Chapter 9 - Graph

- A Graph G consists of a set V, whose members are called the vertices of G, together with a set E of pairs of distinct vertices from V.

- The pairs in E are called the edges of G.

- If the pairs are unordered, G is called an *undirected graph* or a *graph*. Otherwise, G is called a *directed graph or* a *digraph*.

- Two vertices in an undirected graph are called adjacent if there is an edge from the first to the second.

Chapter 9 - Graph

- A path is a sequence of distinct vertices, each adjacent to the next.
- A cycle is a path containing at least three vertices such that the last vertex on the path is adjacent to the first.
- A graph is called connected if there is a path from any vertex to any other vertex.
- A free tree is defined as a connected undirected graph with no cycles.

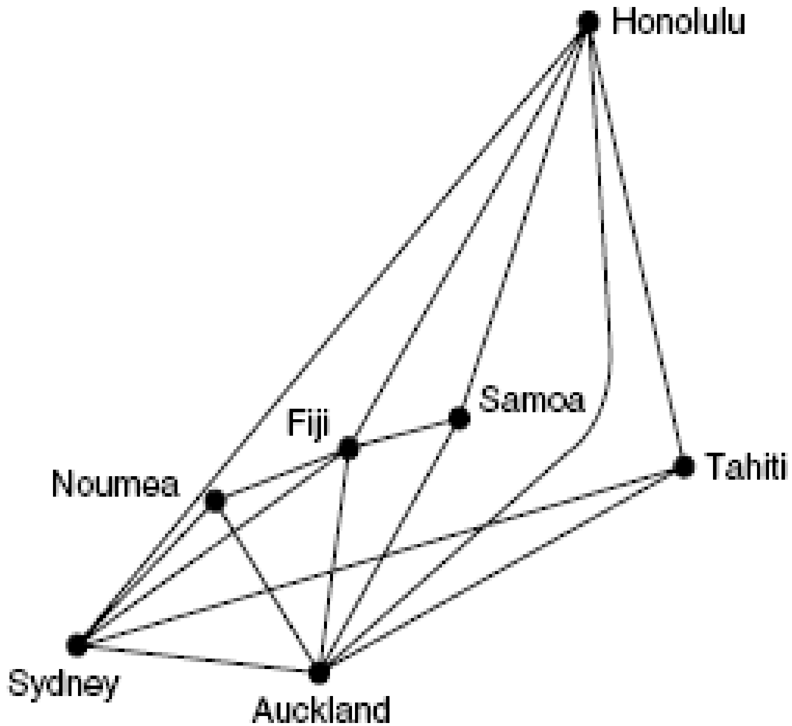
Chapter 9 - Graph

- In a *directed graph* a path or a cycle means always moving in the direction indicated by the arrows.

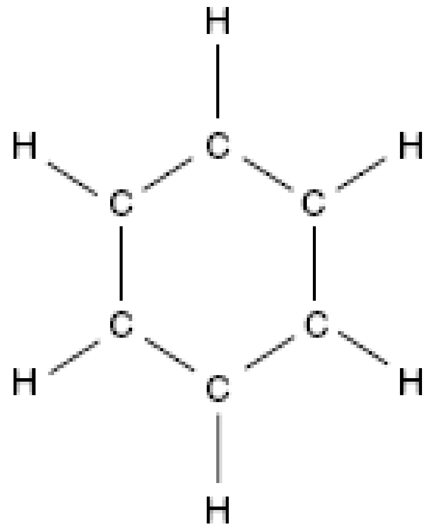
- A directed graph is called strongly connected if there is a directed path from any vertex to any other vertex.

- If we suppress the direction of the edges and the resulting undirected graph is connected, we call the directed graph weakly connected

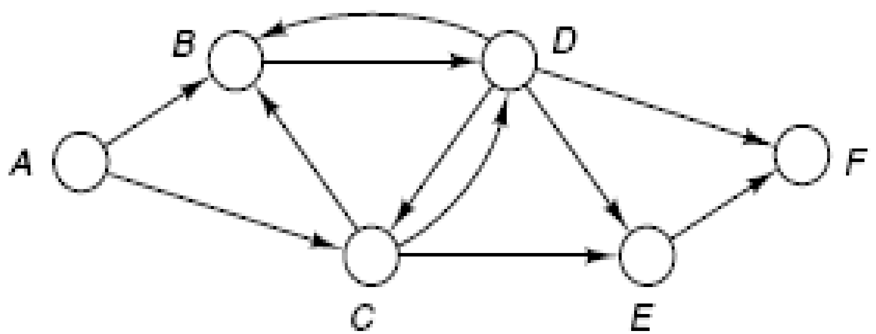
Examples of Graph



Selected South Pacific air routes

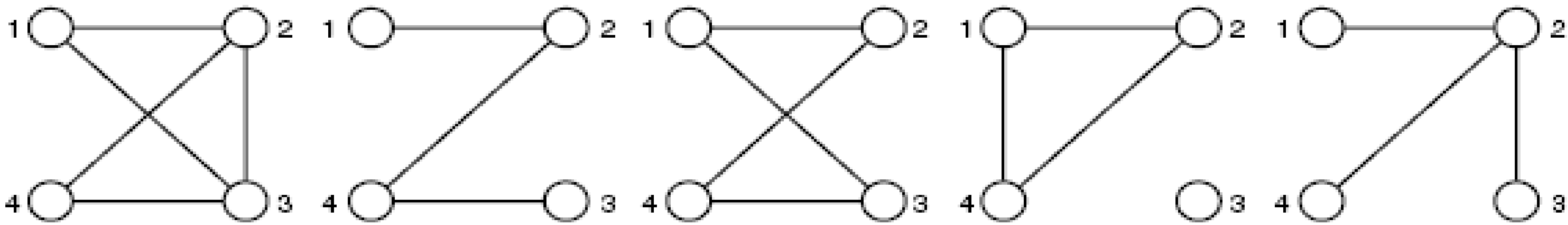


Benzene molecule

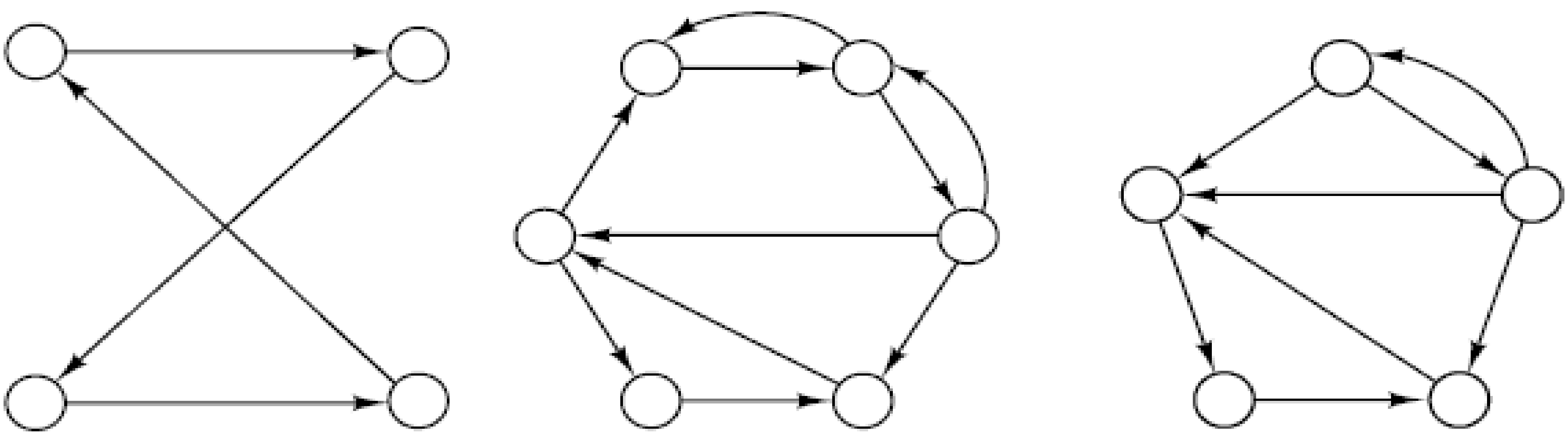


Message transmission in a network

Examples of Graph

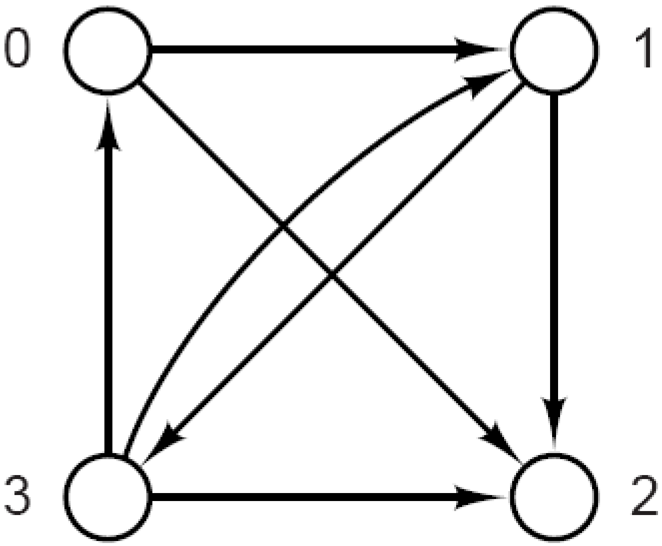


Connected (a) | Path (b) | Cycle (c) | Disconnected (d) | Tree (e)



Directed cycle (a) | Strongly connected (b) | Weakly connected (c)

Digraph as an adjacency table



Directed graph

vertex	Set
0	{ 1, 2 }
1	{ 2, 3 }
2	ø
3	{ 0, 1, 2 }

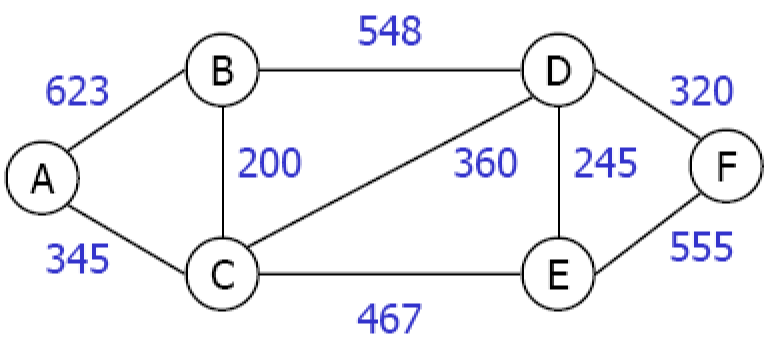
Adjacency set

	0	1	2	3
0	F	T	T	F
1	F	F	T	T
2	F	F	F	F
3	T	T	T	F

Adjacency table

```
Digraph
    count <integer>                                  // Number of vertices
    edge <array of  <array of <boolean> > >          // Adjacency table
End Digraph
```

Weighted-graph as an adjacency table



Weighted-graph

A
B
C
D
E
F

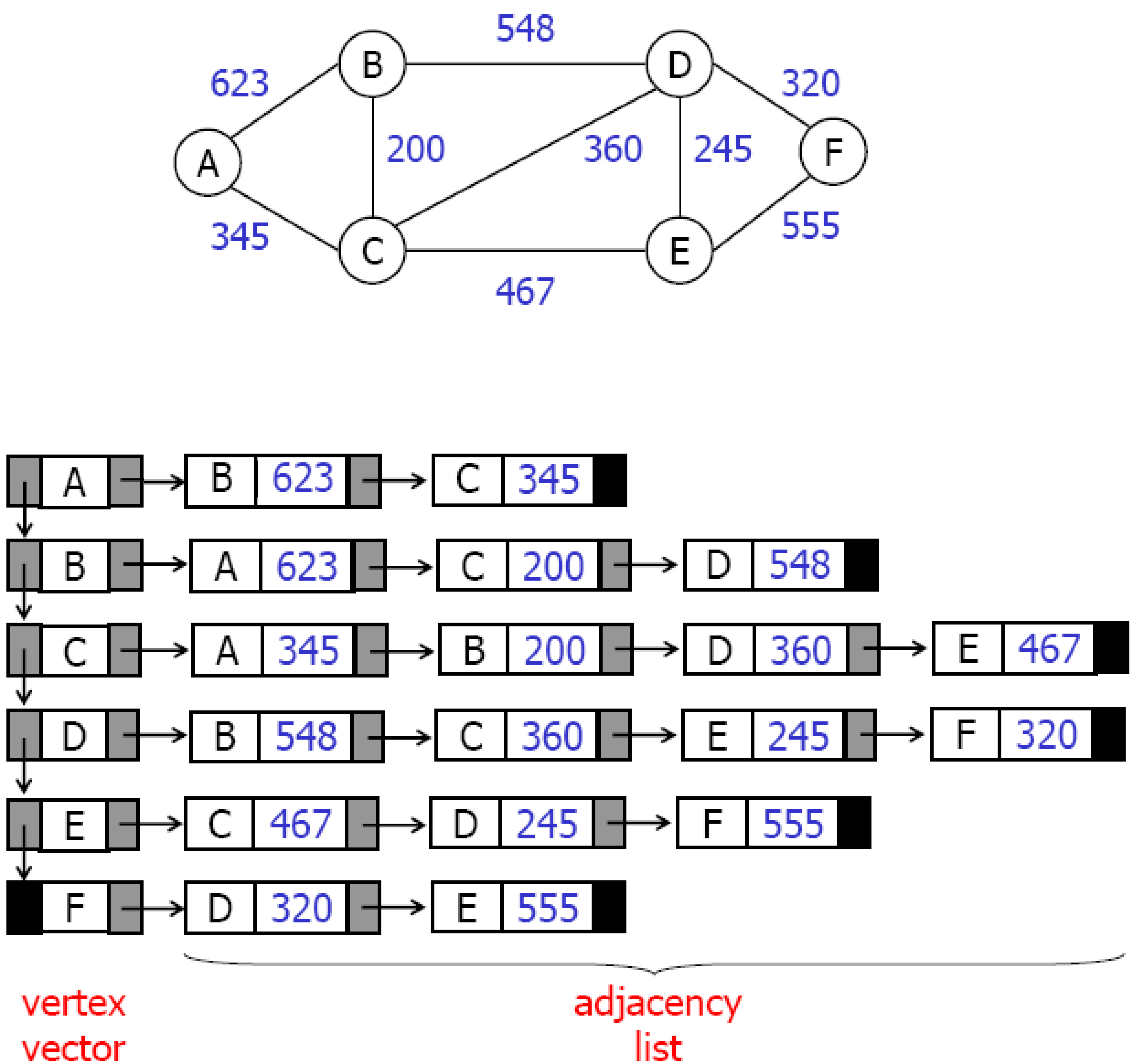
vertex vector

	A	B	C	D	E	F
A	0	623	345	0	0	0
B	623	0	200	548	0	0
C	345	200	0	360	467	0
D	0	548	360	0	245	320
E	0	0	467	245	0	555
F	0	0	0	320	555	0

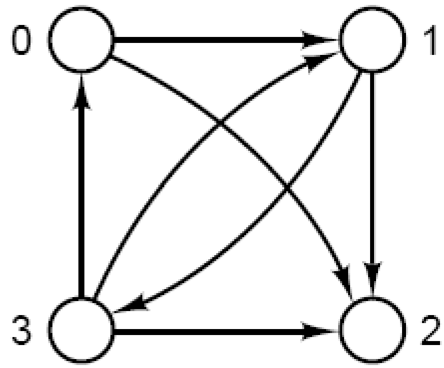
adjacency table

```
WeightedGraph
 count <integer>                                  // Number of vertices
 edge<array of<array of<WeightType>>>             // Adjacency table
End WeightedGraph
```

