International Journal of Theoretical Physics, Vol. 23, No. 8, 1984

# "Time" Replaced by Quantum Correlations[1]

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*Received August 5, 1983*

Abstract coordinate time is not observable, but "clock time" is observable. Therefore, any statement about temporal evolution can be replaced, without loss of observational content, by a statement about correlations between a clock and another physical system. We show that information about such correlations can be represented just as well in a single "timeless" quantum state as in a whole history of states. It is therefore not necessary to include "time" as a basic element in the description of the world.

## 1. THE CONDENSATION OF HISTORY

I would like to argue here that physics can do without the concept of "time," and that the function that "time" performs in physics can be performed just as well by nonlocal quantum correlations (Einstein et al., 1935; d'Espagnat, 1976) particularly correlations between the physical system one is studying and another system which is used as a clock. The essence of the argument is as follows: Any statement we would ordinarily make regarding the time dependence of a system can without loss of observational content be cast in the form, "If the clock is found to be in the state..., then the probability of finding the system in the state...is... ." Such a statement makes no reference to coordinate time. We will see that a single highly correlated quantum state can hold as much of this kind of information as an entire *history* of states, and, in fact, that *any* such history can be replaced by a single state. A similar conclusion has been reached in work on the quantum theory of gravity (DeWitt, 1967; Baierlein et al., 1962; Arnowitt et al., 1962; Wheeler, 1968; Peres, 1968). Here we wish to emphasize that the possibility of representing evolution in a single static

[1]This paper is based on work done by D. N. Page and the author and published in *Phys. Rev. D*, **27**, 2885–2892 (1982).

state is a general feature of quantum theory and is not peculiar to quantum gravity.

One motivation for considering such a "condensation" of history is the desire for economy as regards the number of basic elements of the theory: quantum correlations are an integral part of quantum theory already; so one is not adding a new element to the theory. And yet an old element, time, is being eliminated, becoming a secondary and even approximate concept. [See DeWitt (1967), p. 1137; also Peres (1980).]

Another motivation, more in the spirit of this conference, is this: If "time" can be replaced by correlations, perhaps "space" can be also. Spatial information might somehow be encoded in a complex network of correlations among nonspatial variables, such as spin variables, for example, which would be regarded as logically prior to space and time. Such a repicturing of "space" would obviously provide a new starting point for attempts to construct a complete and workable quantum theory of gravity. Indeed, with the description of space-time being so intimately tied up with quantum theory to begin with, there would likely be nothing left to quantize. Similar hopes of finding a pregeometric (Misner et al., 1973) foundation for space-time have been expressed before, and the goal remains ahead of us. For the rest of this paper we will restrict our attention to reducing "time" to quantum correlations.

## 2. A TWO-PARTICLE UNIVERSE

Let us consider first a very simple model universe: two noninteracting spin-1/2 particles in a magnetic field. Although we speak of this two-particle system as a model *universe*, we will nevertheless assume for now, for heuristic purposes, that there is an outside observer who can make measurements on the "universe". Later on (in Section 5) we will do without the assumption of an outside observer.

In this model universe the magnetic field is uniform and parallel to the positive $z$ axis, and the two particles, which have identical magnetic moments, start out at $t=0$ with both of their spins pointing in the positive $x$ direction. That is, at $t=0$ each particle is in the state

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle)$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenstates of $J_z$. The equation of motion then requires that the particles' spins precess together around the vertical axis, so that the state of each particle at time $t$ is

$$|\xi(t)\rangle=\frac{1}{\sqrt{2}}(e^{i\alpha t}|\uparrow\rangle+e^{-i\alpha t}|\downarrow\rangle)$$

where $\alpha$ is the product of the magnetic moment of a particle and the magnitude of the magnetic field, divided by $\hbar$.

