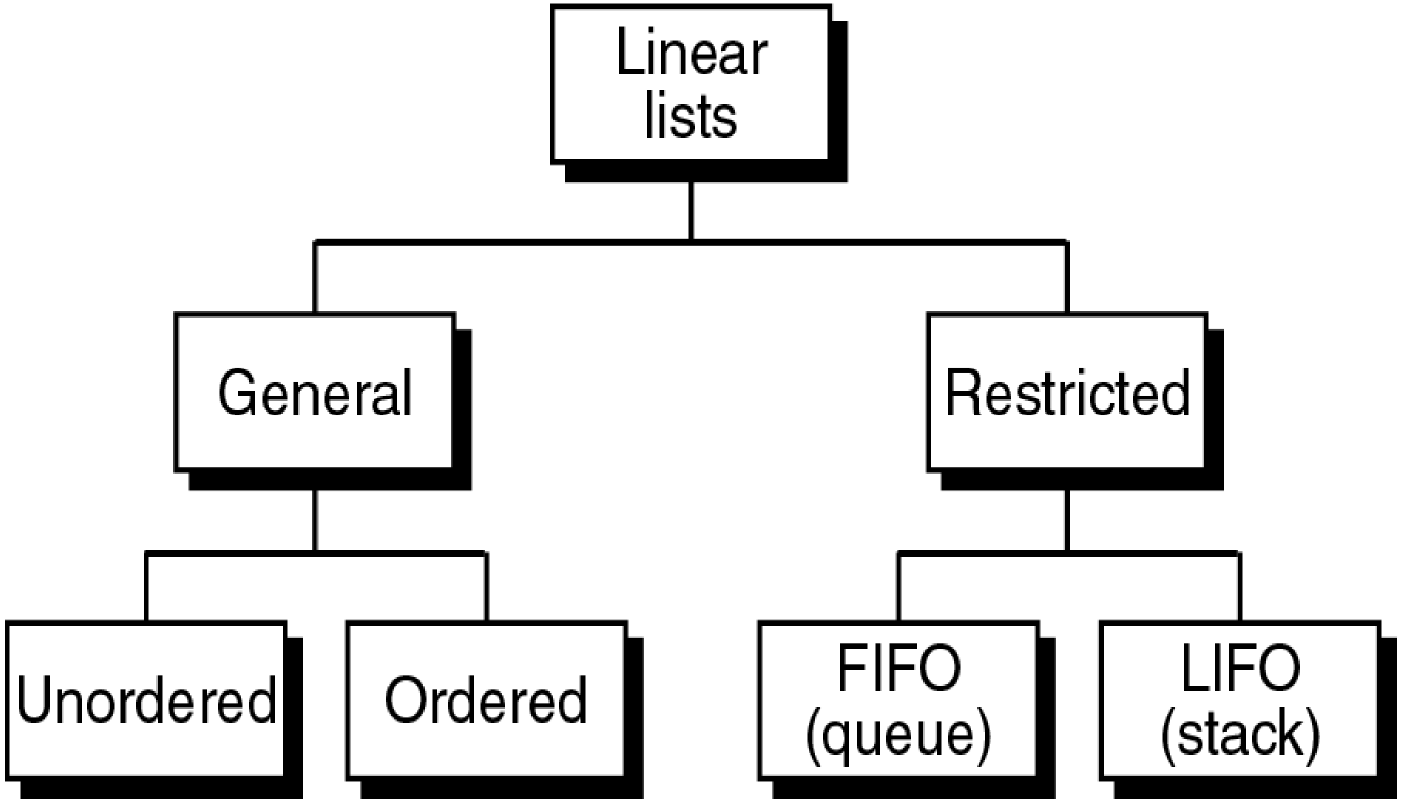
Chapter 2 – LIST

- Linear List Concepts
- List ADT
- Specifications for List ADT
- Implementations of List ADT
- Contiguous List
- Singly Linked List
- Other Linked Lists
- Comparison of Implementations of List

Linear List Concepts

DEFINITION: Linear List is a data structure where each element of it has a unique successor.

Linear List Concepts (cont.)



Linear List Concepts (cont.)

- **General list**:
  - No restrictions on which operation can be used on the list
  - No restrictions on where data can be inserted/deleted.

- Unordered list (random list): Data are not in particular order.
- Ordered list: data are arranged according to a key.

Linear List Concepts (cont.)

- **Restricted list**:
  - Only some operations can be used on the list.
  - Data can be inserted/deleted only at the ends of the list.

- Queue: FIFO (First-In-First-Out).
- Stack: LIFO (Last-In-First-Out).

List ADT

element 1 -- **element 2** -- **element 3** -- . . . . -- **element n**

DEFINITION: A list of elements of type T is a finite sequence of elements of T together with the following operations:

Basic operations:

- *Construct* a list, leaving it empty.
- *Insert* an element.
- *Remove* an element.
- *Search* an element.
- *Retrieve* an element.
- *Traverse* the list, performing a given operation on each element.

List ADT (cont.)

Extended operations:

- Determine whether the list is *empty* or not.
- Determine whether the list is *full* or not.
- Find the *size* of the list.
- *Clear* the list to make it empty.
- *Replace* an element with another element.
- *Merge* two ordered list.
- *Append* an unordered list to another.
- …

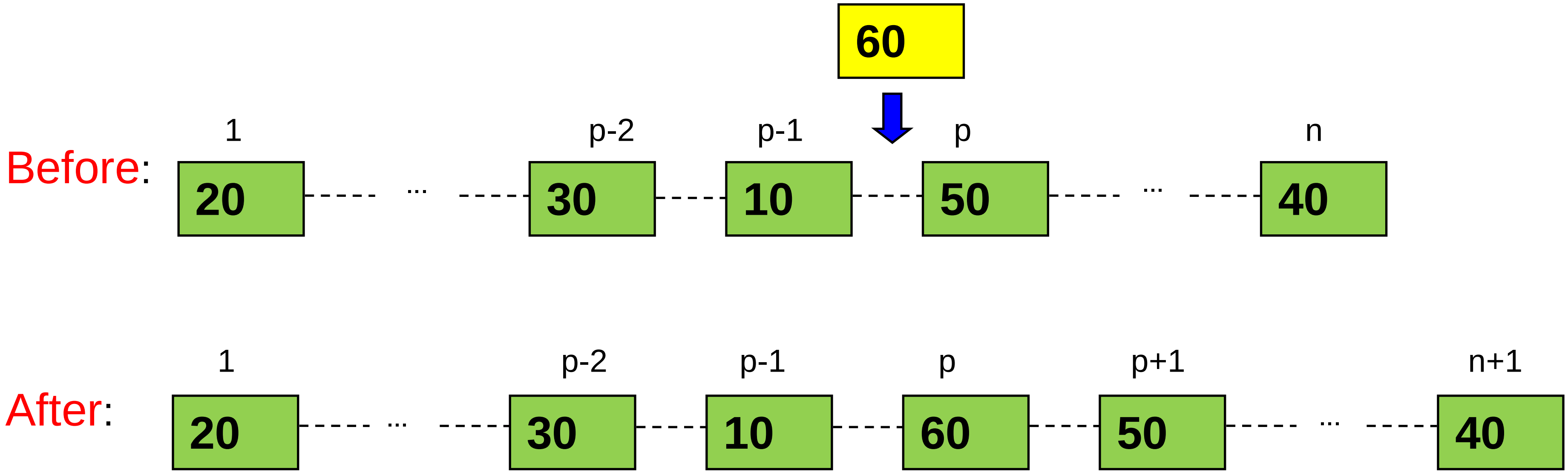
Insertion

- **Insert an element at a specified position p in the list**
  - ✓Only with *General Unordered List*.

- **Insert an element with a given data**
  - ✓*With General Unordered List*: can be made at any position in the list (at the beginning, in the middle, at the end).
  - ✓*With General Ordered List*: data must be inserted so that the ordering of the list is maintained (searching appropriate position is needed).
  - ✓*With Restricted List*: depend on it own definition (FIFO or LIFO).

Insertion (cont.)

Insert an element at *a specified position p* in the list.



Any element formerly at position p and all later have their position numbers increased by 1.

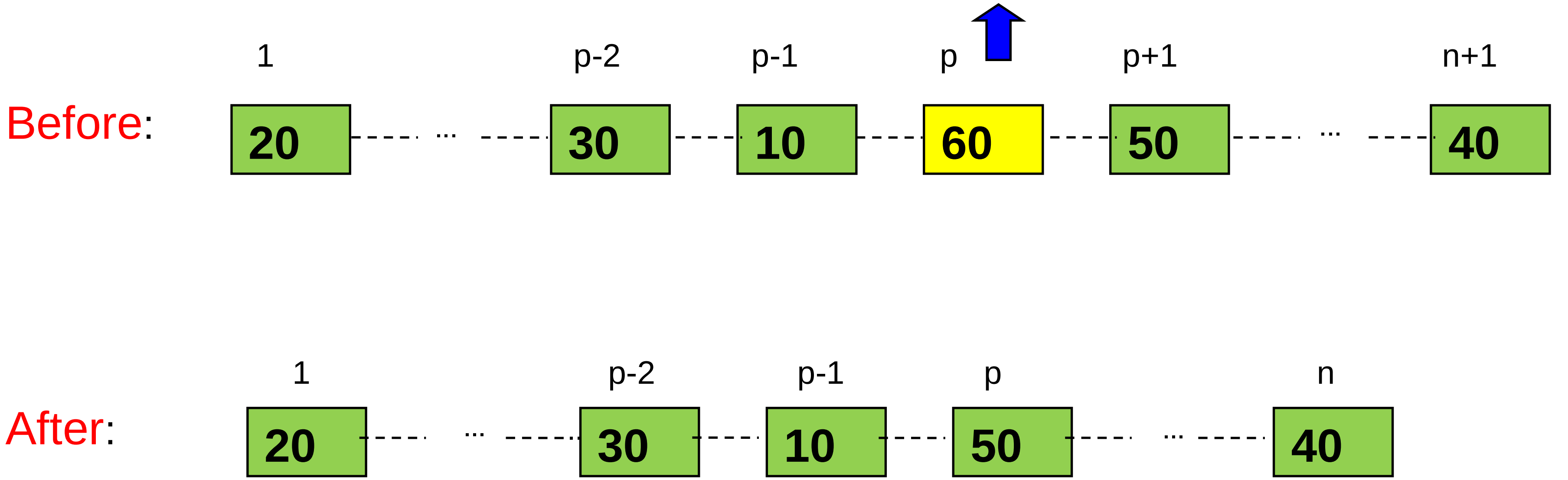
Removal, Retrieval

- Remove/ Retrieve an element at a specified position p in the list
  - ✓With *General Unordered List* and *General Ordered List*.

- Remove/ Retrieve an element with a given data
  - ✓With *General Unordered List* and *General Ordered List*: Searching is needed in order to locate the data being deleted/ retrieved.

Removal

Remove an element at *a specified position p* in the list.



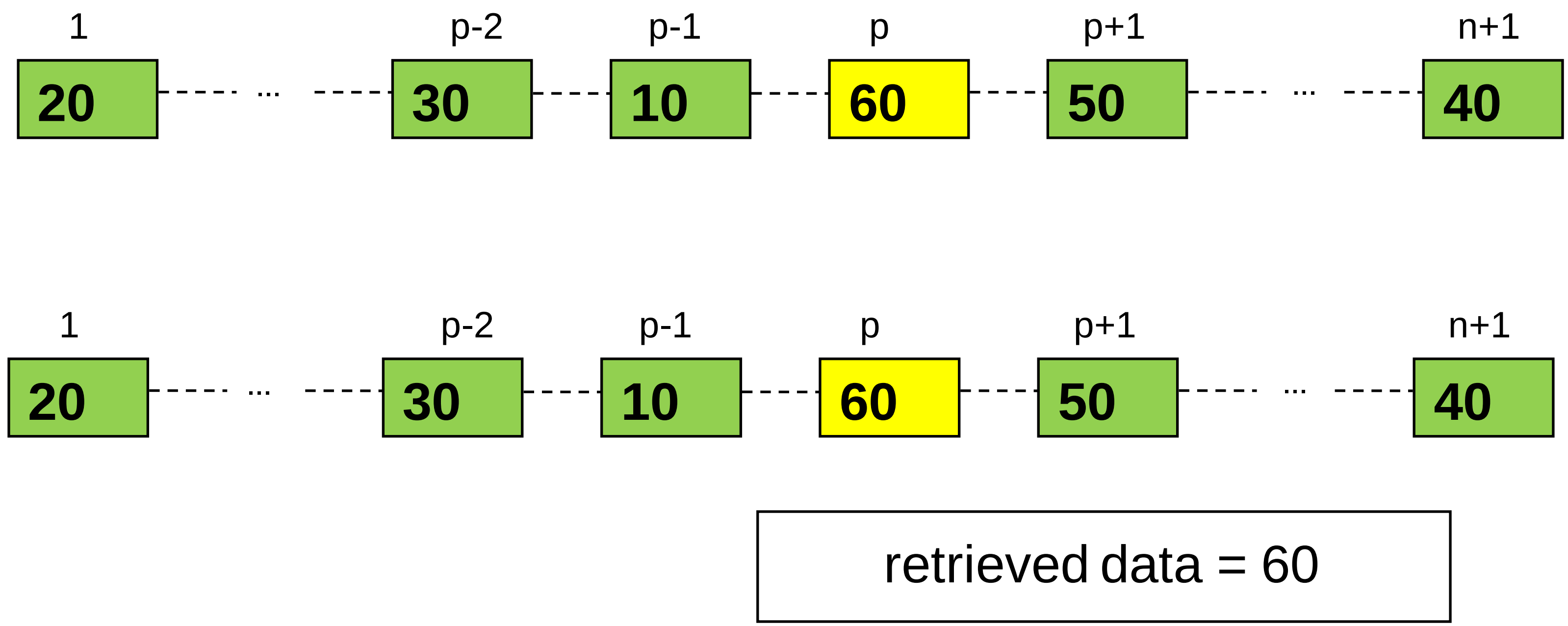
The element at position p is removed from the list, and all subsequent elements have their position numbers decreased by 1

Retrieval

Retrieve an element at *a specified position p* in the list.