Weighted-graph as an adjacency list



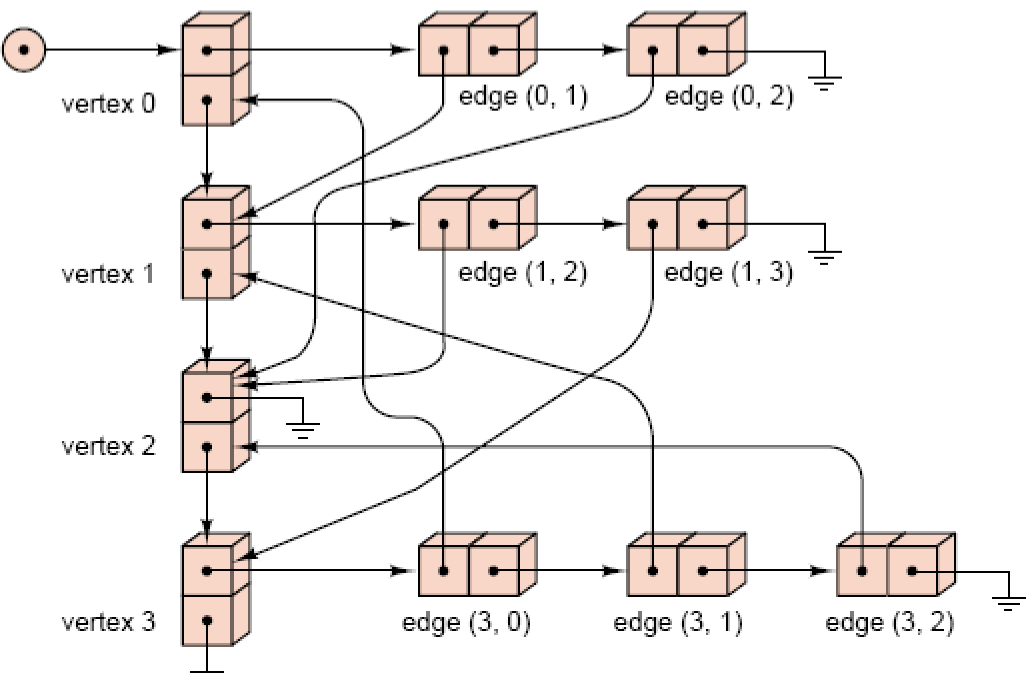
Digraph as an adjacency list



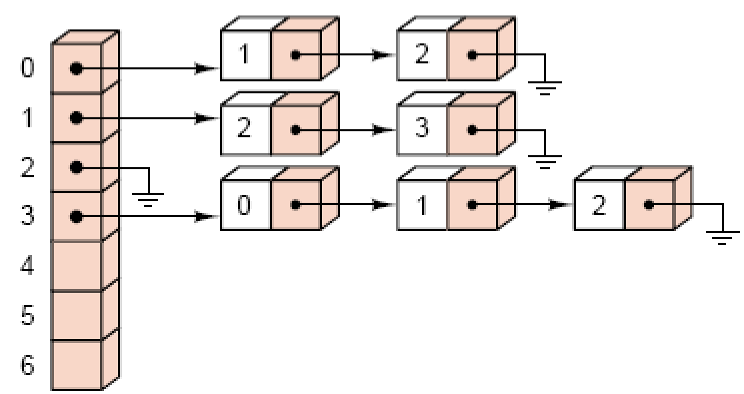
Directed graph

0	1	2	–	–	–	–	–
1	2	3	–	–	–	–	–
2	–	–	–	–	–	–	–
3	0	1	2	–	–	–	–
4	–	–	–	–	–	–	–
5	–	–	–	–	–	–	–
6	–	–	–	–	–	–	–

contiguous structure

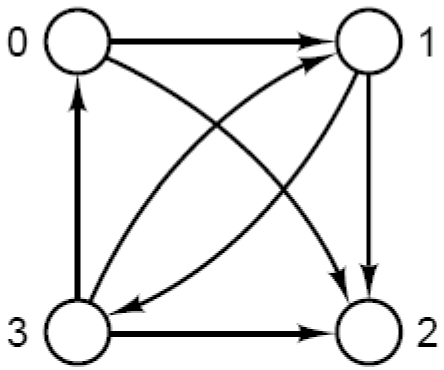


linked structure

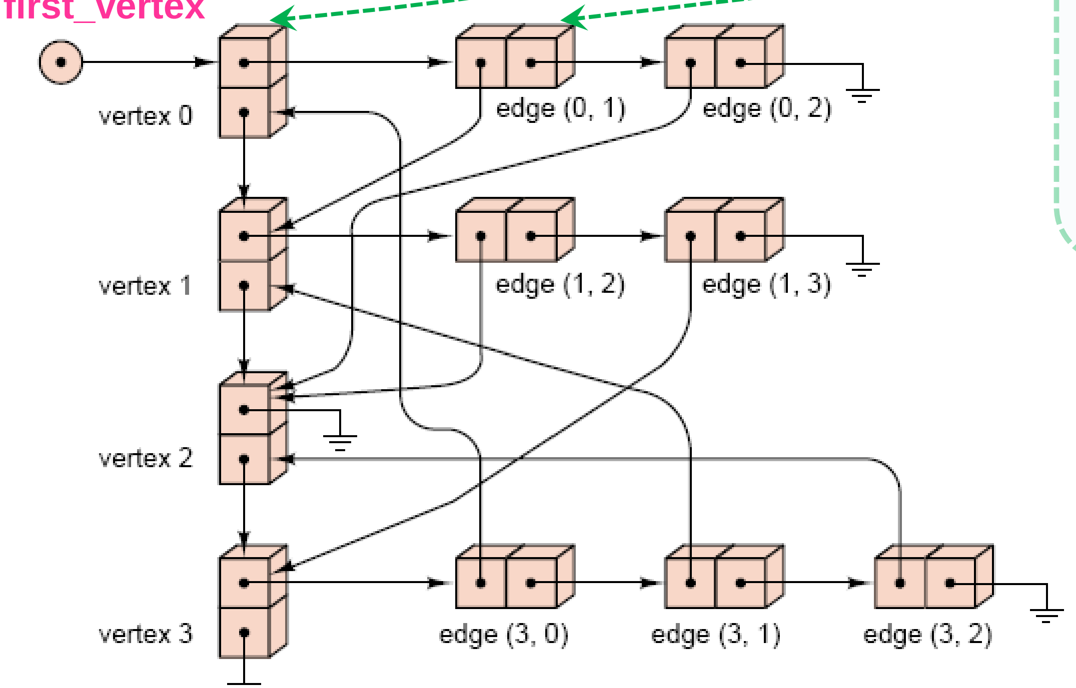


mixed structure

Digraph as an adjacency list (not using List ADT)



Directed graph



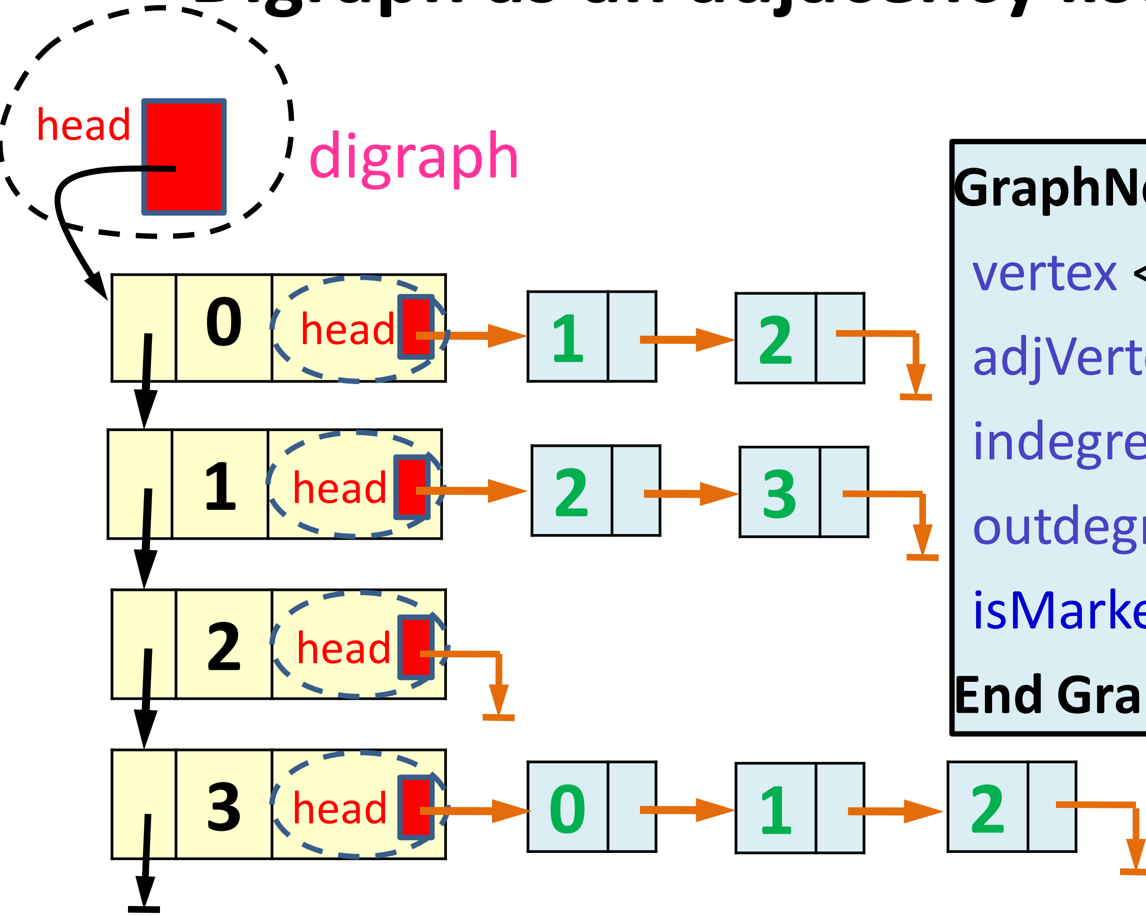
linked structure

VertexNode
    first_edge <pointer to EdgeNode>
    next_vertex<pointer to VertexNode>
End VertexNode

EdgeNode
    vertex_to <pointer to VertexNode>
    next_edge <pointer to EdgeNode>
End EdgeNode

DiGraph
    first_vertex <pointer to VertexNode>
End DiGraph

Digraph as an adjacency list (using List ADT)



GraphNode

vertex <VertexType> *// (key field)*

adjVertex<**LinkedList** of< VertexType >>

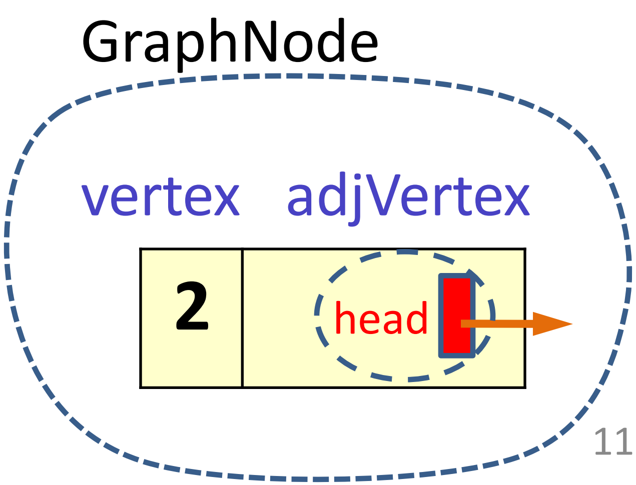
indegree <int>

outdegree <int>

isMarked <boolean>

are hidden from the image below

End GraphNode



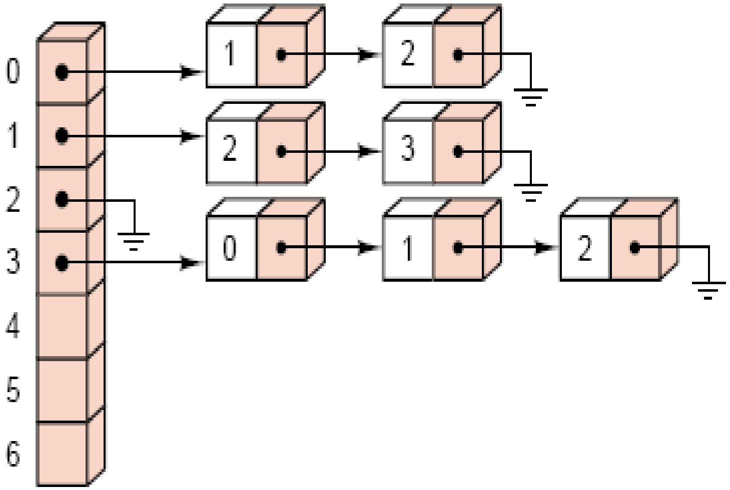
***ADT List is linked list*:**

DiGraph

digraph <**LinkedList**<of<GraphNode>>

End DiGraph

Digraph as an adjacency list (using List ADT)



mixed list

GraphNode

vertex <VertexType> // (*key field*)

adjVertex<**LinkedList** of< VertexType >>

indegree <int>

outdegree <int>

isMarked <boolean>

End GraphNode

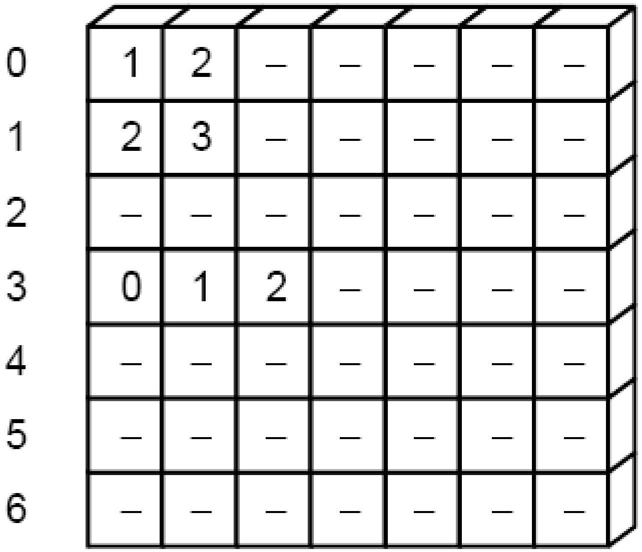
***ADT List is contiguous list*:**

DiGraph

digraph <**ContiguousList**<of<GraphNode>>

End DiGraph

Digraph as an adjacency list (using List ADT)



contiguous list

GraphNode

vertex <VertexType> // (*key field*)

adjVertex<**ContiguousList** of< VertexType >>

indegree <int>

outdegree <int>

isMarked <boolean>

End GraphNode

***ADT List is contiguous list*:**

DiGraph

digraph <**ContiguousList**<of<GraphNode>>

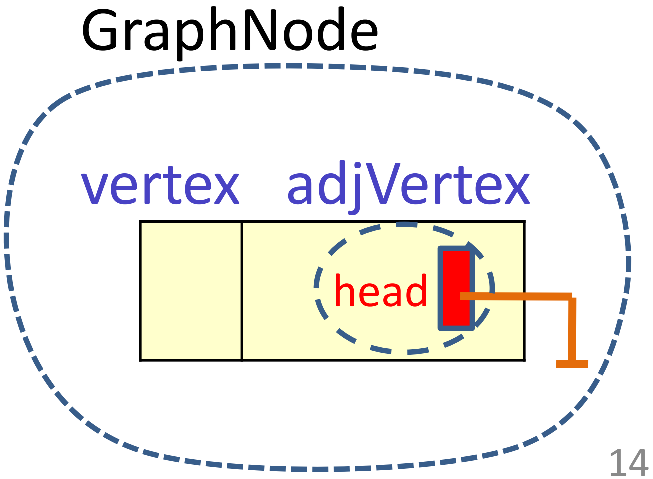
End DiGraph

GraphNode

<void> GraphNode() *// constructor of GraphNode*

1. indegree = 0
2. outdegree = 0
3. adjVertex.clear() *// By default, constructor of adjVertex made it empty.*

End GraphNode



Operations for Digraph

- Insert Vertex
- Delete Vertex
- Insert edge
- Delete edge
- Traverse

Digraph

```
Digraph
    private:
        digraph <List of <GraphNode> > // using of List ADT .
        <void> Remove_EdgesToVertex(val VertexTo <VertexType>)

    public:
        <ErrorCode> InsertVertex (val newVertex <VertexType>)
        <ErrorCode> DeleteVertex (val Vertex <VertexType>)
        <ErrorCode> InsertEdge  (val VertexFrom <VertexType>,
                                  val VertexTo <VertexType>)
        <ErrorCode> DeleteEdge (val VertexFrom <VertexType>,
                                  val VertexTo <VertexType>)

        // Other methods for Graph Traversal.
End Digraph
```

Methods of List ADT

***Methods of Digraph will use these methods of List ADT*:**

<ErrorCode> Insert (val DataIn <DataType>) // (*success*, *overflow*)

<ErrorCode> Search (ref DataOut <DataType>) // (*found*, *notFound*)

<ErrorCode> Remove (ref DataOut <DataType>) // (*success* , *notFound*)

<ErrorCode> Retrieve (ref DataOut <DataType>) // (*success* , *notFound*)

<ErrorCode> Retrieve (ref DataOut <DataType>, position <int>)

// (*success* , *range_error*)

<ErrorCode> Replace (val DataIn <DataType>, position <int>)

// (*success*, *range_error*)

<ErrorCode> Replace (val DataIn <DataType>, val DataOut <DataType>)

// (*success*, *notFound*)

<boolean> isFull()

<boolean> isEmpty()

<integer> Size()

Insert New Vertex into Digraph

<ErrorCode> **InsertVertex** (val newVertex <VertexType>)

Inserts new vertex into digraph.

Pre newVertex is a vertex needs to be inserted.

Post if the vertex is not in digraph, it has been inserted and no edge is involved with this vertex.

