Plurality Voting Versus Proportional Representation in the Citizen-Candidate Model

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Abstract. We contrast the effect of plurality voting and proportional representation (PR) on candidates' entry behavior. We do so using the citizen-candidate paradigm (Osborne and Slivinski, 1996 and Besley and Coate, 1997). To this aim we introduce a new way of modeling PR that takes coalitions explicitly into account and compare the equilibria to those under plurality voting and PR without coalitions (Hamlin and Hjortlund, 2000). We find that (i) taking coalitions into accounts reduces candidate polarization; (ii) for policy-motivated candidates PR leads to more polarized entrants; (iii) for office-motivated candidates PR with coalitions is most conducive to multi-candidate equilibria. We complement the theoretical analysis with data from a laboratory experiment comparing plurality voting and PR without coalitions. In line with theoretical predictions we find that higher costs of entry lead to fewer entrants that are more centrist and that PR leads to weakly more entry than plurality.

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1. Introduction

How does the electoral system influence the entry decision of candidates and parties? Specifically, how does the number of entrants and their polarization differ between plurality voting and proportional representation? To address these questions this paper offers a theoretical analysis comparing plurality voting and proportional representation paying special attention to the coalition formation process typically associated with systems of proportional representation. We complement this analysis by providing results from a controlled laboratory experiment comparing behavior under the two electoral rules.

The relevance and importance of the research question stems from the fact that it forms an integral part of one of the biggest questions in political science and the economics literature on political economy: How does the electoral system influences election outcomes both in terms of who wins and what policy results. This is a fundamental question since many political and economic outcomes are a function of election outcomes and the distribution of power they induce. In the end the electoral outcomes depend on the interaction of candidates, parties and voters. Therefore when studying the effect of the electoral system one needs first to understand the behavioral effects on each of these different groups before combining them and solving for an electoral equilibrium that includes all players.

This paper provides such a first step, by focusing on candidates. The electoral system can influence their behavior in two main ways. First, it can influence which positions candidates adopt when campaigning. Second, the electoral rule can influence how many and what types of candidates (centrist vs. extreme) contest an election. The citizen-candidate paradigm (Osborne and Slivinsky, 1996, Besley and Coate, 1997) offers a tractable model that is able to address both channels at the same time. For this reason, we will employ this paradigm to analyze the effect of the electoral rule on the entry decision.

In the theoretical part of this article we contrast plurality voting to proportional representation with and without coalitions. Our main theoretical results are as follows. First, taking the coalitions associated with proportional representation into account allows for more centrist equilibria compared to proportional representation ignoring coalitions. Second, for policy

motivated candidates proportional representation with and without coalitions supports more polarized equilibria than plurality voting. And third, with Downsian –i.e. office-motivated– candidates, equilibria with many entrants are more likely under proportional representation with coalitions than plurality voting or proportional representation without coalitions.

We complement these theoretical results by running a controlled laboratory experiment testing the theoretical predictions for plurality voting and proportional representation without coalitions.[1] The reason for using a laboratory experiment is that with observational data it is virtually impossible to investigate the influence that the electoral rule has on the entry decision of political parties since electoral rules are not randomly assigned but are the result of the specific country characteristics that most likely also have a direct effect on entry decision. Furthermore, testing a model requires clear theoretical predictions but with real world data we do not know the values of the model parameters which makes it very hard to properly test the performance of the model. In contrast, the researcher controls all relevant parameters of the political system in the laboratory, which enables a true ceteris paribus variation of the electoral rule and allows for a clear benchmark for testing the citizen-candidate model.[2]

Next to the electoral rule the cost of running for office are of central importance in the citizen-candidate model. They have a large influence on the number of candidates running for office and also the effect of the electoral rule is conditional on the costs of running. We therefore also vary the costs of running in the experiment.

The results from the experiment show the theoretically predicted differences between plurality voting and proportional representation (without coalition formation). Proportional representation leads to more entry than plurality if the costs of running for office are low but for high costs there is no difference in entry behavior. This implies that more entry under proportional representation is an equilibrium phenomenon and not just due to some heuristic,

[1] The main reason for not including proportional representation with coalitions in the experiment is that three-way comparisons are difficult to implement in the lab and would have required a much larger number of treatments (see section 3 for a more extensive discussion). In a follow-up study we will compare proportional representation with and without coalitions experimentally.

[2] All these advantages of the experimental method have led to a sharp increase in the use of experiments in political science (Druckerman et al., 2006) and the political economy literature (Palfrey, 2012).

f.i. entering to influence the policy without regard for payoffs. Furthermore, an increase in the costs of running for office reduces the number of entrants. Overall the comparative statics predictions for the experiment are therefore confirmed but entry rates are across the board higher than predicted.

The remainder of the paper is structured as follows. First, we present the general citizen-candidate model for plurality voting and the two types of proportional representation. Section 3 introduces the experimental design and the hypothesis. Next, we present the results from the experiment and section 5 concludes and discusses some avenues for possible future work.

2. The model

General Set-up

The citizen-candidate model introduced by Besley and Coate (1997) and Osborne and Slivinsky (1996) is a model in the Downsian tradition of spatial voting (Downs, 1957). We have an electorate of citizens distributed over a policy space where each citizen is described by her ideal point (i.e. position) in the policy space. The defining feature of the model –and that its name derives from– is that each citizen can run for office by paying a cost c. After simultaneous decisions on whether to run for office, all the candidates and their positions in the policy space are announced[3] and an election takes place. This election determines a policy x^* that will be implemented as well as the allocation of office rents, denoted by b.[4]

The utility for a citizen with ideal point x_i is assumed to take the following form

$$U = -f(|x^* - x_i|) + b * W - c * R,$$

where $f \geq 0;\ f' > 0$; W is a dummy variable that is equal to 1 if the candidate wins the office rents and R is a dummy variable that is equal to 1 if the candidate runs for election.

[3] Grosser and Palfrey (2014) demonstrate how important the assumption of perfect observability is by analyzing the opposite extreme where positions are not observable (or only whether they are to the left/right of the median) and demonstrate that the equilibria change fundamentally.

[4] One can, for instance, think about these office rents as compensation for government work or perks from office but also as an improvement in opportunities upon leaving office.

Contrasting the specific modeling assumptions made by Besley and Coate (1997, henceforth 'B+C'), and Osborne and Slivinsky (1996, 'O+S'), can help to clarify some of the important modeling decisions that have to be made. While O+S assume a continuous distribution of citizens along a one-dimensional policy space, B+C allow for a multi-dimensional policy space and focus on the case of N citizens. Regarding voter behavior, B+C allow for strategic voting while O+S impose sincere voting. Finally, O+S consider the case where candidates might care about office in itself (i.e. $b \geq 0$) while B+C assume that candidates are purely policy-motivated ($b = 0$).