Let us regard one of the particles as a clock, its direction of spin being analogous to the direction of the pointer of an ordinary clock. We can then speak of the other particle (called the "object particle") as precessing with respect to clock time, in the following sense: If we make a right-vs.-left measurement (i.e., positive-$x$ direction vs. negative-$x$ direction) on the clock particle and simultaneously make an identical measurement on the object particle, we will find with high probability that the outcomes of the two measurements are the same. (The reason the probability of agreement is not 100% is that the measurements may be made at a time when the particles are not in eigenstates of $J_x$, in which case each particle will have a nonzero probability of yielding either outcome.) Interpreting the "right" outcome of the measurement on the clock as a clock reading of 12:00, and interpreting the "left" outcome as a clock reading of 6:00, we can say that at 12:00 the object particle has a high probability of yielding the "right" outcome and that at 6:00 it has a high probability of yielding the "left" outcome. Similar statements can be made about the behavior of the object particle at other values of the clock time; one must only imagine measuring other horizontal components of the spin of the clock particle. The behavior expressed in these statements can reasonably be described as "precession with respect to clock time."

This analysis in terms of clock pointer positions may seem to be a long-winded way of saying something which is obvious. After all, the two particles *are* precessing in this example. However, this sort of analysis will be quite necessary in the following example, in which there will be no precession *other than* precession with respect to clock time.

Let us now consider a different initial state of the same two-particle system, namely, the stationary state $|\Psi\rangle$ obtained by integrating the above time-dependent state over time:

$$
\begin{aligned}
|\Psi\rangle &\propto \int_0^{2\pi/\alpha} dt |\,\xi^{(1)}(t)\rangle |\,\xi^{(2)}(t)\rangle = \frac{1}{2}\int_0^{2\pi/\alpha} dt \left(e^{i\alpha t}|\uparrow^{(1)}\rangle + e^{-i\alpha t}|\downarrow^{(1)}\rangle\right) \\
&\quad \times \left(e^{i\alpha t}|\uparrow^{(2)}\rangle + e^{-i\alpha t}|\downarrow^{(2)}\rangle\right) \\
&\propto |\uparrow^{(1)}\downarrow^{(2)}\rangle + |\downarrow^{(1)}\uparrow^{(2)}\rangle
\end{aligned}
$$

Here the superscripts are used to identify the two particles. The normalized state of the two-particle system is thus

$$
|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow^{(1)}\downarrow^{(2)}\rangle + |\downarrow^{(1)}\uparrow^{(2)}\rangle\right)
$$

[$|\Psi\rangle$ can alternatively be characterized as proportional to the projection of $|\xi^{(1)}(t)\rangle|\xi^{(2)}(t)\rangle$ (for any time $t$) onto the zero-energy eigenspace of the system. The projection of any evolving state $|\psi(t)\rangle$ onto the subspace with energy $E$ is equal to $\lim_{T\to\infty}(1/T)\int_0^T dt\, e^{iEt/\hbar}|\psi(t)\rangle$.] Even though $|\Psi\rangle$ is stationary, it has the same property we saw in the above time-dependent state which allowed us to conclude that the object particle was precessing with respect to clock time: the object particle's state is correlated with the clock's pointer position. Indeed, if the clock has been found to be pointing to the right, then the other particle when subjected to a right-vs.-left measurement will certainly also be found pointing to the right. This can be seen by noting that the probability of finding the clock in the "right" state and the other particle in the "left" state is zero:

$$\langle R^{(1)}L^{(2)}|\Psi\rangle = \left[\frac{1}{2}(\langle\uparrow^{(1)}| + \langle\downarrow^{(1)}|)(\langle\uparrow^{(2)}| - \langle\downarrow^{(2)}|)\right] \times\left[\frac{1}{\sqrt{2}}(|\uparrow^{(1)}\downarrow^{(2)}\rangle + |\downarrow^{(1)}\uparrow^{(2)}\rangle)\right] = 0$$

Such perfect agreement between the two particles holds for any horizontal direction. In this sense one can speak of a kind of evolution, namely, evolution with respect to clock time, even within a single stationary state.

