Department of Physics "E. Fermi" University of Pisa

# Entanglement of quantum clocks via gravity

Page and Wootters formalism

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#### Contents



#### **Introduction to Quantum Gravity**

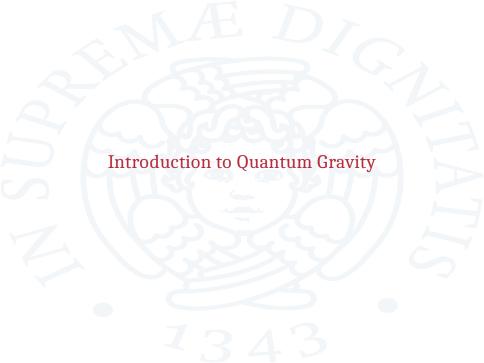
#### **Timeless Quantum Mechanics**

Page and Wootters framework Consequences of the framework More than one quantum clock

Interacting quantum clocks

Conclusion

References



# Introduction to Quantum Gravity Different approaches



When we talk about **Quantum Gravity**, rather than a theory, we refer to a **family of problems** about the relation between our fundamental theories:

Quantum Mechanics and General Relativity

These problems lead physicists to pursue different approaches toward the "complete" picture, but all these paths originated from the same question:

If spacetime is a dynamical field and dynamical fields are quantized, then should we expect a kind of quantum spacetime?<sup>1</sup> or rather there's something wrong in our foundations?

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### Introduction to Quantum Gravity

**Underlying structures** 



In the present picture, our fundamental theories are able to describe with incredible precision a huge amount of natural phenomena, but, in the words of Einstein<sup>2</sup>:

"One is struck [by the fact] that the theory [of special relativity]... introduces two kinds of physical things, i.e., measuring rods and clocks, all other things, e.g., the electromagnetic field, the material point, etc. This, in a certain sense, is inconsistent..."

and so, as pointed out by C. Rovelli<sup>3</sup>

"The search for a quantum theory of gravity raises once more old questions such as: What is space? What is time? What is the meaning of "moving"? Is motion to be defined with respect to objects or with respect to space? And also: What is causality? What is the role of the observer in physics?"

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### Introduction to Quantum Gravity The role of time



Our work is based on a different way of conceiving time, not more as a classical parameter, but defining it from an operative point of view as the observable measured by the clock of an observer.

As pointed out in ref. [Castro-Ruiz et al., 2017]:

"For the sake of consistency, it is natural to assume that the clocks, being physical, behave according to the principles of our most fundamental physical theories."

How can we take care of the physical, observer dependent, nature of time?

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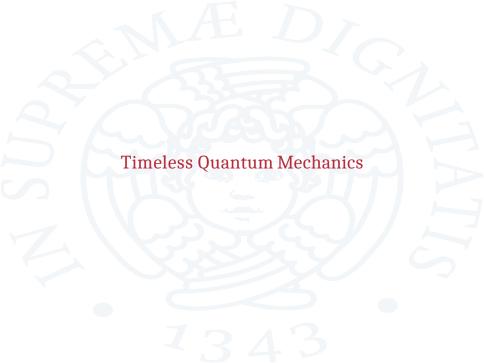


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**Quantum Time** 



Here we follow the approach introduced by Page and Wootters<sup>4</sup>, and then discussed by Giovannetti, Lloyd and Maccone<sup>5</sup> to define the concept of "Time Observable" via correlations.

We can describe a clock as a quantum system C, such that

$$|t\rangle_{\rm C}\in\mathcal{H}_{\rm C}$$
  $\hat{T}|t\rangle_{\rm C}=t|t\rangle_{\rm C}$   $\hat{H}_{\rm C}=\hbar\hat{\Omega}$  (1)

where  $\hat{\Omega}$  is the conjugate operator to  $\hat{\mathcal{T}}$  , i.e.  $[\hat{\mathcal{T}},\hat{\Omega}]=i\mathbb{1}_{\mathcal{C}}.$  We also have

$$_{C}\left\langle t'|t\right\rangle _{C}=\delta(t'-t)$$
 and  $_{C}\left\langle t'|\,\hat{\Omega}\,|t\right\rangle _{C}=\delta(t'-t)\partial_{t}$  (2

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In this framework, we assume that the time measured by **the clock** *C* is used by an observer to make measures on the system *S*.

