simplicity, we assume the setup to be chosen so that S_0 is the same for all σ and becomes a global phase. $S_{\mathcal{F}}$ contains the on–shell contributions of the kinetic terms for the field $\mathcal{F}$ and of the coupling of $\mathcal{F}$ with the masses m_a along their motion x_a: $S_{\mathcal{F}}$ contains the *field mediation*. We define

$$\phi_\sigma = \frac{S^\sigma_{\mathcal{F}}\left[x_a^{s_a}, \mathcal{F}[x_a^{s_a}]\right]}{\hbar}. \tag{5}$$

Given an initially separable state $\left|\Psi^i\right\rangle \propto \left|\psi^i\right\rangle \otimes \sum_\sigma A_\sigma \left|\sigma\right\rangle$ of field, paths and spins, with A_σ complex amplitudes, the final state is given by

$$\left|\Psi^f\right\rangle = U_{i\to f}\left|\Psi^i\right\rangle \propto \left|\psi^f\right\rangle \otimes \sum_\sigma A_\sigma \, e^{i\phi_\sigma} \left|\sigma\right\rangle. \tag{6}$$

Note that boundary states are not entangled with the spin configurations at initial and final times. However, depending on the values of $S^\sigma_{\mathcal{F}}$, entanglement can be produced among the spin degrees of freedom. The phases ϕ_σ are the result of the entanglement production *mediated through* $\mathcal{F}$ [2, 3, 32]. We have shown that the phases ϕ_σ are on–shell actions, therefore they are *manifestly local and gauge invariant*. Differences of ϕ_σ for different σ, the relative phases among branches, are the observables measured by the experiment. We now compute ϕ_σ.

Covariant phases for the gravitational field of moving particles

The action of linearized gravity coupled to matter is gauge invariant. In the Lorenz gauge it reads [35–38]

$$\begin{aligned} S_{\mathcal{F}} =& \frac{c^4}{64\pi G}\int \mathrm{d}^4x \left(-\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{2}\partial^\mu h \partial_\mu h\right) \\ &+ \frac{1}{2}\int d^4x h_{\mu\nu}T^{\mu\nu}. \end{aligned} \tag{7}$$

where $h_{\mu\nu}$ is the metric perturbation and $T_{\mu\nu}$ the energy–momentum tensor. The on–shell action is

$$S_{\mathcal{F}} = \frac{1}{4}\int \mathrm{d}^4x \; h_{\mu\nu}T^{\mu\nu}. \tag{8}$$

where $h_{\mu\nu}$ solves the classical field equations.

The gravitational interaction of point particles with arbitrary timelike trajectories yields an exact solution of the field equations, the gravitational analogue of the Liénard–Wiechert potentials of electromagnetism [39, 40]. Then, the on–shell action (5) is given by

$$S^\sigma_{\mathcal{F}} = \frac{G}{c^4}\sum_{a,b}^{a\neq b}\int \mathrm{d}t \frac{m_a m_b \bar{V}_a^{\mu\nu}(t_{ab})V_{b\mu\nu}(t)}{d_{ab}(t) - \boldsymbol{d}_{ab}(t)\cdot \boldsymbol{v}_a(t_{ab})/c} \tag{9}$$

where $V_a^{\mu\nu} = \gamma_a v_a^\mu v_a^\nu$, $\bar{V}^{\mu\nu} = V^{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}V^{\alpha\beta}$ with $\eta^{\mu\nu}$ the flat metric, $v_a^\mu = (c, \boldsymbol{v}_a)$ with $\boldsymbol{v}_a = d\boldsymbol{x}_a/dt$ the three velocity, and $\gamma_a = (1 - |\boldsymbol{v}_a|^2/c^2)^{-1/2}$ the Lorentz factor. Reinstating the spin configuration indices, the retarded time function t^σ_{ab} is defined as the implicit solution of $t^\sigma_{ab}(t) = t - d^\sigma_{ab}(t)/c$ where $\boldsymbol{d}^\sigma_{ab}(t) = \boldsymbol{x}_b^{s_a}(t) - \boldsymbol{x}_a^{s_b}(t^\sigma_{ab})$ and $d^\sigma_{ab}(t) = |\boldsymbol{d}^\sigma_{ab}(t)|$. The analogous formula in electromagnetism is obtained by replacing $\bar{V}^{a\mu\nu}V_{b\mu\nu} \to v_a^\mu v_{b\mu}$, $m \to q$ and $G/c^4 \to \kappa_e/2c^2$ where q is the charge and κ_e Coulomb's constant. A derivation of (9) for both cases is given in the supplementary material.