It is illuminating to consider a similar system of two spin-$j$ particles in a magnetic field. Again we construct a stationary state $|\Psi\rangle$ by integrating over time a history of states in which the particles precess together, always having their spins parallel and horizontal. More precisely,

$$|\Psi\rangle \propto \int_0^{2\pi/\alpha} dt\,|J_{\alpha t}^{(1)} = j\hbar\rangle|J_{\alpha t}^{(2)} = j\hbar\rangle$$

where $J_\phi$ is the component of angular momentum along the direction $(\cos\phi)\hat{x} + (\sin\phi)\hat{y}$, and $|J_\phi = j\hbar\rangle$ is the eigenstate of $J_\phi$ with eigenvalue $j\hbar$. Upon writing this state out in terms of eigenstates of $J_z$ and performing the integration, one finds that

$$|\Psi\rangle = \binom{4j}{2j}^{-1/2} \sum_{m=-j}^{j} \binom{2j}{j+m} |m^{(1)}, -m^{(2)}\rangle$$

where $|m\rangle$ is the eigenstate of $J_z$ with eigenvalue $m\hbar$, and

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

TABLE I. The Probability of Finding the Object Particle Pointing to the Right Given That the Clock Particle Has Been Found Pointing to the Right: (a) When the Two Particles Are in the Stationary State $|\Psi\rangle$; (b) When the Particles Are Precessing Together in a Time-Dependent State.

| (a) $j$ | (a) $P(R_{\text{obj}}\|R_{\text{clock}})$ | (b) $j$ | (b) $P(R_{\text{obj}}\|R_{\text{clock}})$ |
|---|---|---|---|
| 1/2 | 1 | 1/2 | $3/4 = 0.750$ |
| 1 | $9/10 = 0.900$ | 1 | $35/48 = 0.729$ |
| 3/2 | $25/28 = 0.893$ | 3/2 | $231/320 = 0.722$ |
| ⋮ | ⋮ | ⋮ | ⋮ |
| $\infty$ | $\sqrt{3}/2 = 0.866$ | $\infty$ | $\sqrt{2}/2 = 0.707$ |

Again we ask the question: If the clock particle is found with its spin pointing to the right, by which we mean that it is found in the state $|J_x = j\hbar\rangle$, then what is the probability $P(R_{\text{obj}}|R_{\text{clock}})$ of finding the object particle also pointing to the right? The calculation is straightforward:

$$P(R_{\text{obj}}|R_{\text{clock}}) = \frac{\begin{pmatrix}\text{probability of both}\\ \text{pointing to the right}\end{pmatrix}}{\begin{pmatrix}\text{probability of the clock}\\ \text{pointing to the right}\end{pmatrix}}$$

$$= \frac{|\langle J_x^{(1)} = j\hbar,\, J_x^{(2)} = j\hbar|\Psi\rangle|^2}{\sum_{m=-j}^{j} |\langle J_x^{(1)} = j\hbar,\, J_x^{(2)} = m\hbar|\Psi\rangle|^2}$$

The results for some values of $j$ are given in Table Ia. The lower bound on the conditional probability of agreement $P(R_{\text{obj}}|R_{\text{clock}})$ is about 87%, which is quite high when one considers that "pointing to the right' is only one of $2j+1$ possible orthogonal states in which one could find the object particle. As before, this high degree of agreement holds also for any other horizontal direction, and we can therefore say that the object particle is precessing with respect to clock time.

## 3. COMPARISON WITH THE TIME-DEPENDENT CASE

At this point one might object: In the stationary state $|\Psi\rangle$ the agreement between the two particles is not perfect (unless $j=1/2$), whereas in the original time-dependent example the agreement *is* perfect, since the two spins are exactly parallel at all times. Therefore, some information is lost in the transition to the "time as correlations" picture.