Before:

After:



All elements remain unchanged.

Success of Basic Operations

- **Insertion** is successful when the list is not full.

- **Removal, Retrieval** are successful when the list is not empty.

Specification for List ADT

<void> Create()

<void> Traverse (ref <void> Operation (ref Data <DataType>))

<ErrorCode> Search (ref DataOut <DataType>) *// DataOut contains values need to be found in key field, and will reveive all other values in other fields.*

// ***For Unsorted List***:

<ErrorCode> Insert (val DataIn <DataType>, val position <integer>)

<ErrorCode> Remove (ref DataOut <DataType>,val position <integer>)

<ErrorCode> Retrieve (ref DataOut <DataType>,val position <integer>)

<ErrorCode> Replace (val DataIn <DataType>,
ref DataOut<DataType>, val position <integer>)

(Operations are successful when the required position exists).

Specification for List ADT (cont.)

// ***For Sorted List***:

<ErrorCode> Insert (val DataIn <DataType>)

<ErrorCode> Remove (ref DataOut <DataType>) *// DataOut contains values need to be found in key field, and will reveive all other values in other fields.*

<ErrorCode> Retrieve (ref DataOut <DataType>) *// DataOut contains values need to be found in key field, and will reveive all other values in other fields.*

(Insertion is successful when the list is not full and the key needs to be inserted does not exist in the list.

Removal and Retrieval are successful when the list is not empty and the required key exists in the list).

Specification of List ADT (cont.)

Samples of Extended methods:

<boolean> isFull()

<boolean> isEmpty()

<integer> Size()

<ErrorCode> Sort ()

<ErrorCode> AppendList (ref ListIn <ListType>) *// **For Unordered Lists**. ListIn may be unchanged or become empty.*

<ErrorCode> Merge (ref ListIn1 <ListType>, ref ListIn2 <ListType>) *// **For Ordered Lists**.*

. . .

Specification of List ADT (cont.)

Samples of variants of similar methods:

<void> Create()

<void> Create (ref file <InOutType>) // made a list from content of a file

<ErrorCode> Insert (val DataIn <DataType>, val position <integer>)

<ErrorCode> InsertHead (val DataIn <DataType>)

<ErrorCode> InsertTail (val DataIn <DataType>)

<ErrorCode> Replace (val DataIn <DataType>,
ref DataOut <DataType>, val position <integer>)

<ErrorCode> Replace (val DataIn <DataType>,
ref DataOut <DataType>)

Specification of List ADT (cont.)

Samples of variants of similar methods:

<ErrorCode> Remove (val position <integer>)

<ErrorCode> Remove (ref DataOut <DataType>,
val position <integer>)

<ErrorCode> RemoveHead (val DataOut <DataType>)

<ErrorCode> RemoveTail (ref DataOut <DataType>)

<ErrorCode> Search (val DataIn <DataType>, ref ListOut <ListType>)

// DataIn contains values need to be found in some fields, ListOut will contain all members having that values.

. . .

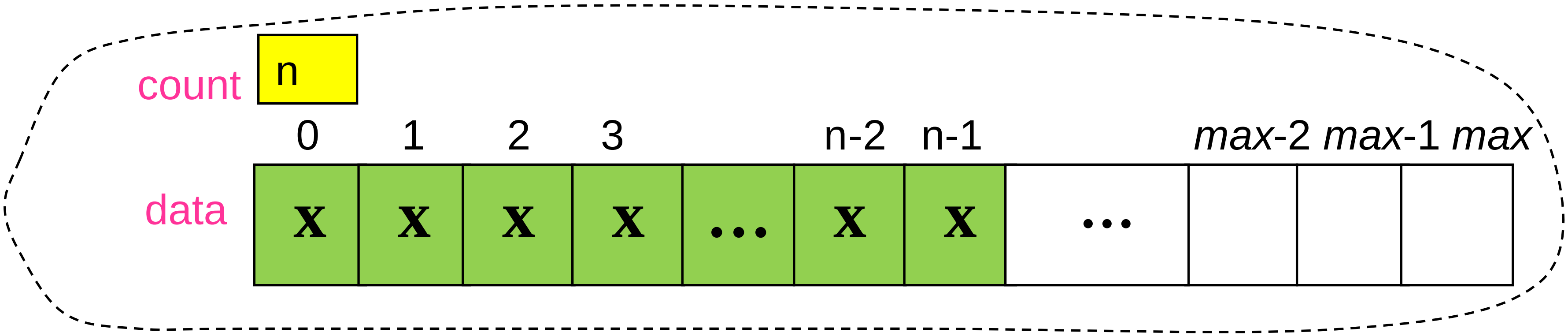
Sample of using List ADT

```
#include <iostream>
#include <List> // uses Unordered List ADT.
int main()
{    List<int> listObj;
     cout << "Enter 10 numbers: \n" <<flush;
     int i, x;
     for (i=0; i<10; i++)
     {       cin >> x;
             listObj.Insert( x, listObj.Size() ); // Insert at the end of the list.
     }
     cout << "Elements in the list: \n";
     for (i=0; i<10; i++)
     {       listObj.Retrieve(x, i);
             cout << x << "\t";
     }
     return 0;
}
```

Implementations of List ADT

- *Contiguous implementation*:
  - Automatically Allocated Array with fixed size.
  - Dynamically Allocated Array with flexible size.
- *Linked implementations*:
  - Singly Linked List
  - Circularly Linked List
  - Doubly Linked List
  - Multilinked List
  - Skip List
  - . . .
- *Linked List in Array*

Automatically Allocated Array



Array with pre-defined maxsize and has n elements.

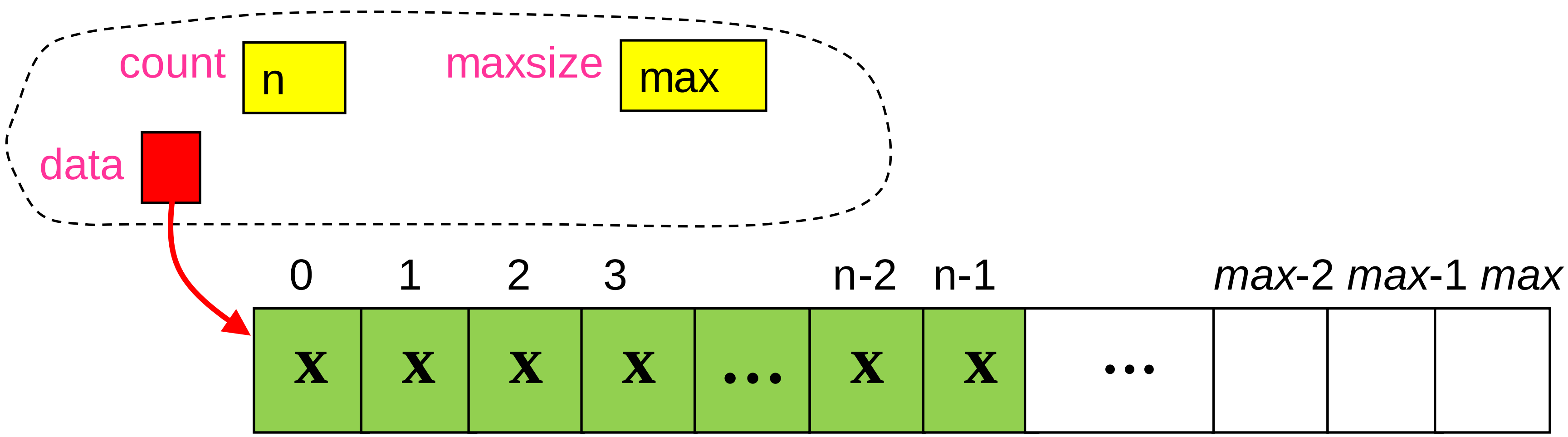
List *// Contiguous Implementation of List*

count <integer> *// number of used elements (mandatory).*

data <array of <DataType> > *// (Automatically Allocated Array)*

End List

Dynamically Allocated Array



List // *Contiguous Implementation of List*

count <integer> // *number of used elements (mandatory).*

data <dynamic array of <DataType> > // *(Dynamically Allocated Array)*

maxsize <integer>

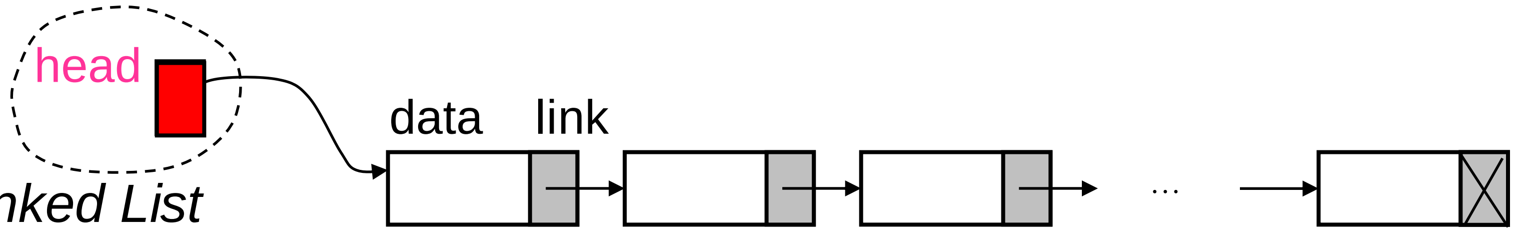
End List

Contiguous Implementation of List

In processing a contiguous list with n elements:

- Insert and Remove operate in time approximately proportional to n (require physical shifting).
- Clear, Empty, Full, Size, Replace, and Retrieve in constant time.

Singly Linked List



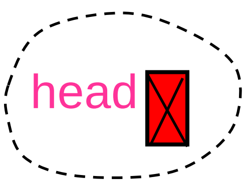
Singly Linked List

List *// Linked Implementation of List (for Singly Linked List)*

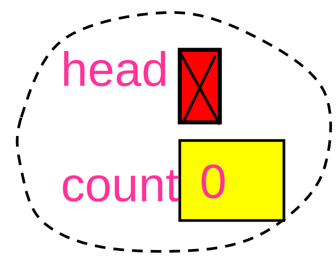
  head <pointer>

  count <integer> *// number of elements (optional).*

End List



An empty Singly Linked List having only head.



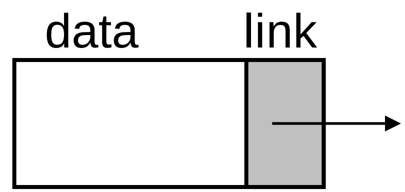
An empty Singly Linked List having head and count.

Singly Linked List (cont.)