The approximation of point particles allows us to use an exact solution of the field equations. So long as the size of the particles is much smaller than their separation this will be a good approximation. Eq. (9) is valid for the spacetime region where $|h_{\mu\nu}| \ll 1$, far from the Schwarzschild radius of the particle (one Planck length for a Planck mass). Realistic smooth mass distributions can also be considered, yielding negligible tidal effects.

The action (9) is a sum of two terms per pair of particles. Each term is the contribution from one particle at coordinate time t interacting with the other causally, that is, with retardation. The causal interaction between matter and the gravitational field is entirely encoded in $S^\sigma_{\mathcal{F}}$. This manifestly Lorentz and gauge invariant quantity gives the observables measured in the experiment.

Slow–motion approximation versus Newtonian limit

When the source is 'slow moving', meaning moving at non–relativistic speeds $|\boldsymbol{v}_a| \ll c$, we have that $\bar{V}_a^{\mu\nu}(t_{ab})V_{b\mu\nu}(t) = c^4 + \mathcal{O}\left(c^3|\boldsymbol{v}_a|\right)$ and (9) approximates

$$S^\sigma_{\mathcal{F}} \approx \frac{1}{2}G\sum_{a,b}^{a\neq b}\int \mathrm{d}t \, \frac{m_a m_b}{d^\sigma_{ab}(t)}. \tag{10}$$

In this regime *the interaction is still local.* The distance $d^\sigma_{ab}(t) = |\boldsymbol{x}_b^{s_b}(t) - \boldsymbol{x}_a^{s_a}(t^\sigma_{ab})|$ depends on the *retarded* time function $t^\sigma_{ab}(t)$. While the speed of light c has cancelled out in the prefactor of (10), it is still present implicitly in the definition of $t^\sigma_{ab}(t)$. Equation (10) can be regarded as the causal version of Newton's law for gravitation.

A different approximation for (9) can be taken when the source's characteristic scale of time variation divided by c is much larger than the distance of the source. Then, retardation in the field can be neglected in the vicinity of the source. This is a near–field approximation, it amounts to replacing the retarded time functions t_{ab} in (9) with the coordinate time t, hence, modelling the gravitational interaction as an instantaneous interaction. The slow–moving and near–field approximations do not imply each other, there are physical regimes when one is applicable and the other is not, and vice versa. When both approximations are applied, they yield the 'Newtonian limit'. Taking in addition d to be constant during a relevant time T, corresponding to considering a static approximation, we recover the formula used in the literature

$$\phi_\sigma \approx \frac{G m_1 m_2 T}{\hbar d}. \tag{11}$$

This expression for the phases ϕ_σ naively models an instantaneous interaction, but it is just an approximation to the manifestly local on–shell action (9) of the joint system of paths, spins and field.

Observable effect of retardation

The effect of retardation can be quantified as the correction to the Newtonian limit (11) by the slow–moving approximation (10). Importantly, a *qualitatively* different behaviour can be observed when the spatial superposition of the particles happens entirely within spacelike separated regions.

Take the particles at rest at a distance d for all times $t < t_1$ and $t > t_2$. Between t_1 and t_2, the particles undergo a spin–dependent motion. The setup is such that $c(t_2 - t_1) < d$, so that the non–stationary parts of the worldlines are spacelike separated. From time $t_3 = t_2 + d/c$, the retarded position of each particle with respect to the other is again constant. See Figure 1. With this setup, no entanglement can be generated.