Our model is closer to O+S in the sense that we focus on a one-dimensional policy space where citizens are uniformly distributed on the interval [0, 1] and we assume sincere behavior by the voters. Regarding the candidates' motivation we analyze both the case of pure policy motivation ($b = 0$) and pure office motivation ($b = \infty$).[5] Finally, we assume that a citizen's utility function is linear in the distance between her ideal point x_i and implemented policy x^*. This gives rise to the following utility function:

$$U = -|x^* - x_i| + b * W - c * R$$

The environment originally studied by B+C and O+S is one of plurality voting. In this case it is straightforward and intuitive to assume that the implemented policy x^* is the ideal point of the candidate receiving the most votes (with ties broken randomly) and that all the office rents are awarded to this candidate. In this paper, we will adopt the same way of modelling plurality voting. An open question is what to assume for the case that no citizen enters the race. We follow O+S in assuming that each citizen receives a payoff of $-\infty$ which can be interpreted as a large loss in utility due to a breakdown of democracy.[6,7]

In the case of proportional representation it is less obvious how to model the mapping from votes to the implemented policy and the distribution of office rents. Hamlin and Hjortlund (2000) –who were the first to model candidate entry under proportional representation in the

[5] Candidates are not purely office motivated since, if office rents are equal for different decisions, they still care about policy. To be precise, they have lexicographic preferences with office rents being the first priority.
[6] B+C assume for this case that an exogenously given default policy will be implemented.
[7] Our results are robust to assuming that instead of breakdown of democracy one citizen will be randomly chosen to form a care-taker government by himself.

citizen-candidate paradigm– assume that the implemented policy is the vote-weighted average of all candidates' positions and that the candidate with most votes is awarded the office rents.[8] While this way of modelling the implemented policy correctly takes into account that plurality is not needed in proportional representation to have an influence on the policy, it cannot capture some other important features of proportional representation. For instance, the important discontinuity that arises when one candidate receives an absolute majority is not accounted for.[9] But most importantly, it does not take account of one of the defining features of systems of proportional representation – coalition governments.

Therefore we propose a different way of modelling proportional representation that takes coalitions explicitly into account.[10]

In our model, the implemented policy is assumed to be the vote-weighted average of the policy positions of the candidate that are part of the governing coalition. Furthermore, the office rents are allocated proportionally within the coalition. The proportional division within the coalition is motivated by Gamson's Law (Gamson, 1961) which states that government portfolios are distributed proportionally within the coalition. Ansolabehere et al. (2005) offer supporting empirical evidence by investigating portfolio allocations in Western Europe from 1946 to 2001.

Coalitions are formed according to the following procedure:[11]

1. If a candidate receives an absolute majority of votes cast, she unilaterally forms a government and the implemented policy x^* is equal to this candidate's policy position.
2. If no candidate receives an absolute majority of votes cast, the candidate with the most votes is assigned the role of 'government formateur'. This candidate then proposes a coalition to the candidate or candidates she wants to cooperate with; if everyone agrees, the coalition is formed.

[8] This can be interpreted as the office rent being the payoff associated with being the prime minister, which is often awarded to the largest party in parliament.
[9] See Indridason (2011), who in a different context shows that it is sufficient to assume that a candidate with an absolute majority can implement her favorite policy to fundamentally change the equilibrium.
[10] Bandyopadhyay and Oak (2004) also study the citizen-candidate model with coalitions. They use a similar set-up but focus on the coalition formation stage and do not analyze the number and type of entrants.
[11] This procedure is taken from Kamm and Schram (2013).

3. If multiple candidates have the most votes, a fair random draw decides which of them is assigned the role of formateur.

4. If the coalition is rejected, bargaining breaks down and every candidate receives a payoff of $-\infty$.

One can think about this as a simplified version of the bargaining approach used in Austen-Smith and Banks (1988) where bargaining breaks down after the first round. An alternative approach would be to have a random formateur (as used in Baron and Ferejohn, 1989) where recognition probabilities are proportional to seat shares. Which of the two assumptions about formateur choice has more external validity is an empirical question. Diermeier and Merlo (2004) analyze government formation in 14 Western European countries and find that the random formateur model fits the data better than recognition in order of seat share but the largest party has a disproportionally high probability of getting the first shot at forming a government. Additionally, Ansolabehere et al. (2005) find that controlling for vote shares the largest party is twice as likely to be the formateur. This provides some support for our simplifying assumption that the largest candidate forms the coalition.[12] Furthermore, while assuming bargaining to be take-it-or-leave is certainly restrictive it greatly simplifies the analysis by avoiding to have to solve for a sub-game perfect equilibrium of the bargaining stage.[13]

Note that we do not impose that coalitions are minimal winning (Riker, 1962) or connected but let the formateur endogenously form the coalition that maximizes her utility (see Bandyopadhyay and Oak, 2008, for a model in a similar spirit).[14]

[12] Additionally, in the case of purely policy-motivated candidates the equilibria do not change if we assume a random formateur rule instead.

[13] Obviously it would be interesting and important to investigate how robust the results are to the specifics of the bargaining protocol. We leave this for future work.

[14] There is also an active empirical literature that tries to establish determinates for the type of coalition observed. Martin and Stephenson (2001), for instance, analyze 220 coalition formation processes in 14 established democracies. They find that both ideology alignment and office rents matter and that minimal winning coalitions are most frequent.

Equilibrium analysis and comparative statics

We offer an analysis for the case of purely policy-motivated ($b = 0$) and the Downsian case of office-motivated ($b = \infty$) candidates.[15] We focus on these two extreme cases that are most often used in the literature to get sharper predictions. While the case of candidates that care substantially for both office rents and policy is certainly very interesting and relevant, it makes the equilibrium analysis intractable. We therefore leave the interaction effect between the two motives for future work. Furthermore, we restrict the equilibrium analysis to one-, two- and three-candidate equilibria since these three cases capture the important cases of uncontested elections, two-candidate elections and multi-candidate elections and they are sufficient to make an argument regarding the number of entrants as well as their polarization across electoral rules.

Since the equilibria for the case of plurality voting and proportional representation without coalitions have been established by Hamlin and Hjortlund (2001)[16] and Osborne and Slivinsky (1996)[17] we will here only present the equilibria for our model of proportional representation with coalitions.[18] All the proofs are relegated to appendix A.[19]

We start with the case of policy-motivated candidates.

