習題 6.8 (Page 252)

There are a number of ways in which the restrictions can be substituted into the model, with each one resulting in a different restricted model. We have chosen to substitute out β_1 and β_3. With this in mind, we rewrite the restrictions as

$$\beta_3 = 1 - 3.8\beta_4$$

$$\beta_1 = 80 - 6\beta_2 - 1.9\beta_3 - 3.61\beta_4$$

Substituting the first restriction into the second yields

$$\beta_1 = 80 - 6\beta_2 - 1.9(1 - 3.8\beta_4) - 3.61\beta_4$$

Substituting this restriction and the first one $\beta_3 = 1 - 3.8\beta_4$ into the equation

$$S_i = \beta_1 + \beta_2 P_i + \beta_3 A_i + \beta_4 A_i^2 + e_i$$

yields

$$S_i = \left(80 - 6\beta_2 - 1.9(1 - 3.8\beta_4) - 3.61\beta_4\right) + \beta_2 P_i + \left(1 - 3.8\beta_4\right) A_i + \beta_4 A_i^2 + e_i$$

Rearranging this equation into a form suitable for estimation yields

$$\left(S_i - A_i - 78.1\right) = \beta_2\left(P_i - 6\right) + \beta_4\left(3.61 - 3.8A_i + A_i^2\right) + e_i$$

習題 9.1 (Page 399)

From the equation for the AR(1) error model $e_t = \rho e_{t-1} + v_t$, we have

$$\text{var}(e_t) = \rho^2 \,\text{var}(e_{t-1}) + \text{var}(v_t) + 2\rho \,\text{cov}(e_{t-1}, v_t)$$

from which we get

$$\sigma_e^2 = \rho^2 \sigma_e^2 + \sigma_v^2 + 0$$

$$\sigma_e^2 \left(1 - \rho^2\right) = \sigma_v^2$$

and hence

$$\sigma_e^2 = \frac{\sigma_v^2}{1 - \rho^2}$$

To find $E(e_t e_{t-1})$ we note that

$$e_t e_{t-1} = \rho e_{t-1}^2 + e_{t-1} v_t$$

Taking expectations,

$$E(e_t e_{t-1}) = \rho E\left(e_{t-1}^2\right) + 0 = \rho \sigma_e^2$$

Similarly,

$$e_t e_{t-2} = \rho e_{t-1} e_{t-2} + e_{t-2} v_t$$

and

$$E(e_t e_{t-2}) = \rho E(e_{t-1} e_{t-2}) + 0 = \rho E(e_t e_{t-1}) = \rho^2 \sigma_e^2$$