Let $x_a^{s_a}(t)$ be the displacement of particle a from its initial position due to the coupling of the external magnetic field B_z with its spin s_a in the spin configuration σ. We remind that $|\sigma\rangle = \otimes_a |s_a\rangle$. Using (10), ϕ_σ is a sum of integrals that can be done by splitting the domain of integration in four. We have

$$\begin{aligned}\int_{t^i}^{t^f} \frac{\mathrm{d}t}{d_{21}^\sigma(t)} &= \int_{t^i}^{t_1} \frac{\mathrm{d}t}{d} + \int_{t_1}^{t_2} \frac{\mathrm{d}t}{d - x_1^{s_1}(t)} \\ &\quad + \int_{t_2}^{t_3} \frac{\mathrm{d}t}{d + x_2^{s_2}\big(t_{21}(t)\big)} + \int_{t_3}^{t^f} \frac{\mathrm{d}t}{d}.\end{aligned} \tag{12}$$

Then, the phases are of the form $\phi_\sigma = C + \phi_{s_a} + \phi_{s_b} + C'$ with terms that depend on at most one spin.[4] Thus, if the initial states of the spins is separable, so will be the final state. If, on the other hand, one calculates the phase in the Newtonian limit with instantaneous interaction, the spins result in an entangled state.

Experimental considerations

The effect described above can in principle be observed experimentally, even though the parameters may be challenging. One possible way to achieve spacelike separation

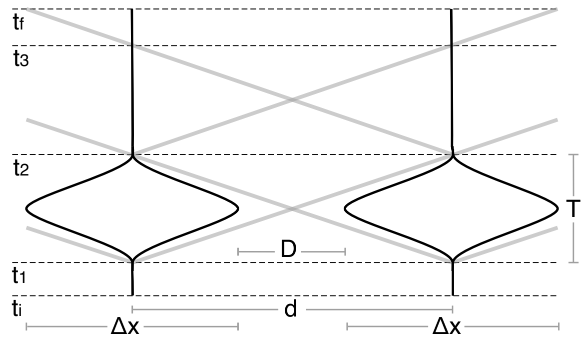

Figure 1. The lightcone structure forbids entanglement when the superposition happens at spacelike separation. This occurs when $d/T \geq c$.

between the two interferometer loops of [2] is to increase the velocity $\boldsymbol{v}$ at which the particles traverse the apparatus. We denote d the initial distance of the particles, Δx the maximum separation of the path superposition, and $D = d - \Delta x$ the minimum distance of the branches at closest approach, see Figure 1.

Using the Newtonian limit (11), and assuming $\Delta x \ll d$, the entanglement is maximal when [2] $\Delta\phi \approx (A/A_P)^2 (\Delta x/d)^2 (cT/D) = \pi$, where $T = t_2 - t_1$, A is the mass m and A_P is the Planck mass $m_P = \sqrt{\hbar c/G}$. For the Coulomb case, A is the charge q and A_P is the Planck charge $q_P = \sqrt{4\pi\epsilon_0 \hbar c}$.

As we showed above, when $d \geq cT$, no entangling interaction can take place between the particles. In other words: one can create a situation in which the Newtonian limit yields $\Delta\phi \sim 1$, while the actual value predicted by (9) and (10) is $\Delta\phi \sim 0$. Fixing the speed v so that $d/T = c$ and assuming $\Delta x \ll d$ at these timescales, achieving such maximum discrepancy would require $A \gg A_P$. For the gravity case this results in magnetic fields and coherence requirements that are not realistic for the foreseeable future (for comparison: current proposals operate in a regime $A \approx 10^{-10} A_P$ at much larger time scales, while the magnetic field requirements for coherent splitting scale with both mass and time).

Tinier quantitative effects of retardation are more easily measured. Let us assume, for the sake of the argument, that one can detect a one part in a thousand deviation from the Newtonian approximation. One can estimate the retarded phases by replacing T with $T - d/c$, which implies a correction $\delta(\Delta\phi) \approx (A/A_P)^2 (\Delta x/d)^2$. This will still require fairly large A/A_P, which is unlikely reachable for the gravity case. For the electromagnetic case however ion and electron interferometry offers a promising path [41]. For a single electron, $A/A_P \approx 0.1$. Assuming $d \approx 1\,\mathrm{cm}$, $T \approx 50\,\mathrm{ns}$ and that the superposition is produced by diffraction with a grating of periodicity $10\,\mathrm{nm}$, it is possible to produce $\Delta x/d \approx 0.3$ and thus the desired $\delta(\Delta\phi) \approx 10^{-3}$. In an interferometer of length $10\,\mathrm{cm}$, this can be achieved with electron veloc-

[4] The same is true if we use the exact expression (9).

ity of $v = 10^{-2}c$, which is reachable in current electron microscopes.