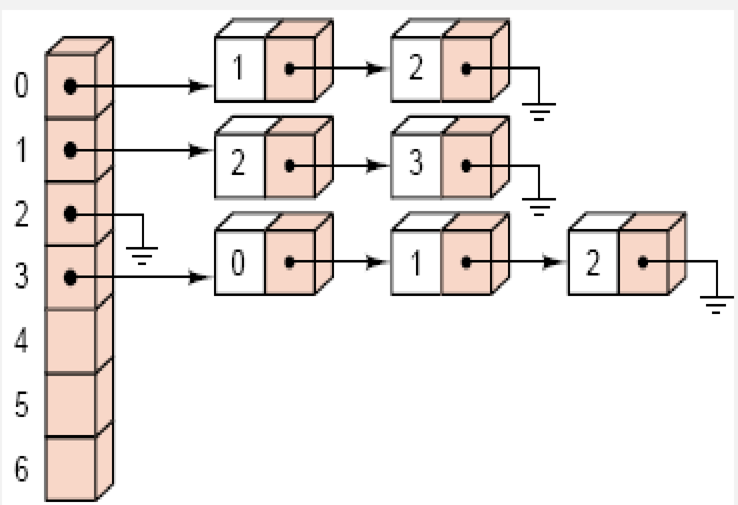
Return *success*, *overflow*, or *duplicate_error*

Insert New Vertex into Digraph

<ErrorCode> **InsertVertex** (val newVertex <VertexType>)

1. DataOut.vertex = newVertex
2. **if** (digraph.Search(DataOut) = *success*)
    1. return *duplicate_error*
3. **else**
    1. return digraph.Insert(DataOut) // *success* or *overflow*

End InsertVertex



GraphNode

vertex <VertexType> // (*key field*)

adjVertex<List of< VertexType >>

indegree <int>

outdegree <int>

isMarked <boolean>

End GraphNode

Delete Vertex from Digraph

<ErrorCode> **DeleteVertex** (val Vertex <VertexType>)

Deletes an existing vertex.

Pre Vertex is the vertex needs to be removed .

Post if Vertex 's indegree <>0, the edges ending at this vertex have been removed. Finally, this vertex has been removed.

Return *success*, or *notFound*

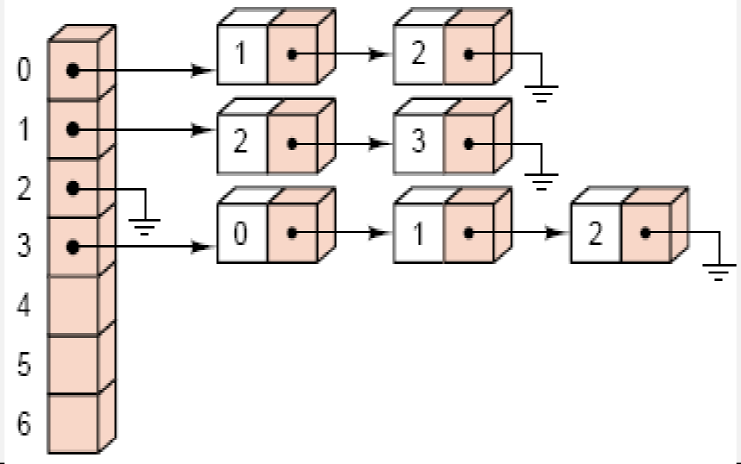
Uses Function Remove_EdgeToVertex.

Delete Vertex from Digraph

<ErrorCode> **DeleteVertex** (val Vertex <VertexType>)

1. DataOut.vertex = Vertex
2. **if** (digraph.Retrieve(DataOut) = *success*)
    1. **if** (DataOut.indegree>0)
        1. digraph.Remove_EdgeToVertex(Vertex)
    2. digraph.Remove(DataOut)
    3. return *success*
3. **else**
    1. return *notFound*

End DeleteVertex



GraphNode

vertex <VertexType> // (*key field*)

adjVertex<List of< VertexType >>

indegree <int>

outdegree <int>

isMarked <boolean>

End GraphNode

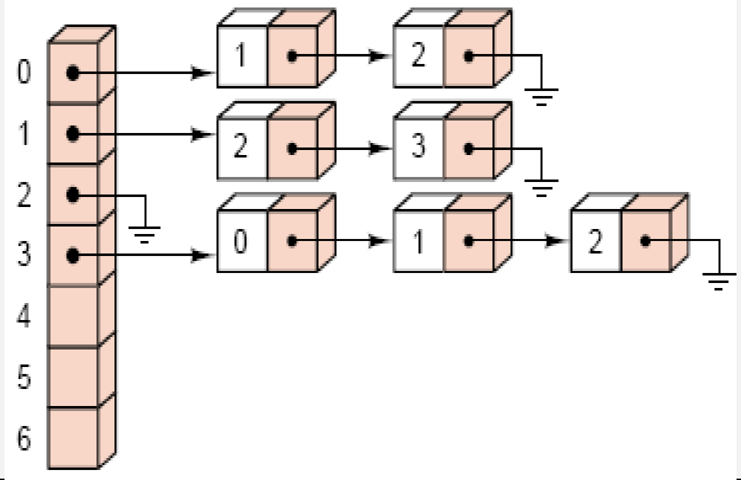
Auxiliary function Remove all Edges to a Vertex

<void> **Remove_EdgesToVertex**(val VertexTo <VertexType>)

Removes all edges from any vertex to VertexTo if exist.

1. position = 0
2. **loop** (digraph.Retrieve(DataFrom, position) = *success*)
   1. **if** (DataFrom.outdegree>0)
      1. **if** (DataFrom.adjVertex.Remove(VertexTo) = *success*)
         1. DataFrom.outdegree = DataFrom.outdegree - 1
         2. digraph.Replace(DataFrom, position)
   2. position = position + 1

End Remove_EdgesToVertex



GraphNode

vertex <VertexType> // (*key field*)

adjVertex<List of< VertexType >>

indegree <int>

outdegree <int>

isMarked <boolean>

End GraphNode

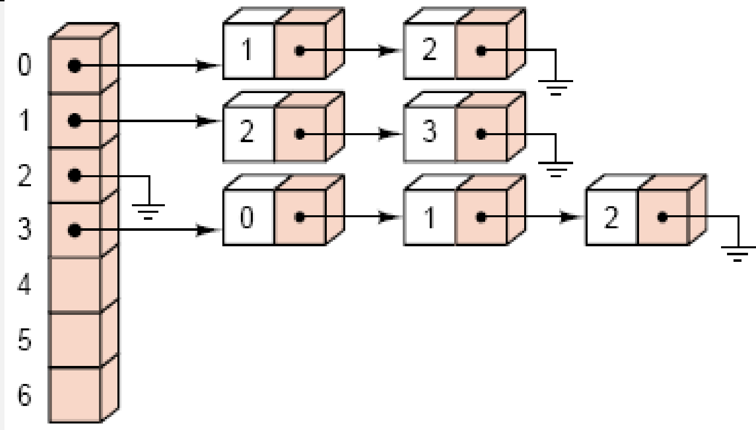
Insert new Edge into Digraph

<ErrorCode> **InsertEdge** (val VertexFrom<VertexType>,
val VertexTo <VertexType>)

Inserts new edge into digraph.

Post if VertexFrom and VertexTo are in the digraph, and the edge from VertexFrom to VertexTo is not in the digraph, it has been inserted.

Return *success*, *overflow*, *notFound_VertexFrom* , *notFound_VertexTo* or *duplicate_error*



1. DataFrom.vertex = VertexFrom
2. DataTo.vertex = VertexTo
3. **if** (digraph.Retrieve(DataFrom) = *success*)
    1. **if** (digraph.Retrieve(DataTo) = *success*)
        1. newData = DataFrom
        2. **if** (newData.adjVertex.Search(VertexTo) = *found*)
            1. return *duplicate_error*
        3. **if** (newData.adjVertex.Insert(VertexTo) = *success*)
            1. newData.outdegree = newData.outdegree +1
            2. digraph.Replace(newData, DataFrom)
            3. return *success*
        4. **else**
            1. return *overflow*
    2. **else**
        1. return *notFound_VertexTo*
4. **else**
    1. return *notFound_VertexFrom*

End InsertEdge

GraphNode

vertex <VertexType> // (*key field*)

adjVertex<List of< VertexType >>

indegree <int>

outdegree <int>

isMarked <boolean>

End GraphNode

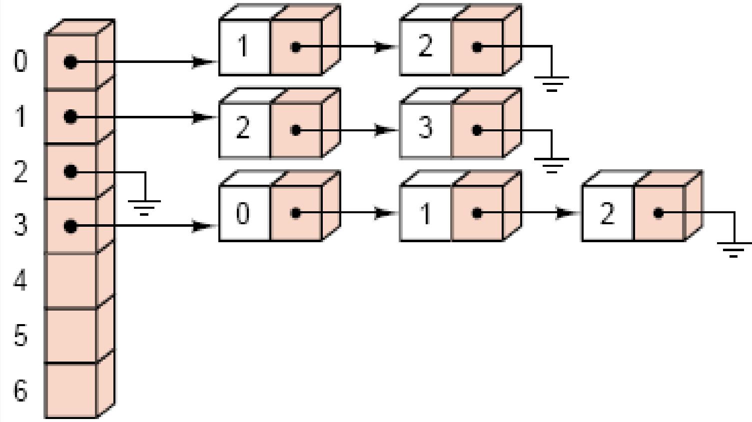
Delete Edge from Digraph

<ErrorCode> **DeleteEdge** (val VertexFrom <VertexType>,
val VertexTo <VertexType>)

Deletes an existing edge in the digraph.

Post	if VertexFrom and VertexTo are in the digraph, and the edge from VertexFrom to VertexTo is in the digraph, it has been removed
Return	*success*, *notFound_VertexFrom* , *notFound_VertexTo* or *notFound_Edge*

```
1.  DataFrom.vertex = VertexFrom
2.  DataTo.vertex = VertexTo
3.  if ( digraph.Retrieve(DataFrom) = success )
    1.  if ( digraph.Retrieve(DataTo) = success )
        1.  newData = DataFrom
        2.  if ( newData.adjVertex.Remove(VertexTo) = success )
            1.  newData.outdegree = newData.outdegree -1
            2.  digraph.Replace(newData, DataFrom)
            3.  return success
        3.  else
            1.  return notFound_Edge
    2.  else
        1.  return  notFound_VertexTo
4.  else
    1.  return notFound_VertexFrom
End DeleteEdge
```

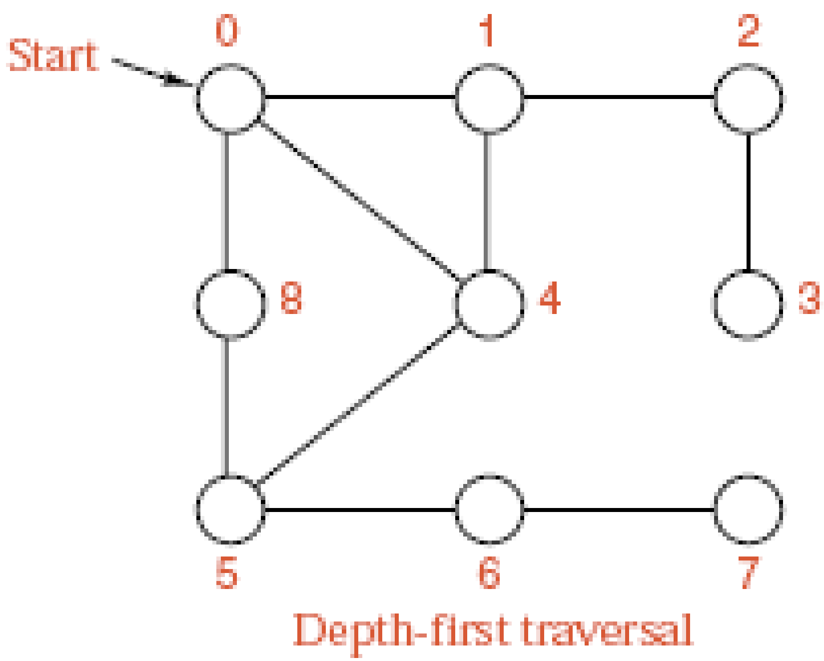


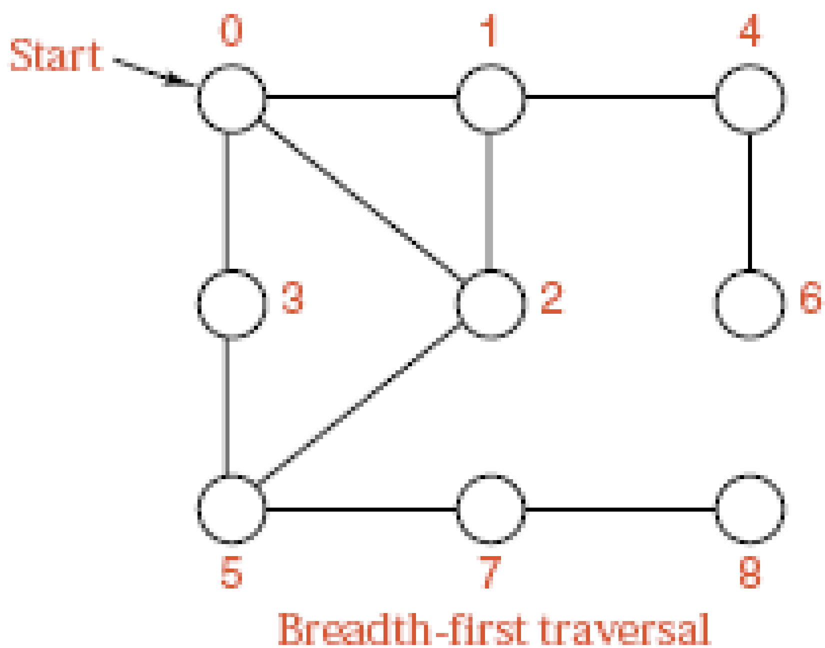

```
GraphNode
  vertex <VertexType> // (key field)
  adjVertex<List of< VertexType >>
  indegree <int>
  outdegree <int>
  isMarked <boolean>
End GraphNode
```

Graph Traversal

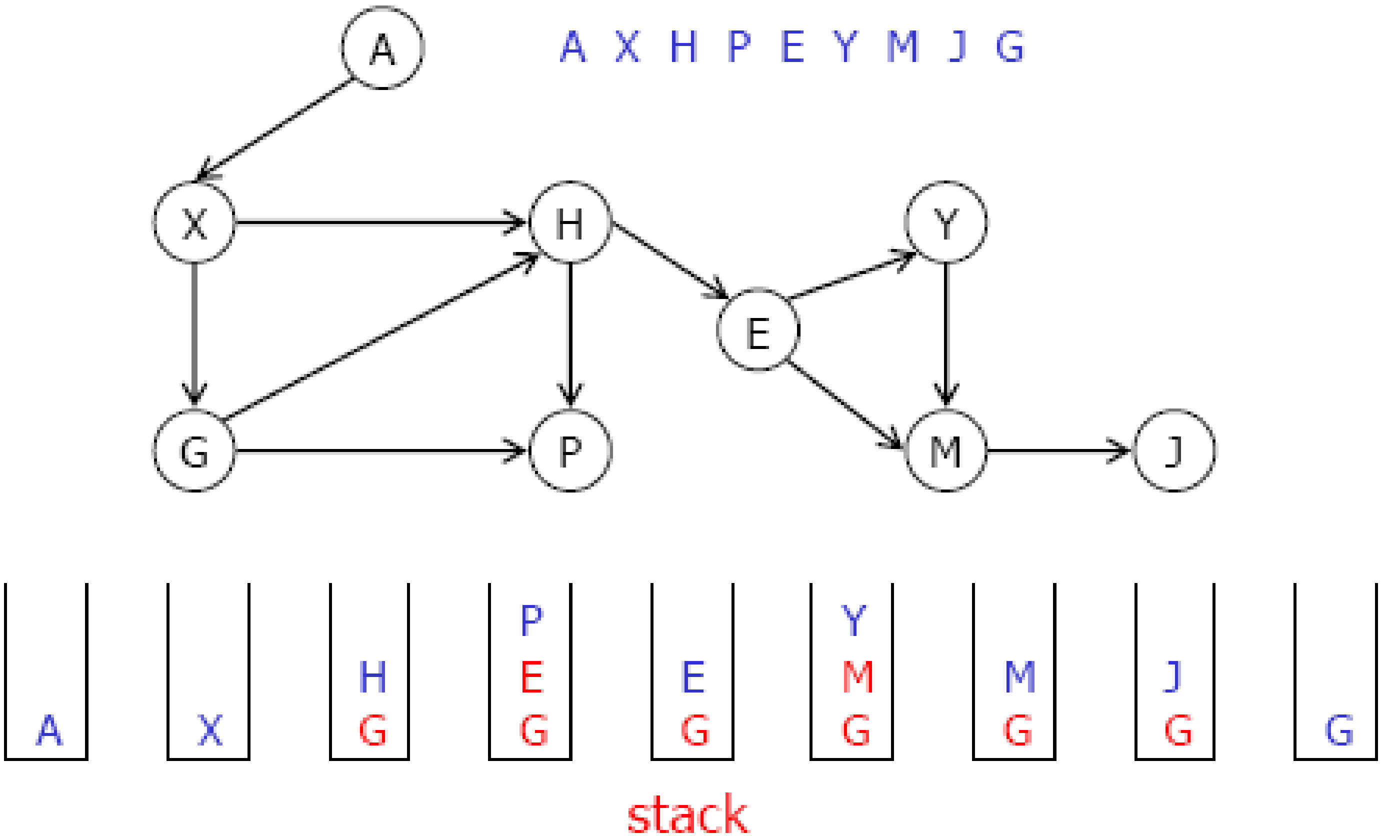
- **Depth-first traversal**: analogous to preorder traversal of an oredered tree.

