Endogenous Candidacy in Plurality Rule Elections: Some Explanations of the Number of Candidates and their Polarization*

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Abstract: This paper surveys the literature that uses endogenous candidacy models of electoral competition to explain the number of candidates and the extent of their polarization in elections held under the plurality rule. The plurality rule is the voting rule under which each voter votes for one candidate, and the candidate who gets the most votes is elected. We organize the different contributions to this literature into three families based on candidates' motivation to contest the election and on which part of the candidate set is endogenous. We argue that endogenous candidacy models offer both theoretical and empirical advantages over the standard Hotelling-Downs model. On the theoretical front, these models can provide a more satisfactory microfoundation for the emergence and/or stability of a two-party system under the plurality rule. On the empirical front, these models offer a better account of the stylized facts regarding the number of candidates and their polarization. We also point to shortcomings of these models and propose some directions for future research.

Key Words: Hotelling-Downs model; Endogenous candidacy; Entry deterrence; Citizen-candidate model; Plurality rule; Polarization; Duverger's law.

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1. INTRODUCTION

Regularly held, and contested, elections are considered to be an essential characteristic of a well functioning democratic system. It is therefore unsurprising that electoral competition is an extensively studied aspect of the political process. In the political economy literature, which seeks to understand the effect of political processes on economic policies, electoral competition is often used as a shorthand for politics – policies are assumed to emerge from elections rather than being chosen by a social planner. The most commonly used model of electoral competition, the Hotelling-Downs model, has become a standard feature of many textbooks.²

This article provides an overview of some of the advances in modeling electoral competition that go beyond this standard textbook model. In particular, we will survey a class of models that can be termed "endogenous candidacy models" of electoral competition. We argue that these models provide a better account of the stylized facts of electoral competition and are also based on more satisfactory theoretical underpinnings. We will limit our analysis to plurality rule, single-seat elections. Another survey – Bol, Dellis and Oak (2016) – looks at the literature on comparative analysis of different electoral rules with endogenous candidacy, also for single-seat elections.

This article is organized as follows: the remainder of this section will review and critique the canonical Hotelling-Downs model of electoral competition; in Section 2 we will present some stylized facts regarding the number and positions of political parties in elections across different countries; in Section 3 we will introduce a three-way classification of the endogenous candidacy models and discuss them in turn; finally, in Section 4, we will compare the insights offered by three families of models and conclude by pointing out directions for future research in the field.

²While there has been over time several variants of the Hotelling-Downs model in the literature, the original model can be attributed to Downs (1957) which took as a starting point the spatial competition model proposed by Hotelling (1929).

1.1. A Brief Overview of the Hotelling-Downs Model

In the canonical Hotelling-Downs model (see Calvert (1986), Duggan (2006) or De Donder and Gallego (2016) for a complete description of the model and its assumptions) the set of feasible policies is represented as points on the left-to-right spectrum of a line. Two vote-maximizing candidates compete by choosing points on the line which represent the policies they commit to implementing if elected. Voters have ideal policy positions along different points on the line and are assumed to have distance preferences, i.e., a voter's utility decreases in the distance between her ideal policy and the implemented policy. An election is held under the plurality rule; each voter votes for one candidate, and the candidate winning the biggest share of votes is elected. The celebrated median voter theorem states that the equilibrium policy positions of the two candidates will be identical to the median voter's ideal policy. The simplicity and intuitive appeal of this result makes it one of the most popular concepts in political economy. As pointed out in Callander (2005; page 1116): "This powerful result has provided the foundation for insight into many areas of political economy, including influential models on the size of government, the nature of redistributive policies, and the rate of economic growth."

However, there are some shortcomings of the Hotelling-Downs model and its prediction, the median voter theorem, which we shall briefly discuss.

1. Mixed Empirical Evidence. The median voter theorem makes two sharp predictions, viz. that the two candidates will adopt the same policy position and this policy position will coincide with the median voter's ideal policy. However, the empirical evidence does not support these predictions. For example, Ansolabehere, Snyder and Stewart (2001) provides evidence that in any given electoral district, the Democratic and Republican candidates to the U.S. House adopt polarized positions. Gerber and Lewis (2004), using individual ballot data from the 1992 general election in Los Angeles County, finds that the state legislators regularly adopt positions that differ from the preferred positions of the median voters in their respective districts.

Not only is there a divergence in the policy platforms vis-à-vis the median voter's position, the degree of divergence also varies across space and time (e.g., Ansolabehere, Snyder and Stewart (2001) and the evidence presented in Section 2). This finding suggests the need for a richer model that incorporates

factors left out of the canonical model.³ We will discuss some further empirical evidence related to this point in the next section.

2. Non-robustness of Equilibrium Existence. The existence of pure-strategy equilibria in the canonical Hotelling-Downs model depends crucially on there being two candidates and one policy dimension. With more than two candidates the existence of a pure strategy equilibrium itself is not guaranteed. Furthermore, when the policy space is multi-dimensional, pure-strategy equilibrium generically does not exist. This makes the Hotelling-Downs model a poor representation of elections that have more than two candidates or which are contested on more than one dimension.

A well known result, Duverger's law, which we discuss in the next section in detail, states that the polities using the plurality rule tend to have a two-party system. It might be tempting to argue, in light of Duverger's law, that the assumption of two parties is without much loss of generality in plurality rule elections.⁴ We will take up this issue in the next subsection.

3. Lack of Satisfactory Micro-foundations. The Hotelling-Downs model takes the number of candidates as exogenously given, restricting them to two in the canonical case. But the decision to contest an election is obviously strategic and a more satisfactory approach needs to take into account the

³The literature has adopted different approaches to generate candidate polarization in the Hotelling-Downs model. One approach, adopted by Downs himself, relaxes the assumption of full voter turnout, allowing for abstention due to alienation, viz. a citizen abstains from voting when candidates are located too far away from her ideal position (see Downs (1957) and, among the most recent contributions, Laussel, Le Breton and Xefteris (2016) and references therein). Another approach relaxes the assumption of complete information. For example, Banks (1990) assumes voters are uncertain about the policy a candidate will implement if elected. Alternatively, Calvert (1985) considers a setting where candidates are policy-motivated and uncertain about the location of the median voter's ideal policy. A third approach relaxes the assumption of unified policy-making, allowing for alternative forms of policy-making (see, among others, Austen-Smith and Banks (1988), Ortuno-Ortin (1997), Gerber and Ortuno-Ortin (1998), and Alesina and Rosenthal (2000)). Other approaches introduce an incumbency advantage (Bernhardt and Ingberman (1985)), the strategic nomination of candidates by parties (Fauli-Oller, Ok and Ortuno-Ortin (2003), and Owen and Grofman (2006)), or interest group influence (Baron (1994)).

⁴For much of our discussion, we will be using the term candidate synonymously with the term political party. As such, much of the literature we discuss can be thought of as adopting a unitary actor model of political parties. In our concluding section we discuss the relevance of distinguishing between candidates and political parties.

incentives for candidates to enter/exit the electoral race. Dutta, Jackson and Le Breton (2001) explores the implications of strategic candidacy for a broad class of voting procedures. In that paper the authors show that the outcomes of standard voting procedures will be affected by the incentives of non-contending candidates (i.e., candidates who cannot win the election) to influence the outcome by entering or exiting the election. This suggests that even when there are only two candidates in an electoral race, there may be other candidates that might potentially run and the threat of their entry can affect the election outcome. In other words, it needs to be verified if the median voter outcome of a two party electoral competition is consistent with the subgame perfect equilibrium of a bigger game involving candidate entry decisions.