Node
  data <DataType>
  link <pointer>
End Node

General DataType:

DataType
  key <KeyType>
  field1 <…>
  field2 <…>
  …
  fieldn <…>
End DataType

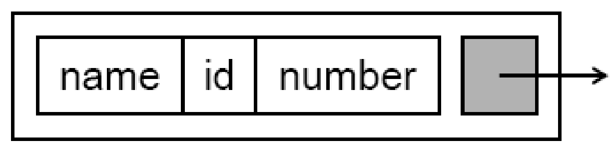


Element in the Singly Linked List

A node with
one data field

A node with
three data fields

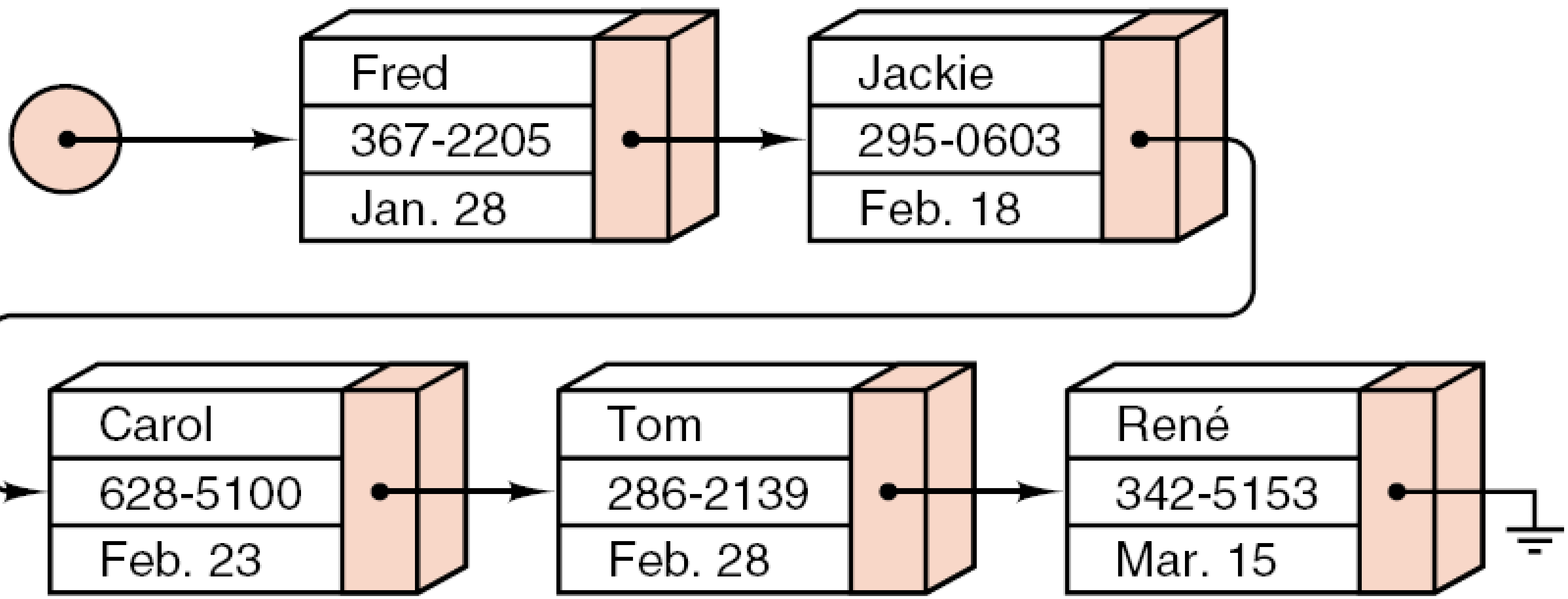
A node with one
structured data field



DataType may be an atomic or a composite data

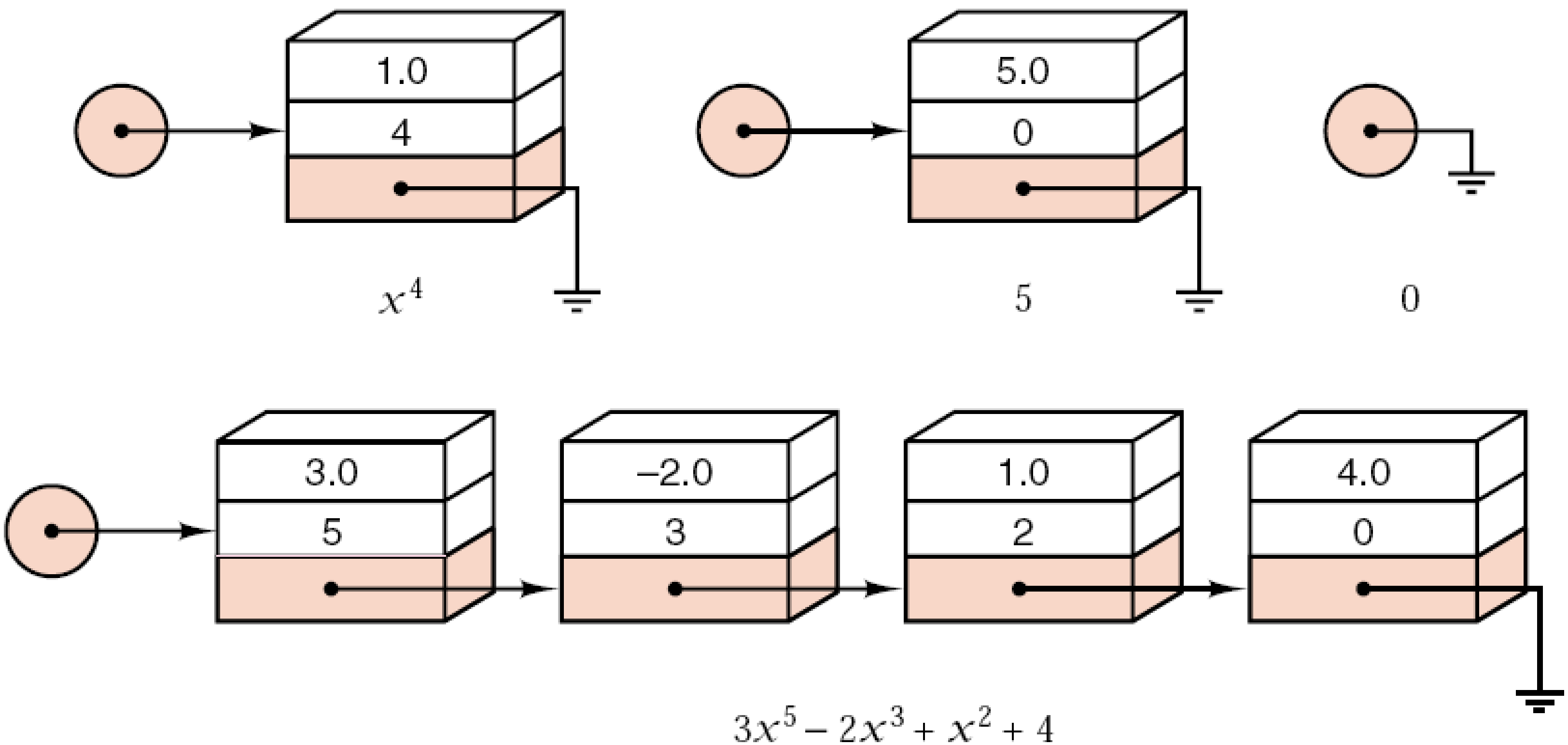
Singly Linked List (cont.)

- Sample:

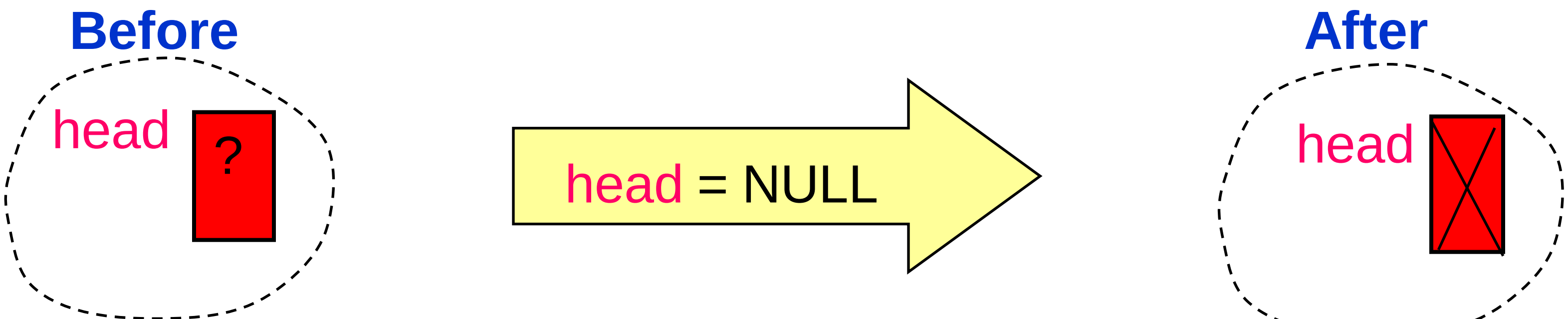


Singly Linked List (cont.)

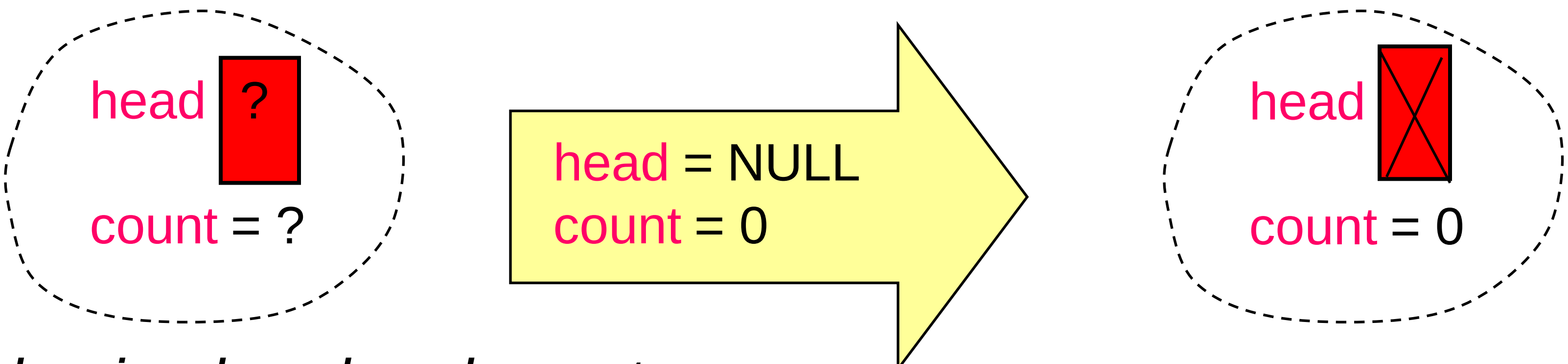
- Sample: list representing polynomial



Create an Empty Linked List



List having head



List having head and count

Create an Empty Linked List (cont.)

<void> **Create**()

Creates an empty link list

Pre none

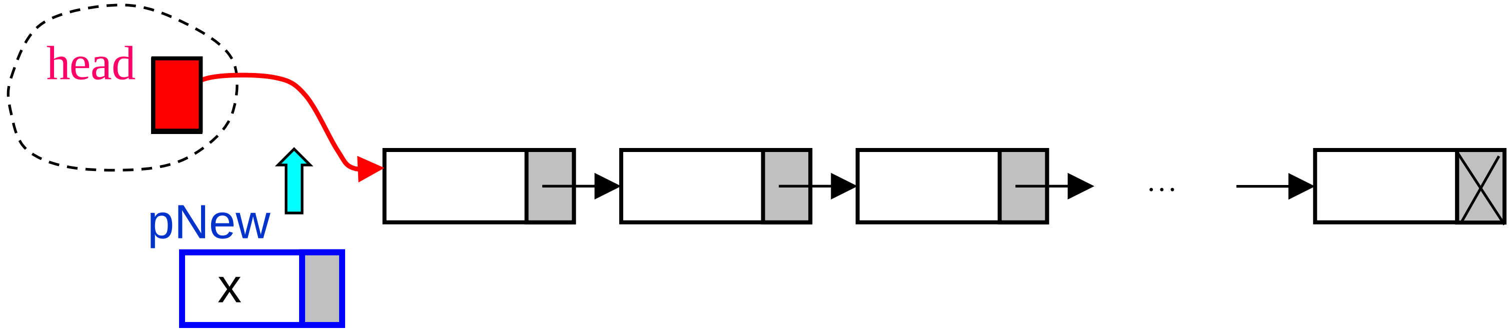
Post An empty linked list has been created.