One possible way of testing for entanglement generation in such a scenario could be indirectly via controllable decoherence and recoherence of the single-electron interference signals [42]: if no entanglement is generated, both interferometers will show full (single-electron) coherence, while any generation of entanglement would decohere the single-electron interference signals. Both scenarios are accessible by changing the velocity of both beams. While this is not an easy experiment to perform, it is plausible for the near future.

Discussion

We have considered experimental proposals aiming at observing the entanglement between two masses (or charges) due to the mediation of their gravitational (or electromagnetic) interaction. Entanglement happens because different quantum branches accumulate different phases. The phases were previously computed using an instantaneous interaction, which in part obscured the relevance of the experiment.

We have computed the phases from first principles and shown they are on–shell actions. They are manifestly Lorentz invariant, hence local, and gauge invariant. We have considered the approximation where the particles' motion is non–relativistic and shown that this is still local as it includes the corrections for retardation. As expected, retardation has an observable effect in the production of mediated entanglement.

As an application of this observation, we have considered an experiment to detect retardedly induced entanglement. This is for the moment a gedanken experiment for gravity. But it is interesting to perform it in the case of electromagnetism, a task which we estimate to be difficult but plausible.

The analysis brings also some clarity on the idea that the mediating degrees of freedom are 'virtual gravitons' (or 'virtual photons') [18, 26–28]. In a sense, this idea overstates the significance of the experiment. The entanglement arises because the gravitational field propagates information, but this is only related to virtual particles in the sense in which the fall of an apple is due to virtual gravitons. More precisely, we have shown that loop corrections are irrelevant: they give corrections to equation (9). It is the quantum superposition of semiclassical field configuration that generates the entanglement.

The physical picture arising from the analysis is that information travels in the quantum superposition of field wavefronts: the mechanism that propagates the quantum information with the speed of light is a quantum superposition of macroscopically distinct dynamical field configurations. Per equation (5), it is this superposition that gives rise to different phases for each quantum branch.

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SUPPLEMENTARY MATERIAL

DETAILED DERIVATION OF EQUATION (9)

Here, we give a pedagogical derivation of the on–shell action $S^{\sigma}_{\mathcal{F}}$ when the field $\mathcal{F}$ is the metric perturbation of linearised gravity sourced by point particles. The electromagnetic case proceeds similarly, see next section.

We use the notation: given a two–indexed tensor $T^{\mu\nu}$, $T=\eta_{\mu\nu}T^{\mu\nu}$ is the trace and $\bar{T}^{\mu\nu}=T^{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}T$ the trace–reversed tensor.

Action for the on–shell field and arbitrary sources

The action for linearised gravity coupled to matter is [35, 36]

$$S_h=\frac{c^4}{64\pi G}\int \mathrm{d}^4x\Big(-\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu}+2\partial_\rho h_{\mu\nu}\partial^\nu h^{\mu\rho}-2\partial_\nu h^{\mu\nu}\partial_\mu h+\partial^\mu h\partial_\mu h\Big)+\frac{1}{2}\int \mathrm{d}^4x\, h_{\mu\nu}T^{\mu\nu}, \tag{13}$$

where $\mathrm{d}^4x=\mathrm{d}t\mathrm{d}^3x$ and $T^{\mu\nu}$ is the energy–momentum tensor for matter. Boundary terms at infinity are taken to vanish. The coordinates $\boldsymbol{x},t$ are standard Minkowski coordinates. Greek indices denote 4–vectors and bold latin letters denote 3–vectors. The full spacetime metric is given by $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ with the metric perturbation satisfying $|h_{\mu\nu}|\ll 1$ and $\eta_{\mu\nu}$ the Minkowski metric. The metric signature is $(-,+,+,+)$. The action S_h is invariant under an infinitesimal change of coordinates $x^\mu\to x'^\mu=x^\mu+\xi^\mu(x)$, under which the metric perturbation transforms as $h'_{\mu\nu}(x')=h_{\mu\nu}(x)-\partial_\mu\xi_\nu(x')-\partial_\nu\xi_\mu(x')$.