1.2. Beyond the Hotelling-Downs Model

In light of the above-discussed shortcomings of the Hotelling-Downs model, researchers have developed alternative formulations of electoral competition that depart from the canonical model in one or more aspects. Osborne (1995) provides a comprehensive survey of spatial models of political competition under plurality rule till the date of its writing. In that paper, the author considers different variants of spatial models which differ in assumptions about candidate motivation, voter preferences, voter sophistication (strategic vs. sincere voting) and information available to participants. He concludes (Osborne (1995); pages 283 and 284): "the basic insight afforded by Hotelling's model—that there is an incentive for candidates in two candidate competitions to adopt similar positions—is rather robust." He goes on to further argue that the presence of more than two potential candidates significantly dilutes the convergent tendencies inherent in the Hotelling-Downs model. The survey concludes by noting that "most of the ideas designed to explain the stylized facts of political competition rely on features that are absent from [the Hotelling-Downs] model ..." (Osborne (1995); page 289). Among these features the author mentions the role of potential entry and endogenous formation of political parties to be important for future research. In this survey we will review the literature that follows this line of research. Some papers in this literature predate Osborne (1995), and have been mentioned in it, while others post-date it.

2. STYLIZED FACTS ABOUT ELECTORAL COMPETITION UNDER PLURALITY RULE

In reviewing the models of electoral competition with endogenous candidacy we emphasize the ability of these models to account for the following two stylized facts. 1) Duverger's law: plurality rule elections tend to favor a two party system; 2) Polarization: the political parties/candidates, not just the fringe players but also the serious contenders, do not always adopt convergent policy platforms. The canonical Hotelling-Downs model sidesteps the first stylized fact by exogenously assuming two parties, whereas its prediction of convergence to the median voter policy is not supported in the data.

We discuss below, in some detail, each of these stylized facts and present some empirical evidence pertaining to each.

2.1. Duverger's Law

In his seminal contribution (see Duverger (1954)), Duverger identified an empirical regularity of elections, namely, that the plurality rule tends to favor a two-party system. Riker (1982) has dubbed this empirical regularity Duverger's law.

In this section, we empirically examine the extent to which we observe two-party systems in countries that use plurality rule. A standard measure of the number of parties that is used in the political science literature is the Effective Number of Parties (hereafter ENP; Laakso and Taagepera (1979)). The ENP is an adjusted measure of the number of parties that accounts for the relative strength of each party. The ENP in a given election is obtained by taking the inverse of the sum of squared vote shares of the participating parties (v_is) . The precise formula is given by:

$$ENP = \frac{1}{\sum_{i=1}^{I} v_i^2}.$$

Thus, the ENP varies from 1 (when one party obtains all the votes) to I (when votes are equally split across all parties) where I is the number of parties participating in the election. The ENP corresponds to the inverse of the Herfindahl-Hirschman index of concentration.

The ENP is the most commonly used indicator of party system fragmentation (Lijphart (1994); page 70, Cox (1997); page 29).⁵ The indicator is used in most

⁵Another, less common, indicator of party system fragmentation is the Second-First ratio. The

observational (Ordeshook and Shvetsova (1994), Amorim Neto and Cox (1997), Clark and Golder (2006), or Carey and Hix (2011)) and experimental studies on the subject (Bol, Blais, and Labbé St-Vincent (2016), and Labbé St-Vincent, Blais and Pilet (2016)). Also many studies use the ENP as predictor of diverse phenomena such as corruption (Chang and Golden (2007)), correct voting (Lau et al. (2014)), or government stability (Colomer (2012)).

Table 1 below presents the average ENP across all districts and national parliamentary elections held between 1946 and 2013 in three democracies that have been using the plurality rule: Canada, the United Kingdom, and the United States.⁶

Table 1: Effective Number of Parties

Country	# of Elections	# of Districts	ENP (s.d.)
Canada	21	6,236	2.62 (0.48)
UK	17	11,466	2.46 (0.45)
US	17	14228	1.85 (0.34)

The numbers obtained are arguably consistent with Duverger's law. Duverger explained the tendency of the plurality rule to favor a two-party system by the strategic behavior of voters as well as by strategic calculations by the party elites in terms of whether to contest elections in a given electoral district. However, the formal models of Duverger's law in the exogenous candidacy paradigm have, naturally, focused on strategic voter behavior. Under the plurality rule, a voter who anticipates that her most-preferred candidate has no chance of winning the election would be wasting her vote if she cast it sincerely and would therefore have an incentive to vote for another candidate whom she may like less but whose electoral prospects are better. This incentive is known as the wasting-the-vote effect. This effect induces voters to concentrate their votes on only two candidates.

Second-First ratio is obtained by dividing the vote share of second losing party with the vote share of the first losing party. In single-member district elections, the first losing party is the party that comes second in the district, and the second losing party is the party that comes third in the district.

⁶The numbers presented in Tables 1 and 2 are based on our own calculations and data from the constituency-level election archive's dataset (Kollman *et al.* (2016)). The dataset can be accessed at http://www.electiondataarchive.org/. We had to drop the districts for which the vote share of at least one candidate/party was missing. In total, less than 1% of all districts was dropped.

Defenders of the canonical Hotelling-Downs model of electoral competition might argue that in light of Duverger's law assuming two party competition is not too bad an assumption to make for studying plurality rule elections. This would then vindicate the use of the median voter theorem, at least for the case of one-dimensional competition. However, Duverger's law is usually explained in terms of strategic behavior on part of the voters whereas most of the models assume sincere voting, in particular, since most models treat the set of voters as a continuum. This being the case, we need to seek explanation for Duverger's law elsewhere. One obvious candidate is strategic behavior of the political parties. As Cox (1997; page 151) pointed out in his seminal contribution, many researchers, including Duverger, have considered this possibility: "Duverger, and many after him, have argued that elite anticipation of strategic voting should lead to prudent withdrawals and hence a reduction in the number of competitors entering the field of battle." As we argue below, endogenous candidacy models have the potential to provide another channel for Duverger's law, one that does not even rely on voter's strategic behavior.

2.2. Polarization in Plurality Rule Elections

The median voter result, while being a sharp prediction of much convenience for tractable models of political economy, is not always supported by data. In Table 2 we present, for the same set of countries and time period as above, the average degree of polarization across elections. Polarization is a measure of how different the platforms of the competing parties are, along the left-right dimension. A party system is said to be heavily polarized if all the parties are located at the extremes. Polarization is said to be absent if all the parties are located at the center (Sartori (1976)). In this paper, we use the index developed by Taylor and Herman (1971), which is widely used in the empirical literature (e.g., Curini and Hino (2012), Dalton (2008), Lachat (2008), Lupu (2015), or Matakos, Troumpounis, and Xefteris

⁷We use manually coded data from the comparative manifesto dataset (Volkens *et al.* (2015)). In this dataset, each sentence is coded according to its theme, in particular, a left-wing theme (e.g., a sentence in favor of multiculturalism) and a right-wing theme (e.g., a sentence in favor of maintaining law and order). The left-right position of each party is calculated by taking the difference between the number of right sentences and the number of left sentences, and dividing this difference by the sum of left and right sentences altogether. This measure of the left-right position varies between -1 (extreme left) to +1 (extreme right). The dataset can be accessed at https://manifestoproject.wzb.eu/.