1. head = NULL
2. Return

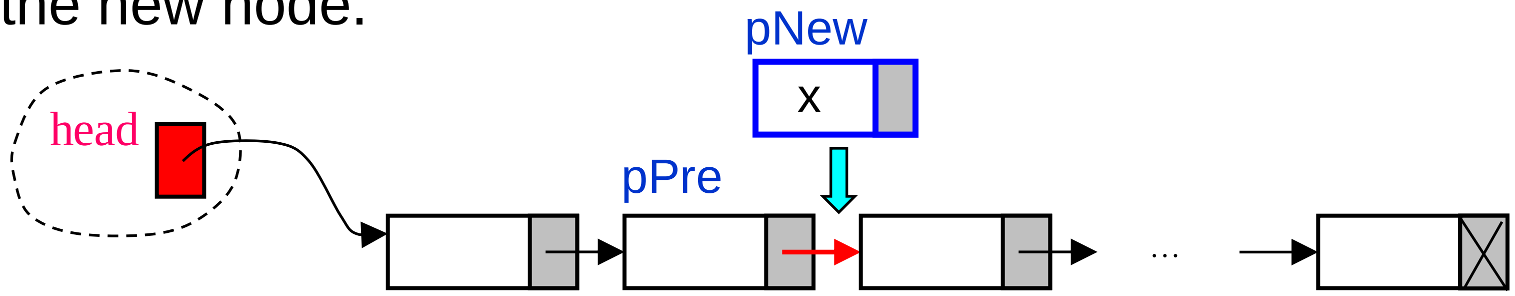
end Create

Insert Node to a Linked List

1. Allocate memory for the new node and set up data.
2. Locate the pointer p in the list, which will point to the new node:
   - If the new node becomes the first element in the List: p is head.



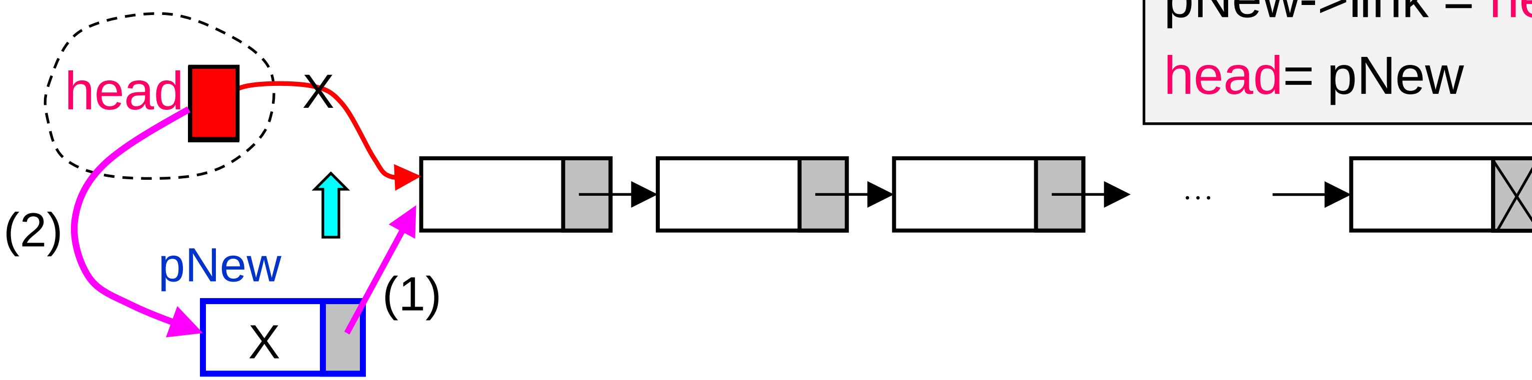
   - Otherwise: p is pPre->link, where pPre points to the predecessor of the new node.

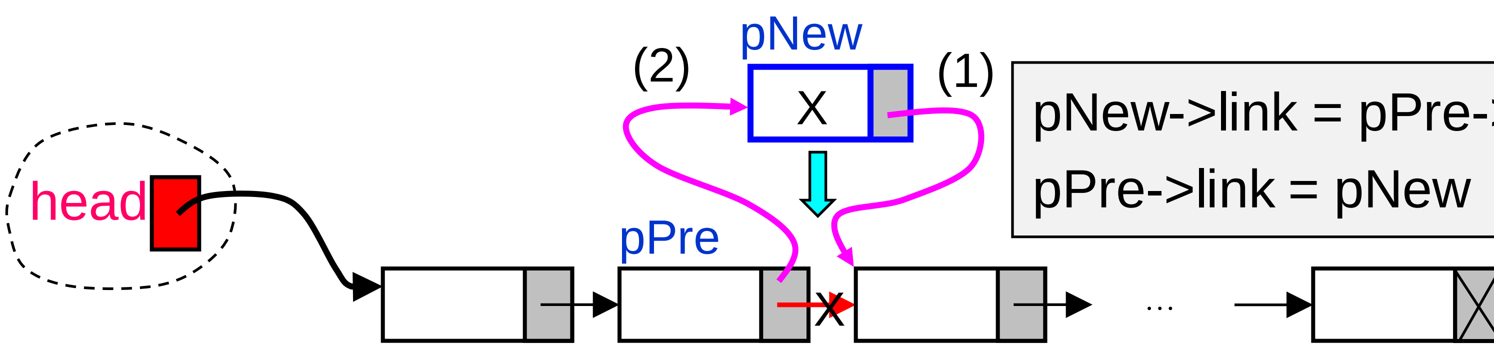


Insert Node to a Linked List (cont.)

3. Update pointers:
   - Point the new node to its successor.
   - Point the pointer p to the new node.

```
pNew->link = head     (1)
head= pNew            (2)
```





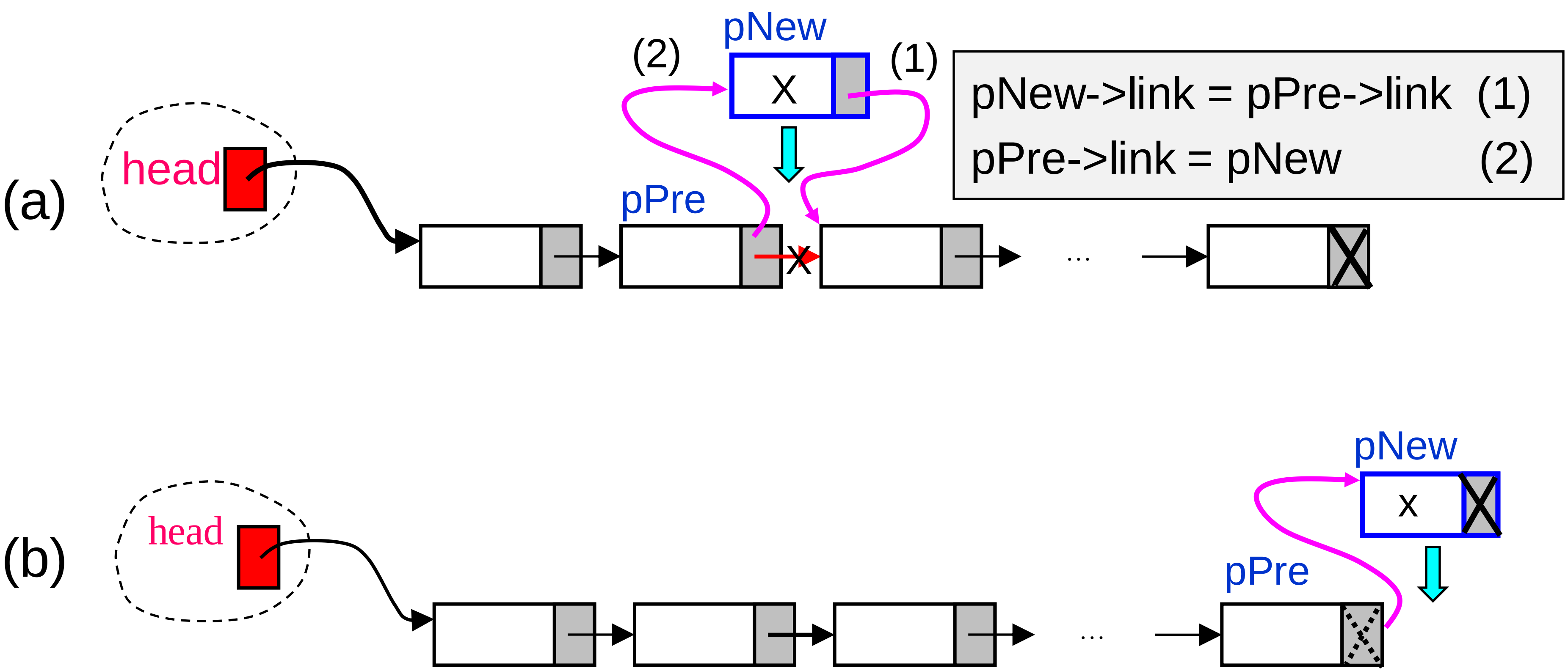

```
pNew->link = pPre->link  (1)
pPre->link = pNew        (2)
```

Insert Node to a Linked List (cont.)

- Insertion is successful when allocation memory for the new node is successful.

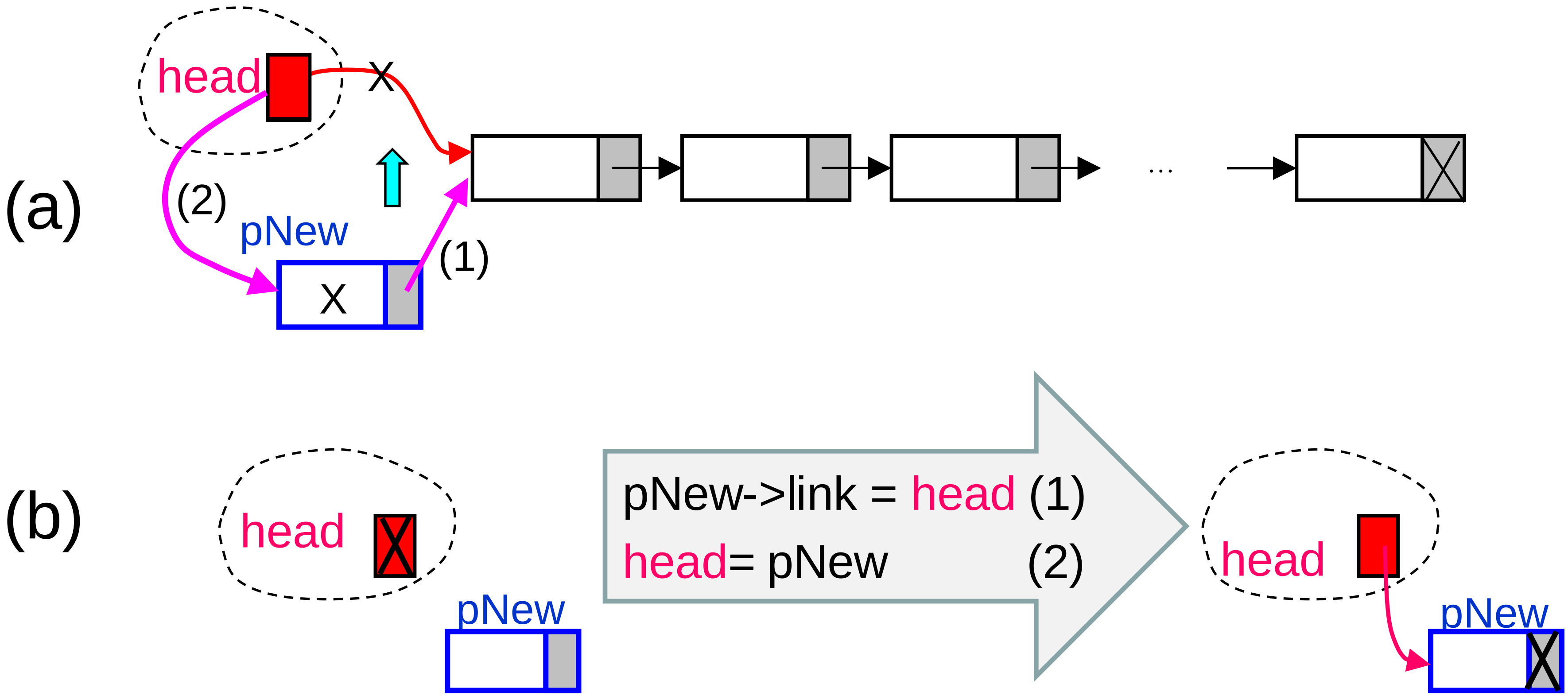
Insert Node to a Linked List (cont.)

- There is no difference between
  - ✓ insertion in the middle (a) and insertion at the end of the list (b)



Insert Node to a Linked List (cont.)

- There is no difference between
  - ✓ insertion at the beginning of the list (a) and insertion to an empty list (b).



Insert Algorithm

<ErrorCode> **Insert** (val DataIn <DataType>)

// ***For ordered list.***

Inserts a new node in a singly linked list.

Pre DataIn contains data to be inserted

Post If list is not full, DataIn has been inserted; otherwise, list remains unchanged.

Return *success* or *overflow*.