- **Breadth-first traversal**: analogous to level-by-level traversal of an ordered tree.

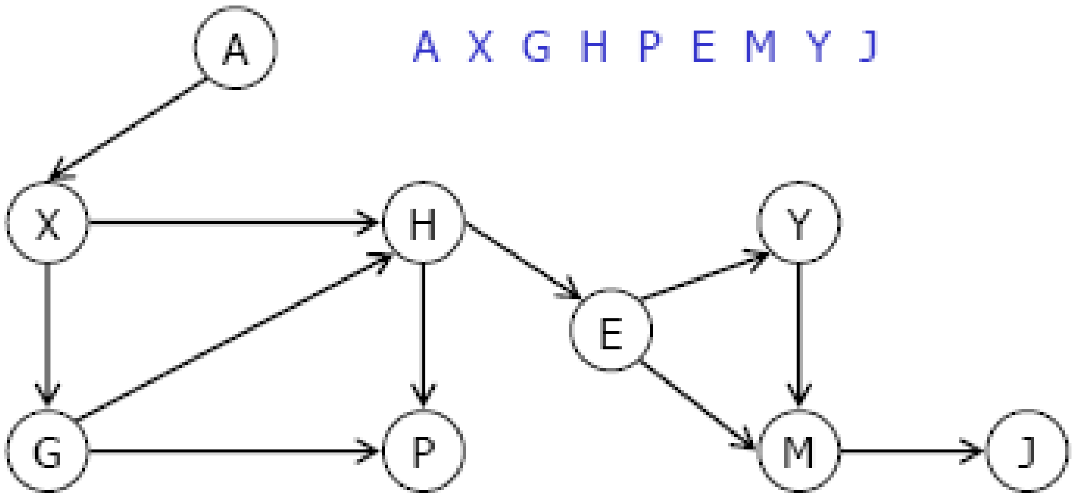




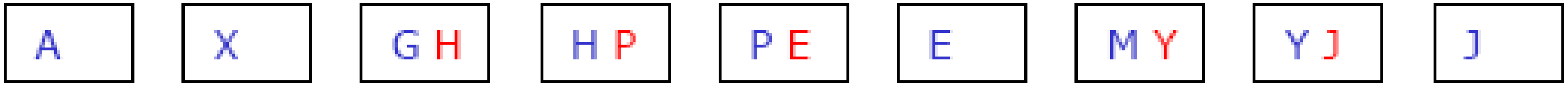
Depth-first traversal



Breadth-first traversal



A X G H P E M Y J



queue

Depth-first traversal

<void> **DepthFirst**

(ref <void> Operation (ref Data <DataType>))

Traverses the digraph in depth-first order.

Post The function Operation has been performed at each vertex of the digraph in depth-first order.

Uses Auxiliary function recursiveTraverse to produce the recursive depth-first order.

Depth-first traversal

<void> **DepthFirst**

(ref <void> Operation (ref Data <DataType>))

1. **loop** (more vertex v in Digraph)
    1. unmark (v)
2. **loop** (more vertex v in Digraph)
    1. **if** (v is unmarked)
        1. recursiveTraverse (v, Operation)

End DepthFirst

Depth-first traversal

<void> **recursiveTraverse** (ref v <VertexType>,

ref <void> Operation (ref Data <DataType>))

Traverses the digraph in depth-first order.

Pre v is a vertex of the digraph.

Post The depth-first traversal, using function Operation, has been completed for v and for all vertices that can be reached from v.

Uses function recursiveTraverse recursively.

Depth-first traversal

```
<void> recursiveTraverse(ref v <VertexType>,
                         ref <void> Operation ( ref Data <DataType>) )
1.  mark(v)
2.  Operation(v)
3.  loop (more vertex w adjacent to v)
    1.  if (vertex  w is unmarked)
        1.  recursiveTraverse (w, Operation)
End Traverse
```

Breadth-first traversal

<void> **BreadthFirst**

(ref <void> Operation (ref Data <DataType>))

Traverses the digraph in breadth-first order.

Post The function Operation has been performed at each vertex of the digraph in breadth-first order.

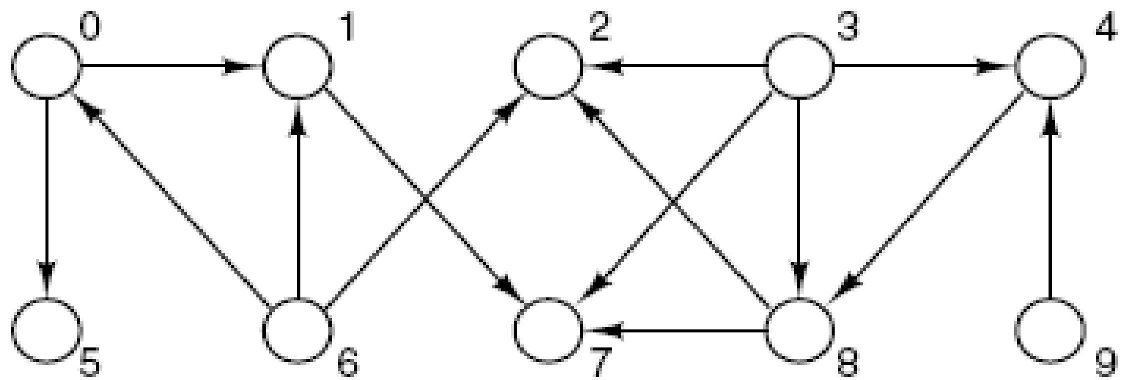
Uses Queue ADT.

```
// BreadthFirst
1.  queueObj <Queue>
2.  loop (more vertex  v in digraph)
    1.  unmark(v)
3.  loop (more vertex v in Digraph)
    1.  if (vertex v is unmarked)
        1.  queueObj.EnQueue(v)
        2.  loop (NOT queueObj .isEmpty())
            1.  queueObj.QueueFront(w)
            2.  queueObj.DeQueue()
            3.  if (vertex w is unmarked)
                1.  mark(w)
                2.  Operation(w)
                3.  loop (more vertex x adjacent to w)
                    1.  queueObj.EnQueue(x)
End BreadthFirst
```

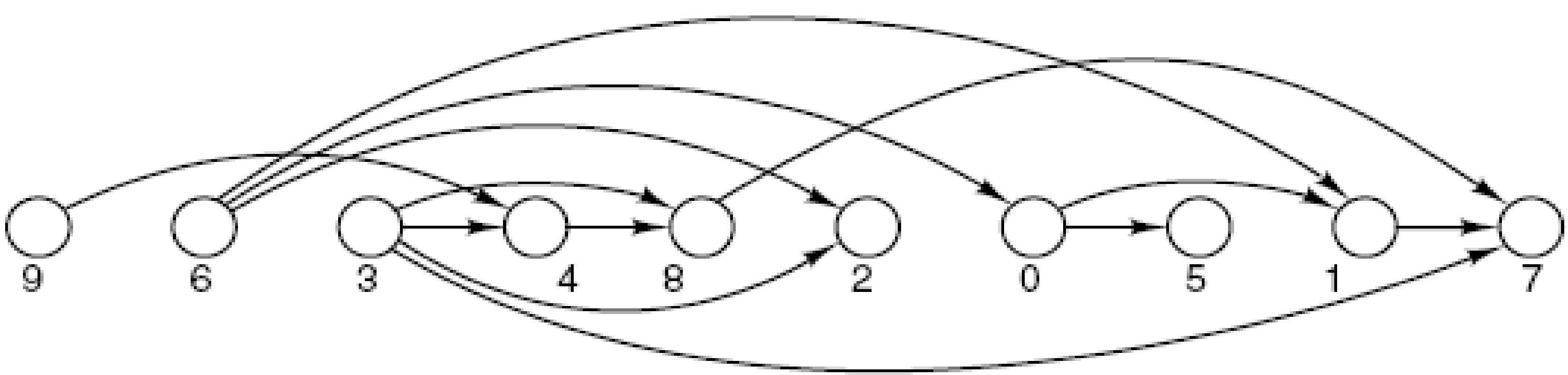
Topological Order

A topological order for G, a directed graph with no cycles, is a sequential listing of all the vertices in G such that, for all vertices v, w $\in$ G, if there is an edge from v to w, then v precedes w in the sequential listing.

Topological Order

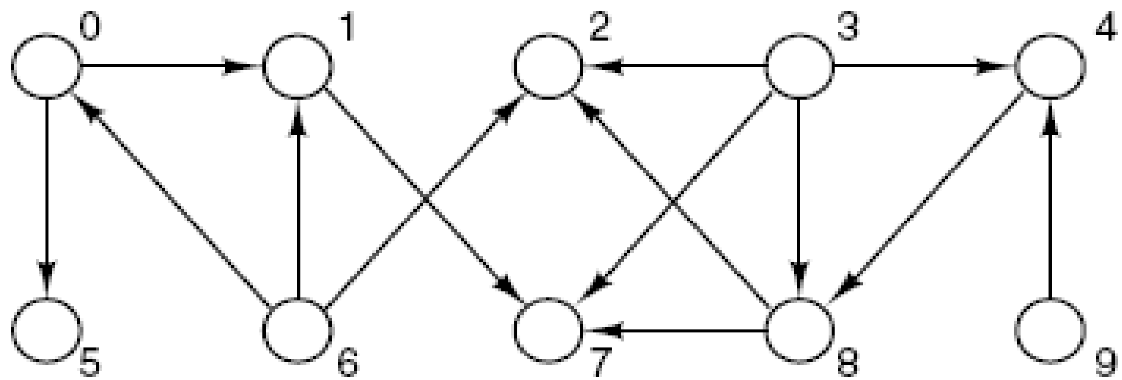


Directed graph with no directed cycles

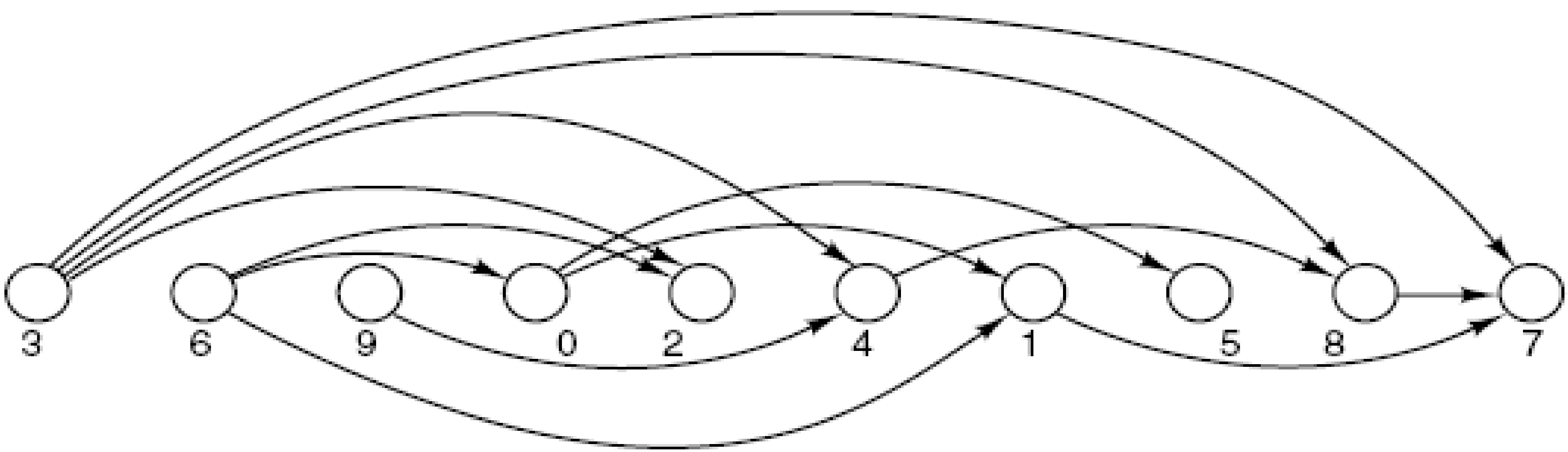


Depth-first ordering

Topological Order



Directed graph with no directed cycles



Breadth-first ordering

Applications of Topological Order

Topological order is used for:

- Courses available at a university,
  - Vertices: course.
  - Edges: (v,w), v is a prerequisite for w.
  - A topological order is a listing of all the courses such that all perequisites for a course appear before it does.

- A glossary of technical terms: no term is used in a definition before it is itself defined.

- The topics in the textbook.

Topological Order

<void> **DepthTopoSort** (ref TopologicalOrder <List>)

Traverses the digraph in depth-first order and made a list of topological order of digraph's vertices.

Pre Acyclic digraph.

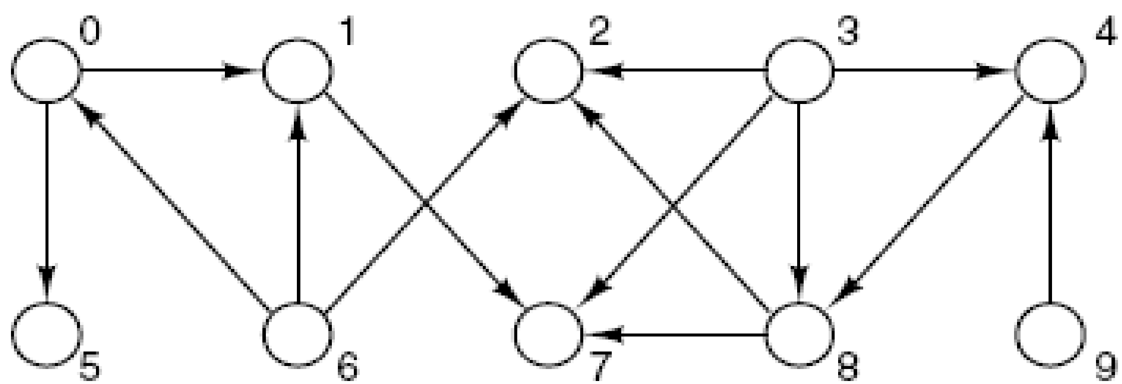
Post The vertices of the digraph are arranged into the list TopologicalOrder with a depth-first traversal of those vertices that do not belong to a cycle.

Uses List ADT and function recursiveDepthTopoSort to perform depth-first traversal.

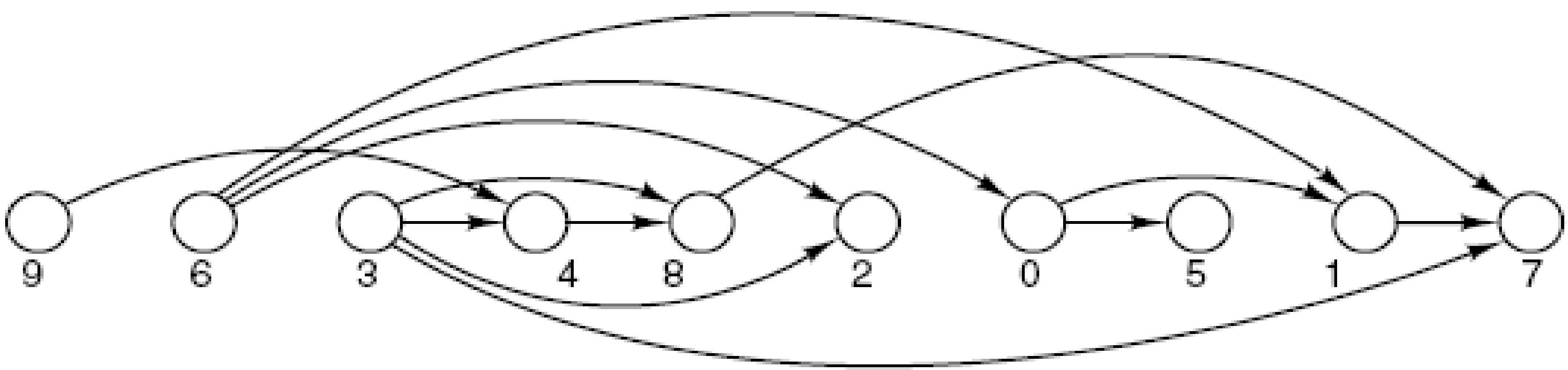
Idea:

- *Starts by finding a vertex that has no successors and place it last in the list.*
- *Repeatedly add vertices to the beginning of the list.*
- *By recursion, places all the successors of a vertex into the topological order.*
- *Then, place the vertex itself in a position before any of its successors.*

Topological Order



Directed graph with no directed cycles



Depth-first ordering

Topological Order

```
<void> DepthTopoSort (ref TopologicalOrder <List>)

1.    loop (more vertex  v in digraph)
      1.    unmark(v)
2.    TopologicalOrder.clear()
3.    loop (more vertex v in Digraph)
      1.    if (vertex v is unmarked)
            1.    recursiveDepthTopoSort(v, TopologicalOrder)
End DepthTopoSort
```

Topological Order

<void> **recursiveDepthTopoSort** (val v <VertexType>,

ref TopologicalOrder <List>)

Pre	Vertex v in digraph does not belong to the partially completed list TopologicalOrder.
Post	All the successors of v and finally v itself are added to TopologicalOrder with a depth-first order traversal.
Uses	List ADT and the function recursiveDepthTopoSort.

