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Entanglement of quantum clocks via gravity

Page and Wootters formalism

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Contents

Introduction to Quantum Gravity

When we talk about **Quantum Gravity**, rather than a theory, we refer to a **family of problems** about the relation between our fundamental theories:

Quantum Mechanics and General Relativity

These problems lead physicists to pursue different approaches toward the "complete" picture, but all these paths originated from the same question:

If spacetime is a dynamical field and dynamical fields are quantized, then should we expect a kind of quantum spacetime?[1]
there's something wrong in our foundations?

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In the present picture, our fundamental theories are able to describe with astonishing precision a huge amount of natural phenomena, but, in the words of Einstein[2]:

> *"One is struck [by the fact] that the theory [of special relativity]... introduces two kinds of physical things, i.e., measuring rods and clocks, all other things, e.g., the electromagnetic field, the material point, etc. This, in a certain sense, is inconsistent..."*

and so, as pointed out by C. Rovelli[3]:

> *"The search for a quantum theory of gravity raises once more old questions such as: What is space? What is time? What is the meaning of "moving"? Is motion to be defined with respect to objects or with respect to space? And also: What is causality? What is the role of the observer in physics?"*

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As pointed out in ref. [Castro-Ruiz et al., 2017]:

> *"For the sake of consistency, it is natural to assume that the clocks, being physical, behave according to the principles of our most fundamental physical theories."*

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How can we take care of the physical, observer dependent, nature of time?

Timeless Quantum Mechanics

PaW framework
Quantum Time

Here we follow the approach introduced by Page and Wootters[4], and then modified by Giovannetti, Lloyd and Maccone[5] to define the concept of "Quantum Time" as the one measured via the "Time Observable".

We can describe a clock as a quantum system C, such that

$$|t\rangle_C \in \mathcal{H}_C \qquad \hat{T}\,|t\rangle_C = t\,|t\rangle_C \qquad \hat{H}_C = \hbar\hat{\Omega} \tag{1}$$

where $\hat{\Omega}$ is the conjugate variable to $\hat{T}$, i.e. $[\hat{T}, \hat{\Omega}] = i\mathbb{1}_C$. We also have

$$_C\langle t'|t\rangle_C = \delta(t'-t) \quad and \quad _C\langle t'|\,\hat{\Omega}\,|t\rangle_C = \delta(t'-t)\partial_t \tag{2}$$

[4] Cfr. ref. [Page and Wootters, 1983, Wootters, 1984, Marletto and Vedral, 2016] for the original approach and further specifications.

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In this framework, we assume that the time measured by **the clock** C is used by an observer to make measures on the system S.

It follows that C **has to be regarded as a part of the quantum system** $S + C$, subject to a global constraint $\hat{H}$ in the form of a **Wheeler-DeWitt Equation**, such that

$$|\Psi\rangle \in \mathcal{H} := \mathcal{H}_C \otimes \mathcal{H}_S \qquad \hat{H}\,|\Psi\rangle = 0 \ \ with \ \ \hat{H} = \hat{H}_C \otimes \mathbb{1}_S + \mathbb{1}_C \otimes \hat{H}_S \tag{3}$$

This allows an observer-dependent and timeless description of QM!

Note that we are making several assumptions such as that C is a perfect clock, i.e. isomorphic to a particle on a line, and $\hat{H}_S$ is time-independent.

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Indeed, we recover the **Schrödinger Equation** from equation 3 conditioning on time via projection on the generalized eigenvectors of the $\hat{T}$ operator. Using equations 2, we find:

$$ {}_C\langle t|\,\hat{H}\,|\Psi\rangle = 0 \iff \partial_t\,|\psi(t)\rangle_S = \hat{H}_S\,|\psi(t)\rangle_S \tag{4} $$

where $|\psi(t)\rangle_S := {}_C\langle t|\Psi\rangle$ is the **Schrödinger State** we know from QM.[6]

A general solution of equation 3 is called **history state**[7] and can be written as

$$ |\Psi\rangle = \int \mathrm{d}t\,|t\rangle_C \otimes |\psi(t)\rangle_S \tag{5} $$

It incorporates all of the information of the quantum system.

[6] Cfr. ref. [Giovannetti et al., 2015]. Some definitions may vary because of the introduction of a normalization factor into the time generalized eigenvectors. Note that here they're improper states.