Proposition 1 (PR with coalitions and policy-motivated candidates):

(a) A single entrant at position X is an equilibrium if

   a. $0 \leq \mathrm{X} < 0.5$ and $1 - 2\mathrm{X} \leq \mathrm{c}$ **OR**

   b. $X = 0.5$ **OR**

   c. $0.5 < \mathrm{X} \leq 1$ and $2\mathrm{X} - 1 \leq \mathrm{c}$

(b) Two candidates entering at position L and R is an equilibrium if:

   a. $R = 1 - L$ and $\frac{\sqrt{29}-1}{28} \leq L < \frac{1}{2}$ and $c < 0.5 - L$ **OR**

[15] As mentioned above, the latter candidates do care about policy but only if multiple decisions give them the same office rent.
[16] See their proportions 1-3 and claims 1 and 2.
[17] See their propositions 1-3.
[18] Additionally, in appendix A we describe the equilibria with plurality and proportional representation for our exact model specification.
[19] All appendices can be found here.

b. $R = 1 - L$ and $L < \frac{\sqrt{29}-1}{28}$ and $\frac{0.5-2L-14L^2}{1+2L} < c < 0.5 - L$

(c) There does not exist an equilibrium with three candidates.

What is the intuition for these equilibria and their structure?

In the case of a one-candidate equilibrium all entrants that are located closer to the median than the current entrant could implement their favorite policy and therefore the entry costs have to be high enough such that these citizens prefer accepting a sub-optimal policy over implementing their favorite policy by entering. The equilibrium where the candidate is located at the median position is a special case since the median candidate cannot be beaten by any candidate and therefore this equilibrium exists independent of the level of entry costs.

The two-candidate equilibria have to be symmetric since otherwise one of the two candidates has an absolute majority which means that the other candidate has no influence on the policy and prefers to stay out of the race. Furthermore, for very low entry costs the equilibrium cannot be too polarized. The reason is that if the candidates are very polarized there is an opening for a moderate candidate to become formateur of the coalition and she would enter.

Finally, there cannot be a three-candidate equilibrium.[20] The reason is that always one of the extreme candidates prefers an absolute majority of the center candidate over the situation where the other extreme candidate is part of the government.

Proposition 2 (comparative statics with policy-motivated candidates):

(a) Proportional representation, with and without coalitions, allows for more polarized two-candidate equilibria than plurality rule.

(b) Proportional representation with coalitions allows more centrist two-candidate equilibria than proportional representation without coalitions.

The intuition for part (a) of the proposition stems from the fact that with proportional representation an entrant has a different influence on the policy than under plurality voting. For a high degree of polarization an entrant can win a substantial fraction of the votes by

[20] This is also the case for plurality voting and proportional representation without coalitions.

entering in the center of the policy space. With plurality voting this entrant can implement her favorite policy and therefore has a strong incentive for entry. Under proportional representation she will have a smaller effect on the policy since at least one other candidate will also have an influence on the policy. This reduces the likelihood that polarization will be reduced to avoid entry at the center of the policy space.[21]

The intuition for the second part of the proposition is that with coalitions an extreme entrant (who would win a lot of votes if the candidates are both centrist) will not be part of the coalition. The reason is that the formateur (which the entrant will never be) prefers a coalition with the closely aligned candidate and not the extreme entrant. Without coalitions on the other hand the extreme entrant has an influence on the policy and therefore a two-candidate equilibrium with little polarization is not sustainable.

Next, we turn to the classical Downsian case of office motivated candidates.

Proposition 3 (PR with coalitions and office-motivated candidates):

(a) A single entrant at position X is an equilibrium if
   a. $\mathrm{X} = 0.5$
(b) There does not exist an equilibrium with two candidates entering.
(c) Three entrants entering at position L, C and R is an equilibrium if:
   a. $L = \frac{1}{6}$ and $C = \frac{1}{2}$ and $R = \frac{5}{6}$
   b. $C = \frac{1}{2}$ and $R = 1 - L$ and $L > \frac{1}{3}$

The reason that in the (only) one-candidate equilibrium the median citizen enters is that given the large office rents every citizen that can win against the candidate will enter. The only candidate that cannot be beaten is the median candidate.

The reason for the non-existence of a two-candidate equilibrium is that there is always an entrant that can join the coalition and which therefore would enter.

[21] This is in line with theoretical work by Cox (1990) and recent empirical work by Matakos et al. (2013).

Finally, it is noteworthy that in all three-candidate equilibria the extreme candidates have a weakly higher vote share than the center candidate. The intuition is that if the center candidate is the sole formateur there is always an incentive for a moderate candidate to enter between the extreme and the center candidate to become part of the coalition.

Proposition 4 (comparative statics with office-motivated candidates):

(a) Multi-candidate elections are most likely under proportional representation with coalitions.
(b) Compared with plurality voting, proportional representation with coalitions allows for more centrist outcomes in three-candidate equilibria.
(c) Compared with plurality voting, proportional representation without coalitions leads to more polarized outcomes.
(d) Proportional representation with coalitions leads to more centrist three-candidate equilibria than proportional representation without coalitions.

Part (a) of the proposition follows from the fact that with coalitions there does not exists an equilibrium with two candidates and therefore apart from the case where only the median citizen enters the equilibrium is a multi-candidate outcome. Part (b) and (c) of the proposition highlight how taking coalitions into account changes the equilibrium structure. Coalitions allow a lower degree of polarization compared to plurality voting since in this case under plurality voting the center candidate has no influence on the outcome while with coalitions she is an attractive coalition partner. If on the other hand we do not take coalitions into account, we find that proportional representation leads to more polarized outcomes. The intuition is that under plurality voting all candidates have to receive office rents which implies that the center candidate cannot win a strict plurality of the votes which is only possible if polarization is not too high. Under proportional representation the opposite is true and the center candidate needs to win office rents for him to enter which is only possible for a high degree of polarization. The final part of the proposition follows from the fact that with coalitions a large degree of polarization opens the door for moderate entrants that are very attractive coalition

partners while if coalitions are not taken account, these moderates would only enter if they could win a plurality of the votes.

Table 1 sums up the comparison of the different electoral rules which leads to the following main findings: First, due to the majoritarian aspect inherent in coalition government proportional representation with coalitions allows for more centrist outcomes. Second, proportional representation without coalitions allows more (less) polarized outcomes than plurality voting when candidates are office (policy) motivated. Third, for office motivated candidates proportional representation with coalitions is most conducive to multi-candidate outcomes.

Table 1: Comparison of electoral rules

<table>
<tr><th></th><th>Policy motivated</th><th>Office motivated</th></tr>
<tr><td>PR+C vs. PR</td><td>PR+C allows for more centrist 2-candidate equilibria</td><td rowspan="2">3-candidate equilibria most likely in PR+C and can be more centrist</td></tr>
<tr><td>PR+C vs. PL</td><td rowspan="2">PR and PR+C allow for more centrist 2-candidate equilibria</td></tr>
<tr><td>PR vs. PL</td><td>PR more polarized outcomes</td></tr>
</table>

Notes. PR+C (PR; PL) denotes proportional representation with coalitions (proportional representation without coalitions; plurality voting).