Idea:

- *Performs the recursion, based on the outline for the general function traverse.*
- *First, places all the successors of v into their positions in the topological order.*
- *Then, places v into the order.*

Topological Order

```
<void> recursiveDepthTopoSort (val v <VertexType>,
                                        ref TopologicalOrder <List>)

1.   mark(v)
2.   loop (more vertex w adjacent to v)
    1.   if (vertex w is unmarked)
        1.   recursiveDepthTopoSort(w, TopologicalOrder)
3.   TopologicalOrder.Insert(0, v)
End recursiveDepthTopoSort
```

Topological Order

<void> **BreadthTopoSort** (ref TopologicalOrder <List>)

Traverses the digraph in depth-first order and made a list of topological order of digraph's vertices.

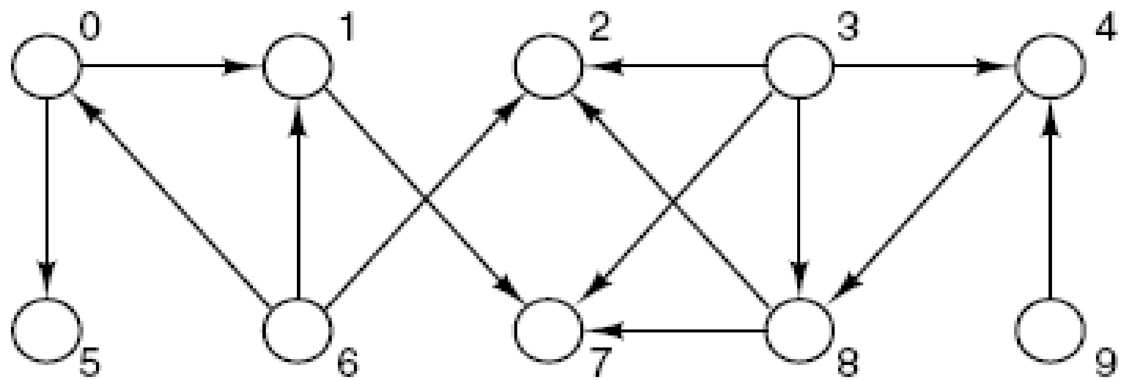
Post The vertices of the digraph are arranged into the list TopologicalOrder with a breadth-first traversal of those vertices that do not belong to a cycle.

Uses List and Queue ADT.

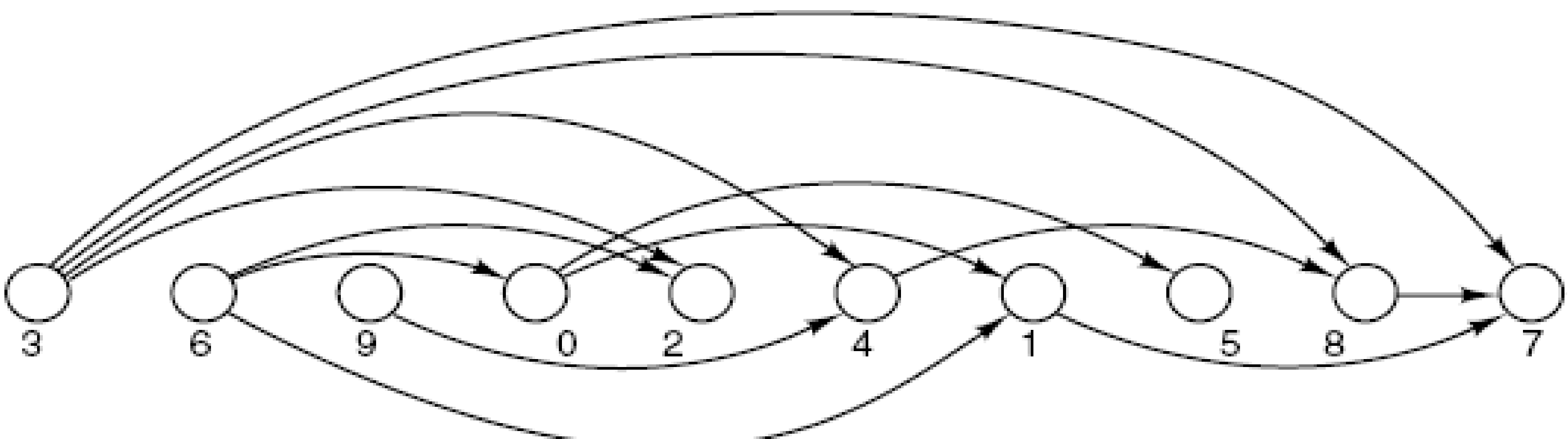
Idea:

- *Starts by finding the vertices that are not successors of any other vertex.*
- *Places these vertices into a queue of vertices to be visited.*
- *As each vertex is visited, it is removed from the queue and placed in the next available position in the topological order (starting at the beginning).*
- *Reduces the indegree of its successors by 1.*
- *The vertex having the zero value indegree is ready to processed and is places into the queue.*

Topological Order



Directed graph with no directed cycles



Breadth-first ordering

```
<void> BreadthTopoSort (ref TopologicalOrder <List>)
1.  TopologicalOrder.clear()
2.  queueObj <Queue>
3.  loop (more vertex v in digraph)
    1.  if (indegree of v = 0)
        1.  queueObj.EnQueue(v)
4.  loop (NOT queueObj.isEmpty())
    1.  queueObj.QueueFront(v)
    2.  queueObj.DeQueue()
    3.  TopologicalOrder.Insert(TopologicalOrder.size(), v)
    4.  loop (more vertex w adjacent to v)
        1.  decrease the indegree of w by 1
        2.  if (indegree of w = 0)
            1.  queueObj.EnQueue(w)
End BreadthTopoSort
```

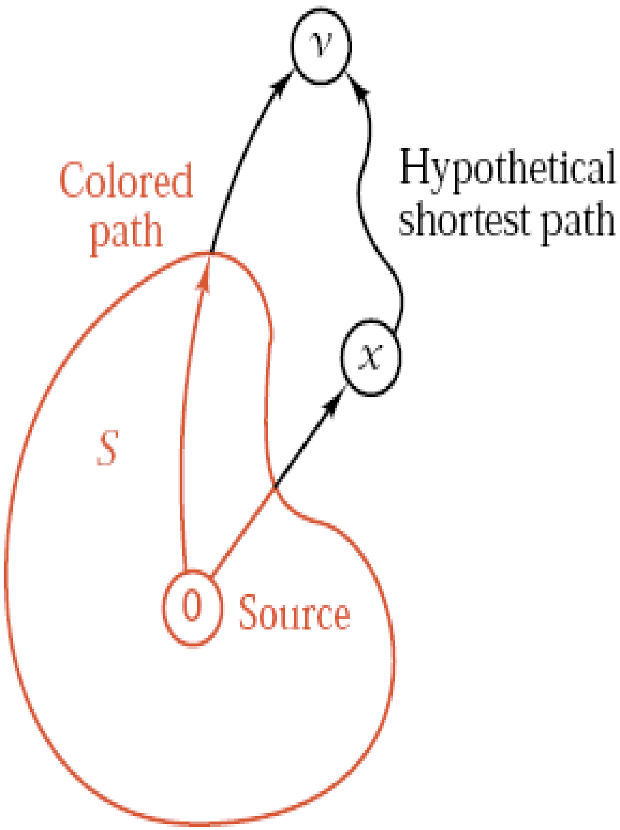
Shortest Paths

- Given a directed graph in which each edge has a nonnegative weight.
- Find a path of least total weight from a given vertex, called the source, to every other vertex in the graph.

- A greedy algorithm of Shortest Paths: Dijkstra's algorithm (1959).

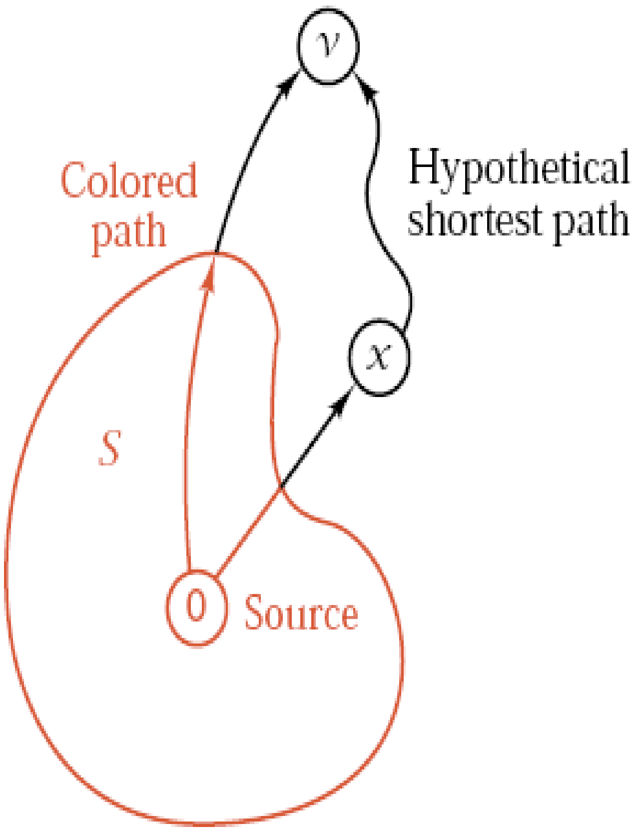
Dijkstra's algorithm

- Let tree is the subgraph contains the shotest paths from the source vertex to all other vertices.
- At first, add the source vertex to the tree.
- Loop until all vertices are in the tree:
  - Consider the adjacent vertices of the vertices already in the tree.
  - Examine all the paths from those adjacent vertices to the source vertex.
  - Select the shortest path and insert the corresponding adjacent vertex into the tree.

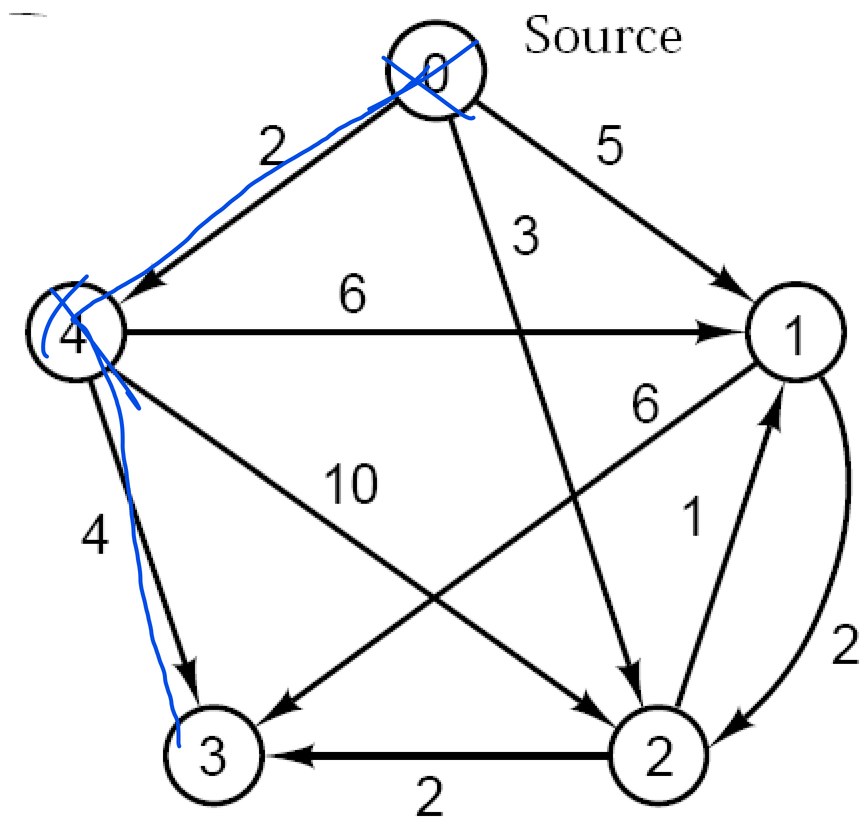


Dijkstra's algorithm in detail

- S: Set of vertices whose closest distances to the source are known.
- Add one vertex to S at each stage.
- For each vertex v, maintain the distance from the source to v, along a path all of whose vertices are in S, except possibly the last one.
- To determine what vertex to add to S at each step, apply the greedy criterion of choosing the vertex v with the smallest distance.
- Add v to S.
- Update distance from the source for all w not in S, if the path through v and then directly to w is shorter than the previously recorded distance to w.