(2016)). It is calculated as the weighted sum of squared distances between each party's position on a left-right scale and the center of gravity of the party system, which is itself the weighted average of all the parties' positions on the left-right scale. Thus, we have

$$Polarization = \sum_{i=1}^{I} v_i \cdot (p_i - \bar{p})^2$$

where v_i is party i's vote share and p_i is the policy position of party i on the leftright policy spectrum with the center of gravity, denoted by \bar{p} , which is obtained by the formula

$$\bar{p} = \sum_{i=1}^{I} v_i \cdot p_i.$$

According to this index, a party system has a polarization of zero when the parties (receiving votes) are all located at the center of gravity. By contrast, a party system has a polarization of 1 when two conditions are satisfied: (1) all the parties are located at one of the two extremes, and (2) the left-wing and right-wing camps both have a vote share of 50%. The advantage of this index is that it accounts for parties' sizes. For example, systems in which two large parties are located at the two extremes would be considered as more polarized than systems in which large parties are located at the center but small parties are located at the two extremes.⁸

Table 2: Polarization

Country	# of Elections	Polarization (s.d.)
Canada	21	0.10 (0.05)
UK	17	0.10 (0.05) 0.15 (0.10) 0.08 (0.06)
US	17	0.08 (0.06)

The above table shows that, on average, it is usual to see between 10% to 20% polarization (100% being complete polarization). This is a non-negligible degree of

⁸ Another index consists in taking the left-right distance between the two most distant parties. However, this index is less common, as it does not account for parties' sizes (e.g., see Matakos, Troumpounis and Xefteris (2016)).

polarization.⁹ Across different elections that number can vary as well.

The canonical Hotelling-Downs model with its celebrated result, the median voter theorem, is unable to explain these two stylized facts. In light of this short-coming, researchers have sought to develop a richer model that would explain these stylized facts. One approach is to endogenize candidacy. The next section discusses various models of endogenous candidacy and their success (or lack thereof) in explaining the stylized facts discussed above.

3. MODELS WITH ENDOGENOUS CANDIDACY

We classify the models with endogenous candidacy into three families. The different families are classified along two dimensions: 1) whether the candidates are Downsian or Partisan, and 2) whether the entire set of candidates or only a part of it is endogenous. The second dimension is rather self-explanatory.

The first dimension concerns the motivation of candidates to contest the election. A key assumption of the Hotelling-Downs model is the pure office-motivation of candidates. In fact, candidates that are purely office-motivated are referred to as Downsian candidates and the models assuming pure office-motivation of the candidates are said to belong to the *Downsian paradigm*. Empirical evidence as well as common sense suggests that there is more than rents from office that motivates individuals to pursue a career in politics. An alternative paradigm, the *partisan paradigm*, assumes that candidates are policy-motivated, i.e., they intrinsically care about the policy outcome (see, for instance, Wittman (1983) and Calvert (1985)).¹⁰ Whether candidates are office- or policy-motivated has important implications for the credibility of their policy positions, at least in one-shot election games. Purely office-motivated parties do not intrinsically care about the policies they implement;

⁹For instance, even in the most moderate case of the US, the polarization index of 0.08 means that party manifestos have, on average, an imbalance of about 28% of sentences towards left or right compared to the "center of gravity" of the party system. Otherwise stated, if the two parties, the Democratic party and the Republican party, were to lie symmetrically around zero and tie for first place, then we have that $\bar{p}=0$ and a polarization of 0.08 would mean that the Democratic party would stand at -0.28 while the Republican party would stand at +0.28.

¹⁰This is not to deny the existence of rents from office. The defining characteristic of models in the partisan paradigm is the presence of policy motivation of the candidates, not the absence of rents from office.

rather, policies are instruments towards gaining political office. This allows the Downsian candidates to announce any feasible policy, and to credibly implement it if elected. For policy-motivated candidates, on the other hand, there is an expost incentive to deviate from the announced policy and instead implement their most favored policy. Absent any commitment device (such as reputational concerns), this limits the candidates' ability to credibly commit to implementing any feasible policy (see Alesina (1988)). Lee, Moretti and Butler (2004) provides empirical evidence supporting the premise that candidates cannot credibly commit to moderating their policies, voters essentially electing policies rather than influencing candidates' choices of policy positions (as in the Downsian paradigm).

Note that issue of the motivations of political candidates, like human motivations in any field, is a complex one. While highlighting the Downsian-partisan dichotomy we do not intend to suggest this is an exhaustive or exclusive categorization. The survey is intended to present the main strands of the existing literature, with focus on models that represent each paradigm in its canonical form. With this caveat in mind, Table 3 summarizes our classification of models of electoral competition with endogenous candidacy.

Table 3: Classification of Endogenous Candidacy Models

	Downsian paradigm	Partisan paradigm
Established candi-	Models with a threat of	
dates face threat of	entry (family 1)	
entry		
Entire set of candi-	Hotelling-Downs model	Citizen-candidate model
dates is endogenous	with endogenous entry	(family 3)
	(family 2)	

Models in the first two families belong to the Downsian paradigm while those in the third family belong to the partisan paradigm. The first family of models treats some parties as already established in the political arena while other parties are potential entrants. The focus is on the effect that (the threat of) entry can have on the positioning of the established candidates. The other two families of models do not privilege any group with an incumbency advantage and treat all parties as capable of entering or staying out.

To the best of our knowledge, there is no model of electoral competition that

fits the "fourth family", i.e., one that uses the partisan paradigm to consider the threat of potential entry to established candidates. We can offer some conjectures for this lacuna in the literature.

In order to set up such a model one would need to make a modelling choice as to whether or not a candidate can credibly commit to any policy other than his most preferred one. In the case where policy commitment is not allowed, the model will have two established candidates running on their ideal positions with the possibility of a challenger deciding whether to enter at his own preferred position. The problem of such a model is that it places very little structure on what might emerge as equilibrium outcome. Depending on the (assumed) ideal policies of the candidates, one could get one of the established candidates always assured of victory, or one could get a tied race, or one could get the challenger entering with or without a chance of success. Similarly, there is little restriction on the degree of polarisation which is obtained in equilibrium. Thus, such a model would be vacuous in terms of making empirically testable predictions.

One could, on the other hand, write down a model that assumes candidates can credibly commit to any policy, not just their ideal one. Such a model is potentially interesting and will have considerably complex strategic behavior. However, we conjecture that such a model might run into the same problem as the second family of models, viz. an equilibrium in pure strategies might not exist.

In our discussion of these models we will highlight the predictions they make on both the degree of polarization and the number of candidates running for election, and compare the models in terms of the intuition driving the results.

3.1. A Unifying Framework

We start by providing a simple spatial model of electoral competition that will serve as a unifying framework for all three families of models.

A community must elect a single policymaker to choose and implement policy. The set of policy alternatives, X, is the closed unit interval [0,1]. The community consists of a unit mass of voters, \mathcal{N} . Each voter $n \in \mathcal{N}$ has preferences over X that can be represented by a utility function $u^n(x) = -|x - x_n|$, where $|x - x_n|$ is the distance between x and x_n , and where $x_n \in X$ is voter n's ideal policy.

¹¹We assume linear-loss utility functions to simplify exposition.

Voters' ideal policies are distributed according to a distribution $F: X \to [0, 1]$, with associated density function f. We assume F to be continuously differentiable and strictly increasing on [0, 1], with F(0) = 0 and F(1) = 1. We refer to $m \equiv F^{-1}(1/2)$ as the median voter's ideal policy. To further simplify exposition, we assume f to be single-peaked and symmetric around m.