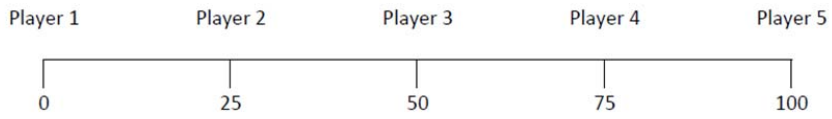
3. Experimental design

The general set-up

For the experiment the model has to be somewhat simplified since we cannot implement the infinite number of potential candidates implied by a continuous distribution. Instead we choose five fixed positions with one potential candidates each as shown in figure 1.

Furthermore, there is a continuum of uniformly distributed voters whose voting behavior is automated in the experiment since our focus is on candidate behavior. We assume voters vote sincerely for the candidate located closest to their positions.

Figure 1: The players' positions



Notes. The figure shows the positions of the potential candidates.

The subject's payoff function is given by: [22]

$$100 - |x^* - x_i| + b \text{ (if winning the office rents)} - c \text{ (if running for office)},$$

where x^* denotes the policy implemented after the election, x_i is the subjects position in the policy space, b are the office rents and c are the costs of running for office. The constant is used to make losses unlikely so that we do not need to worry about loss aversion.

In the experiment we will compare plurality voting and proportional representation. Under plurality voting the position of the candidate that receives the most votes is the implemented policy and this candidate receives all the office rents (ties are broken randomly). For modeling proportional representation we follow Hamlin and Hjortlund (2000) and assume that the implemented policy is the vote-weighted average of the candidates' positions and the office rents are awarded to the candidate that receives the most votes (ties are broken randomly).

We do not include the case of proportional representation in our experiment. The main reason is that it would complicate the experimental design substantially. To have interesting comparative statics for a three-way comparison of electoral rules we would need at least three different cost level leading to a three-by-three design, i.e. more than double the conditions than in the two-by-two design employed in our analysis. Furthermore, while proportional representation is more complex than plurality voting adding coalitions makes the game even more complicated for subjects. Therefore exploring proportional representation without coalitions is the natural first step before exploring the additional effect of coalitions. In a follow-

[22] This implies that parties are not Downsian, i.e. completely office motivated. For an experiment that uses purely office-motivated candidates see Bol et al. (2014).

up study that we are currently designing we will compare proportional representation with and without coalitions in the laboratory.

Treatments

The experiment consists of four between-subject treatments that are organized in a two-by-two structure as shown in table 2. On the first dimension the costs of running for office will be either low (8 points) or high (40 points). In the second dimension the voting rule will either be plurality voting or proportional representation. Should nobody enter, in both cases one candidate will randomly be chosen, whose position will be implemented as the policy. The office rents *b* are 25 points and are awarded to the candidate that receives the most votes (ties broken randomly). Should no player enter, no office rents are awarded.

Table 2: Treatments

	Plurality	Proportional Representation
Low cost	PL-low	PR-low
(c=8)	N=12	N=11
	EQ: 50 OR 25 and 75	EQ: 25 and 75 OR 0, 50 and 100
High cost	PL-high	PR-high
(c=40)	N=12	N=12
	EQ: 50	EQ: 50 OR 0 and 100

Notes. Cell entries give the treatment acronym used throughout this paper, the number of independent observations (N=# groups) and the symmetric pure-strategy Nash equilibria (EQ) for each treatment.

Varying the electoral rule enables us to investigate whether there is a difference in entry behavior when comparing proportional representation and plurality. Varying the cost of running for office achieves two things. First, it enables us to test the internal logic of the citizen-candidate model, i.e. whether a change in the parameters leads to the predicted change in behavior, and whether the performance of the model is different for distinct electoral rules. Second, it makes it possible to see whether a difference in entry behavior between the two electoral rules interacts with costs of entry (as is predicted for the parameters we chose). This

can shed light on the question whether a difference in behavior is an equilibrium phenomenon or due to some non-equilibrium heuristic employed by the subjects. For instance, we expect higher entry under proportional representation only for low costs (see hypothesis (c) below). Should we observe higher entry for both costs levels this would indicate that the difference in entry is due a heuristic, f.i. entering to influence the policy without regard for payoffs.

Hypothesis

These treatments give rise to the equilibrium predictions shown in table 2 (we focus on symmetric[23] Nash equilibria in pure strategies).[24] These predictions imply three main treatment effects:

(a) Cost effect: An increase in the costs of running for office leads to fewer players entering and less polarized entrants.

(b) System effect: Under proportional representation (weakly) more players enter than under plurality voting.

(c) Interaction effect: When costs are high the difference in entry between proportional representation and plurality voting is (weakly) smaller than when costs are low.

Hypothesis (a) and (b) follow straightforward from table 2. Hypothesis (c) is a conjecture based on the fact that the second equilibrium for proportional representation and high costs is quite unstable (entry for the extremes is only profitable if neither the median nor the moderate players enter) and therefore given the noise to be expected in the experiment it seems quite unlikely that this equilibrium will be selected.[25]

[23] The reason for focusing on symmetric equilibria is that coordination on asymmetric equilibria is difficult given that positions are reassigned every round. Furthermore, only in treatment 3 (low costs and proportional representation) do asymmetric pure strategy equilibria exist where players 1, 3 and 4 or 2, 3 and 5 enter. In the experiment no group coordinated on this equilibrium.

[24] Equilibria were computed using Gambit 13.1.2, McKelvey et al. 2014.

[25] Additionally, if we use quantal response equilibrium (McKelvey and Palfrey, 1995), which is a noisy best-response concept that has a better track record than Nash in explaining binary choice data in experiments (see Goerre and Holt, 2004), the 'principal branch' of the quantal response equilibrium (computed using Gambit, see fn. 24) selects the equilibrium with the median player entering over the polarized equilibrium. For PL-low also the median equilibrium is selected while for PR-low the two-candidate equilibrium is selected.

Besides these comparative statics predictions we also investigate whether the subjects behave in line with the equilibrium predictions and (in case of multiplicity) which of the equilibria are selected in the lab.

Experimental Protocol

The experiment was conducted at the CREED laboratory at the University of Amsterdam in February 2014 and implemented using php/mysql.[26] Participants were recruited using CREED's subject database. In each of eight sessions, 25 or 30 subjects participated. Most of the 235 subjects in the experiment were undergraduate students of various disciplines.[27] Earnings in the experiment are in 'points', which are converted to euros at the end of the experiment at an exchange rate of 100 points = 1€. The experiment lasted on average 75 minutes and the average earnings were 19€ (including a 7€ show-up fee).