But in fact no information is lost if we concern ourselves only with the outcomes of measurements. If we take seriously the idea that there are no clocks in the universe other than the two particles, then even in the time-dependent case the agreement between measurements made on the two particles is not perfect. (We have seen this before in our discussion of the two spin-1/2 particles.) Suppose, for example, that the outside observer checks both particles to see if they are in the state $|J_x = j\hbar\rangle$. If he makes this measurement at a time when the particles are in fact in that state, then he will of course find perfect agreement. However, he may make the measurement at a time when both particles are in the state $|J_\phi = j\hbar\rangle$ with $\phi \neq 0$, i.e., when their spins have precessed away from the "right" position, in which case there will be a nonzero probability of finding one particle in the state $|J_x = j\hbar\rangle$ while the other is found in some orthogonal state. Assuming the measurement is made at a random time, we can calculate the conditional probability of agreement:

$$P(R_{\text{obj}}|R_{\text{clock}}) = \frac{\int_0^{2\pi/\alpha} dt\, |\langle J_x^{(1)} = j\hbar, J_x^{(2)} = j\hbar | J_{\alpha t}^{(1)} = j\hbar, J_{\alpha t}^{(2)} = j\hbar\rangle|^2}{\int_0^{2\pi/\alpha} dt |\langle J_x^{(1)} = j\hbar | J_{\alpha t}^{(1)} = j\hbar\rangle|^2}$$

Upon doing the integrals one finds that

$$P(R_{\text{obj}}|R_{\text{clock}}) = \frac{(8j-1)!!(4j)!!}{(8j)!!(4j-1)!!}$$

This result is illustrated for a few values of $j$ in Table Ib. Comparing this result with that of Table Ia we see that the correlation is actually *better* when the particles are in the stationary state $|\Psi\rangle$ than when they are precessing together with respect to the unobservable coordinate time.

The fact that the stationary state gives better correlations is easily understood. The conditional probability of agreement, calculated as above in the time-dependent case, can be written in the form

$$P(R_{\text{obj}}|R_{\text{clock}}) = \frac{\int_0^{2\pi/\alpha} dt\, \mathrm{Tr}[P_{RR}\rho(t)]}{\int_0^{2\pi/\alpha} dt\, \mathrm{Tr}[P_R\rho(t)]}$$

where $P_{RR}$ is the projection operator onto the state in which both particles are pointing to the right, $P_R$ is the projection onto the subspace in which the clock particle is pointing to the right, and $\rho(t)$ is the instantaneous density operator of the two particles:

$$\rho(t) = | J_{\alpha t}^{(1)} = j\hbar, J_{\alpha t}^{(2)} = j\hbar\rangle\langle J_{\alpha t}^{(1)} = j\hbar, J_{\alpha t}^{(2)} = j\hbar |$$

We can rewrite $P(R_{\text{obj}}|R_{\text{clock}})$ once again to get

$$P(R_{\text{obj}}|R_{\text{clock}}) = \frac{\text{Tr}[P_{RR}\rho]}{\text{Tr}[P_R\rho]}$$

where $\rho$ is the time-averaged density operator

$$\rho \propto \int_0^{2\pi/\alpha} dt\, \rho(t)$$

Thus the degree of correlation between the two particles when they are precessing together in the time-dependent case is exactly the same as when they are in the stationary mixed state $\rho$.

The state $|\Psi\rangle$ is a *pure* state chosen for its high degree of correlation, and it is one of several orthogonal pure states which are represented in $\rho$, the others being less highly correlated than $|\Psi\rangle$. It is therefore not surprising that the two particles exhibit a greater degree of agreement when they are in the state $|\Psi\rangle$ than when they are in the state $\rho$, and hence than when they are in the observationally equivalent time-dependent state.

The above equivalence between the time-dependent case and the time-averaged stationary mixed state $\rho$ generalizes to any closed system. Furthermore, one can always find a stationary pure state $|\Psi\rangle$ in which the correlations among the parts of the system are at least as strong as in the stationary mixed state $\rho$. Therefore, as far as these correlations are concerned, one does not lose any information by going from a time-dependent description to a description in terms of a single stationary state.