It follows that C has to be regarded as a part of the quantum system S+C, subject to a global constraint  $\hat{H}$  in the form of a Wheeler-DeWitt Equation, such that

$$|\Psi\rangle\in\mathcal{H}:=\mathcal{H}_C\otimes\mathcal{H}_S \quad \hat{H}\,|\Psi\rangle=o \text{ with } \hat{H}=\hat{H}_C\otimes\mathbb{1}_S+\mathbb{1}_C\otimes\hat{H}_S \quad \text{(3)}$$

This allows an observer-dependent and timeless description of QM!

Note that we are making several assumptions such as that C is a perfect clock, i.e. omorphic to a particle on a line, and  $\hat{H}_{S}$  is time-independent, i.e. not interacting.



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Indeed, we recover the **Schrödinger Equation** from equation 3 conditioning on time via projection on the generalized eigenvectors of the  $\hat{T}$  operator. Using equations 2, we find:

$$_{\text{C}}\left\langle t\right|\hat{H}\left|\Psi\right\rangle =\text{O}\iff\partial_{t}\left|\psi(t)\right\rangle _{\text{S}}=\hat{H}_{\text{S}}\left|\psi(t)\right\rangle _{\text{S}}$$
 (4)

where  $|\psi(t)\rangle_{\rm S}:={}_{\rm C}\,\langle t|\Psi\rangle$  is the **Schrödinger State** we know from QM. $^6$ 

A general solution of eq. 3 is called **history state<sup>7</sup> and can be written as** 

$$|\Psi\rangle = \int \mathrm{d}t \, |t\rangle_{\mathcal{C}} \otimes |\psi(t)\rangle_{\mathcal{S}}$$
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It incorporates all of the information of the quantum system.

Cir. ref. [Castro-kuiz et al., 2020], equation 15, for an afternative definition.

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# Consequences of the framework

#### At this point:

- ► How to describe measurements and probabilities?
- ► Which criticisms have been argued?
- ► How to interpret the emerging picture?



In ref. [Giovannetti et al., 2015], the **correct expression** for the n-times **propagator** has been recovered. For the derivation, they start adopting the **von Neumann formulation of the measurement apparatus** and then they cast the outcome in the time dependent PaW framework.

It is a "purification" of the procedure, introducing a memory system M for each measure. This allows us to write the probability of n-outcomes in n-times accordingly to the Bayes rule for conditional probabilities.

As an example, the conditional probability of getting b at time t given a at time  $t>t_0$  reads

$$P(b, t|a, t_o) = \frac{P(b, a, t)}{P(a, t_o)} = \frac{\|(c \langle t| \otimes_{M_b} \langle b| \otimes_{M_a} \langle a|) |\Psi\rangle\|^2}{\|(c \langle t_o| \otimes_{M_a} \langle a|) |\Psi\rangle\|^2}$$
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(6)



### Here we describe the **von Neumann formulation** of the measurement apparatus.<sup>8</sup>

We consider the system *S* as the sum of the **subsystem of interest** *Q* and a **memory system** *M*. Then, we describe the **process of measuring** as an **instantaneous transformation** which induces the unitary mapping

$$|\psi(\mathsf{t}_{\mathsf{I}})\rangle_{\mathsf{Q}}\otimes|\mathsf{ready}\rangle_{\mathsf{M}}\longmapsto\sum_{a}\hat{\mathsf{K}}_{a}|\psi(\mathsf{t}_{\mathsf{I}})\rangle_{\mathsf{Q}}\otimes|a\rangle_{\mathsf{M}}$$
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in which  $\{\hat{K}_a\}$  are called **Kraus operators** and fulfill the normalization condition  $\sum_a \hat{K}_a^{\dagger} \hat{K}_a = 1$ . The transformation can be expressed as an **interaction term** between Q and M in the Hamiltonian of S as