There is a finite set, \mathcal{M} , of $M \geq 2$ (potential) candidates who must decide whether and/or at which position to contest the election. We denote (potential) candidate i's decision by $e_i \in \mathcal{E}_i \subseteq X \cup \{\emptyset\}$, where $e_i = \emptyset$ if potential candidate i does not run for election, and $e_i = x \in X$ if he runs for election at position x.¹² Depending on the family of models, candidates are Downsians (families 1 and 2) or Partisans (family 3).

The policymaking process is modeled as a three stage game. At stage 1, each (potential) candidate makes his candidacy decision. At stage 2, each voter casts a vote for one candidate. In this survey, we maintain the assumption that each voter votes sincerely, viz. for the candidate whose position is closer to her ideal policy. The election winner is the candidate who obtains the most votes. All ties are broken equiprobably. Finally, at stage 3, the elected candidate implements policy. In case no one runs for election, a default policy, x_0 , is implemented. To facilitate exposition, we follow Osborne and Slivinski (1996) and assume x_0 provides utility $-\infty$ to every potential candidate.

The solution concept is pure-strategy (subgame perfect) Nash equilibrium.¹³

3.2. Models with a Threat of Entry (and Entry Deterrence)

This family of models considers Downsian candidates who seek to maximize their vote shares or their probability of winning. Its defining attribute is the distinction it makes between two types of candidates: the established candidates and the

¹²As we shall see below, the set of feasible choices, \mathcal{E}_i , varies from one family of models to another. In the first family, there are two types of candidates: the established candidates, for whom $\mathcal{E}_i = X$ (i.e., each established candidate chooses a position on X); and a potential candidate, for whom $\mathcal{E}_i = X \cup \{\emptyset\}$ (i.e., he chooses whether or not to enter the race and, if he enters, at which position on X). In the second family, $\mathcal{E}_i = X \cup \{\emptyset\}$ for every potential candidate. Finally, in the third family, $\mathcal{E}_i = \{x_i, \emptyset\}$ for every potential candidate (i.e., a potential candidate chooses between entering the race at his ideal policy x_i or stay out of the race).

¹³Since stage 1 is the only stage at which there are strategic interactions, we look for equilibria of the stage-1 game.

(potential) entrants. Specifically, the canonical model in this family considers an election with two established candidates, called L and R, who choose simultaneously their policy positions $x_i \in \mathcal{E}_i = X$ for i = L, R, and are followed by one (potential) entrant, called E, who makes his candidacy decision $x_E \in \mathcal{E}_E \subseteq X \cup \{\emptyset\}$ after having observed the positions adopted by the two established candidates.

The seminal papers in this family are Palfrey (1984) and Weber (1992).^{14,15} These papers consider settings in which the entrant always stands for election (formally, $\mathcal{E}_E = X$). The substantively different (to Hotelling-Downs) conclusion reached by these papers is that entry induces the two established candidates to assume divergent positions on either side of the median, i.e., $x_L < m < x_R$. These models are thus better able to capture the policy divergence observed in real-world political races. However, these models also exhibit the contentious feature that the entrant systematically enters, even though he is sure to lose the election.

Weber (1997) relaxes the assumption of systematic entry, assuming the potential entrant enters the race if and only if he can obtain at least a share $\delta \in [0, 1]$ of the votes.¹⁶ In other words, Weber (1997) extends the Palfrey-Weber line of work by replacing systematic entry ($\mathcal{E}_E = X$) with a threat of entry ($\mathcal{E}_E = X \cup \{\emptyset\}$). By varying δ one can obtain, as special cases, the Hotelling-Downs model ($\delta > 1/2$) and the Palfrey-Weber model ($\delta = 0$). Depending on the value of δ , the model generates equilibria in which the established candidates converge/do not converge and deter/do not deter the potential entrant from entering the race. We now present and discuss a simplified version of Weber (1997).

Each candidate i seeks to maximize his vote share. We denote by v_i (x_i ; { x_L , x_R } \cup { x_E }) candidate i's vote share (i = L, R, E) where the potential entrant makes candidacy decision $x_E \in X \cup \{\emptyset\}$ and the two established candidates choose to stand at x_L and x_R , respectively.¹⁷ The potential entrant enters if and only if there is a po-

 $^{^{14}}$ Weber (1992) generalizes Palfrey (1984) in two ways. First, it allows for any quasi-concave or single-peaked density function of voters' ideal points, f, whereas Palfrey (1984) restricts the analysis to symmetric densities. Secondly, Weber (1992) proposes a different solution concept for which existence is guaranteed for a broader class of densities, f, and which coincides with Palfrey's (1984) equilibrium concept when f is symmetric.

¹⁵Brams and Straffin (1982) is an earlier attempt. In that paper the authors take the established candidates' positions as exogenous and determine how much these positions can be polarized and still prevent an entrant from winning the election.

¹⁶Weber (1997) further relaxes the assumption of only two established candidates.

¹⁷If necessary we relabel the two established candidates such that $x_L \leq x_R$.

sition $x \in X$ at which $v_E(x; \{x_L, x_R\} \cup \{x\}) > \delta$. However, it is important to observe that $\max_{x \in [0,1]} v_E(x; \{x_L, x_R\} \cup \{x\})$ may not exist for some $\{x_L, x_R\}$. This occurs when the positions chosen by the two established candidates, x_L and x_R , are such that the greatest support the entrant can get is obtained by standing outside $[x_L, x_R]$; in this case the entrant can always increase his support by moving slightly closer to x_L or x_R while staying outside $[x_L, x_R]$. Weber addresses this issue by considering ε -best responses. Formally, the space of established candidates' positions $\{x_L, x_R\}$ can be partitioned into three subsets:

- 1. $I_{\alpha}(\delta)$ where $\sup_{x \in [0,1]} v_E(x; \{x_L, x_R\} \cup \{x\}) \leq \delta$, in which case the potential entrant does not enter.
- 2. $I_{\beta}(\delta)$ where $\max_{x \in [0,1]} v_E(x; \{x_L, x_R\} \cup \{x\})$ exists and is bigger than δ , in which case the potential entrant enters and his (unique) best-response is well-defined.
- 3. $I_{\gamma}(\delta)$ where $\max_{x \in [0,1]} v_E(x; \{x_L, x_R\} \cup \{x\})$ does not exist and $\sup_{x \in [0,1]} v_E(x; \{x_L, x_R\} \cup \{x\}) > \delta$, in which case the potential entrant enters but his best-response is not well-defined.