## 4. AN $N$-PARTICLE UNIVERSE

In the real world there are more than two systems whose states appear to be changing in time. We take this fact into account in our final example, in which the universe consists of $N$ spin-$j$ particles instead of just two. We construct our stationary state in the same way as before: Start with all the particles pointing to the right; let them precess in the uniform vertical magnetic field; integrate the state of the whole $N$-particle system over time to get the highly correlated stationary state $|\Psi\rangle$. (As we mentioned above, doing the integral amounts to projecting onto the $E = 0$ subspace. If $N$ and $2j$ are both odd, then zero is not an eigenvalue of energy, and one has to project onto a different eigenspace of energy. As long as the eigenvalue is close to zero, the resulting stationary state will have strong correlations among the particles.)

In this stationary state $|\Psi\rangle$, one has a description of several clocks all keeping the same time, or alternatively, of several dynamical systems all evolving together. Of course, if you make a measurement on just one of the particles, the outcome will be completely random; in a sense you are trying to look at the whole history of that particle all at once. In order to see the evolution, you need to look at at least two particles and let one of them serve as a clock. Indeed this is what we do in real life: when we speak of a system changing in time we are always really comparing two systems, an object system and a clock. In most everyday cases the clock is nothing more than the physical system that corresponds to our own internal sense of time.

For very large $N$, the degree of correlation between any two particles in the state $|\Psi\rangle$ turns out to be the same as what is shown in Table Ib. In this sense the information content of the stationary state is equal to that of the time-dependent state, rather than being greater as in the previous example. Of course, the main point is that in this example, as in any case, no information is *lost* in replacing a history of states by an appropriate stationary state.

## 5. EVOLUTION WITHIN A SINGLE STATE

We are thus led to consider the view that there is one single $|\Psi\rangle$ describing the whole history of the universe. We now discuss very briefly the interpretation of such a $|\Psi\rangle$ in terms of measurements made by observers within the universe, and we ask what properties $|\Psi\rangle$ should have in order for the observed evolution to conform to the usual equation of motion.

An observer within the universe is somewhat analogous to one of the particles in the above $N$-particle example, in the sense that his own state is highly correlated with the state of the rest of the world. When he makes a measurement on the world around him, he also makes a measurement, without even trying, on some of his own internal variables. This combined measurement, being made entirely within the universe, does not collapse the state of the universe as a whole but only gives the observer the experience of being in one of the many different states which are possible for him within the state $|\Psi\rangle$. Given that he is in this state, the correlations inherent in $|\Psi\rangle$ place strong restrictions on the result of his measurement of the world around him. The correlations he thus observes between his own state and the state of the world around him, as well as the correlations among the various parts of the outside world, are interpreted by him as the passage of time.

In this picture there is clearly no need for an equation of motion of the state $|\Psi\rangle$, since $|\Psi\rangle$ is fixed once and for all. However, if *any* $|\Psi\rangle$ were allowed, then the world as we observe it would not necessarily exhibit any regularity in its evolution: the observed evolution is determined by correlations in $|\Psi\rangle$, and one can construct a $|\Psi\rangle$ with whatever correlations one pleases. We must therefore introduce a restriction on the set of possible $|\Psi\rangle$'s. It is clear that we will get agreement with the usual equation of motion if we insist that $|\Psi\rangle$ be an eigenstate of the Hamiltonian of the universe, since such a stationary $|\Psi\rangle$ is allowed even in the usual picture, in which one does have a coordinate time. Thus one can replace the equation of motion of the universe by the requirement that the state of the universe be an eigenstate of the Hamiltonian.