After all subjects had arrived at the laboratory, each was randomly assigned to one of the computers. Once everyone was seated they were shown the instructions on their screen. After everyone had read these and the experimenter had privately answered questions, a summary of the instructions was distributed. This summary also contained a table that for all possible combination of entry decisions specified what vote share each candidate receives and which policy will be implemented. Then, all subjects had to answer quiz questions that tested their understanding of the instructions. After everyone had successfully finished this quiz, the experiment started. At the end of the session, all subjects answered a short questionnaire and were subsequently privately paid their earnings.

In each session the subjects participated in fifteen rounds of play and had do decide whether to run for office or not.[28] Given the multiplicity of equilibria learning and coordination are very important and therefore subjects stay in the same group of five subjects throughout the whole session (partner matching). Furthermore, in each round it was randomly determined which

[26] For screenshots of the interface as well as the text of the instructions and the summary handout, see appendix B.
[27] 149 of the 233 participants that gave information on their field of study were students in business or economics.
[28] We decided to not use a neutral frame but talk about coalitions to make it easier for subjects to understand the task.

subject is located at which position in the policy space, i.e., positions were reallocated in each round.

The specific task in each round is presented as follows: subjects are informed in every round about their position in the policy space. In all treatments we give the subjects the option to see the complete history in which they took part by clicking on a button.[29] Hence, they can see what they did in the past for different positions, what the other players' entry decision were and what the resulting implemented policy was. Furthermore, we provide them with a payoff calculator such that they can compute the payoffs they would get from different decisions by them and the other players, given their position in the current round.

After everyone has decided whether to enter, the computer casts the votes (according to a uniform distribution) and shows each subject the entry decision by all players, what vote share each candidate received, what policy is implemented, who received the office rents and the payoff from the current round as well as the accumulated payoffs from past rounds.

4. Results

We start with presenting results for each treatment separately to investigate the degree to which the Nash equilibrium predictions are supported and, in case of multiple equilibria, which equilibrium was selected.[30] Subsequently, we turn to the comparative statics predictions. An analysis of subject behavior on the individual level is presented in appendix C.

Within treatment analysis

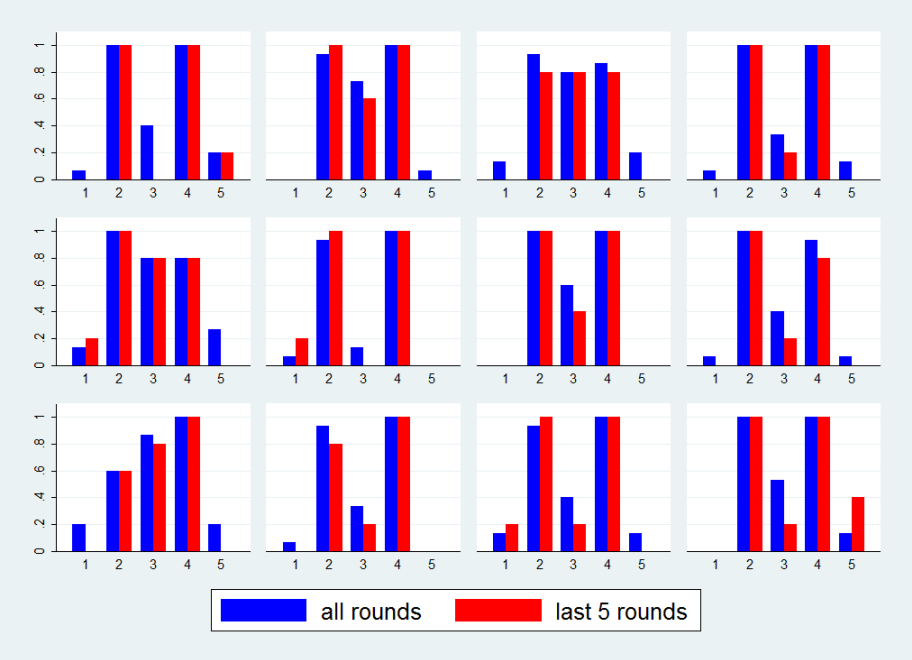
Plurality voting with low costs

The theoretical prediction is that either only player 3 (the median player) enters or that the moderate players 2 and 4 enter. As we can see from figure 2 the one-candidate equilibrium is clearly not selected. Instead the moderate players have the highest entry rates. Using a Wilcoxon rank-sum test with the group average as unit of observation we find that the moderates have significantly higher entry rates than the median player ($p<0.01$) and the

[29] Subjects did not use this option very much. Overall, subjects only had a look at the history in 5% of the rounds.
[30] Figures showing entry decisions of over time for every group separately are available upon request from the author.

median player has a significantly higher entry rate than the extreme players ($p<0.01$).[31] Figure 2 also clearly shows that the two-candidate equilibrium is not perfectly attained since the median player enters quite often. In fact only 37% of rounds (50% in the last 5 rounds) correspond perfectly to the two-candidate equilibrium. The likely reason for this over-entry by the median player is that given the low costs of entry the median player enters in the hopes that one of the moderates will not enter which would make the median player the winner of the election (this only works in 5% of the elections).

Figure 2: Entry under plurality voting with low costs



Notes. The figure shows the average entry rates by position (1 is position 0, 2 is position 25, 3 is position 50, 4 is position 75 and 5 is position 100) and group.

As we can see from the difference between the entry rates for all rounds and for the last five rounds depicted in figure 2 some learning took place. Employing a Wilcoxon signed-rank test we find that the median player is significantly less likely to enter in the last five rounds

[31] This result (as well as all other results for the within treatment analysis if not stated otherwise) are also significant at the 5%-level when restricting attention to the last five rounds after learning has taken place.

compared to the first five rounds ($p<0.01$).[32] Therefore behavior gets closer over time to the two-candidate equilibrium.

Regarding the election outcomes we find that in 95% of the elections, one of the moderate candidates wins the election while in the remaining cases the median player does.[33] This leads to an average policy of 49.0 and an expected distance of the policy to the median's preferences of 23.8 (very close to 25 predicted for the two-candidate equilibrium). Therefore, the election outcomes are in line with the theoretical predictions both in terms of the winner of the election and the policy outcomes.

Plurality voting with high costs

For this parameter configuration the unique pure strategy equilibrium is for only the median player to enter. Figure 3 shows that this is predominantly the case for four of the nine groups (1,3,8 and 10) and in three more groups (7, 9, 11) the median player has the highest entry rate but behavior is quite heterogeneous across groups. Overall a Wilcoxon rank-sum test with the average per group as the unit of analysis shows that the median player has a significantly higher entry rate than the moderate players ($p=.03$) and the extreme players have a significantly lower entry rate than the moderate players ($p<.01$).

Again the behavior does not perfectly correspond to the prediction since only in 27% (for the last 5 rounds 36%) of the rounds the median player is the sole entrant.[34] The reason for the deviation is that the moderates enter quite often. This could be attributed to a joy of winning. Given that the extreme players rarely enter, an entering moderate will win the election regularly (37%, see below), which might lead players to enter even though for the high entry costs it would be better to stay out and let the median player obtain the office rents.