Based on this partition, Weber defines the payoff of established candidate i as

$$\pi_{i}(x_{L}, x_{R}) = \begin{cases} v_{i}(x_{i}; \{x_{L}, x_{R}\} \cup \{\emptyset\}) & \text{if } \{x_{L}, x_{R}\} \in I_{\alpha}(\delta) \\ v_{i}(x_{i}; \{x_{L}, x_{R}\} \cup \{x\}) & \text{if } \{x_{L}, x_{R}\} \in I_{\beta}(\delta) \text{ and } A_{0}^{\delta}(x_{L}, x_{R}) = \{x\} \\ u_{i}(x_{L}, x_{R}) & \text{if } \{x_{L}, x_{R}\} \in I_{\gamma}(\delta) \end{cases}$$

where

$$A_{\varepsilon}^{\delta}\left(x_{L}, x_{R}\right) = \left\{ \begin{array}{ll} x_{E} \in [0, 1] : v_{E}\left(x_{E}; \left\{x_{L}, x_{R}\right\} \cup \left\{x_{E}\right\}\right) \geq \\ & \max \left\{\delta, \sup_{x \in [0, 1]} v_{E}\left(x; \left\{x_{L}, x_{R}\right\} \cup \left\{x\right\}\right) - \varepsilon \right\} \right\}$$

is the set of ε -best responses that guarantee the entrant a vote share of δ or more, and

$$u_i\left(x_L, x_R\right) = \lim_{\varepsilon \downarrow 0} \frac{\int_{A_\varepsilon^{\delta}(\left\{x_L, x_R\right\})} v_i\left(x_i; \left\{x_L, x_R\right\} \cup \left\{x_E\right\}\right) \ dx_E}{\int_{A^{\delta}(x_L, x_R)} dx}.$$

Define $\Delta \equiv \inf_{\{x_L, x_R\}} \sup_{x \in [0,1]} v_E(x; \{x_L, x_R\} \cup \{x\})$, which is, roughly speaking, the smallest biggest vote share the entrant can get. Weber (1997) shows the following:

PROPOSITION 1. An equilibrium $(\overline{x}_L, \overline{x}_R, \overline{x}_E)$ exists and is unique. Furthermore,

1. an entry-deterring equilibrium exists if and only if $\delta \geq \Delta$. If $\delta \geq \Delta$, $\overline{x}_E = \emptyset$ while \overline{x}_L and \overline{x}_R are given by

$$(\overline{x}_L, \overline{x}_R) = \begin{cases} (m, m) & \text{if } \delta > 1/2 \\ (F^{-1}(\delta), F^{-1}(1 - \delta)) & \text{if } \Delta \leq \delta \leq 1/2. \end{cases}$$

2. an entry-accomodating equilibrium exists if and only if $\delta < \Delta$. If $\delta < \Delta$, \overline{x}_L and \overline{x}_R are such that $\overline{x}_L < m < \overline{x}_R$ and

$$F\left(\overline{x}_{L}\right)=1-F\left(\overline{x}_{R}\right)=\sup_{x\in\left[\overline{x}_{L},\overline{x}_{R}\right]}v_{E}\left(x;\left\{\overline{x}_{L},\overline{x}_{R}\right\}\cup\left\{x\right\}\right).$$

Furthermore, the potential entrant enters at $\overline{x}_E = m$ and his vote share is such that

$$v_E\left(\overline{x}_E; \{\overline{x}_L, \overline{x}_R\} \cup \{\overline{x}_E\}\right) \le \pi_L\left(\overline{x}_L, \overline{x}_R\right) = \pi_R\left(\overline{x}_L, \overline{x}_R\right).$$

First, if the potential entrant enters only when he can get an absolute majority of votes ($\delta > 1/2$), the two established candidates converge to the median voter's ideal policy m without triggering entry. This is the Hotelling-Downs outcome. At the polar opposite case of $\delta = 0$, the equilibrium outcome coincides with that of the Palfrey-Weber models, where the potential entrant always enters, independently of the positions chosen by the established candidates.

Second, for cases in-between the two polar cases of systematic entry ($\delta = 0$) and of entry occurring only when it guarantees a majority ($\delta > 1/2$), there will be entry by the potential entrant if the two established candidates converged to the median voter's ideal policy. In this case, the threat of entry generates a centrifugal force, the two established candidates seeking to deter entry on their flanks by adopting polarized positions. Equilibrium is reached when this centrifugal force and the centripetal force of the Hotelling-Downs model counterbalance each other. In this range, even when there is no entry in equilibrium ($\delta \in [\Delta, 1/2]$), the threat of entry generates divergence in the equilibrium positions of the two established candidates, i.e., the median voter theorem no longer holds even though there may end up being only two candidates in the race. Moreover, the extent of divergence increases as δ decreases.

Thus, models with entry are capable of simultaneously producing polarization and a stable two-candidate equilibrium. However, in any entry-accommodating equilibrium, the potential entrant enters in-between the positions of the two established candidates and does not win the election. This prediction raises two

questions. First, as noted in Shepsle and Cohen (1990; page 30), empirically we often observe third-party candidates standing at extreme positions, not at centrist ones. Second, one may question the potential entrant's decision to enter the race even though he anticipates he will come last in the election. The question of why the entrant does enter although he (correctly) anticipates that he will be defeated is a key question. Indeed, equilibrium would not exist if the entrant were restricted to enter only if he can be elected with a positive probability. This follows because of a discontinuity in the entrant's probability of election at the point where the two established candidates stand at the median voter's ideal policy. As long as the two established candidates stand at not-too-polarized positions around the median voter's ideal policy, the entrant cannot be elected. Indeed, the largest support he could obtain is by locating on the left (resp. right) flank of the established candidate positioned on the left (resp. right) of the median. But then the other established candidate would receive a majority of votes and would be elected outright. The two established candidates have thus an incentive to converge towards the median. But as soon as the two established candidates stand at the median, the entrant would win if he were to enter just on the left or just on the right of the median. The two established candidates would thus have an incentive to deviate and slightly polarize. Hence there is no pair of positions for the established candidates that would constitute an equilibrium.

By deviating from the standard assumptions about voter and/or candidate objectives or information, one can obtain equilibria which exhibit polarization and have the incentives for the third candidate to enter on the flanks. For instance, Callander and Wilson (2007) allows for voter abstention due to alienation, which generates a centrifugal force that induces established candidates to polarize. Moreover, the potential entrant is shown to enter at the extremes, assuming he chooses to enter, which is consistent with empirical observations. Another justification for the potential entrant's decision to enter the race could be that the entrant's candidacy is expressively motivated. Alternatively, Palfrey (1984) conjectures that the introduction of candidates' uncertainty about the distribution of voters' ideal policies might permit the entrant to be elected with a positive probability in equilibrium.

Callander (2005) departs from the Palfrey-Weber models by considering a multidistrict, instead of a single-district, election. In his model, multiple national parties (the analogs to the established candidates of the Palfrey-Weber model) choose simultaneously and independently whether to stand for election and at which po-The position of a national party is identical in all districts. National parties seek to maximize their share of districts won. After national parties have made their candidacy decisions, district-specific potential entrants, one per district, choose whether to enter in their respective district and at which position. National parties and potential entrants enter if and only if they anticipate winning seat(s). Each district election is held under the plurality rule. Consistent with Duverger's law, Callander shows that a two-party system can prove stable. Specifically, for intermediate levels of heterogeneity across districts, equilibrium exists in which two national parties adopt polarized positions and all potential entrants are deterred from entering. Consistent with deviations to Duverger's law, an equilibrium with multiple national parties can exist when there is sufficient, but not excessive, heterogeneity across districts. Thus, Callander's model has the appealing property of being able to generate predictions that are consistent with Duverger's law as well as predictions that are consistent with the deviations to Duverger's law. Key to explaining polarization is that when districts are heterogeneous in the position of their respective median voter, the national parties being positioned symmetrically in one district implies they are asymmetrically positioned in another district. If the national parties are not sufficiently polarized, the latter district-specific potential entrant can then enter on a flank and win the seat.

3.3. Hotelling-Downs Models with Endogenous Candidacy

One criticism of the models discussed above, at least in their canonical form, is that they are stacked in favor of the Duvergerian prediction by assuming two established candidates. Put otherwise, these models can explain the stability of a two-party system, but not its emergence. The second family of models breaks this asymmetry of treatment between established and potential candidates, and investigates the emergence of a two-party system.