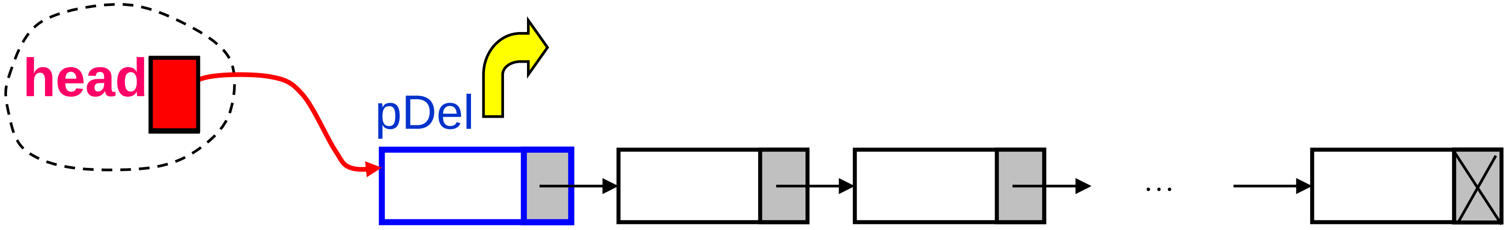
InsertNode Algorithm (cont.)

```
<ErrorCode> Insert (val DataIn <dataType>)
1.  Allocate pNew
2.  if (memory overflow)
      1. return overflow
3.  else
      1. pNew->data = DataIn
      2. Locate pPre // pPre remains NULL if Insertion at the beginning
         or to an empty list
      3. if (pPre = NULL) // Adding at the beginning or to an empty list
           1. pNew->link = head
           2. head = pNew
      4. else // Adding in the middle or at the end of the list
           1. pNew->link = pPre->link
           2. pPre->link = pNew
      5. return success
end Insert
```

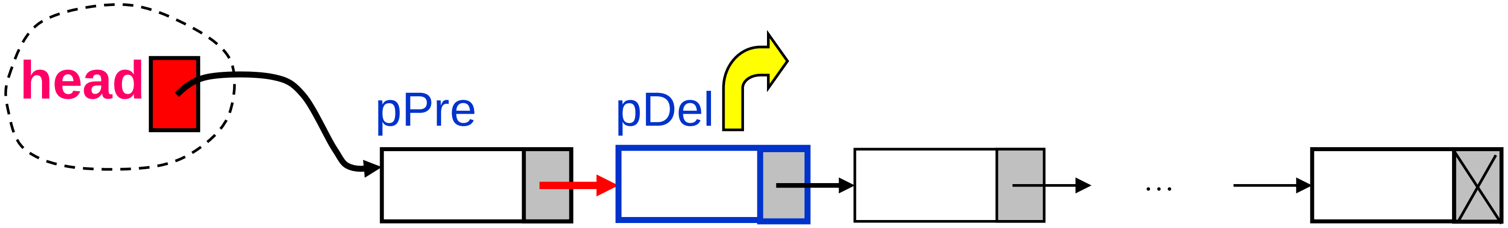
Remove Node from a Linked List

1. Locate the pointer p in the list which points to the node to be deleted (pDel will hold the node to be deleted).

   ➢ If that node is the first element in the List: p is head.



   ➢ Otherwise: p is pPre->link, where pPre points to the predecessor of the node to be deleted.

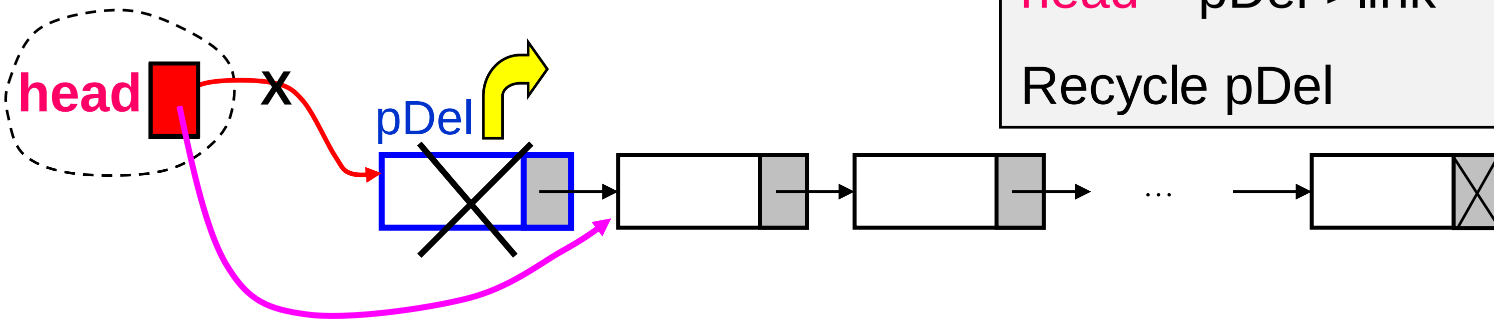


Remove Node from a Linked List (cont.)

2. Update pointers: p points to the successor of the node to be deleted.

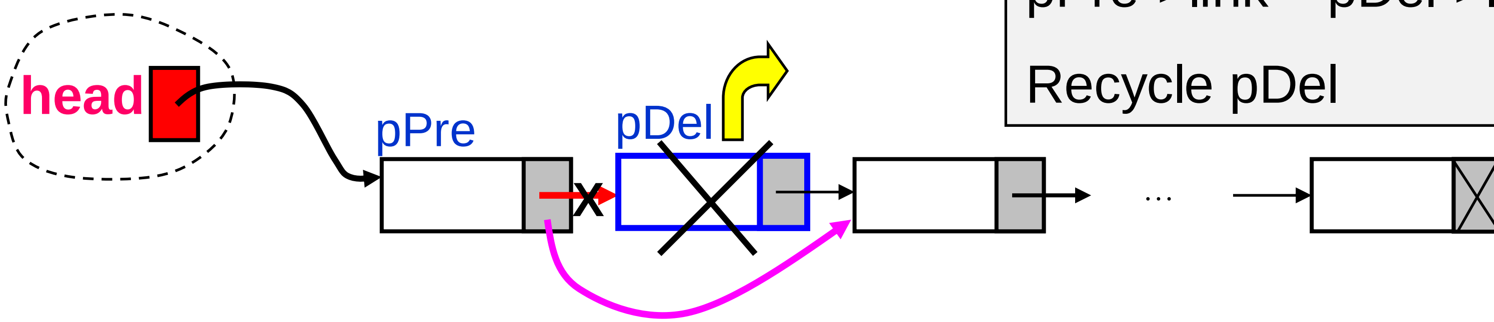
```
head = pDel->link

Recycle pDel
```




```
pPre->link = pDel->link

Recycle pDel
```



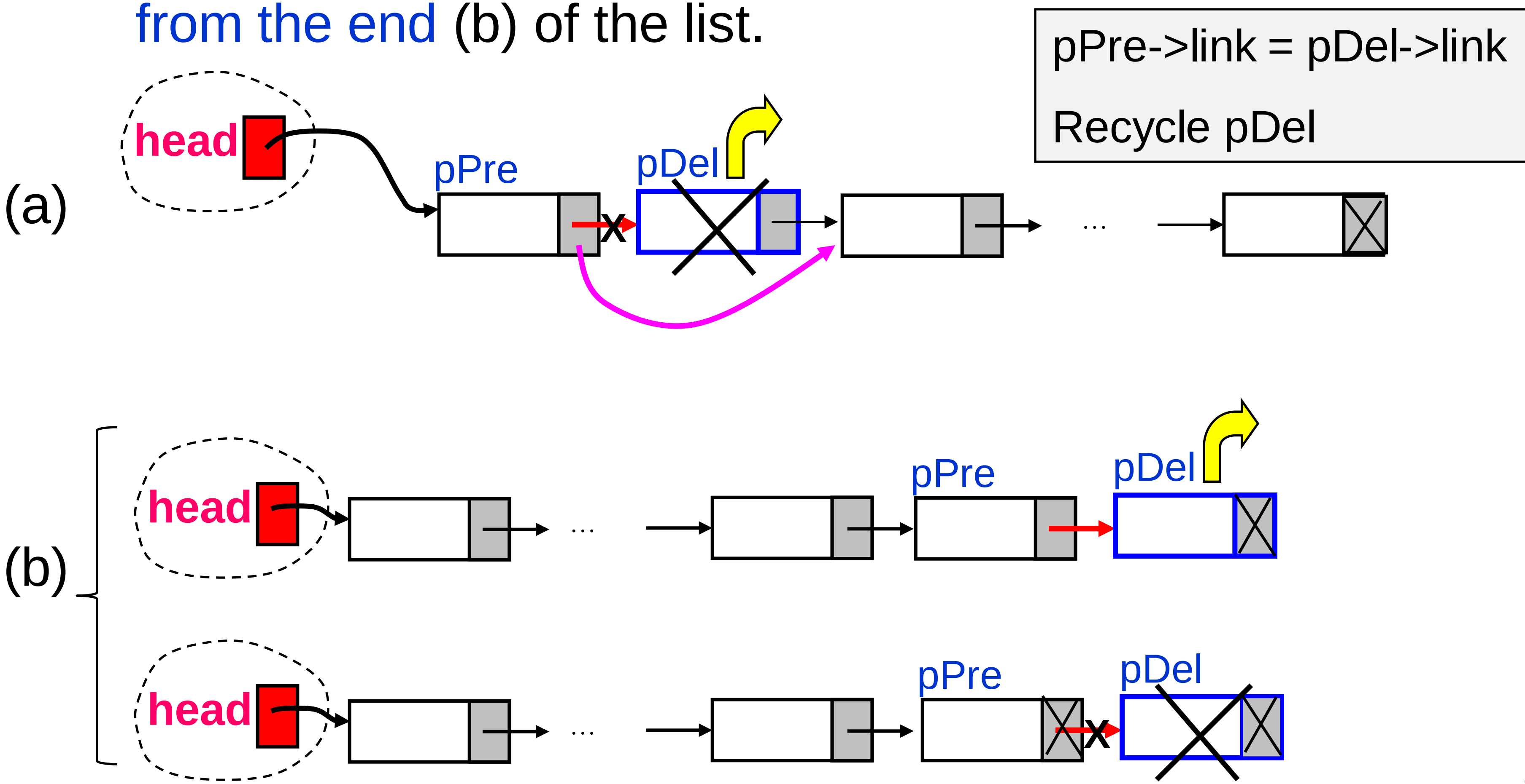
3. Recycle the memory of the deleted node.

Remove Node from a Linked List (cont.)

- Removal is successful when the node to be deleted is found.

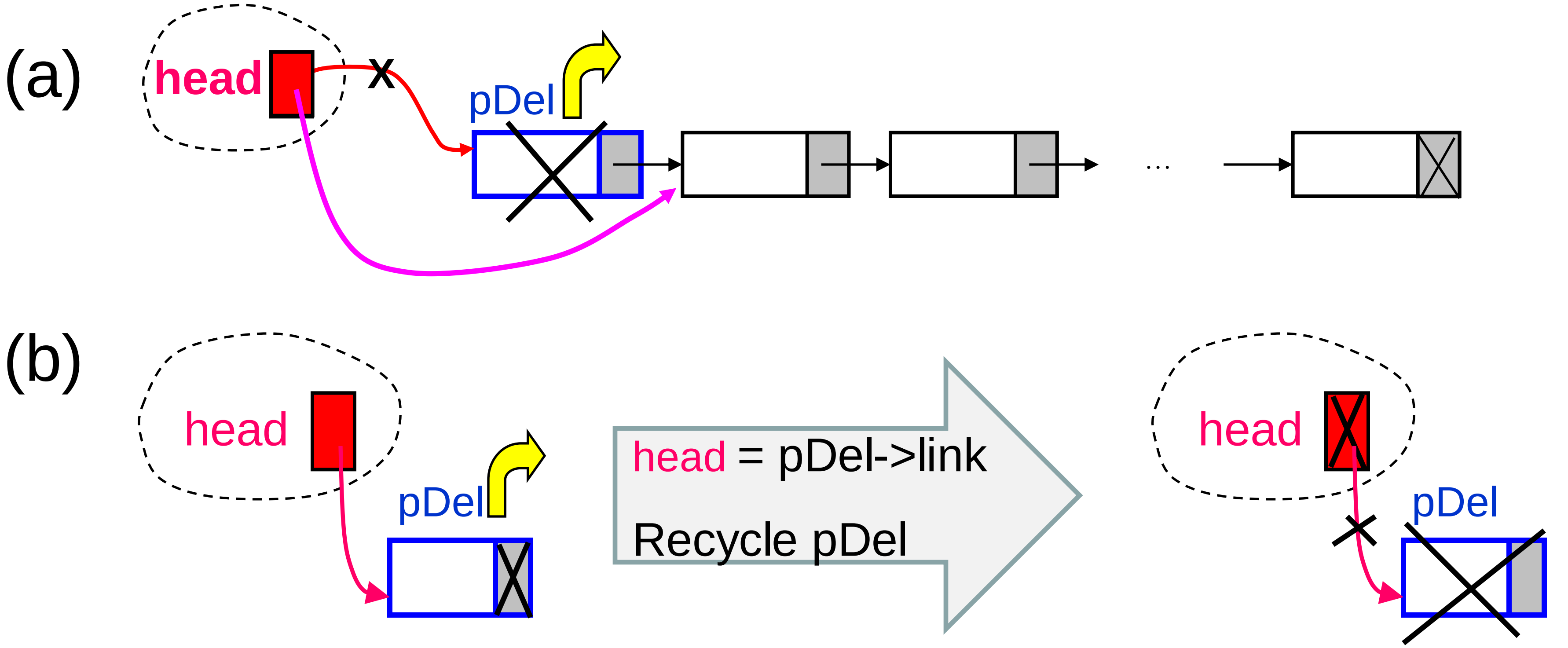
Remove Node from a Linked List (cont.)