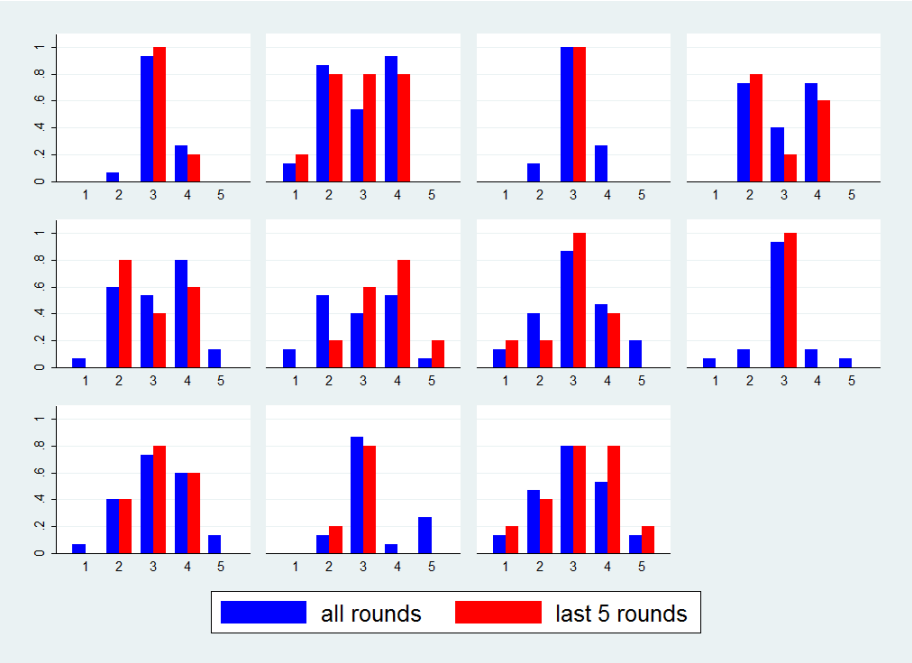
[32] For the extreme ($p=.12$) and moderate ($p=.62$) players no significant difference between entry rates in the first five and last five rounds is detected.

[33] Given that for a risk-neutral player at the median position entry is only worthwhile if she wins more than 32% of the elections the observed over-entry by the median player is not a best-response to the behavior observed in the experiment.

[34] In this treatment no significant learning seems to be happening since the entry rates do not significantly differ between the first five and last five rounds. The p-values of the Wilcoxon signed-rank test are $p=.13$ for the extreme players, $p=.14$ for the moderate players and $p=.59$ for the median.

Regarding the election outcomes, we find that the average implemented policy is 51.1 and the observed distance to the median player's position is 10.3. While the average policy is therefore close to the predicted value the variance of the policy is larger than predicted. Furthermore, we find that the median player wins only 59% of the elections and not all of them as the Nash equilibrium predicts. In almost all the remaining elections (37% of the elections) a moderate wins.

Figure 3: Entry under plurality with high costs



Notes. The figure shows the average entry rates by position (1 is position 0, 2 is position 25, 3 is position 50, 4 is position 75 and 5 is position 100) and group.

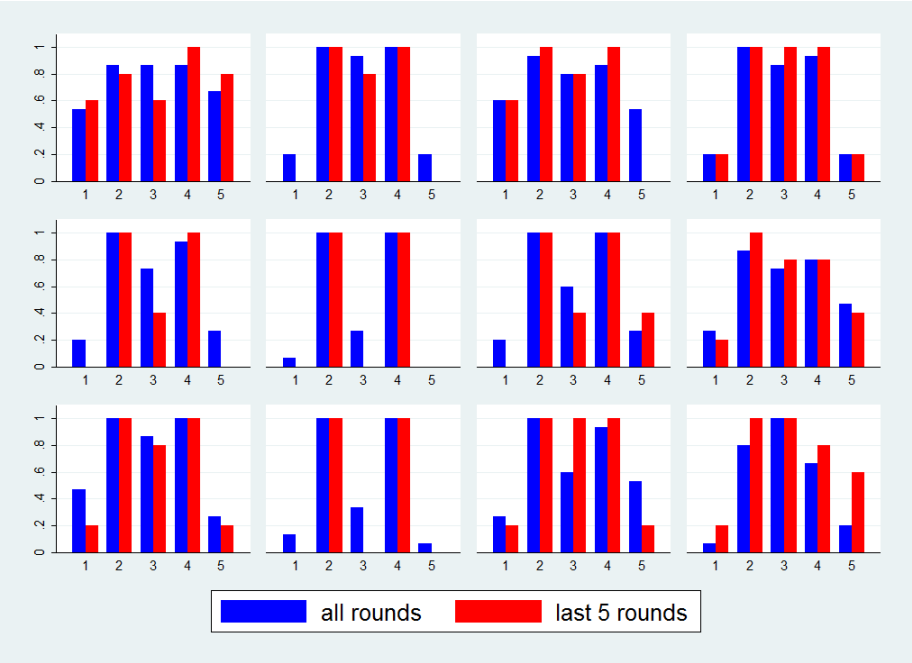
Proportional representation with low costs

For this treatment the prediction is either a two-candidate equilibrium with the moderate players entering or a three-candidate equilibrium where the extremes and the median player enter.[35] Inspecting figure 4 shows no support for the three-candidate equilibrium since the extremes enter less than half the time. At the same time there is some (albeit quite weak) evidence for the two-candidate equilibrium since two groups (6 and 8) exhibit high entry rates

[35] There are also two mixed strategy equilibria where either players 1, 3 and 4 or players 2, 3, and 5 enter.

by the moderates and low entry rates by the other players. In most other groups both the median and the moderate players enter frequently with the extremes having lower entry rates. Furthermore, a Wilcoxon rank-sum test shows that the moderates have significantly higher entry rates than the median player ($p<0.01$) which is in line with the structure of the two candidate equilibrium. Overall 15% (for the last 5 rounds 27%) of rounds exhibit entry by only the moderate players. The reason for this low rate of equilibrium play is the substantial entry by the median player. It is difficult to rationalize this behavior since the moderates have entry rates close to 100% which implies that the median player will not win the office rents and she has no influence on the implemented policy.

Figure 4: Entry under proportional representation with low costs



Notes. The figure shows the average entry rates by position (1 is position 0, 2 is position 25, 3 is position 50, 4 is position 75 and 5 is position 100) and group.

Analyzing learning over time by comparing behavior in the first five and last five rounds using a Wilcoxon signed-rank test shows that the extreme players significantly reduce their entry rates over time ($p=.01$) and the median player enters weakly significantly ($p=.09$) less often in later

rounds while the moderates significantly increase their entry rates (p=.04). These findings taken together imply that behavior may be converging towards the two-candidate equilibrium.