The canonical model in this family, due to Osborne (1993), assumes a finite number $M \geq 3$ of potential Downsian candidates who choose, simultaneously and independently, whether and at which position to stand for election, i.e., $e_i \in \mathcal{E}_i = X \cup \{\emptyset\}$ for each $i \in \mathcal{M} = \{1, 2, ..., M\}$. They make their candidacy decisions seeking

to maximize their respective probability of winning, and enter the race only if they expect to be elected with a positive probability.¹⁸ Thus, the key difference with the previous family of models is the elimination of the distinction between established candidates and potential entrants; in other words, all potential candidates are now ex ante identical.

Osborne (1993) shows the following:

Proposition 2. We have that:

- if M = 3, then for any distribution of voters' ideal policies, F, the game has no pure-strategy Nash equilibrium.
- 2. if $M \geq 4$, then for almost any F, the game has no pure-strategy Nash equilibrium.

Thus, with more than two potential candidates $(M \geq 3)$, pure-strategy Nash equilibria are non-generic. This result is consistent with the findings of Cox (1987), which studies multi-candidate elections in the Hotelling-Downs model with exogenous candidacy. This result follows because of the restrictions that centripetal and centrifugal forces impose on the distribution of voters' ideal policies.

To understand the intuition underlying this result, observe that a potential candidate runs for election only if he anticipates to be elected with positive probability. It follows that in equilibrium, assuming one exists, at least two candidates must be standing for election. Furthermore, an equilibrium must be non-convergent, meaning not all standing candidates are located at the same position, with the leftmost and rightmost positions at which candidates are standing lying on either side of the median, i.e., $\underline{x} < m < \overline{x}$ where \underline{x} (resp. \overline{x}) is the leftmost (resp. rightmost) position with a standing candidate. In addition, the centripetal force of the

¹⁸More precisely, Osborne (1993) assumes that a candidate prefers to win than tie for first place, prefers to tie for first place with another candidate than stay out of the race, and prefers to stay out of the race than enter the race and lose.

¹⁹If nobody was running for election, then a candidate entering the race would be elected outright. If only one candidate was running for election, then a second candidate could enter at the same position and tie for first place.

 $^{^{20}}$ If all standing candidates were located at the same position (i.e., $\underline{x} = \overline{x}$), then either one of them could improve his electoral prospects by instead standing next to this position, or another potential candidate could enter next to this position and win outright. If all candidates were standing on the same side of the median (i.e., $\underline{x} < \overline{x} \le m$ or $m \le \underline{x} < \overline{x}$), then one of the candidates could improve his electoral prospects by standing instead at, or next to, the median

Hotelling-Downs model imposes that in equilibrium, two candidates are standing at \underline{x} and two other candidates are standing at \overline{x} .²¹ These last two observations rule out existence of pure-strategy equilibria when M=3. To understand why pure-strategy equilibria are non-generic when $M\geq 4$, observe that there must be as many voters located on the left of \underline{x} (resp. \overline{x}) voting for a candidate at \underline{x} (resp. \overline{x}) as there are voters located on the right of \underline{x} (resp. \overline{x}) voting for a candidate at \underline{x} (resp. \overline{x}).²² Since the same must be true at any position with two standing candidates, for almost any distribution F there cannot be more than one standing candidate at each of the positions on the right of \underline{x} , including \overline{x} , which contradicts that there must be two candidates standing at \overline{x} .

It is clear from proposition 2 that some additional structure is required for the (generic) existence of a pure-strategy equilibrium. To this end, Osborne (1993) subsequently alters the original model by considering a setting with an infinite sequence of periods. At each period, every potential candidate who has not yet chosen a position can decide to enter this period or wait one more period. Osborne shows that in this setting, an equilibrium exists for cases where M=3, 4 or 5. In these cases the equilibrium outcome involves entry of one (M=3) or two candidates (M = 4,5) occurring at the median, $m.^{23}$ Hence, while the model succeeds in getting equilibrium existence, we get back the median voter outcome, the lack of empirical validity of which we have already discussed earlier. Osborne (2000) remedies this shortcoming by adding uncertainty over the position of the median voter's ideal policy to the mix. Considering only the specific case where M=3 and assuming candidates seek to maximize their probability of winning, Osborne shows that a divergent equilibrium (possibly with only two candidates) can exist. As in the Palfrey-Weber models, polarization occurs as a way to deter or limit the impact of further entry.

m.

²¹If there was only one candidate at, say, \underline{x} , then this candidate could improve his electoral prospects by locating instead slightly further on the right since he would still get all the votes on his left and would, in addition, get extra votes on his right. If there were more than two candidates at, say, \underline{x} , then one of them could improve his electoral prospects by locating instead next to \underline{x} , thus capturing at least half of the votes that he would otherwise have shared with two or more other candidates at \underline{x} .

²²Otherwise, one of the two candidates at the position could improve his electoral prospects by locating instead next to the position, on the side with the most voters.

²³The cases where $M \geq 6$ remain an open question.

Sengupta and Sengupta (2008) shows that the threat of exit (as opposed to the threat of entry as in the Palfrey-Weber models) can also produce pure-strategy equilibria with divergent policy positions. The authors add a second candidacy stage before the election, at which each candidate can decide whether to withdraw from the race and save some fraction of the entry cost. They show that the option of withdrawing from the race can generate a centrifugal force that leads candidates to polarize. Interestingly, it is the possibility of exit that can serve to deter potential candidates from deviating from their policy positions at the entry stage.

Feddersen, Sened and Wright (1990) studies a Downsian model of endogenous candidacy but, in contrast to all the papers previously discussed, assumes the voting behavior to be strategic rather than sincere. The authors show that an equilibrium always exists and has the entrants locating at the median voter's ideal policy. Thus we get, once again, the median voter result. Moreover, as was noted by the authors themselves, the result relies crucially on a potential candidate being deterred from entering at a non-median position by the (correct) anticipation that all the voters who prefer the median to the deviator's position will coordinate on one of the candidates standing at the median, thereby defeating the deviator. Thus, the result relies on an implausible assumption of a rather fine degree of voter coordination.

To sum up, the Hotelling-Downs models with endogenous candidacy can, under suitable assumptions to ensure equilibrium existence, explain the emergence as well as the stability of a two-party system \grave{a} la Duverger. This marks an improvement over the first family of models. However, these models get a mixed grade for explaining policy divergence. In particular, Duvergerian equilibria tend to be convergent, and non-convergent equilibria tend to be non-Duvergerian (Osborne (2000), and Sengupta and Sengupta (2008) being the notable exceptions). The family of models we consider next has the potential to explain both stylized facts simultaneously. In considering this family we will be moving away from the Downsian paradigm to the partisan paradigm.

3.4. The Citizen-candidate Models

In contrast to the two previous families of models, the third one, called citizencandidate models, considers potential candidates who are Partisans, i.e., who have preferences over the policy that will be implemented. Formally, this family of models assumes that potential candidates are voters $(\mathcal{M} \subset \mathcal{N})$,²⁴ and, as all voters, have preferences over policy. The models in this family have been used for investigating several issues. These include the issues we focus on in this review, viz. the number of candidates and the extent of polarization, but also other issues such as equilibrium (non) genericity, comparison of electoral systems, the efficiency of policy outcomes, the effects of lobbying or the identity of politicians (e.g., their policy preferences or their quality).