- There is no difference between
  - ✓ Removal a node from the middle (a) and removal a node from the end (b) of the list.



Remove Node from a Linked List (cont.)

- There is no difference between
  - ✓ removal the node from the beginning (a) of the list and removal the only-remained node in the list (b).



RemoveNode Algorithm

<ErrorCode> **Remove** (ref DataOut <DataType>)

Removes a node from a singly linked list.

Pre DataOut contains the key need to be removed.

Post If the key is found, DataOut will contain the data corresponding to it, and that node has been removed from the list; otherwise, list remains unchanged.

Return *success* or *failed*.

RemoveNode Algorithm (cont.)

<ErrorCode> **Remove** (ref DataOut <DataType>)

1. Allocate pPre, pDel *// pPre remains NULL if the node to be deleted is at the beginning of the list or is the only node.*
2. **if** (pDel is not found)
    1. return *failed*
3. **else**
    1. DataOut = pDel->data
    2. **if** (pPre = NULL) *// Remove the first node or the only node*
        1. head = pDel->link
    3. **else** *// Remove the node in the middle or at the end of the list*
        1. pPre->link = pDel->link
    4. recycle pDel
    5. return *success*

end Remove

Search Algorithm for Auxiliary Function in Class

- This search algorithm is not a public method of List ADT.
- Sequence Search has to be used for the linked list.
- This studying for the case: List is ordered accordingly to the key field.

Search Algorithm for Auxiliary Function in Class

- *Public method Search of List ADT*:

<ErrorCode> Search (ref DataOut <DataType>)

Can not return a pointer to a node if found.

- *Auxiliary function Search of List ADT:*

<ErrorCode> **Search** (val target <KeyType>,
ref pPre <pointer>,
ref pLoc <pointer>)

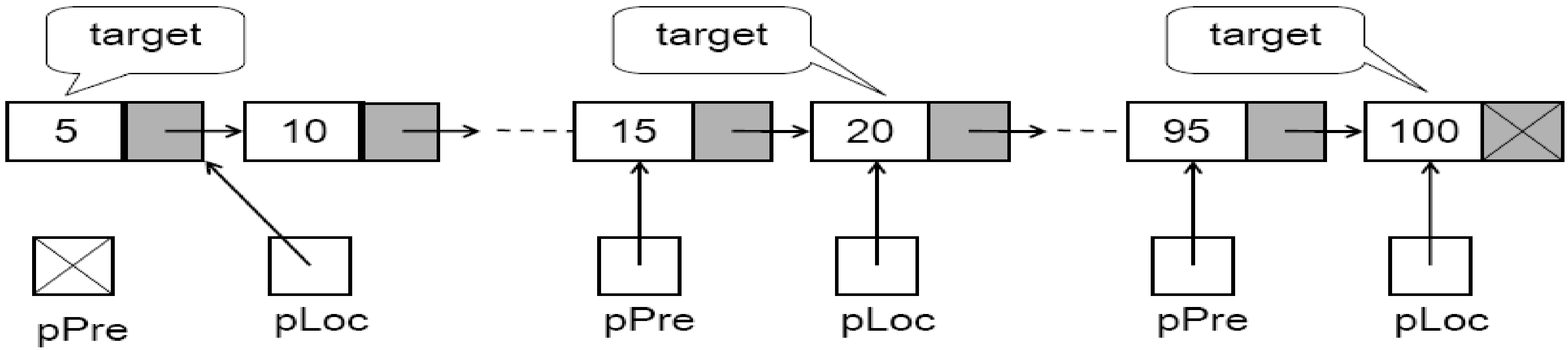
Searches a node and returns a pointer to it if found.

Successful Searches

Located first

Located middle

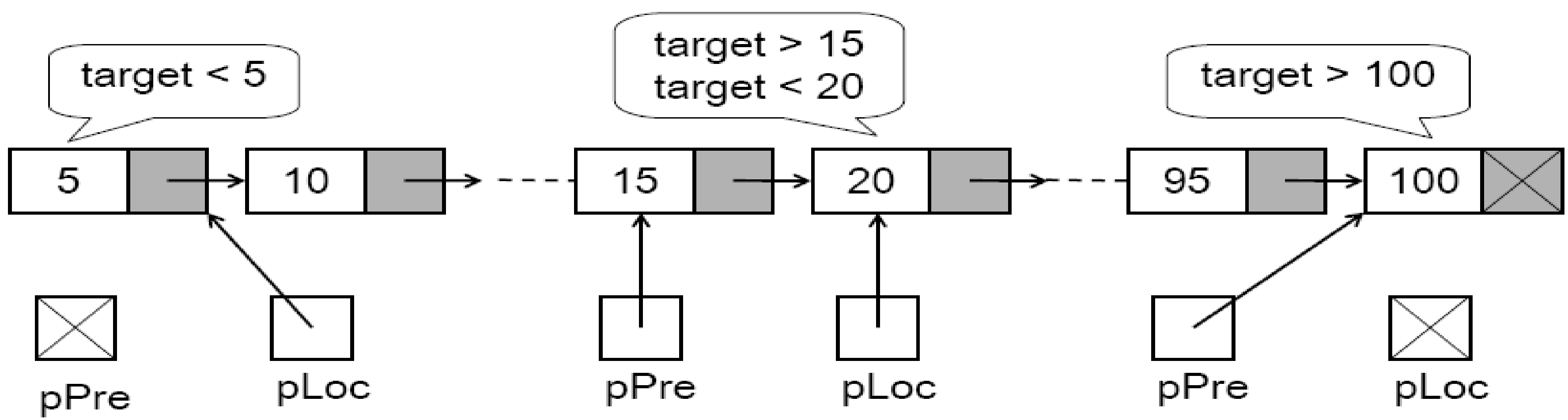
Located last



Unsuccessful Searches

Less than first

Greater than last



Search Algorithm (cont.)

<ErrorCode> **Search** (val target <KeyType>,

ref pPre <pointer>,

ref pLoc <pointer>)

Searches a node in a singly linked list and return a pointer to it if found.

// For Ordered List

Pre target is the key need to be found

Post pLoc points to the first node which is equal or greater than key, or is NULL if target is greater than key of the last node in the list.

pPre points to the largest node smaller than key, or is NULL if target is smaller than key of the first node.

Return *found* or *notFound*

Search Algorithm (cont.)

```
<ErrorCode> Search (val target <KeyType>,
                    ref pPre <pointer>,
                    ref pLoc <pointer>)
// For Ordered List
1.  pPre = NULL
2.  pLoc = head
3.  loop ( (pLoc is not NULL) AND (target > pLoc ->data.key) )
    1. pPre = pLoc
    2. pLoc = pLoc ->link
4.  if (pLoc is NULL)
    1. return notFound
5.  else
    1. if (target = pLoc ->data.key)
        1. return found
    2. else
        1. return notFound
end Search
```

Retrieve Algorithm

- Using Search Algorithm to locate the node
- Retrieving data from that node

Retrieve Algorithm (cont.)

<ErrorCode> **Retrieve** (val target <KeyType>,

ref DataOut <DataType>)

Retrieves data from a singly linked list

Pre target is the key its data need to be retrieved

Post if target is found, DataOut will receive data

Return *success* or *failed*

Uses Auxiliary function Search of class List ADT.

RetrieveNode Algorithm (cont.)

```
<ErrorCode> Retrieve (val target <KeyType>,
                                    ref DataOut <DataType>)

1.   errorCode = Search (target, pPre, pLoc)
2.   if (errorCode = notFound)
     1.  return failed
3.   else
     1.  DataOut = pLoc->data
     2.  return success
end Retrieve
```

Traverse List

Traverse Module controls the loop:

Calling a user-supplied operation to process data

<void> **Traverse** (ref <void> Operation (ref Data <DataType>))

Traverses the list, performing the given operation on each element.

Pre Operation is user-supplied.

Post The action specified by Operation has been performed on every element in the list, beginning at the first element and doing each in turn.

1. pWalker = head
2. **loop** (pWalker is not NULL)
    1. Operation(pWalker->data)
    2. pWalker = pWalker->link

end Traverse

Traverse List (cont.)

User controls the loop:

Calling GetNext Algorithm to get the next element in the list.

<ErrorCode> **GetNext** (val fromWhere <boolean>,
ref DataOut <DataType>)

Traverses the list, each call returns data of an element in the list.

Pre fromWhere is 0 to start at the first element, otherwise, the next element of the current needs to be retrieved.

Post According to fromWhere, DataOut contains data of the first element or the next element of the current (if exists) in the list. That element becomes the current.

Return *success* or *failed*.

Traverse List (cont.)

User controls the loop:

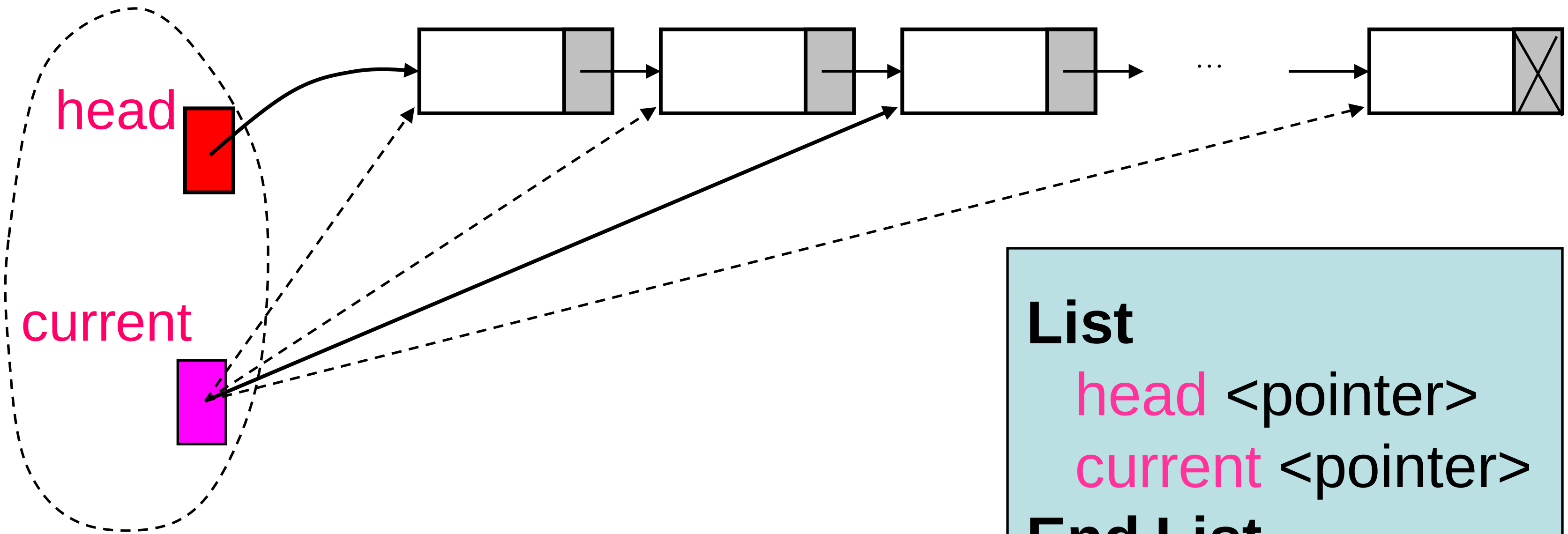
Calling GetNext Algorithm to get the next element in the list.

<void> **Operation** (ref Data <DataType>)// the function needs to apply to each element in the list.