We use the gauge–invariance of S_h to simplify computations by writing the the Lagrangian in Lorenz gauge. In this gauge, the field $h_{\mu\nu}$ satisfies $\partial^\nu\bar{h}_{\mu\nu}=0$ and the action (13) simplifies to

$$S_h=\frac{c^4}{64\pi G}\int \mathrm{d}^4x\,\big[-\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu}+\frac{1}{2}\partial^\mu h\partial_\mu h\big]+\frac{1}{2}\int d^4x\; h_{\mu\nu}T^{\mu\nu}. \tag{14}$$

The Euler-Lagrange equations for the field are

$$\Box h_{\mu\nu}=-\frac{16\pi G}{c^4}\bar{T}_{\mu\nu}. \tag{15}$$

When the field is taken on–shell, we can integrate by parts the terms with two derivatives of $h_{\mu\nu}$ in (14) to obtain two terms of the form $h_{..}\Box h^{..}$ and use (15) to get

$$S_h=\frac{1}{4}\int \mathrm{d}^4x\; h_{\mu\nu}T^{\mu\nu}. \tag{16}$$

The field of N point masses

Let us now consider the gravitational interaction of N point particles of masses m_a. The use of point particles is an approximation that allows to use an explicit solution of the field equations. So long as the size of the two matter distributions is much smaller than their separation, so that finite size effects can be neglected, the use of point charges will be a good approximation. The field solution we derive will be a valid approximation for the spacetime region outside any realistic body of mass m_a.

The solution obtained here is the gravitational analogue of the well–known Liénard–Wiechert potential of electromagnetism [40]. These solutions are already known, see for example [39]. We include a full derivation here for completeness, as we are not aware of a published solution of the gravitational case.

The stress–energy tensor for N point masses is

$$T^{\mu\nu}(t,\boldsymbol{x})=\sum_{a=1}^{N}m_a\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}_a(t))V_a^{\mu\nu}(t) \tag{17}$$

where

$$V_a^{\mu\nu}(t)=\gamma_a(t)v_a^\mu(t)v_a^\nu(t) \tag{18}$$

with $v_a^\mu(t) = (c, \boldsymbol{v}_a)$, where $\boldsymbol{v}_a = d\boldsymbol{x}_a/dt$ is the velocity of particle a and $\gamma_a(t) = (1-|\boldsymbol{v}_a(t)|^2/c^2)^{-1/2}$ the corresponding Lorentz factor. The retarded solution of the wave equation (15) for all times is

$$h_{\mu\nu}(t, \boldsymbol{x}) = \frac{4G}{c^4} \int \mathrm{d}x'^3 \, \frac{\bar{T}_{\mu\nu}(\boldsymbol{x}', t_r)}{|\boldsymbol{x} - \boldsymbol{x}'|}, \tag{19}$$

with the retarded time $t_r = t_r(t, \boldsymbol{x}, \boldsymbol{x}')$ defined by $ct_r = ct - |\boldsymbol{x}' - \boldsymbol{x}|$. Plugging in the expression for the energy–momentum tensor, we obtain

$$h_{\mu\nu}(t, \boldsymbol{x}) = \frac{4G}{c^4} \sum_a m_a \int d^3x' \frac{\delta^{(3)}\big(\boldsymbol{x}' - \boldsymbol{x}_a(t_r)\big)\bar{V}_a^{\mu\nu}(t_r)}{|\boldsymbol{x} - \boldsymbol{x}'|}. \tag{20}$$

Before being able to perform the space integration, one needs to first eliminate the awkward dependence of the retarded time t_r on $\boldsymbol{x}'$ by help of an integration in a dummy time variable t' and a delta function:

$$h_{\mu\nu}(t, \boldsymbol{x}) = \frac{4G}{c^4} \sum_a m_a \int \mathrm{d}^3x' \int \mathrm{d}t' \frac{\bar{V}_{\mu\nu}(t')\delta\big(\boldsymbol{x}' - \boldsymbol{x}_a(t')\big)\delta(t' - \tilde{t})}{|\boldsymbol{x} - \boldsymbol{x}'|}, \tag{21}$$

where $\tilde{t} = \tilde{t}(\boldsymbol{x}, t, t') = t - |\boldsymbol{x} - \boldsymbol{x}_a(t')|/c$. We can now do the $\boldsymbol{x}'$ integration to get

$$h_{\mu\nu}(t, \boldsymbol{x}) = \frac{4G}{c^4} \sum_a m_a \int \mathrm{d}t' \frac{\bar{V}_{\mu\nu}(t')\delta\big(t' - \tilde{t}(\boldsymbol{x}, t, t')\big)}{|\boldsymbol{x} - \boldsymbol{x}_a(t')|}. \tag{22}$$