$$\hat{H}_S(t) = \hat{H}_Q + \hat{f}(t) \qquad \hat{f}(t) = \delta(t - t_I)\hat{h}_{QM}$$
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Let's define the unitary operator  $\hat{V}_{QM}:=e^{-\frac{i}{\hbar}\hat{h}_{QM}}$ . 9 It is responsible of the mapping in equation 7 and its action can be expressed as

$$\hat{V}_{QM} |\psi(t)\rangle_{Q} \otimes |r\rangle_{M} = \sum_{a} \langle a|_{M} \hat{V}_{QM} |r\rangle_{M} |\psi(t)\rangle_{Q} \otimes |a\rangle_{M} = \sum_{a} \hat{K}_{a} |\psi(t)\rangle_{Q} \otimes |a\rangle_{M} = \sum_{a} \sqrt{P(a,t)} |\phi_{a}\rangle_{Q} \otimes |a\rangle_{M} \quad (9)$$

in which, according to equation 6,  $|\phi_a
angle_Q$  is the state of Q after the collapse

$$\ket{\phi_a}_Q := rac{1}{\sqrt{P(a,t)}} \hat{\mathsf{K}}_a \ket{\psi(t)}_Q \qquad \quad \mathsf{P}(a,t) := \left\|\hat{\mathsf{K}}_a \ket{\psi(t)}_Q 
ight\|^2$$

<sup>&</sup>lt;sup>9</sup>Its relation with the experimental apparatus may be far away simple.



The evolution of the system is then described by the unitary operator

$$\hat{U}_{S}(t, t_{o}) = \begin{cases} \hat{U}_{Q}(t, t_{o}) & \forall t < t_{I} \\ \hat{U}_{Q}(t, t_{I})\hat{V}_{QM}\hat{U}_{Q}(t_{I}, t_{o}) & \forall t > t_{I} \end{cases}$$

$$(10)$$

Putting together equations 7, 9 and 10, the history state in 5 becomes

$$\begin{split} |\Psi\rangle &= \int_{-\infty}^{t_{\rm I}} \mathrm{d}t \, |t\rangle_{\rm C} \otimes |\psi(t)\rangle_{\rm Q} \otimes |r\rangle_{\rm M} \, + \\ &\int_{t_{\rm I}}^{+\infty} \mathrm{d}t \, |t\rangle_{\rm C} \otimes \hat{\sf U}_{\rm Q}(t,t_{\rm I}) \sum_{a} \hat{\sf K}_{a} \, |\psi(t_{\rm I})\rangle_{\rm Q} \otimes |a\rangle_{\rm M} \quad \text{(11)} \end{split}$$



#### **Problems:**

- Instantaneous transformations are unrealistic and a generalization to short-time interaction is necessary for a realistic description.
   Introducing an interaction like in ref. [Ozawa, 1997] could be a solution.
- an approximation of a n-level system.

  Indeed, a lower bounded Hamiltonian leads to closed time-like curves and for a realistic description one may look at different clock models, like in ref. [Wootters. 1984. Moreva et al., 2013].
- The introduction of an interaction term leads to the "clock ambiguity". As pointed out in ref. [Marletto and Vedral, 2016], in order to recover a unique Schrödinger Equation, the clock must be weakly-interacting.



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- the external observer, who sees the whole universe as a static system whose state is an eigenstate of its global Hamiltonian, i.e. equation 3.
- the internal observer, who can anyway observe a dynamics, i.e. time dependence, and Born-rule induced wave-function collapses.



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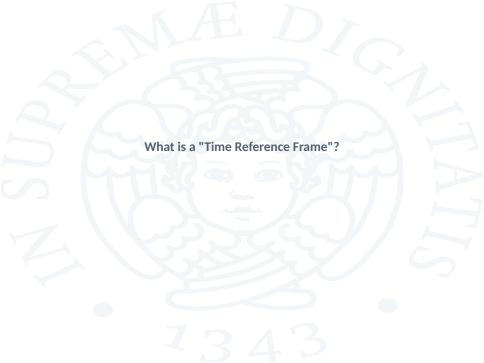
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This is a kind of generalization of the **relativity principle**, which lead us to the introduction of the concept of **Time Reference Frame**.