User controls the loop:

1. errorCode = **GetNext**(0, DataOut)
2. **loop** (errorCode = *success*)
   1. **Operation**(DataOut)
   2. errorCode = **GetNext**(1, DataOut)

GetNext Algorithm



List
  head <pointer>
  current <pointer>
End List

- Singly Linked List has additional attribute current pointing to the current element (the last element has just been processed).
- All additional attributes must be updated when necessary!.

GetNext Algorithm (cont.)

```
<ErrorCode> GetNext (val fromWhere <boolean>,
                     ref DataOut <DataType>)
1.  if (fromWhere is 0)
    1.  if (head is NULL)
        1.  return failed
    2.  else
        1.  current = head
        2.  DataOut = current->data
        3.  return success
2.  else
    1.  if (current->link is NULL)
        1.  return failed
    2.  else
        1.  current = current->link
        2.  DataOut = current->data
        3.  return success
end GetNext
```

Clear List Algorithm

<void> **Clear**()

Removes all elements from a list.

Pre none.

Post The list is empty.

1. loop (head is not NULL)
    1. pDel = head
    2. head = head->link
    3. recycle pDel
2. return

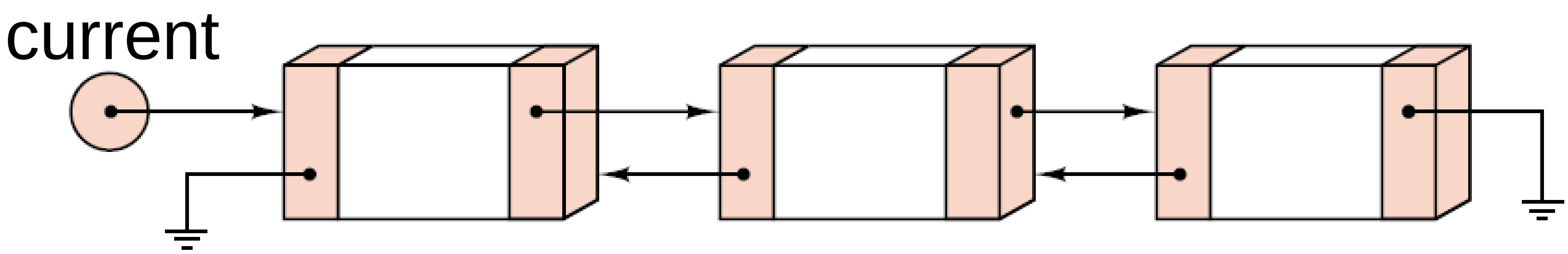
end Clear

Comparison of Implementations of List

- Contiguous storage is generally preferable
  - When the entries are individually very small;
  - When the size of the list is known when the program is written;
  - When few insertions or deletions need to be made except at the end of the list; and
  - When random access is important.

- Linked storage proves superior
  - When the entries are large;
  - When the size of the list is not known in advance; and
  - When flexibility is needed in inserting, deleting, and rearranging the entries.

Doubly Linked List



Doubly Linked List allows going forward and backward.

Node

  data <DataType>

  next <pointer>

  previous <pointer>

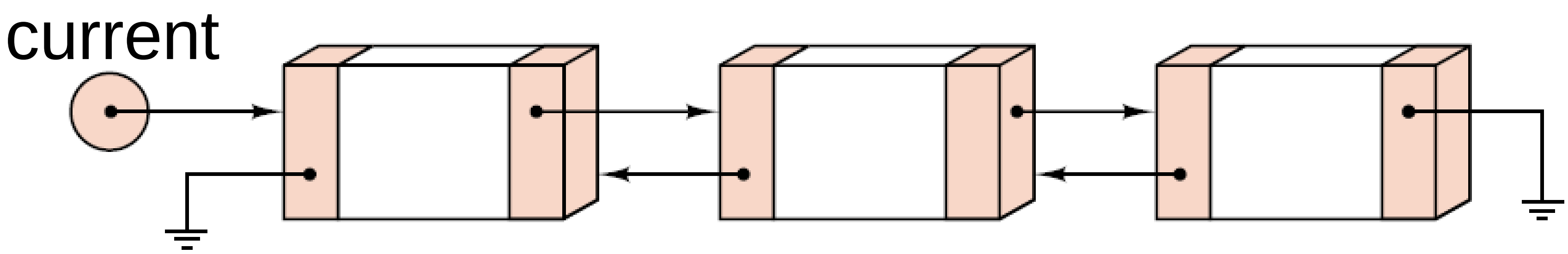
End Node

List

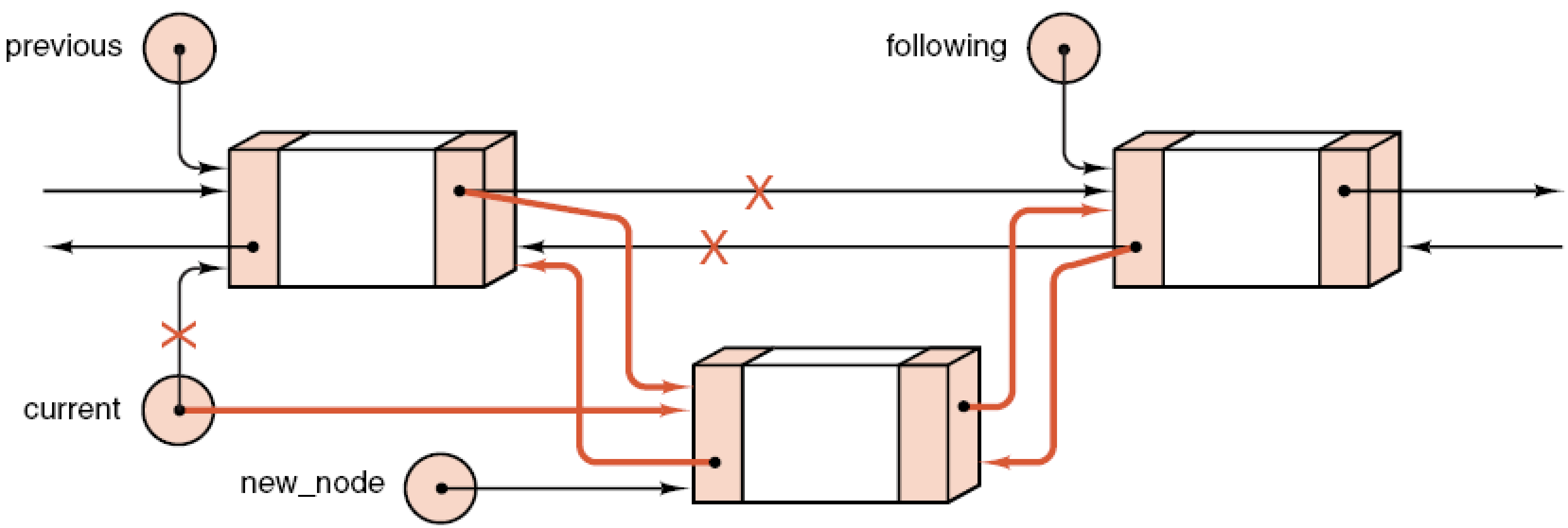
  current <pointer>

End List

Doubly Linked List

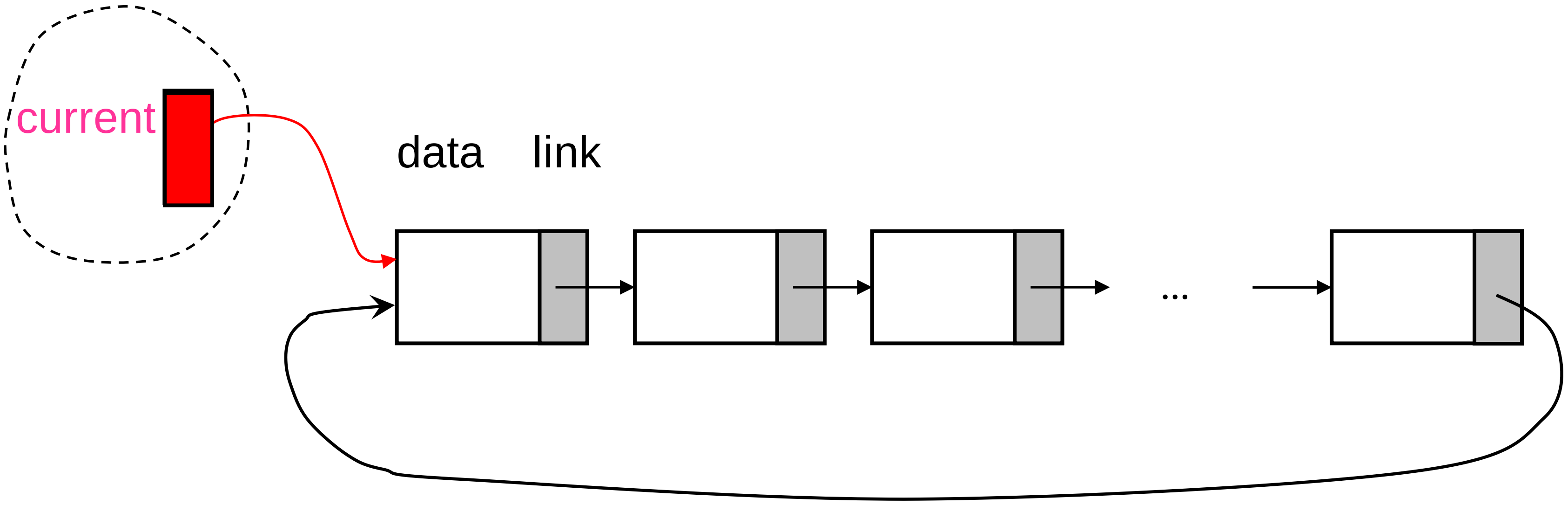


Doubly Linked List allows going forward and backward.