For the remaining integration in t', we make use of the identity

$$\delta(f(y)) = \sum_i \frac{\delta(y - y_i)}{|\partial_y f(y_i)|}, \tag{23}$$

where y_i are zeros of $f(y)$. It follows that

$$\delta\big(t' - \tilde{t}(\boldsymbol{x}, t, t')\big) = \frac{\delta(t' - t_a)}{1 - \boldsymbol{d}_a \cdot \boldsymbol{v}_a(t_a)/(d_a c)}, \tag{24}$$

where the retarded time t_a is implicitly defined as a function of t and $\boldsymbol{x}$ as satisfying

$$c(t - t_a) = |\boldsymbol{x} - \boldsymbol{x}_a(t_a)|; \tag{25}$$

t_a is the time at which the past lightcone of the event $(t, \boldsymbol{x})$ intersects the worldline of particle a. We also defined the retarded displacement

$$\boldsymbol{d}_a = \boldsymbol{d}_a(t, \boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{x}_a(t_a), \tag{26}$$

and its magnitude $d_a = |\boldsymbol{d}_a|$. One then obtains the following solution of the field equations:

$$h^{\mu\nu}(t, \boldsymbol{x}) = \frac{4G}{c^4} \sum_a \frac{m_a \bar{V}_a^{\mu\nu}(t_a)}{d_a - \boldsymbol{d}_a \cdot \boldsymbol{v}_a(t_a)/c}. \tag{27}$$

Note that the values of the field at any given spacetime point $(t, \boldsymbol{x})$ depend exclusively on the behaviour of the particles on the past lightcone of $(t, \boldsymbol{x})$.

The action of N interacting point masses

Let us start by plugging in the energy–momentum tensor (17) for N point particles into (16) and performing the space integration:

$$S_h = \frac{1}{4} \sum_{b=1}^{N} \int \mathrm{d}t \; m_b V_b^{\mu\nu}(t) h_{\mu\nu}\big(t, x_b(t)\big). \tag{28}$$

Next, make use of the solution (27) to obtain

$$S_h = \frac{G}{c^4} \sum_{a,b} \int \mathrm{d}t \frac{m_a m_b \bar{V}_a^{\mu\nu}(t_{ab}) V_{b\mu\nu}(t)}{d_{ab} - \boldsymbol{d}_{ab} \cdot \boldsymbol{v}_a(t_{ab})/c}. \tag{29}$$

We denoted by $t_{ab} = t_{ab}(t)$ the time at which the past lightcone of the event $\big(t, \boldsymbol{x}_b(t)\big)$ intersects the timelike wordline of particle a, defined implicitly by

$$c(t - t_{ab}) = |\boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab})|. \tag{30}$$

We also defined the retarded displacement

$$\boldsymbol{d}_{ab} = \boldsymbol{d}_{ab}(t) = \boldsymbol{x}_b(t) - \boldsymbol{x}_a(t_{ab}), \tag{31}$$

and its magnitude $d_{ab} = |\boldsymbol{d}_{ab}|$. (Compare with equations (25) and (26).

When plugged in the formula (5) for the phases $\phi^\sigma = S^\sigma_{\mathcal{F}}/\hbar$ derived in the main text, one obtains (9). Note that the terms of the sum where $a = b$ are dropped, as they just contribute an overall (infinite) phase to the state.