As an illustration of this point, consider a universe consisting of two noninteracting parts: (i) a clock and (ii) the rest of the universe. The Hamiltonian is of the form

$$H = H_c \otimes I_r + I_c \otimes H_r$$

where $c$ and $r$ stand for "clock" and "the rest." The state $|\Psi\rangle$ of the universe is an eigenstate of $H$, and for simplicity we take the eigenvalue to be zero: $H|\Psi\rangle = 0$. Let $|\xi_c(0)\rangle$ be a state of the clock chosen to correspond to the zero of clock time, and for each $\tau$ define $|\xi_c(\tau)\rangle = \exp(-iH_c\tau)|\xi_c(0)\rangle$ as the state possessing the value $\tau$ of clock time. Finally, let $|\xi_r(\tau)\rangle$ be the relative state (Everett, 1957) of the rest of the universe when the clock is in the state $|\xi_c(\tau)\rangle$. That is, $|\xi_r(\tau)\rangle$ is the state from which probabilities for the rest of the universe can be calculated given that the clock has been found in the state $|\xi_c(\tau)\rangle$. Mathematically, $|\xi_r(\tau)\rangle$ is the coefficient of $|\xi_c(\tau)\rangle$ in an expansion of $|\Psi\rangle$ as a linear combination of orthogonal clock states. We will use the compact notation

$$|\ \xi_r(\tau)\rangle = \langle \xi_c(\tau)|\ \Psi\rangle\rangle$$

The $\rangle\rangle$ indicates that $|\Psi\rangle$ is in a larger Hilbert space than $|\xi_c(\tau)\rangle$. We want now to find the evolution of $|\xi_r\rangle$ with respect to clock time $\tau$. We have

$$\begin{aligned}
i\hbar\frac{d}{d\tau}|\ \xi_r(\tau)\rangle &= i\hbar\frac{d}{d\tau}\langle \xi_c(\tau)|\ \Psi\rangle\rangle \\
&= -\langle \xi_c(\tau)|H_c|\ \Psi\rangle\rangle \\
&= -\langle \xi_c(\tau)|\ H - H_r|\Psi\rangle\rangle \\
&= \langle \xi_c(\tau)|\ H_r|\Psi\rangle\rangle \\
&= H_r\langle \xi_c(\tau)|\ \Psi\rangle\rangle \\
&= H_r|\ \xi_r(\tau)\rangle
\end{aligned}$$

Thus, as long as $|\Psi\rangle$ is an eigenstate of $H$, and as long as there exists a clock, then the rest of the universe automatically follows the usual law of evolution with respect to clock time.

In our own world there is not just one clock but many clocks, which are all correlated with each other in much the same way as the $N$ spin-$j$ particles were correlated with each other in our earlier example. (Any system whose state is observed to be changing can be counted as such a clock.) In that example, the state $|\Psi\rangle$ we chose was a very special state: not only was it an energy eigenstate; it also had a special kind of correlation among the particles. These correlations guaranteed that one could guess fairly well the state of one particle by observing another particle, just as in our own world we can guess the state of one clock by looking at another. On the other hand, a *typical* state of the $N$-particle system, even a typical energy eigenstate, would not have this property. In a typical state, there would be complicated many-particle correlations but very few two-particle correlations, so that one could not say much about the reading of one "clock" by looking at another. In such a case the particles would not be very useful at all as clocks. Each one could be regarded as keeping its own "private" time, with respect to which the rest of the universe could be said to be evolving, but there would be no common time which they all shared. Thus, although any energy eigenstate $|\Psi\rangle$ gives rise to the usual evolution with respect to any given clock, it takes a very special $|\Psi\rangle$ to allow the existence of a large number of clocks keeping a common time.

In conclusion, it is not necessary to include "time" as an *a priori* element of physics. Clock time, which makes sense even within a stationary state, is the only kind of time that can be observed. Furthermore, even the existence of clock time is only a contingent property of the world, especially the kind of clock time we are familiar with, which is common to many different clocks. Only certain states of the universe admit such a universal time. In this respect *time itself* has the same status as that which we normally attribute to the *existence of a preferred direction* of time: we can imagine a world without it, and we must even regard our universe as special because it has it.

## ACKNOWLEDGMENTS

I would like to thank Lee Link for several helpful discussions and for his calculations on the $N$-particle case, which helped lead to the conclusions stated in the paper. I would also like to thank David Park for his careful reading of the manuscript and for many useful suggestions. This work was supported by a grant from Research Corporation.

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