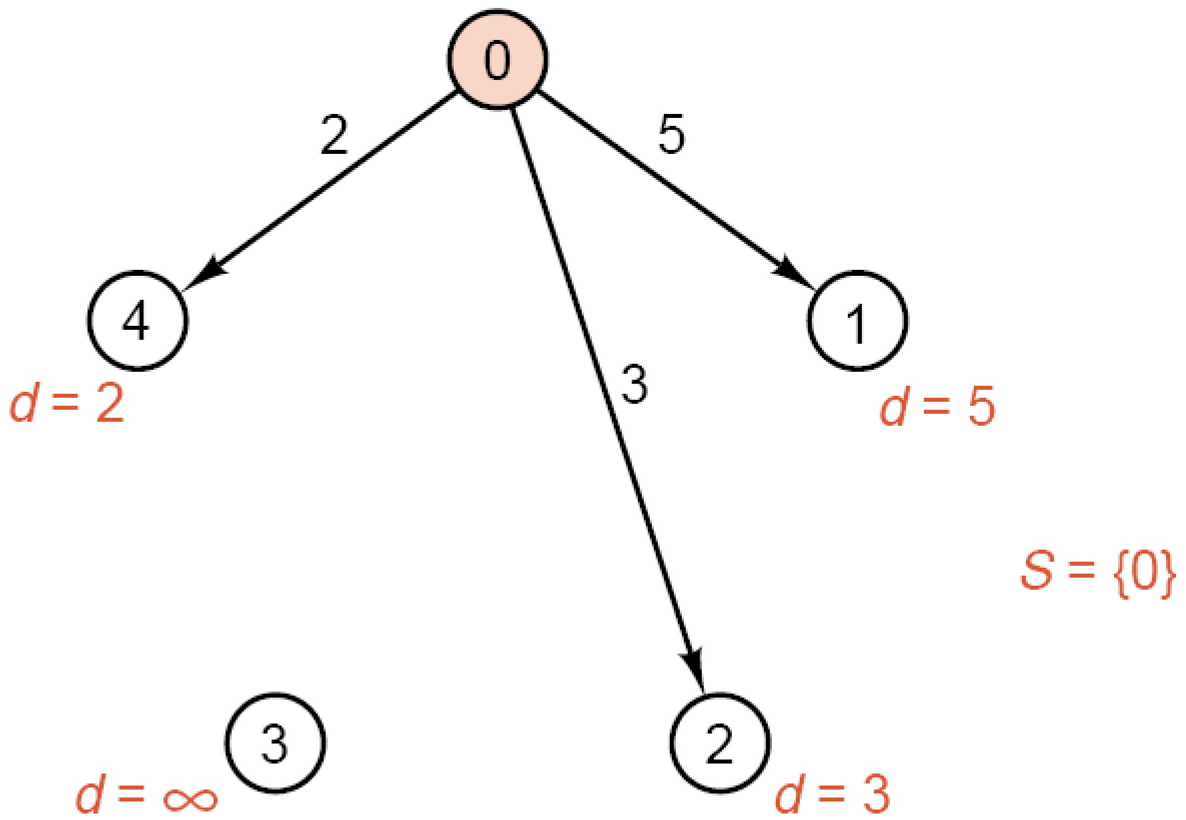


Dijkstra's algorithm



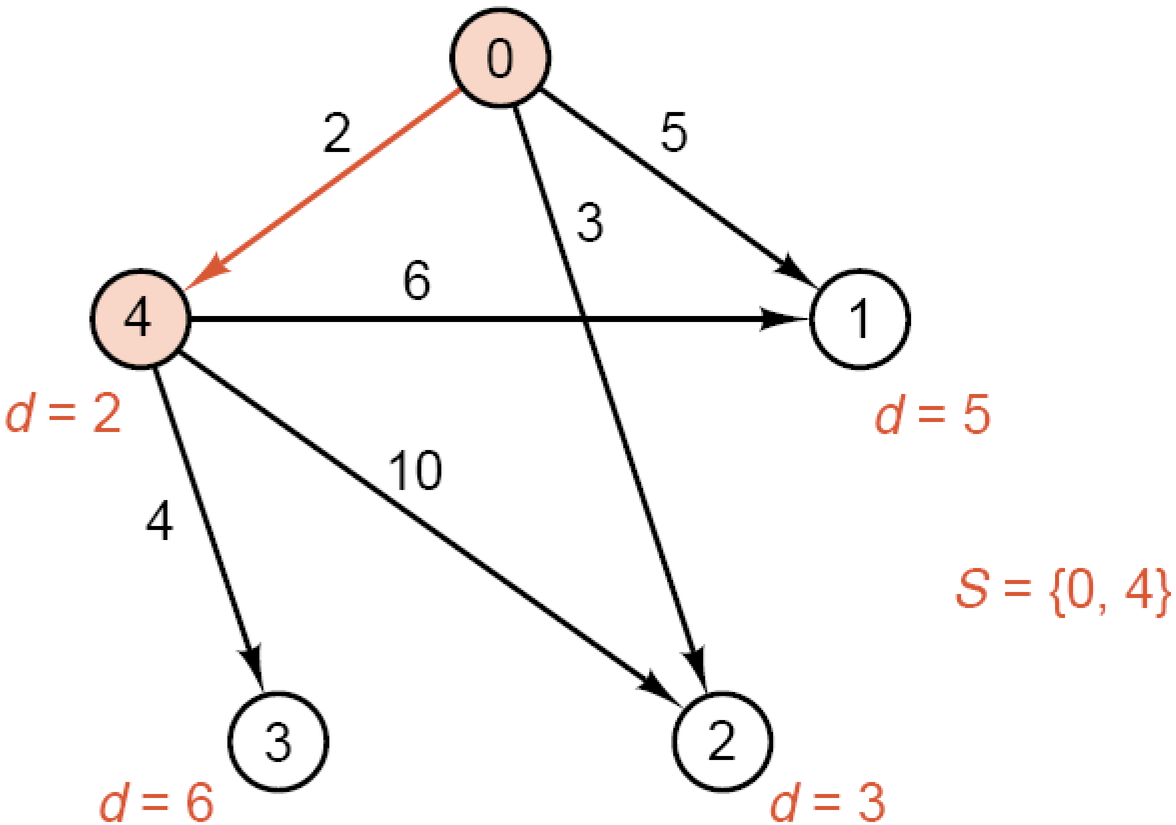
(a)

Dijkstra's algorithm



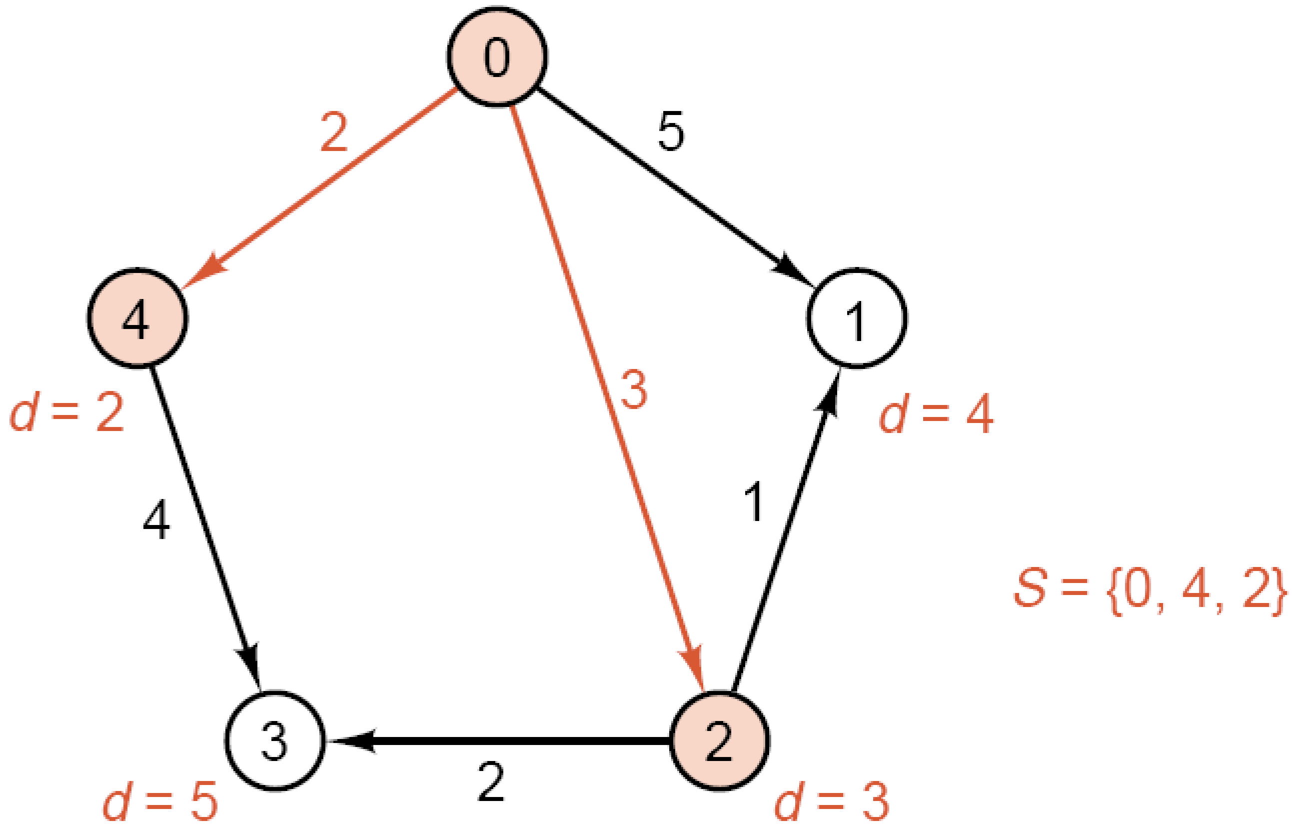
(b)

Dijkstra's algorithm



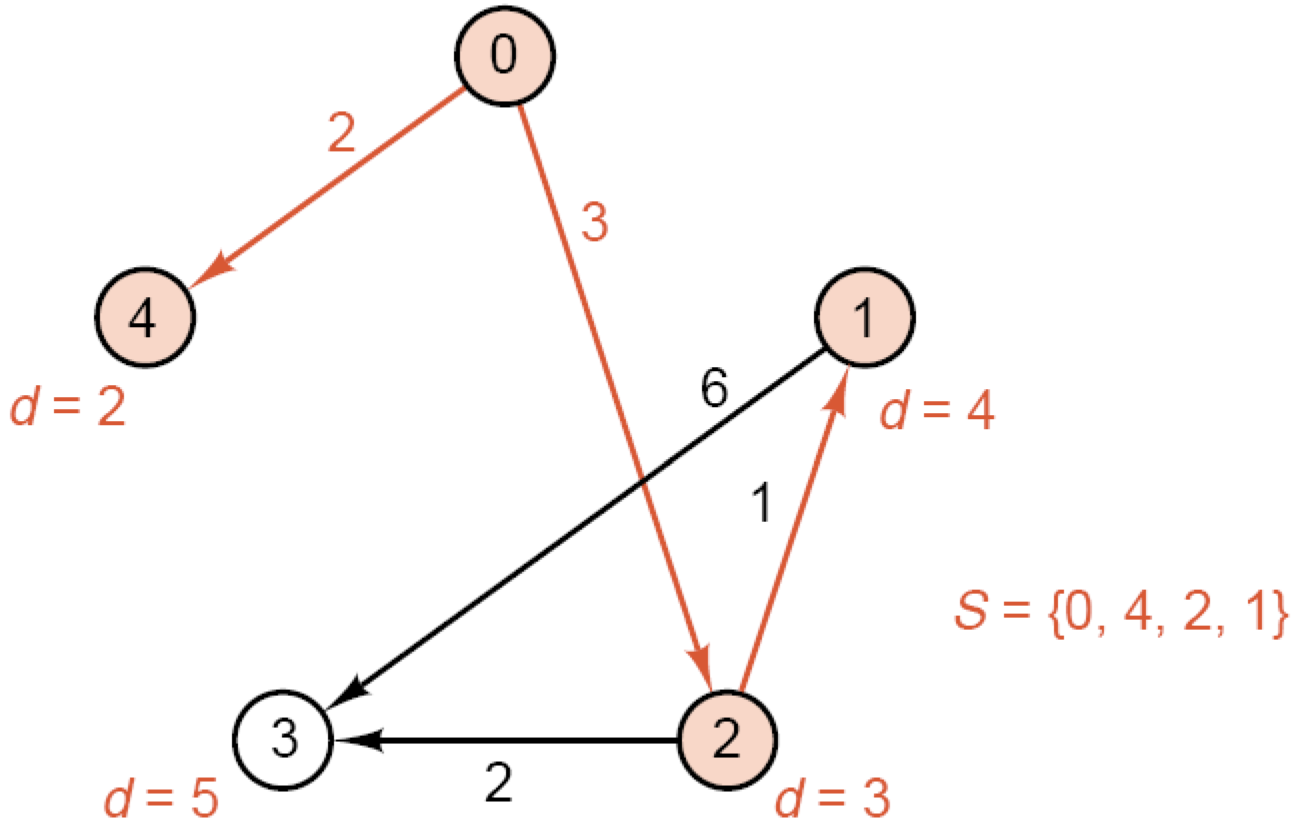
(c)

Dijkstra's algorithm



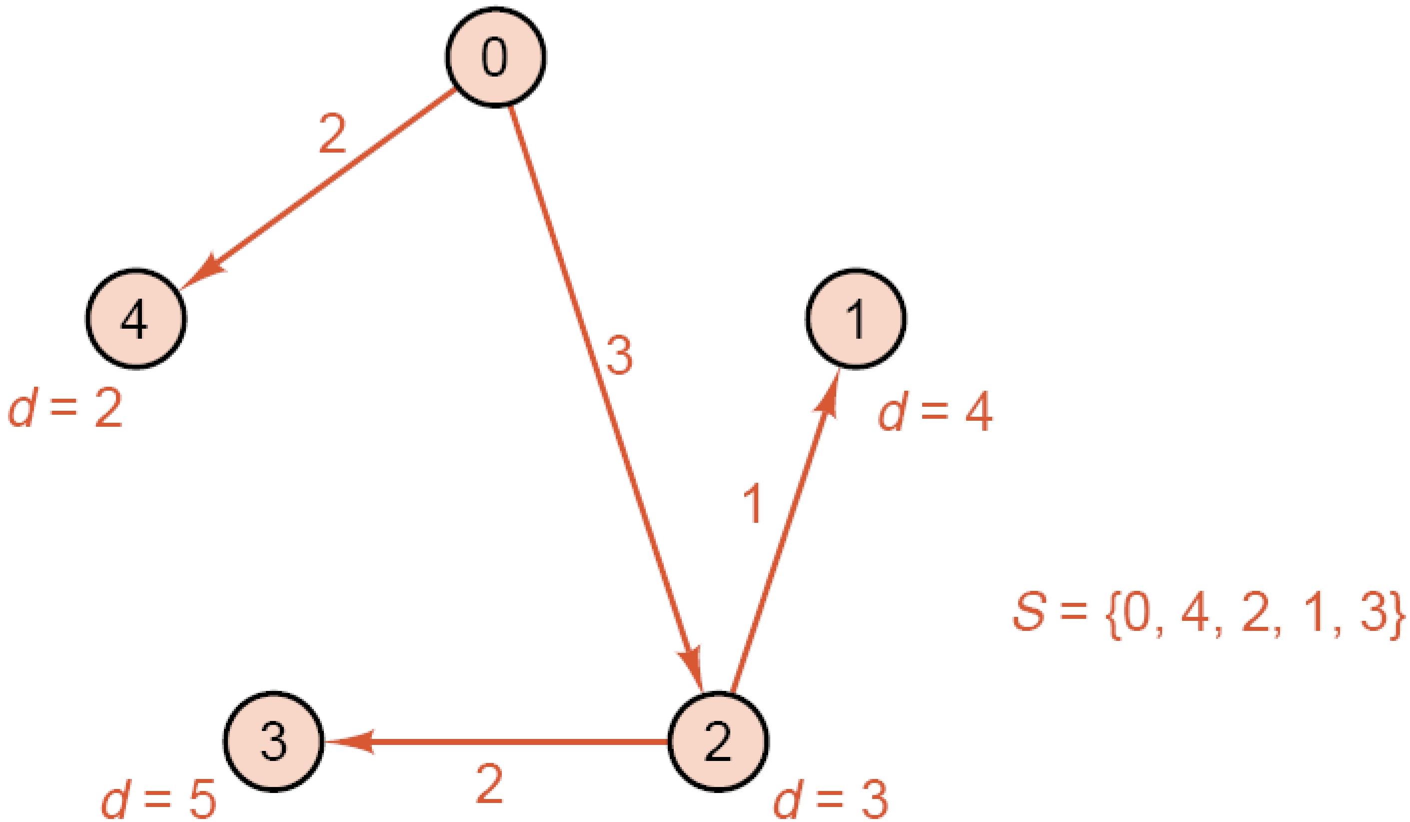
(d)

Dijkstra's algorithm



(e)

Dijkstra's algorithm



(f)

Dijkstra's algorithm

<void> **ShortestPath** (val source <VertexType>,
ref listOfShortestPath <List of <DistanceNode>>)

Finds the shortest paths from source to all other vertices in digraph.

Post Each node in listOfShortestPath gives the minimal path weight from vertex source to vertex destination in distance field.

```
DistanceNode
        destination <VertexType>
        distance <int>
End DistanceNode
```

// ShortestPath

1. listOfShortestPath.clear()
2. Add source to set S
3. **loop** (more vertex v in digraph) *// Initiate all distances from source to v*
    1. distanceNode.destination = v
    2. distanceNode.distance = weight of edge(source, v) *// = infinity if*
       // edge(source,v) isn't in digraph.
    3. listOfShortestPath.Insert(distanceNode)
4. **loop** (more vertex not in S) *// Add one vertex v to S on each step.*
    1. minWeight = infinity *// Choose vertex v with smallest distance.*
    2. **loop** (more vertex w not in S)
        1. Find the distance x from source to w in listOfShortestPath
        2. **if** (x < minWeight)
            1. v = w
            2. minWeight = x
    3. Add v to S.

```
DistanceNode
        destination <VertexType>
        distance <int>
End DistanceNode
```

// ShortestPath (continue)

4. **loop** (more vertex w not in S) *// Update distances from source*

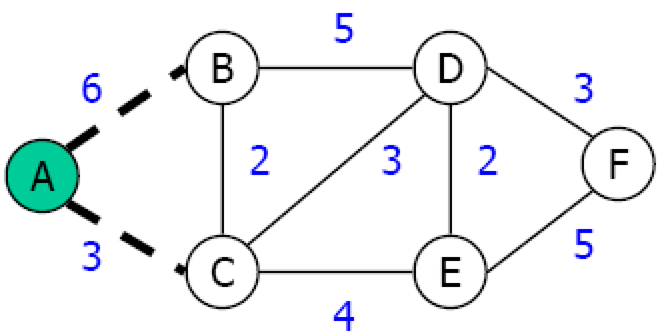
// to all w not in S

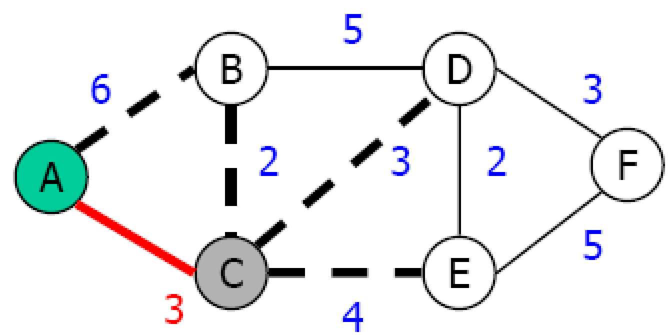
1. Find the distance x from source to w in listOfShortestPath
2. **if** ((minWeight + weight of edge from v to w) < x)
   1. Update distance from source to w in listOfShortestPath to (minWeight + weight of edge from v to w)