Definition and basic concepts



#### According to ref. [Castro-Ruiz et al., 2020],

- We define a "Time Reference Frame" as a quantum temporal reference frame associated to a quantum clock.
- ► Temporal localization of events, i.e. quantum measurements on the system, is defined by the time reference frame of an observer.
- ► In the presence of more than one clock, each of them describes the evolution of the rest system according to its "proper time".



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Suppose to have a system made up of two clocks I = A, B and a subsystem of interest S with a trivial Hamiltonian  $\hat{H}_S = o$ . No interaction is considered between A, B, S.

Accordingly to equations 3 and 8, we introduce in S an experimenta apparatus with two memory systems, each associated to one clock

$$\hat{H} = \hat{H}_A \otimes \mathbb{1}_{\bar{A}} + \hat{H}_B \otimes \mathbb{1}_{\bar{B}} + \hat{f}_A(\hat{T}_A) + \hat{f}_B(\hat{T}_B)$$
(12)

in which, defining  $R_l$  as the rest of  $l + M_l + Q$ , we have

$$\hat{f}_I(\hat{T}_I) = |t_I\rangle\!\langle t_I| \otimes \hat{h}_{QM_I} \otimes \mathbb{1}_{\hat{F}}$$



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### Time Reference Frames

Change of Time Reference Frame



Expressing equation 5 in terms of I = A, B reference frames, we have

$$|\Psi\rangle = \int dt_I \, |t_I\rangle_I \otimes |\psi_I(t_I)\rangle_{\overline{I}} = \int dt_I \, |t_I\rangle_I \otimes \hat{U}(t_I) \, |\psi_I(o)\rangle_{\overline{I}}$$

Using the relation

$$|\psi_{\mathsf{A}}(\mathsf{t}_{\mathsf{A}})\rangle_{ar{\mathsf{A}}} = \hat{\mathsf{U}}_{ar{\mathsf{A}}}(\mathsf{t}_{\mathsf{A}})\hat{\mathsf{S}}_{\mathsf{A}\mathsf{B}}\hat{\mathsf{U}}_{ar{\mathsf{B}}}^{\dagger}(\mathsf{t}_{\mathsf{B}})|\psi_{\mathsf{B}}(\mathsf{t}_{\mathsf{B}})\rangle_{ar{\mathsf{B}}}$$

in which

$$\hat{S}_{AB} = \int dt_B \left| t_B \right\rangle_B \otimes \left\langle t_A = o \right|_A \hat{U}_{\bar{B}}(t_B) e^{i\hat{T}_A \hat{H}_S}$$

We can express them in terms of each others as

$$|\Psi\rangle = \int \mathrm{d}t_\mathrm{A} \left|t_\mathrm{A}\right\rangle_\mathrm{A} \otimes e^{-it_\mathrm{A}\hat{H}_\mathrm{B}} \Im\{e^{-i\int_\mathrm{o}^{t_\mathrm{A}}\mathrm{d}s(\hat{f}_\mathrm{A}(s)+\hat{f}_\mathrm{B}(s+\hat{T}_\mathrm{B}))}\} \left|\psi_\mathrm{A}(\mathrm{O})\right\rangle_{\bar{A}}$$

## **Time Reference Frames**

Change of Time Reference Frame



Expressing equation 5 in terms of I = A, B reference frames, we have

$$|\Psi\rangle = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes |\psi_I(t_I)\rangle_{\overline{I}} = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes \hat{U}(t_I) \, |\psi_I(o)\rangle_{\overline{I}}$$