A canonical citizen-candidate model has three stages. In the first stage, potential candidates decide simultaneously and independently whether to become a candidate by incurring a candidacy cost $\delta > 0$; candidates cannot commit to the policy they will implement once elected. In the second stage, an election is held over the set of candidates to decide the winner. In the third stage, the winner chooses and implements a policy. Since this is a one-shot election, and the winning candidate has a preferred policy, subgame perfection requires that he chooses his ideal policy. This makes the third stage decision trivial, and it reduces the stage one decision to one about whether to run, but not which position at which to run. Formally, each potential candidate $i \in \mathcal{M}$ makes candidacy decision $e_i \in \mathcal{E}_i = \{x_i, \emptyset\}$, where x_i is potential candidate i's ideal policy.

The two seminal contributions in this literature are Osborne and Slivinski (1996), and Besley and Coate (1997). These contributions differ in two important ways. Osborne and Slivinski consider a unidimensional policy space and assume voting behavior to be sincere. Besley and Coate, on the other hand, allow the policy space to be multidimensional and assume voting behavior to be strategic. In order to facilitate a comparison with the canonical models of the two previous families, which assume sincere voting and a unidimensional policy space, we shall focus on the Osborne-Slivinski version of the citizen-candidate model. Also, to offer a sharp contrast with the two previous families of models, we shall consider the opposite polar case of purely policy-motivated candidates, i.e., candidates who care only about the policy outcome.²⁵

The equilibrium set can be partitioned into three subsets, with equilibria involving one, two and multiple candidates.

²⁴The other two families of models assume instead that the set of (potential) candidates is disjoint from the set of voters $(\mathcal{M} \cap \mathcal{N} = \emptyset)$.

²⁵Note that the results we present below are robust to adding small rents from office.

Proposition 3. We have that:

- 1. One-candidate equilibria. An equilibrium in which $i \in \mathcal{M}$ runs unopposed exists if and only if each of the following two conditions holds:
 - (a) $\delta \ge |x_i x_j|$ for each $j \in \mathcal{M}$ with $|x_j m| < |x_i m|$; and
 - (b) $\delta \geq \frac{1}{2}|x_i x_j|$ for each $j \in \mathcal{M}$, $j \neq i$, with $|x_j m| = |x_i m|$.
- 2. **Two-candidate equilibria.** An equilibrium in which $i, j \in \mathcal{M}$, $i \neq j$, run against each other exists if and only if each of the following three conditions holds:
 - (a) $x_i < m < x_j \text{ and } |m x_i| = |m x_j|$;
 - (b) $\frac{1}{2}|x_i x_j| \ge \delta$; and
 - (c) either (i) $F\left(\frac{x_i+m}{2}\right) > 1/3$; or (ii) $F\left(\frac{x_i+m}{2}\right) = 1/3$ and $\delta \ge \frac{1}{3}|x_i-m|$; or (iii) $F\left(\frac{x_i+m}{2}\right) < 1/3$ and $\delta \ge |x_i-m|$.
- 3. Multi-candidate equilibria. There is no pure-strategy equilibrium with three or more candidates running for election.

In one-candidate equilibria, the candidate must be located at a position sufficiently close to the median m so that no other potential candidate wants to enter the race. A candidate whose ideal policy lies further away from the median voter's ideal policy would be defeated and would not want to enter the race. A candidate whose ideal policy lies closer to the median (condition 1.a), being preferred by the median voter, would get a majority of votes, and be elected outright. A candidate whose ideal policy is as far away from the median (condition 1.b) would leave the median voter indifferent, and tie for first place. In the latter two cases, a second potential candidate is deterred from entering the race if his expected utility gain from implementing his ideal policy is smaller than the candidacy cost δ .

In two-candidate equilibria, each candidate must tie for first place, otherwise the losing candidate would be better off not running since he would save the candidacy cost without changing the policy outcome. By the same logic, the two candidates must be standing at two different positions, otherwise one of them would be better off not running; he would save the candidacy cost without changing the policy outcome. Given that the two candidates must be tying for first place while standing at two different positions, their ideal policies must be located symmetrically around the median so that they split equally the votes and tie for first place (condition 2.a). In a two-candidate equilibrium neither of the two candidates should be better off

not running and no other potential candidate should want to enter the race. The former happens when the two candidates are sufficiently polarized, so that their expected utility gain from adopting their ideal policy exceeds the candidacy cost (condition 2.b). The latter happens when the two candidates are close enough to each other so that a potential candidate entering in-between would be defeated or would face a candidacy cost that exceeds his expected utility gain from being elected and adopting his ideal policy (condition 2.c). Observe that potential candidates with more extreme ideal policies are necessarily deterred from entering the race since they would split votes with the candidate on their side of the median, thereby triggering the outright election of the other, less preferred, candidate.

Pure-strategy equilibria with more than two candidates do not exist.²⁶ If a multi-candidate equilibrium were to exist, the leftmost candidate or the rightmost candidate (or both) would be better off not running since his votes would then go to the candidate(s) located closest to him. This vote transfer would improve the electoral prospects of the latter candidate(s) while worsening the electoral prospects of the other candidates. This rules out the existence of pure-strategy equilibria with three or more candidates running for election.

To sum up, the canonical citizen-candidate model is capable of simultaneously generating the emergence of a Duvergerian two-candidate race as well as divergence away from the median, $m.^{27}$ Moreover, the existence of a pure-strategy equilibrium is not an issue in this family of models, even beyond the one-dimensional policy space. Another strength of the citizen-candidate models is their ability to explain the existence of *spoiler candidates*. These are candidates who run to spoil the election prospects of another candidate even though they do not stand a chance to

²⁶This result holds as long as the rents from office are not too large and the potential candidates are either risk neutral or risk averse.

²⁷Using a citizen-candidate model with two political parties holding primary elections, Cadigan and Janeba (2002) shows that party primaries can lead to even more polarization. On the use of the citizen-candidate model to study candidate nomination by parties, see also Jackson, Mathevet and Mattes (2007).

 $win.^{28,29}$

While citizen-candidate models avoid the problem of equilibrium non-genericity, they typically suffer from the opposite problem of equilibrium multiplicity. The existence of multiple pure-strategy equilibria raises the tricky question of which equilibrium eventually emerges. Different approaches have been taken to tackle this issue. For example, Grosser and Palfrey (2014) obtains a unique (symmetric) equilibrium by assuming that potential candidates' ideal policies are private information. Messner and Polborn (2004) gets a unique equilibrium as well, but by introducing a privately known individual cost/rent of serving as policymaker. Alternatively, Eguia (2007) introduces aggregate uncertainty on the vote count, each vote being recorded with probability less than one. Yet another approach consists in refining the solution concept for citizen-candidate models à la Besley and Coate, i.e., models which allow strategic voting behavior (e.g., De Sinopoli (2004)).

4. DISCUSSION AND CONCLUDING REMARKS

In the previous section we discussed three families of endogenous candidacy models of elections under plurality rule. We argued that the need for these models arose due to both theoretical and empirical shortcomings of the Hotelling-Downs model. In particular, we discussed the need to explain two stylized facts: 1) the emergence and stability of the two-party system under plurality rule, 2) the existence of polarization (sometimes substantial) in the policy positions of the contending candidates. We found that there exist models in each family that could explain the above stylized facts, but different models generate these results through different channels. We will now provide a comparative perspective on these differences across models.

²⁸Equilibria with spoiler candidates are non-Duvergerian and divergent, meaning they involve more than two candidates standing at different positions. In addition, these equilibria require strong office-motivation on the part of the candidates such that the leftmost candidate or the rightmost candidate does not have an incentive to deviate and stay out of the race in an attempt at improving the electoral prospects of his neighboring candidate.