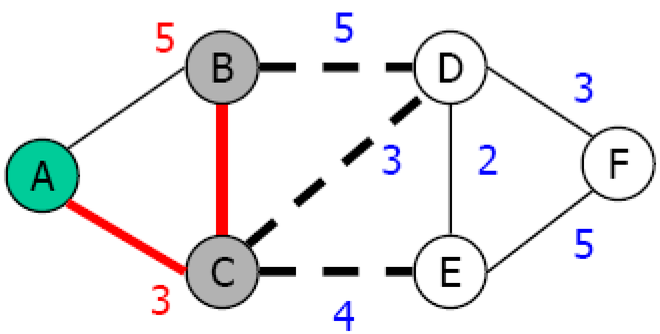
End ShortestPath

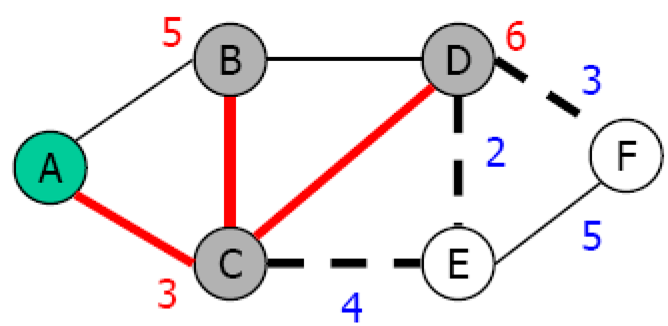
```
DistanceNode
        destination <VertexType>
        distance <int>
End DistanceNode
```

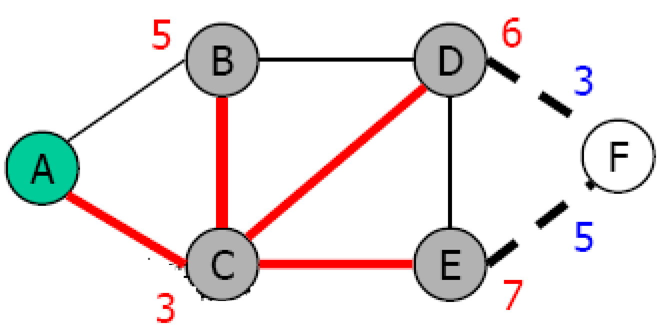
Another example of Shortest Paths

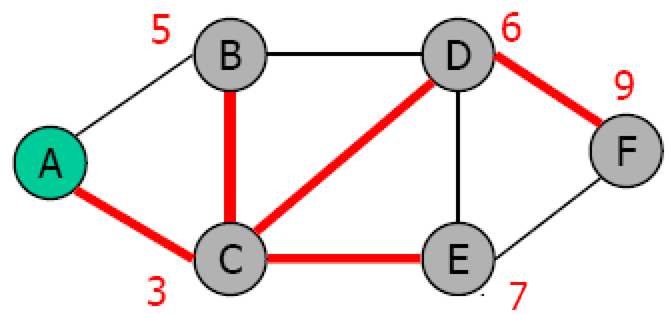












Select the adjacent vertex having minimum path to the source vertex

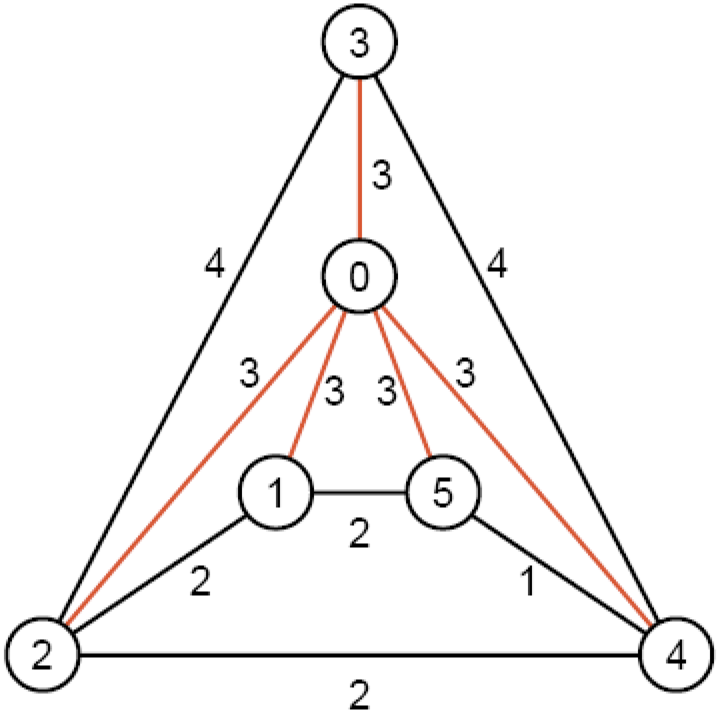
Minimum spanning tree

DEFINITION:

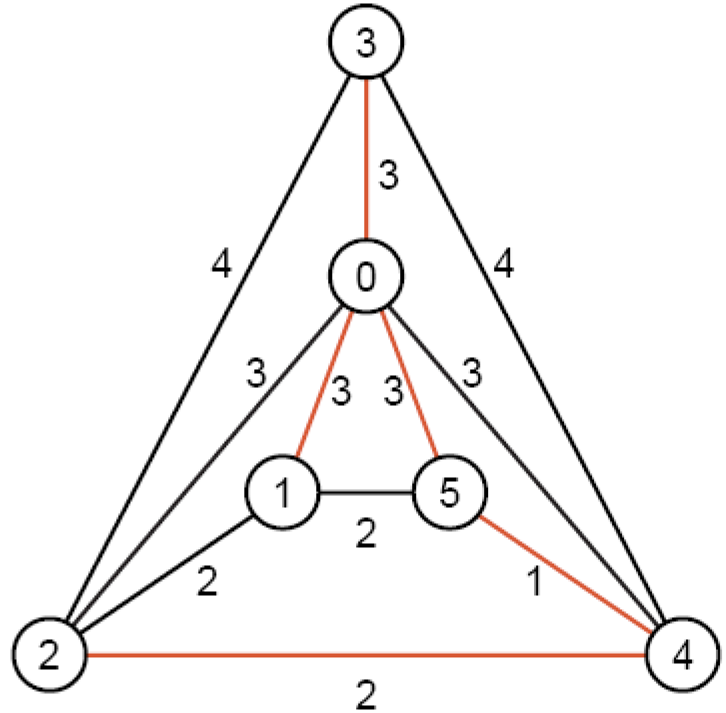
Spanning tree: tree that contains all of the vertices in a connected graph.

Minimum spanning tree: spanning tree such that the sum of the weights of its edges is minimal.

Spanning Trees



Weight sum of tree = 15
(a)



Weight sum of tree = 12
(b)

Two spanning trees in a network

A greedy Algorithm: Minimum Spanning Tree

- Shortest path algorithm in a connected graph found an its spanning tree.
- What is the algorithm finding the minimum spanning tree?
- A small change to shortest path algorithm can find the minimum spanning tree, that is Prim's algorithm since 1957.

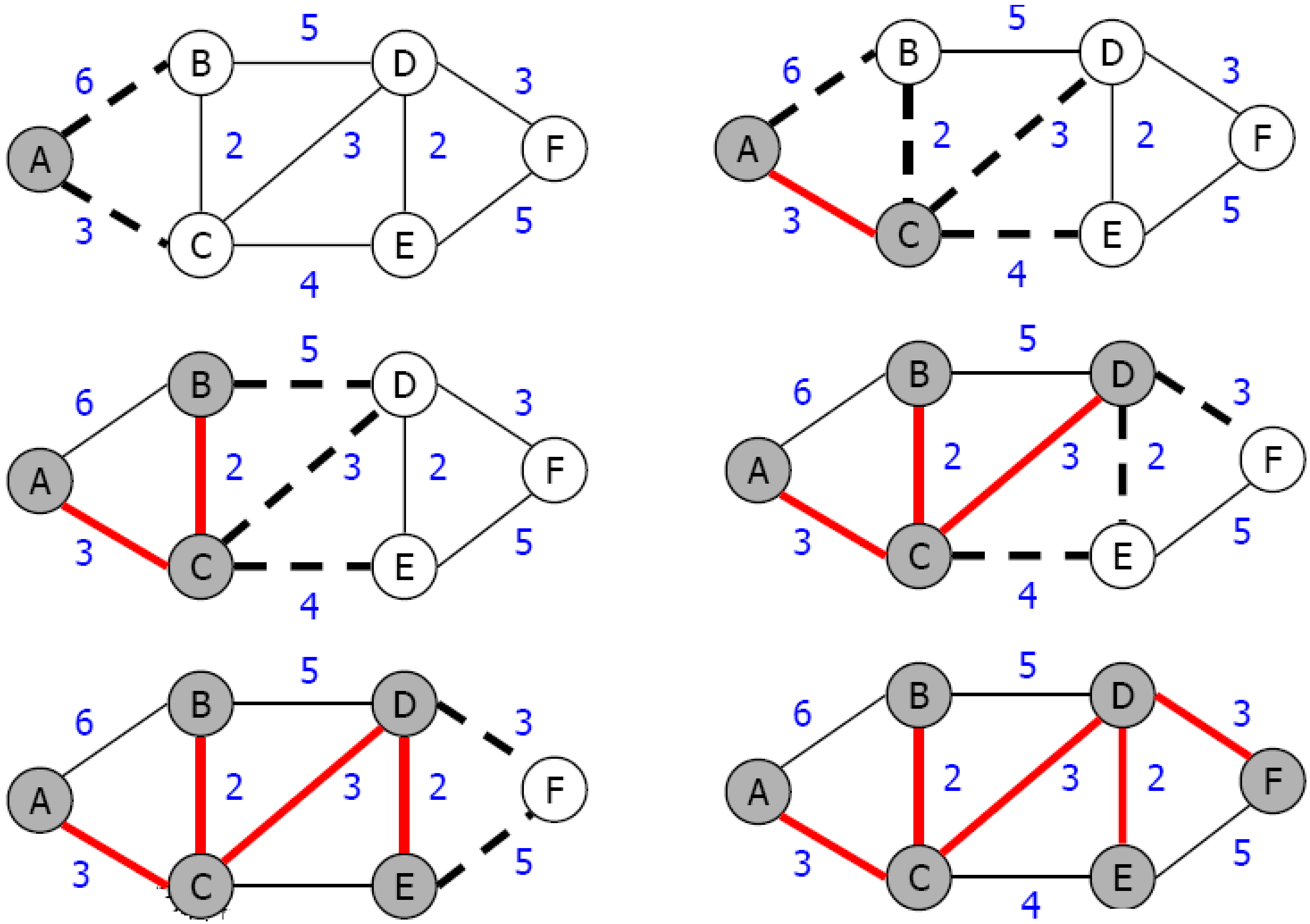
Prim's algorithm

- Let tree is the minimum spanning tree.
- At first, add one vertex to the tree.
- Loop until all vertices are in the tree:
  - Consider the adjacent vertices of the vertices already in the tree.
  - Examine all the edges from each vertices already in the tree to those adjacent vertices.
  - Select the smallest edge and insert the corresponding adjacent vertex into the tree.

Prim's algorithm in detail

- Let S is the set of vertices already in the minimum spanning tree.
- At first, add one vertex to S.
- For each vertex v not in S, maintain the distance from a vertex x to v, where x is a vertex in S and the edge(x,v) is the smallest in all edges from another vertices in S to v (this edge(x,v) is called the distance from S to v). As usual, all edges not being in graph have infinity value.
- To determine what vertex to add to S at each step, apply the greedy criterion of choosing the vertex v with the smallest distance from S.
- Add v to S.
- Update distances from S to all vertices v not in S if they are smaller than the previously recorded distances.

Prim's algorithm



Select the adjacent vertex having minimum edge to the vertices already in the tree.

Prim's algorithm

<void> **MinimumSpanningTree** (val source <VertexType>,

ref tree <Graph>)

Finds the minimum spanning tree of a connected component of the original graph that contains vertex source.

Post tree is the minimum spanning tree of a connected component of the original graph that contains vertex source.

Uses local variables:

- Set S
- listOfDistanceNode
- continue <boolean>

```
DistanceNode
        vertexFrom <VertexType>
        vertexTo <VertexType>
        distance <WeightType>
End DistanceNode
```

1. tree.clear()
2. tree.InsertVertex(source)
3. Add source to set S
4. listOfDistanceNode.clear()
5. distanceNode.vertexFrom = source
6. **loop** (more vertex v in graph)*//Initiate all distances from source to v*
    1. distanceNode.vertexTo = v
    2. distanceNode.distance = weight of edge(source, v) *// = infinity if*
       // edge(source,v) isn't in graph.
    3. listOfDistanceNode.Insert(distanceNode)

```
DistanceNode
        vertexFrom <VertexType>
        vertexTo <VertexType>
        distance <WeightType>
End DistanceNode
```

7. continue = TRUE
8. **loop** (more vertex not in S) and (continue) *//Add one vertex to S on*
   // each step
    1. minWeight = infinity *//Choose vertex v with smallest distance toS*
    2. **loop** (more vertex w not in S)
        1. Find the node in listOfDistanceNode with vertexTo is w
        2. **if** (node.distance < minWeight)
            1. v = w
            2. minWeight = node.distance

```
DistanceNode
    vertexFrom <VertexType>
    vertexTo <VertexType>
    distance <WeightType>
End DistanceNode
```

3. **if** (minWeight < infinity)
    1. Add v to S.
    2. tree.InsertVertex(v)
    3. tree.InsertEdge(v,w)
    4. **loop** (more vertex w not in S) *// Update distances from v to*
       // all w not in S if they are smaller than the
       // previously recorded distances in listOfDistanceNode
        1. Find the node in listOfDistanceNode with vertexTo is w
        2. **if** (node.distance > weight of edge(v,w))
            1. node.vertexFrom = v
            2. node.distance = weight of edge(v,w))
            3. Replace this node with its old node in listOfDistance
4. **else**
    1. continue = FALSE

End MinimumSpanningTree

```
DistanceNode
        vertexFrom <VertexType>
        vertexTo <VertexType>
        distance <WeightType>
End DistanceNode
```

Maximum flows

- A network of water pipelines from one source to one destination.
- Water is pumped thru many pipes with many stations in between.
- The amount of water that can be pumped may differ from one pipeline to another.

Maximum flows

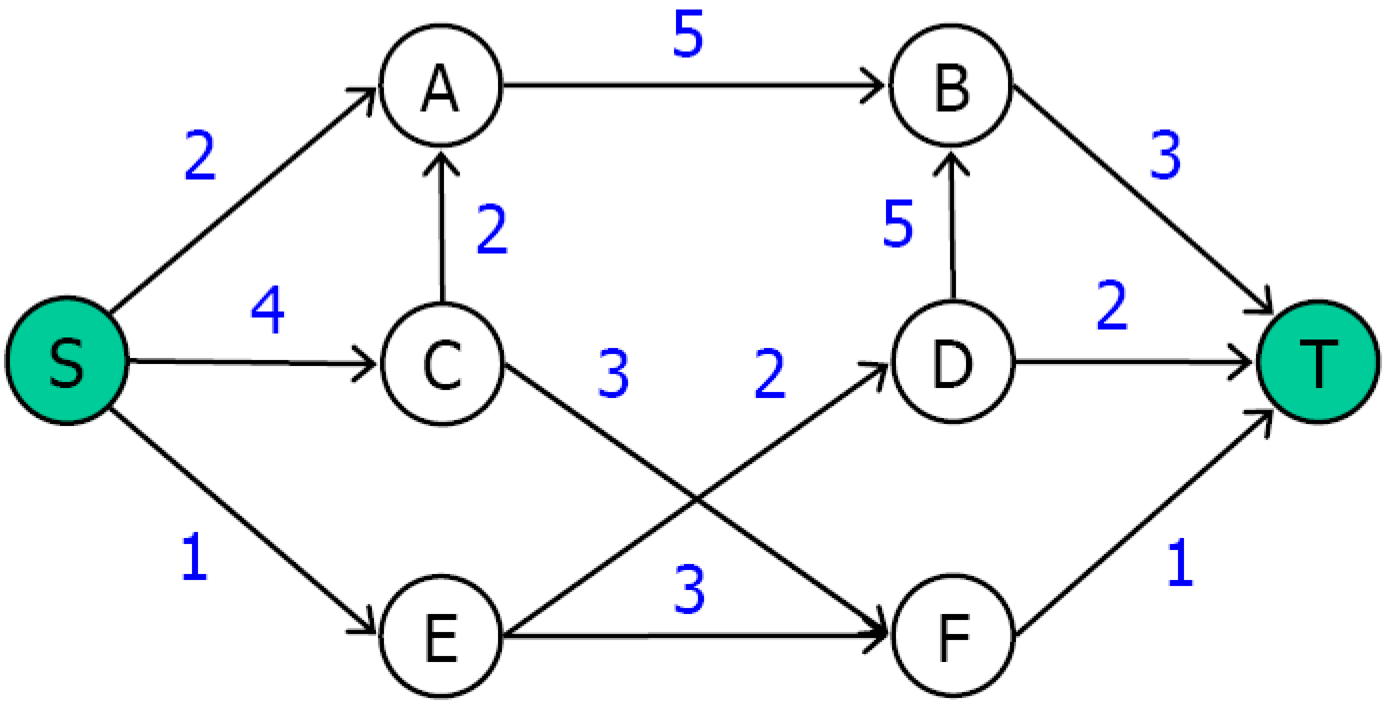
- The flow thru a pipeline cannot be greater than its capacity.
- The total flow coming to a station is the same as the total flow coming from it.