The election outcomes are broadly in line with the theory since a moderate win 86% of the elections while the median only wins 13% of the elections.[36] Furthermore, the mean policy of 50.0 and the very low variance with a mean distance between the policy and the median of 2.0 are also in line with the theoretical prediction.

Proportional representation with high costs

In this treatment two pure strategy equilibria exist. In one only the median enters while in the other the two extreme players enter. Figure 5 clearly shows that the polarized equilibrium is not selected but five of the twelve groups (1,2,3,5,6,12) converge to a situation where the median player has the highest entry rates. Overall the entry rates of the median player are significantly higher than of the moderates (p=0.04) but in the last five rounds this difference is only weakly significant (p=0.09).

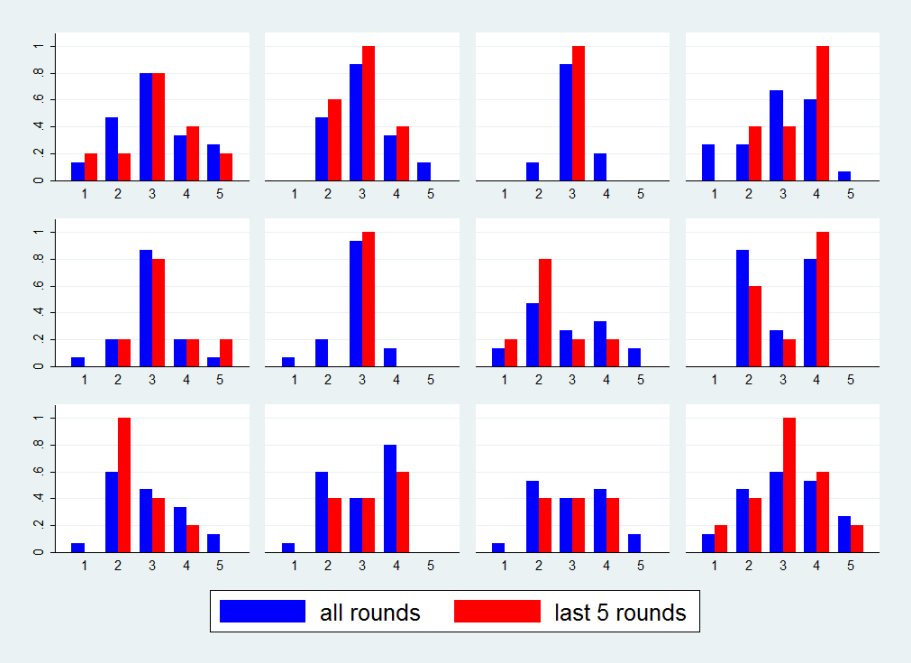
In 25% (30% for the last 5 rounds) of the rounds behavior corresponds to the equilibrium and most of the off-equilibrium behavior is due to very low entry rates across the board which might be due to the high entry costs and the complexity of the situation.

Investigating behavior over time we find that the moderates and the median players do not significantly change their behavior going from the first five to the last five rounds (p=.17 for moderates and p=0.84 for the median player) while the extreme players reduce their entry rates over time (p=.02) which indicates that over time behavior gets closer to the one-candidate equilibrium.

The average policy of 49.4 is in line with the Nash equilibrium but the observed distance between implemented policy and the median is 9.4 which is larger than predicted. Furthermore the median player wins only 52% of the elections while a moderate player wins 42% of the elections.

[36] This winning rate is clearly below the cut-off of winning 32% of the elections that would make entry beneficial for a median player.

Figure 5: Entry under proportional representation with high costs



Notes. The figure shows the average entry rates by position (1 is position 0, 2 is position 25, 3 is position 50, 4 is position 75 and 5 is position 100) and group.

Overall picture

Combining the results across treatments yields two conclusions. First, over time behavior converges towards equilibrium, which is an indication of learning. Second, entry rates are substantially higher than theoretically predicted. This is a common finding in experiments on entry decisions, such as market entry games (Fischbacher and Thöni, 2008) or contest games (Cason et al., 2010), and was also found in previous experiments on the citizen-candidate model (Cadigan, 2004, Elbitar and Gamberg, 2008).[37]

[37] The two most prominent explanations for this finding of over-entry are risk-aversion and joy of winning. We did not measure risk-aversion in this experiment but given that in treatment 3 (PR-low) the median player enters even though the entry rates of the moderates are close to 100%, risk-aversion cannot be the whole story (which is in line with Fischbacher and Thöni who find that risk-aversion does not predict individual behavior in their experiment).

Comparative statics across treatments

The cost effect predicts that entry rates with high costs are lower than with low costs. From table 3 we see that this is the case in our experiment and a Wilcoxon rank-sum test shows that this difference is highly significant ($p<0.01$).[38] Furthermore, we expect more polarized outcomes for low costs. Table 4 shows that this is indeed the case with the difference being significant using a Wilcoxon rank-sum test (p-value=0.01 for both proportional representation and plurality).

The system effect predicts that proportional representation leads to (weakly) higher entry while the interaction effect posits that the effect is positive for low costs and zero for high costs. Both predictions are supported since there is a significant difference for low costs ($p<0.01$) and no significant difference for high costs ($p=0.39$).[39]

Table 3: Average number and polarization of entrants by treatment

	aver. number of entrants		Polarization	
	All rounds	Last 5 rounds	All rounds	Last 5 rounds
PL-low	2.6	2.4	.88	.90
PL-high	1.8	1.6	.58	.48
PR-low	3.2	3.0	.93	.92
PR-high	1.7	1.6	.65	.58

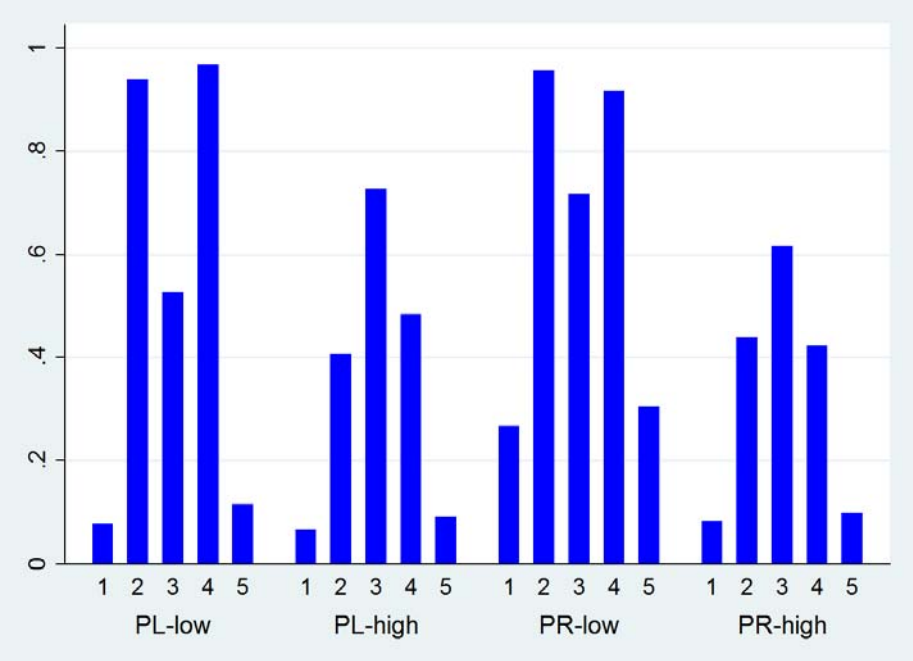
Notes. This table shows average number of entrants and their across treatments. Polarization is defined as the average distance of an entrants' positions to the median voter's location.