²⁹ In this survey, we have considered the Osborne-Slivinski variant of the citizen-candidate model. The other variant, studied in Besley and Coate (1997), is also capable of generating two-candidate equilibria with positions diverging from the median as well as equilibria with spoiler candidates.

4.1. A Comparative Analysis of the Three Families

We start with the issue of polarization. In the Palfrey-Weber models, the established candidates face two conflicting forces. With the established candidates located on the two sides of the median, each can gain by moving closer to the median—this is the centripetal force. However, if one candidate, say the leftist, were to move too close to the median, he would invite the new entrant to enter at a position slightly left of him. This threat generates the centrifugal force. The equilibrium is obtained at locations where the two forces, the centripetal force and the centrifugal force, are in balance; such a point has both candidates locating at positions that are divergent from the median. By contrast, the citizen-candidate model does not have the centripetal or centrifugal forces, since the candidates are immobile. In this family of models, the polarized positions occur because of the mutually reinforcing presence of a pair of candidates—the presence of, say, the leftist candidate on the opposite side makes the rightist candidate not want to quit the race because doing so will mean a loss in utility due to a distant policy being implemented. This insight also sheds light on why the citizen-candidate models suffer from a multiplicity of equilibria, each created by mutually reinforcing incentives for each candidate generated by the presence of the opposite side's candidate. On the other hand, the second family of models are not particularly successful in generating polarized equilibria.

Contrasting the Downsian paradigm of the first two families with the partisan paradigm of the third, we can see the role played by policy commitment in driving the convergence result. Polarization arises in citizen-candidate models because of the inability of candidates to commit to policies, which eliminates the centripetal force that would induce two candidates on either side of the median to converge. The role of policy commitment in generating polarization is confirmed in Dellis and Oak (2007), and Brusco and Roy (2011). These two contributions allow each candidate to commit to policies, specifically, any policy in Dellis and Oak (2007) and only policies that are ε -close to their ideal policy in Brusco and Roy (2011). In this context, all pure-strategy equilibria are one-candidate equilibria which, for a low candidacy cost, involve a candidate at the median running unopposed.

Secondly, all three types of models can generate Duvergerian outcomes. However on this front, the first family, i.e., the Palfrey-Weber models, are not particularly satisfactory because, in their canonical form, they assume the presence of two established candidates. Thus, while these models can succeed in showing the stability of a two-party system, they do not consider its emergence. However, there also exist entry-accommodating equilibria, i.e., equilibria with more than two candidates. These equilibria are a mixed bag. On the one hand, they could be bought as showing the presence of spoiler candidates, an empirically documented phenomenon. However, on the other hand, this result is driven by the vote share maximization assumption. Moreover, with a few exceptions discussed earlier, the third candidate enters at the center, rather than at the flanks of the established candidates, which does not match the stylized facts since sometimes new parties enter on the extremes as well. In the second family of models, with the exception of Osborne (2000) and Sengupta and Sengupta (2008), Duvergerian equilibria are also convergent. The existence of polarized, Duvergerian equilibria arises rather naturally in the citizen-candidate models. Moreover, unlike the first family, the emergence of this outcome is entirely endogenous. One important insight provided by these models is that Duverger's law need not arise due to the wasting-the-vote effect as argued by Duverger but it can also arise due to strategic behavior of endogenous candidates. However, as Cox (1997) pointed out, the argument based on strategic candidacy exhibits several of the same limitations as the argument based on the wasting-the-vote effect, viz. the requirement of a rather fine degree of coordination among players (potential candidates or voters) and the assumption that players are concerned with the current election, but not with future elections.

4.2. Other Avenues of Research: Current and Future

The endogenous candidacy models open up avenues for addressing other questions that naturally arise once we treat candidates as endogenous. We did not cover some of these in our survey while others are in their nascent stage and are worthy of future research.

One issue, analyzed in a related survey paper (Bol, Dellis and Oak (2016)), is that of comparative properties of alternative voting rules. In particular, the citizen-candidate model has been used to compare the extent of polarization that occurs across different electoral rules. For instance, different voting rules can be shown to affect the identity of candidates and have an effect on the policy outcome

even when the equilibrium has only two candidates running in the election. This illustrates the strength of the endogenous candidacy approach, since a model with two exogenously given candidates will not be able to distinguish between most electoral rules in this situation, as they will be equivalent; see for instance, Dellis and Oak (2006, 2007, 2016), Dellis (2009), Dellis, Gauthier-Belzile and Oak (2016). As discussed earlier, the importance of endogenizing candidacy when comparing the properties of electoral systems was forcefully raised in Dutta, Jackson and Le Breton (2001).

Similarly, there is a small literature studying the effect of the "district magnitude" (i.e., single vs. multiple seat districts) when candidacy is endogenous. For an early contribution in this area see Greenberg and Shepsle (1987). Indeed, this line of inquiry promises to offer a comprehensive "mechanism design" approach to electoral systems where both aspects of the ballot, the voting rule as well as district magnitude, may be varied and the resulting equilibria compared using, for example, the citizen-candidate framework. In this vein, Morelli (2004) compares voting procedures (using a citizen-candidate model), but with multi-member districts, comparing the plurality rule (with single-seat districts) with proportional representation (with a multi-seat district). Another example is De Sinopoli and Iannantuoni (2007), which investigates the cases where Duverger's hypothesis does not apply under proportional representation.

Another interesting question that endogenous candidacy approach is naturally amenable to is regarding the attributes of the candidates. For instance, what attracts people into politics – rents from office or policy considerations? With endogenous candidacy we can study endogenous emergence of Downsian or partisan candidates rather than assuming candidates to be of one type or another. See for instance, Callander (2008). One can also look at other attributes such as honesty or ability that get selected into political office. See for instance, Caselli and Morelli (2004), Messner and Polborn (2004), and Mattozzi and Merlo (2008), which investigate these questions using citizen-candidate models. One can also use these endogenous candidacy models as a foundation to empirically study policy choices. See for example Chattopadhyay and Duflo (2004), and Beath, Christia, Egorov and Enikolopov (2016).

Finally, one issue deserving further attention is one of the formation of political

parties. In the literature reviewed in this survey, there was essentially no distinction between a party and a candidate. However, much of the political science literature, going back to the work of Duverger, looks seriously at the formation and evolution of political parties and issues surrounding them. These issues include how parties are formed, how their internal functioning affects selection of policies and candidates. Also, the existence of political parties creates reputational concerns both across time and constituencies. Hence, a more satisfactory treatment of political parties is required for building more satisfactory models of political competition.³⁰ The need to articulate the relationship between candidates and political parties is particularly significant in the citizen-candidate models, where candidates are endowed with policy preferences. Roemer points out forcefully the need to incorporate political parties into the analysis of electoral competition. Speaking about the citizen-candidate model, he writes: "... parties organize political competition, and discipline candidates to commit to policies which are, in general, not their ideal policies" (Roemer, 2006; page 423). Several contributions have attempted to introduce parties in the citizen-candidate approach, with parties acting as commitment device for implementing announced policies (e.g., Morelli (2004) and Levy (2004)) or parties acting as cost-sharing organizations, allowing the candidacy cost to be shared between party members (e.g., Riviere (1999)). We believe that this is a key area of research which deserves further study.

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³⁰See Crutzen and Sahuguet (2016) for a survey of the existing literature on political parties in political economy models.

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