Using the relation

$$|\psi_{\mathsf{A}}(t_{\mathsf{A}})\rangle_{\bar{\mathsf{A}}} = \hat{\mathsf{U}}_{\bar{\mathsf{A}}}(t_{\mathsf{A}})\hat{\mathsf{S}}_{\mathsf{A}\mathsf{B}}\hat{\mathsf{U}}_{\bar{\mathsf{B}}}^{\dagger}(t_{\mathsf{B}})|\psi_{\mathsf{B}}(t_{\mathsf{B}})\rangle_{\bar{\mathsf{B}}}$$

in which

$$\hat{S}_{AB} = \int \mathrm{d}t_B \left| t_B \right\rangle_B \otimes \left\langle t_A = o \right|_A \hat{U}_{\bar{B}}(t_B) e^{i\hat{T}_A \hat{H}_S}$$

We can express them in terms of each others as

$$|\Psi\rangle = \int \mathrm{d}t_\mathrm{A} \, |t_\mathrm{A}\rangle_\mathrm{A} \otimes e^{-it_\mathrm{A}\hat{H}_\mathrm{B}} \Im\{e^{-i\int_\mathrm{O}^{t_\mathrm{A}} \mathrm{d}s(\hat{f}_\mathrm{A}(s)+\hat{f}_\mathrm{B}(s+\hat{f}_\mathrm{B}))}\} \, |\psi_\mathrm{A}(\mathrm{O})\rangle_{\bar{\mathrm{A}}}$$



Expressing equation 5 in terms of I = A, B reference frames, we have

$$|\Psi\rangle = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes |\psi_I(t_I)\rangle_{\bar{I}} = \int \mathrm{d}t_I \, |t_I\rangle_I \otimes \hat{U}(t_I) \, |\psi_I(o)\rangle_{\bar{I}}$$

Using the relation

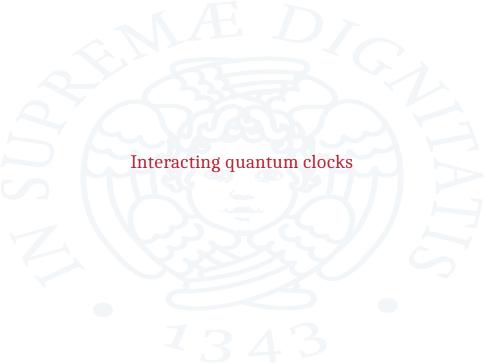
$$|\psi_{\mathsf{A}}(t_{\mathsf{A}})
angle_{ar{\mathsf{A}}}=\hat{\mathsf{U}}_{ar{\mathsf{A}}}(t_{\mathsf{A}})\hat{\mathsf{S}}_{\mathsf{A}\mathsf{B}}\hat{\mathsf{U}}_{ar{\mathsf{B}}}^{\dagger}(t_{\mathsf{B}})\,|\psi_{\mathsf{B}}(t_{\mathsf{B}})
angle_{ar{\mathsf{B}}}$$

in which

$$\hat{S}_{AB} = \int \mathrm{d}t_B \left| t_B \right\rangle_B \otimes \left\langle t_A = o \right|_A \hat{U}_{\bar{B}}(t_B) e^{i\hat{T}_A \hat{H}_S}$$

We can express them in terms of each others as

$$|\Psi\rangle = \int \mathrm{d}t_{\mathsf{A}} \left|t_{\mathsf{A}}\right\rangle_{\mathsf{A}} \otimes e^{-it_{\mathsf{A}}\hat{H}_{\mathsf{B}}} \mathfrak{I}\{e^{-i\int_{\mathsf{o}}^{t_{\mathsf{A}}} \mathrm{d}s(\hat{f}_{\mathsf{A}}(s)+\hat{f}_{\mathsf{B}}(s+\hat{T}_{\mathsf{B}}))}\} \left|\psi_{\mathsf{A}}(\mathsf{o})\right\rangle_{\bar{\mathsf{A}}}$$



# Interacting quantum clocks



Ref. PNAS [Castro-Ruiz et al., 2017].



# Conclusion



What we achieved:

something

## Conclusion



What we achieved:

something

What we learned:

something else

## Conclusion



What we achieved:

something

▶ In the end!

What we learned:

something else



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Thank you for your attention!