EQUATION (9) AND (10) FOR ELECTROMAGNETISM

The action for electromagnetism coupled to a four current j^μ is of the form $S = S_M + S_A$ with

$$S_A = \int \mathrm{d}^4x \left(-\frac{c^2}{16\pi k_e} F_{\mu\nu} F^{\mu\nu} + A_\mu j^\mu \right). \tag{32}$$

Here, $d^4x = dtd^3x$, S_M is the free matter action that also includes the B_0 coupling to the spins, A_μ is the four-potential, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength and k_e Coulomb's constant. We use greek indexed latin letters for 4–vectors and bold latin letters for 3–vectors. The metric signature is $(-,+,+,+)$ and $\eta_{\mu\nu}$ is the Minkowski metric.

The action S_A is gauge invariant. We will express the Lagrangian in the Lorentz gauge $\partial_\mu A^\mu = 0$ to simplify calculations. Boundary terms at infinity are taken to vanish. Integrating by parts, the action then reduces to

$$S_A = \int \mathrm{d}^4x \left(-\frac{c^2}{8\pi k_e} \partial_\mu A_\nu \partial^\mu A^\nu + j_\mu A^\mu \right), \tag{33}$$

and the equations of motion are

$$\Box A^\mu = -\frac{4\pi k_e}{c^2} j^\mu, \tag{34}$$

where $\Box = \partial^\mu \partial_\mu$. Now, we obtain the on–shell action by integrating again (33) by parts and using (34) to get:

$$S^\sigma_A = \frac{1}{2} \int \mathrm{d}^4x \; j_\mu A^\mu. \tag{35}$$

The entire contribution of the electromagnetic field to the on–shell action is encoded in this expression. As we saw in the main text, S^σ_A is the central object of interest for mediated entanglement: this Lorentz covariant and gauge invariant quantity is the observable that would be measured in an experiment aiming to observe mediated entanglement.

We now consider the electromagnetic interaction of N point charges. The four-current is given by

$$j^\mu(x) = \sum_a q_a v^\mu_a(t) \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_a(t)), \tag{36}$$

where $v^\mu_a(t) = (c, d\boldsymbol{x}_a/dt) = (c, \boldsymbol{v}_a)$. The potential of this charge configuration in the Lorenz gauge is the well–known Liénard–Wiechert potential [40]

$$A^\mu(t, \boldsymbol{x}) = \frac{k_e}{c^2} \sum_a \frac{q_a v^\mu_a(t_a)}{d_a - \boldsymbol{d}_a \cdot \boldsymbol{v}_a(t_a)/c}, \tag{37}$$

where t_a is the *retarded* time, defined implicitly as the solution of

$$c(t-t_a)=|\boldsymbol{x}-\boldsymbol{x}_a(t_a)|. \tag{38}$$

The retarded time t_a is the time at which the worldine of particle a intersects the past light-cone of $(t,\boldsymbol{x})$. We also defined, for convenience, the retarded displacement $\boldsymbol{d}_a=\boldsymbol{d}_a(t,\boldsymbol{x})=\boldsymbol{x}-\boldsymbol{x}_a(t_a)$, and its magnitude $d_a=|\boldsymbol{d}_{\boldsymbol{a}}|$.

By placing (36) and (37) in (35) and performing the space integration, we get an explicit expression for the on–shell action giving the interaction between the two charges

$$S^\sigma_A=\frac{k_e}{2c^2}\sum_{a,b}^{a\neq b}\int \mathrm{d}t\;\frac{q_a\,q_b\,v_a^\mu(t_{ab})\,v_{b\mu}(t)}{d_{ab}-\boldsymbol{d}_{ab}\cdot\boldsymbol{v}_a(t_{ab})/c}. \tag{39}$$

Here, the retarded time t_{ab} is defined as the implicit solution of

$$c(t-t_{ab})=|\boldsymbol{x}_b(t)-\boldsymbol{x}_a(t_{ab})|, \tag{40}$$

and we also defined $\boldsymbol{d}_{ab}=\boldsymbol{d}_{ab}(t)=\boldsymbol{x}_b(t)-\boldsymbol{x}_a(t_{ab})$, and $d_{ab}=|\boldsymbol{d}_{ab}|$.

In the slow moving approximation $|\boldsymbol{v}_a|\ll c$ the exact expression (39) approximates to the retarded Coulomb interaction

$$S^\sigma_{\mathcal{F}}\approx-\frac{1}{2}k_e\sum_{a,b}^{a\neq b}\int \mathrm{d}t\;\frac{q_aq_b}{d_{ab}}. \tag{41}$$