Maximum flows

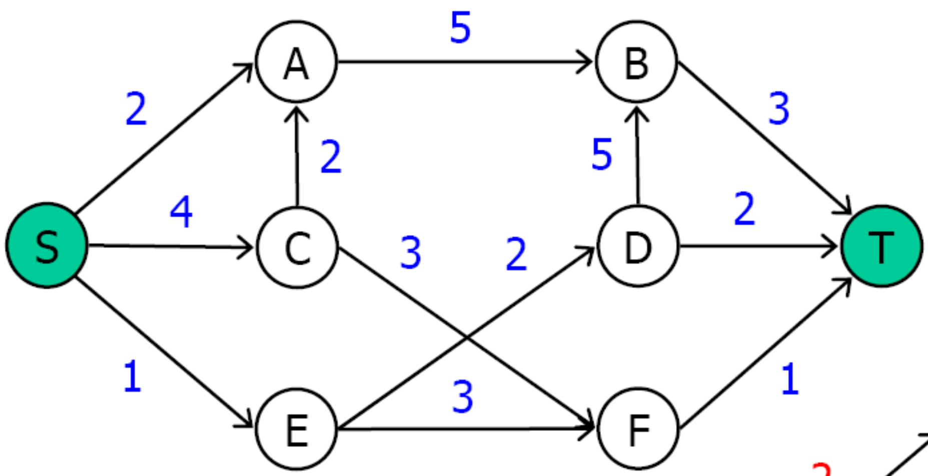
- The flow thru a pipeline cannot be greater than its capacity.
- The total flow coming to a station is the same as the total flow coming from it.

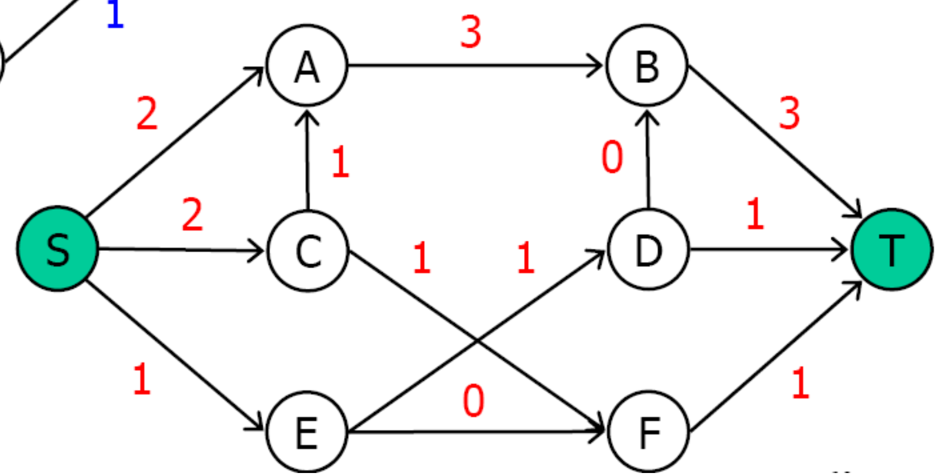
The problem is to maximize the total flow coming to the destination.

Maximum flows



Maximum flows





Matching

- Applicants: p q r s t
- Suitable jobs: a b c | b d | a e | e | c d e
- No applicant is accepted for two jobs, and no job is assigned to two applicants.

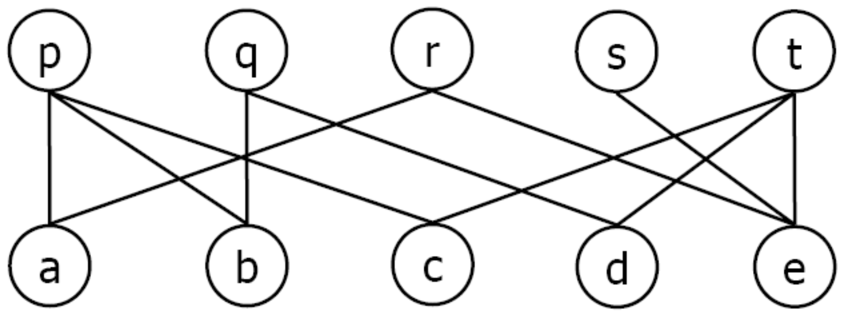
Matching

- Applicants: p q r s t
- Suitable jobs: a b c | b d | a e | e | c d e
- No applicant is accepted for two jobs, and no job is assigned to two applicants.

The problem is to find a worker for each job.

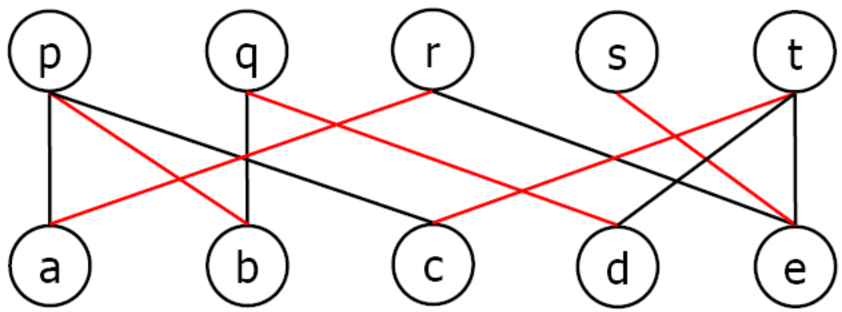
Matching

- Applicants: p q r s t
- Suitable jobs: a b c | b d | a e | e | c d e



Matching

- Applicants: p q r s t
- Suitable jobs: a b c | b d | a e | e | c d e



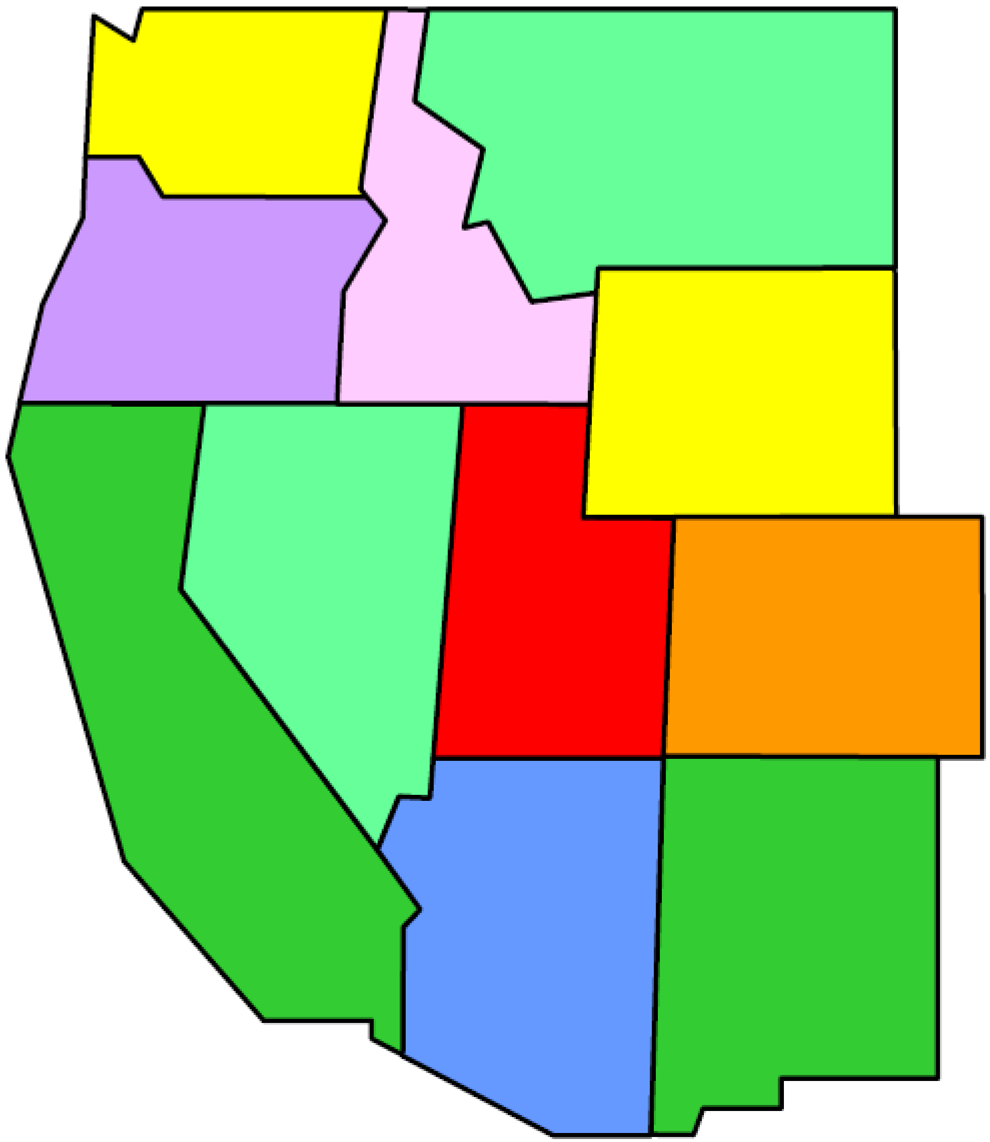
Matching

- Maximum matching: as many pairs of worker-job as possible.
- Perfect matching (marriage problem): no worker or job left unmatched.

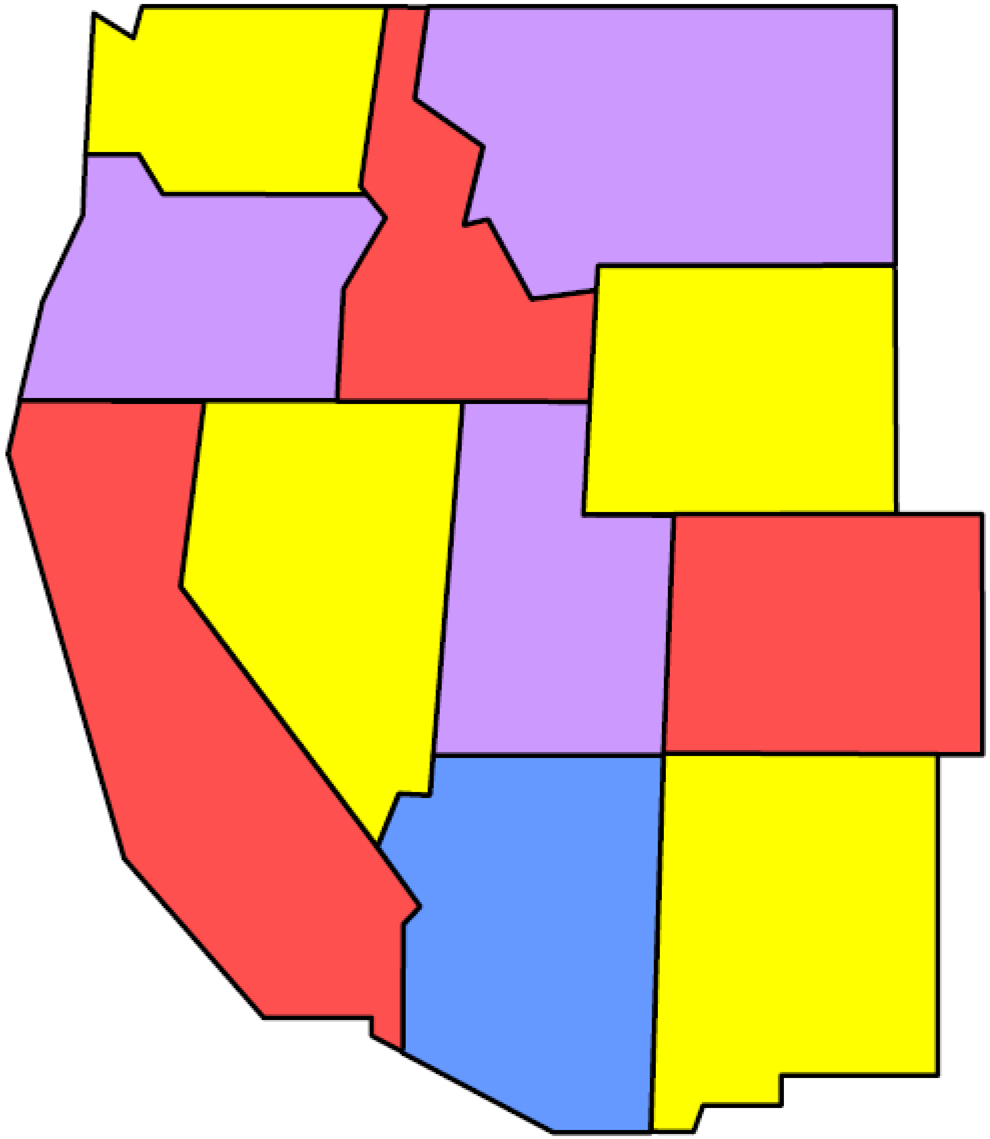
Graph coloring

- Given a map of adjacent regions.
- Find the minimum number of colors to fill the regions so that no adjacent regions have the same color.

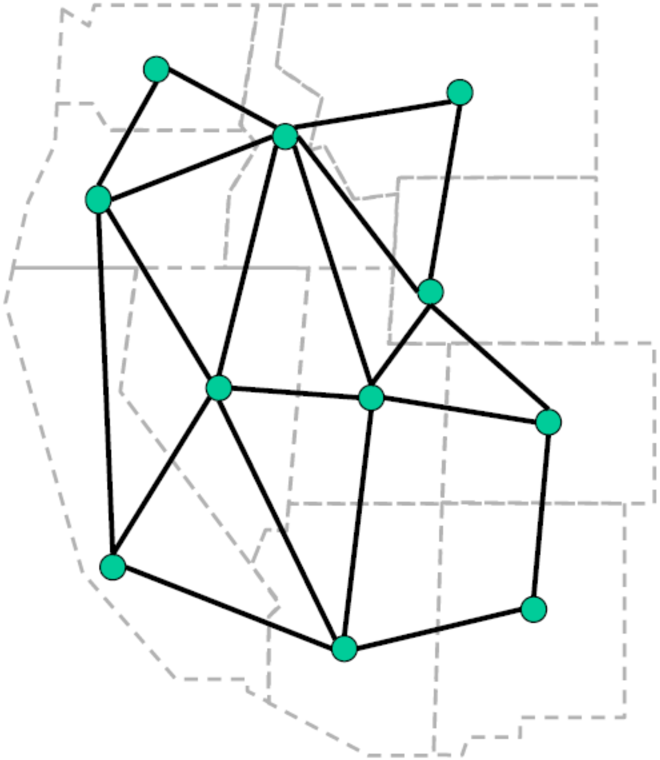
Graph coloring



Graph coloring

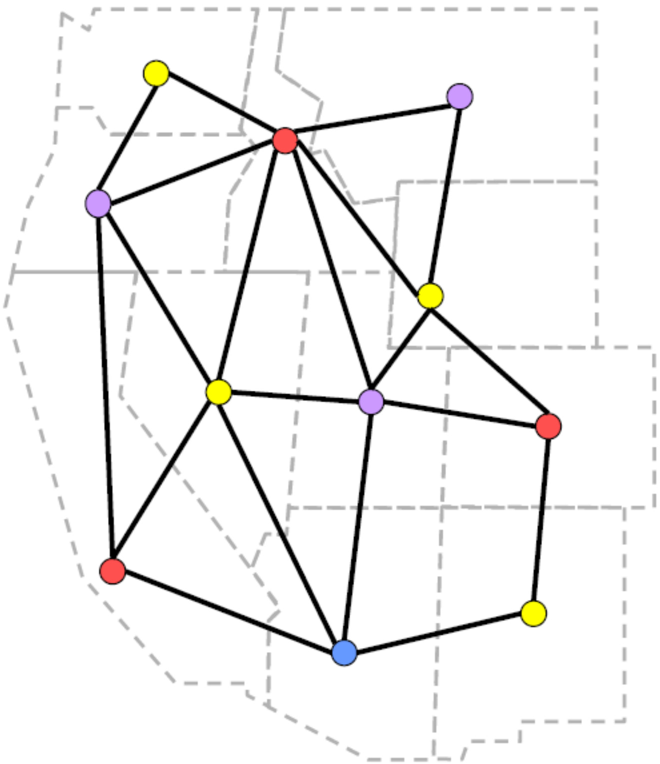


Graph coloring



The problem is to find the minimum number of sets of non-adjacent vertices.

Graph coloring



The problem is to find the minimum number of sets of non-adjacent vertices.