[7] Cfr. ref. [Castro-Ruiz et al., 2020], equation 15, for an alternative definition.

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At this point:

- How to describe measures and probabilities in this framework?
- Which criticisms have been raised about the clock model?
- Which is the interpretation of the emerging picture?

Timeless QM
Conditional probabilities

In ref. [Giovannetti et al., 2015], the **correct expression** for the n-times **propagator** has been recovered. For the derivation, they start adopting the **von Neumann formulation of the measurement apparatus** and then they cast the outcome in the time dependent PaW framework.

It is a "purification" of the procedure, **introducing a memory system** M for each measure. This allows us to **write the probability** of n-outcomes in n-times **accordingly to the Bayes rule** for conditional probabilities.

As an example, the conditional probability of getting b at time t given a at time $t > t_0$ reads

$$P(b,t|a,t_0) = \frac{P(b,a,t)}{P(a,t_0)} = \frac{\left\|\left({}_C\langle t| \otimes {}_{M_b}\langle b| \otimes {}_{M_a}\langle a|\right)|\Psi\rangle\right\|^2}{\left\|\left({}_C\langle t_0| \otimes {}_{M_a}\langle a|\right)|\Psi\rangle\right\|^2} \tag{6}$$

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Here we describe the **von Neumann formulation** of the measurement apparatus.[8]

We consider the system S as the sum of the **subsystem of interest** Q and a **memory system** M. Then, we describe the **process of measuring** as an **instantaneous transformation** which induces the unitary mapping

$$|\psi(t_I)\rangle_Q \otimes |ready\rangle_M \longmapsto \sum_a \hat{K}_a |\psi(t_I)\rangle_Q \otimes |a\rangle_M \tag{7}$$

in which $\{\hat{K}_a\}$ are called **Kraus operators** and fulfill the normalization condition $\sum_a \hat{K}_a^\dagger \hat{K}_a = \mathbb{1}$. The transformation can be expressed as an **interaction term** between Q and M in the Hamiltonian of S as

$$\hat{H}_S(t) = \hat{H}_Q + \hat{f}(t) \qquad \hat{f}(t) = \delta(t - t_I)\hat{h}_{QM} \tag{8}$$

[8] Cfr. ref. [Giovannetti et al., 2015], section "Measurements", for more details. Eq. 7 defines the statistical properties of a Positive Operator Valued Measure (POVM).

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Let's define the unitary operator $\hat{V}_{QM} := e^{-\frac{i}{\hbar}\hat{h}_{QM}}$. [9] It is responsible of the mapping in equation 7 and its action can be expressed as

$$\hat{V}_{QM} \left|\psi(t)\right\rangle_Q \otimes \left|r\right\rangle_M = \sum_a \left\langle a\right|_M \hat{V}_{QM} \left|r\right\rangle_M \left|\psi(t)\right\rangle_Q \otimes \left|a\right\rangle_M = \sum_a \hat{K}_a \left|\psi(t)\right\rangle_Q \otimes \left|a\right\rangle_M = \sum_a \sqrt{P(a,t)} \left|\phi_a\right\rangle_Q \otimes \left|a\right\rangle_M \quad (9)$$

in which, according to equation 6, $\left|\phi_a\right\rangle_Q$ is the state of Q after the collapse

$$\left|\phi_a\right\rangle_Q := \frac{1}{\sqrt{P(a,t)}}\hat{K}_a \left|\psi(t)\right\rangle_Q \qquad P(a,t) := \left\|\hat{K}_a \left|\psi(t)\right\rangle_Q\right\|^2$$

[9] Its relation with the experimental apparatus is far away simple. This is one of the motivations for the introduction of the Kraus operators.

The evolution of the system is the described by the unitary operator

$$\hat{U}_S(t, t_0) = \begin{cases} \hat{U}_Q(t, t_0) & \forall t < t_I \\ \hat{U}_Q(t, t_I)\hat{V}_{QM}\hat{U}_Q(t_I, t_0) & \forall t > t_I \end{cases} \tag{10}$$

Putting together equations 7, 9 and 10, the history state in 5 becomes

$$\begin{aligned} |\Psi\rangle = \int_{-\infty}^{t_I} \mathrm{d}t \, |t\rangle_C \otimes |\psi(t)\rangle_Q \otimes |r\rangle_M + \\ \int_{t_I}^{+\infty} \mathrm{d}t \, |t\rangle_C \otimes \hat{U}_Q(t, t_I) \sum_a \hat{K}_a \, |\psi(t_I)\rangle_Q \otimes |a\rangle_M \end{aligned} \tag{11}$$

Instantaneous transformations are experimentally impossible and a generalization to **short-time interaction** is necessary for a realistic description. We may introduce Ozawa here.