Comparing the predicted entry rates for the distinct positions (which are shown in figure 6) we would expect that increasing the entry costs under plurality rule has no effect on the extremes' entry decisions (since their entry is not part of the equilibrium under either cost of entry), it decreases the entry by moderates (since with high costs their entry is off-equilibrium) and it increases the entry for the median player (since with high costs him entering is the unique

[38] This reproduces the finding in Cadigan (2004) who also found less entry for higher costs of entry.
[39] There is no significant difference in the polarization of entry across electoral rules for either cost level.

equilibrium). All three predictions are confirmed since the p-values are .41 for the extremes, (one-sided) p<.01 for the moderates and (one-sided) p=.02 for the extremes.

Figure 6: Entry by position and treatment



Notes. The figure shows the entry rates by position (1 is position 0, 2 is position 25, 3 is position 50, 4 is position 75 and 5 is position 100) averaged over rounds and groups for each treatment.

Increasing costs under proportional representation should decrease the entry for the moderates since their entry is not part of an equilibrium with high costs. We expect no effect for the extremes since the equilibrium in the high cost case where they would enter is quite unstable. For the median we would then expect more entry under high costs since the others are entering less. While for the moderates (one-sided p<.01) our expectation are confirmed for the extremes and for the median we do not find the predicted effect. For the extremes we find a significant reduction in entry (one-sided p<.01) and for the median the sign of the effect is opposite to the prediction, though the effect is insignificant (one-sided p=.82). A reason might be that there is a general tendency to reduce entry when costs increase and this dominates the effect of the change in equilibrium.

Comparing now entry rates across electoral rules we find that the higher entry rates under proportional representation compared to plurality rule when the costs of entry are low are driven by an increase in the entry rates of the extreme (one-sided $p<.01$) and median (one-sided $p=.03$) while there is no difference for the moderate players ($p=.41$).

When entry costs are higher there is no difference across electoral rule in the entry rate for any of the positions. The p-values are .69 for the extremes, .94 for the moderates and .25 for the median.

In summary, we find support for our three main hypothesis. With high costs of running for office, entry is lower and entrants are less polarized (cost effect), and proportional representation leads to more entry (system effect) but only if costs are low (interaction effect).

5. Conclusions

In this article we employed the citizen-candidate paradigm to investigate the question of how distinct electoral systems influence the number of candidates running for office and the polarization of their policy positions.

In the theoretical analysis we introduce a way of modelling proportional representation that takes coalition governments explicitly into account. We find that compared to ignoring coalitions, proportional representation with coalitions governments taken into account lead to candidate positions that are less polarized. Furthermore, for office-motivated candidates proportional representation with coalitions is more conducive to multi-candidate equilibria than proportional representation without coalitions and plurality voting. Additionally, we find that plurality voting leads to more centrist outcomes than proportional representation without coalitions.

Overall, the theoretical results show that the dynamics associated with coalition formation have important implications for entry under proportional representation that have to be reckoned with. Our analysis offers a first step to understanding how coalitions influence entry and how this depends on the institutional environment but a lot of work still has to be done. One avenue for future research is to allow for candidates that care about both policy and office-rents. One

could then investigate how the comparative statics across electoral rules change as candidates relative concerns for office rents increase vis-à-vis their concerns for policy. Another very important question is how the electoral equilibrium that results from the interplay between candidates and voters reacts to changes in the electoral system. To be able to answer this question one would need to allow for strategic voting[40] and abstention. Especially allowing for abstention might be interesting since it has direct implications for the polarization of outcomes.[41]

On a more general level the theoretical analysis reiterates the point made in the literature that ignoring the coalitions associated with proportional representation is not without effect on the equilibria and the comparative statics across electoral rules. Going forward we should therefore continue to integrate coalition governments into our models of proportional representation not only in models that want to describe election outcomes per se but also and especially in models that try to answer how electoral rules influence other outcomes like taxation and redistribution.

In the second part of the paper we implement the citizen-candidate model in the laboratory with the aim of answering how candidate entry behavior differs between proportional representation without coalitions and plurality voting. We find that (as predicted by the model) with low entry costs, proportional representation increases entry while it has no effect for high costs. This indicates that the observed higher entry under proportional representation is due to equilibrium play and not due to using some simple heuristic like entering to influence the policy without respect for payoffs. Furthermore, we find support for the prediction of a cost effect since for higher entry costs fewer candidates enter and their positions are less polarized.

By-and-large, the experimental results are in line with the citizen-candidate model (notwithstanding the substantial over-entry), which supports the usefulness of the citizen-

[40] De Sinopoli and Iannantuoni (2007) show that in the model of proportional representation without coalitions when strategic voting is allowed the voters engage in 'policy-balancing' (Kedar, 2009) which leads to only the most extreme candidates receiving votes.

[41] In a paper focusing on voter behavior in a three-candidate setting with proportional representation Kamm and Schram (2013) find that compared to mandatory voting endogenous turnout increases the vote share for extreme parties.

candidate approach. Obviously, experiments can only be one part of an empirical evaluation of the predictive power of the paradigm. But given the advantages of laboratory control and the difficulties involved with testing the model in the field (especially given the multiplicity of equilibria), experiments can play an important role by offering a test bed for different institutional environments. In this sprit our paper offers the first experimental analysis of proportional representation in the citizen-candidate paradigm but there are many more aspects of the citizen-candidate paradigm left to explore. Future experimental work should therefore try to investigate the robustness of the models predictions to changes in the underlying assumptions, f.i. the information[42] or the institutional framework. A natural next step from this paper that we will take in a follow-up project is to implement the model of proportional representation with coalitions in the experimental design and investigate which of the rich pattern of predicted comparative statics is observed when human subjects participate such laboratory experiments.

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[42] A good starting point are the results found by Großer and Palfrey (2014).

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