Furthermore, we are introducing an **interaction term** and this is a not trivial problem, as pointed out in ref. [Marletto and Vedral, 2016].

At last, the **clock model** we are using in this work **is unrealistic** and has to be intended as an approximation of a n-level system. Otherwise, a lower bounded Hamiltonian leads to closed time-like curves and for a more realistic description one may look at ref. [Wootters, 1984, Moreva et al., 2013].

Timeless QM
Overcoming criticism

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Finally, as a criticism, some physicists argued that this framework describes a **stationary universe**. The most interesting observation pointed out by the authors of ref. [Wootters, 1984, Giovannetti et al., 2015] is that we can define two points of view:

- the **external observer**, who sees the whole universe as a static system whose state is an eigenstate of its global Hamiltonian, i.e. equation 3.
- the **internal observer**, who can anyway observe a dynamics, i.e. time dependence, and Born-rule induced wave-function collapses.

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- the **internal observer**, who can anyway observe a dynamics, i.e. time dependence, and Born-rule induced wave-function collapses.

This is a kind of generalization of the **relativity principle**, which lead us to the introduction of the concept of **Time Reference Frame**.

What is a "Time Reference Frame"?

Time Reference Frames
Definition and basic concepts

According to ref. [Castro-Ruiz et al., 2020],

- We define a "Time Reference Frame" as a quantum temporal reference frame associated to a quantum clock.
- Temporal localization of events, i.e. quantum measurements on the system, is defined by the time reference frame of an observer.
- In the presence of more than one clock, each of them describes the evolution of the rest system according to its "proper time".

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Suppose to have a system made up of two clocks $I = A, B$ and a subsystem of interest S with a trivial Hamiltonian $\hat{H}_S = 0$. No interaction is considered between A, B, S.

Accordingly to equations 3 and 8, we introduce in S an experimental apparatus with two memory systems, each associated to one clock

$$\hat{H} = \hat{H}_A \otimes \mathbb{1}_{\bar{A}} + \hat{H}_B \otimes \mathbb{1}_{\bar{B}} + \hat{f}_A(\hat{T}_A) + \hat{f}_B(\hat{T}_B) \tag{12}$$

in which, defining R_I as the rest of $I + M_I + Q$, we have

$$\hat{f}_I(\hat{T}_I) = |t_I\rangle\langle t_I| \otimes \hat{h}_{QM_I} \otimes \mathbb{1}_{\bar{R}_I}$$

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Time Reference Frames
Change of Time Reference Frame

Expressing equation 5 in terms of $I = A, B$ reference frames, we have

$$|\Psi\rangle = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes |\psi_I(t_I)\rangle_{\bar{I}} = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes \hat{U}(t_I) \, |\psi_I(0)\rangle_{\bar{I}}$$

Using the relation

$$|\psi_A(t_A)\rangle_{\bar{A}} = \hat{U}_{\bar{A}}(t_A) \hat{S}_{AB} \hat{U}^\dagger_{\bar{B}}(t_B) \, |\psi_B(t_B)\rangle_{\bar{B}}$$

in which

$$\hat{S}_{AB} = \int \mathrm{d}t_B \, |t_B\rangle_B \otimes \langle t_A = 0|_A \, \hat{U}_{\bar{B}}(t_B) e^{i\hat{T}_A \hat{H}_S}$$

We can express them in terms of each others as

$$|\Psi\rangle = \int \mathrm{d}t_A \, |t_A\rangle_A \otimes e^{-it_A \hat{H}_B} \mathcal{T}\{e^{-i\int_0^{t_A} \mathrm{d}s(\hat{f}_A(s) + \hat{f}_B(s + \hat{T}_B))}\} \, |\psi_A(0)\rangle_{\bar{A}}$$

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Interacting quantum clocks

Interacting quantum clocks

Ref. PNAS [Castro-Ruiz et al., 2017].

Perspectives

Perspective

Future research directions are:

- classical limit

Acknowledgments and References

References I

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Thank you for your attention!