Insert an element in Doubly Linked List

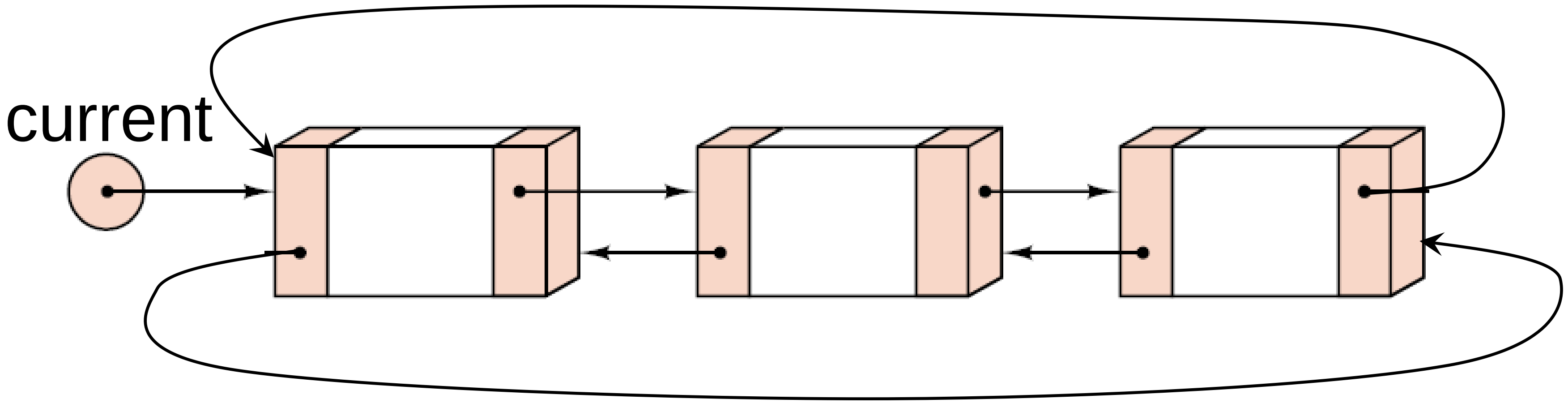
Circularly Linked List



Node
data <DataType>
link <pointer>
End Node

List
current <pointer>
End List

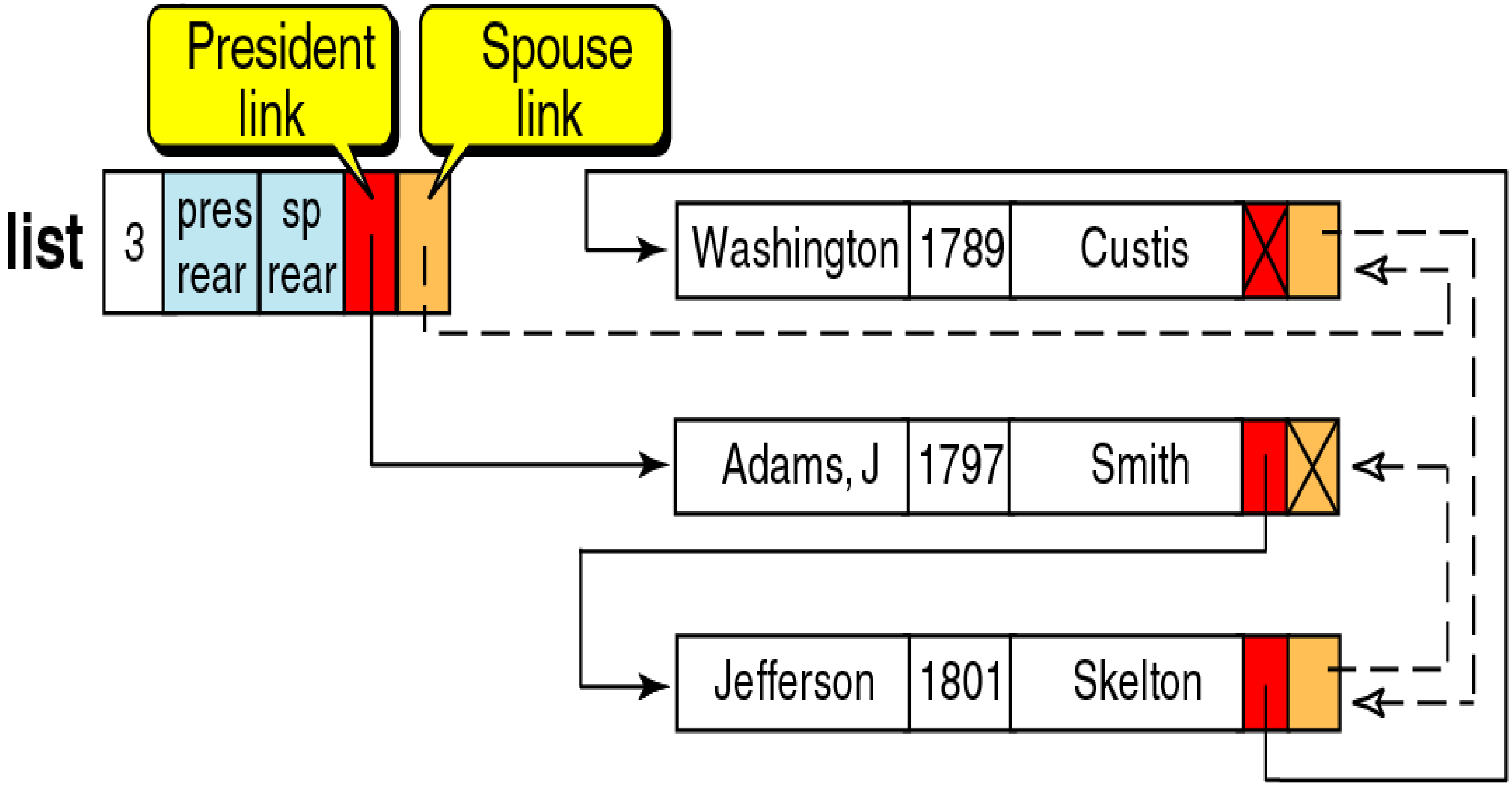
Double Circularly Linked List



Node
    data <DataType>
    next <pointer>
    previous <pointer>
End Node

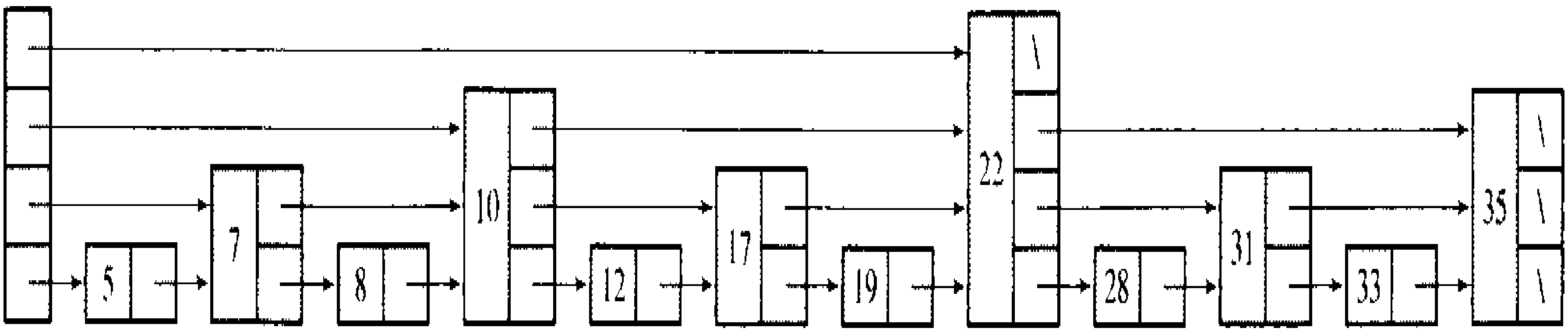
List
    current <pointer>
End List

Multilinked List

- 

Multilinked List allows traversing in different order.

Skip List



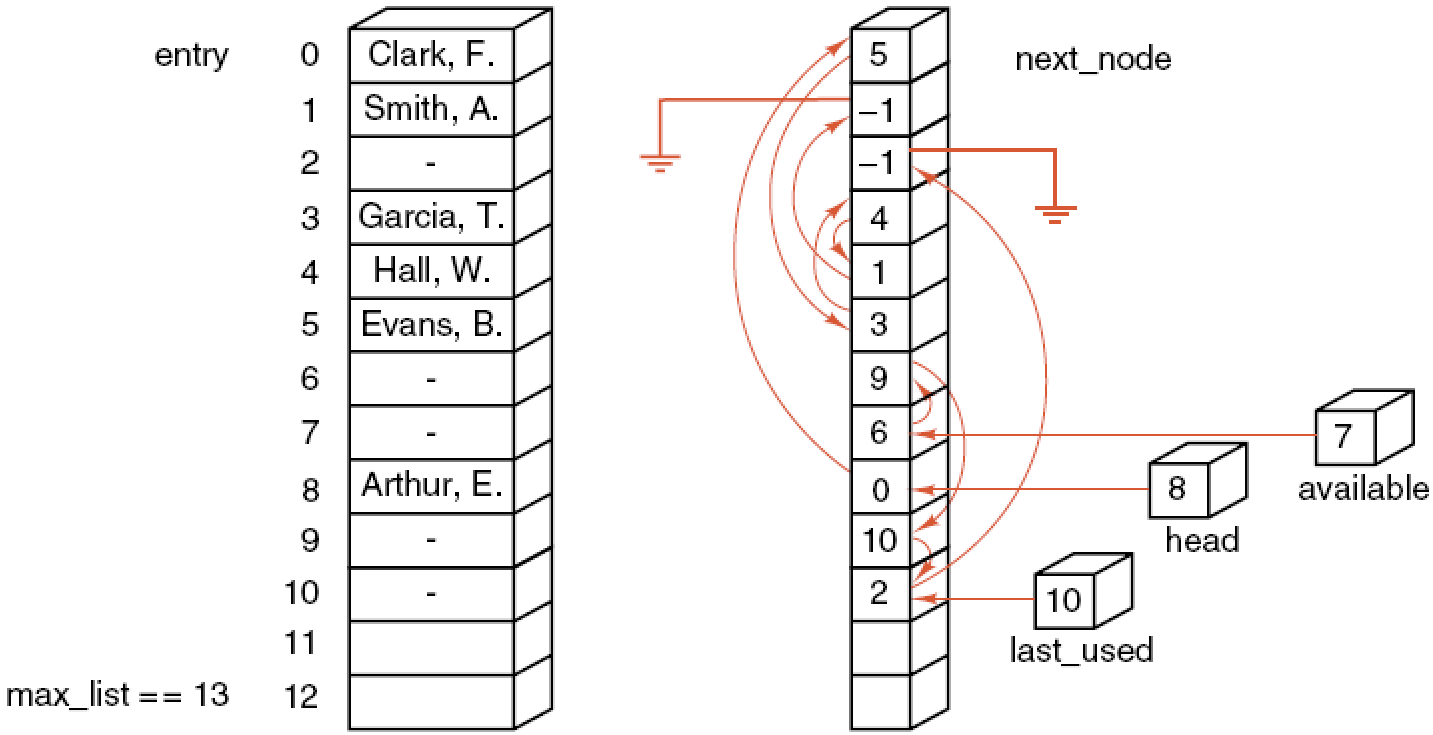
Skip List improves sequential searching.

Choice of variants of Linked List

To choose among linked Implementations of List, consider:

- Which of the operations will actually be performed on the list and which of these are the most important?
- Is there locality of reference? That is, if one entry is accessed, is it likely that it will next be accessed again?
- Are the entries processed in sequential order? If so, then it may be worthwhile to maintain the last-used position as part of list.
- Is it necessary to move both directions through the list? If so, then doubly linked lists may prove advantageous.

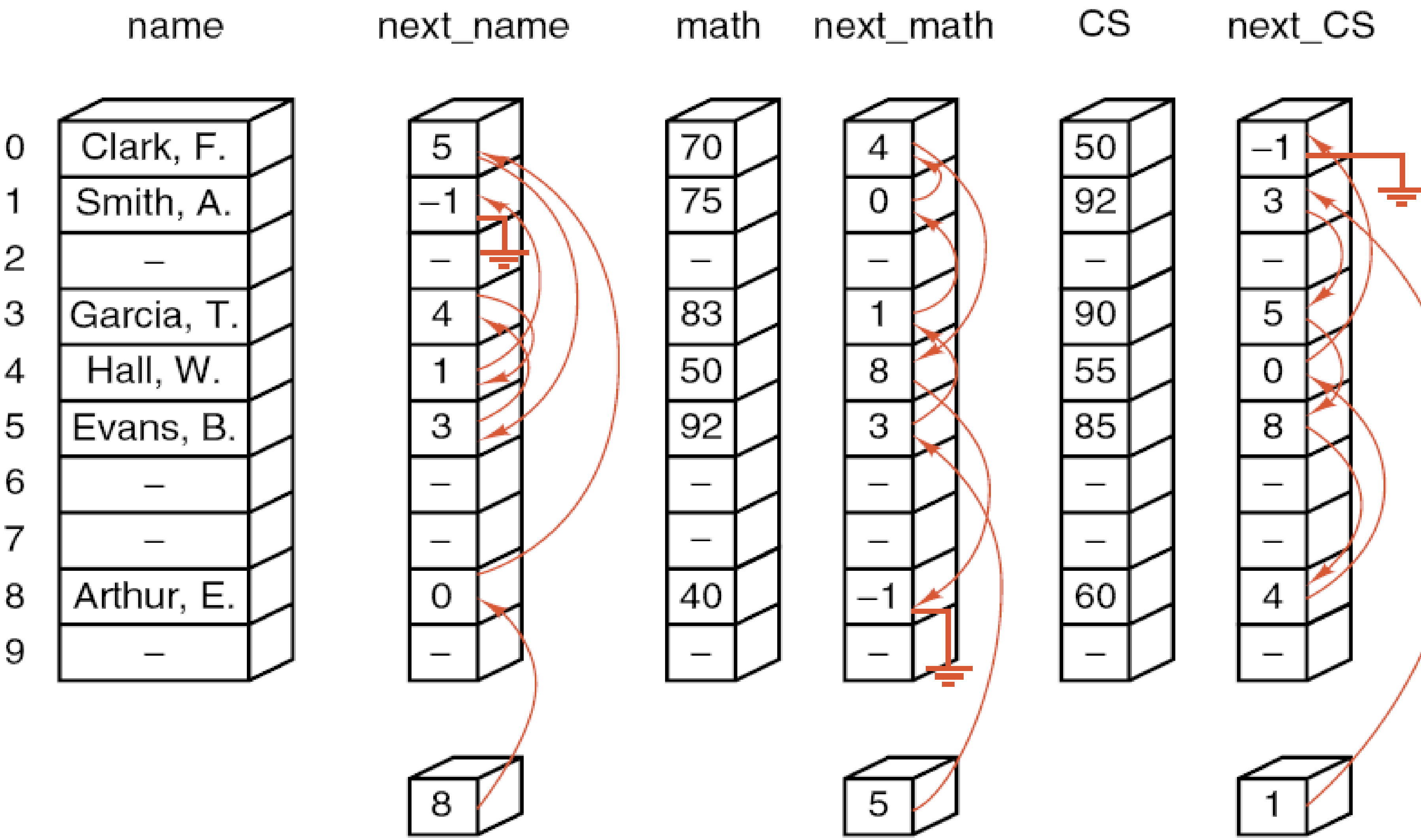
Linked List In Array



There are two linked lists in array:

- One (head) manages used entries.
- Another (available) manages empty entries (have been used or not yet)

Multilinked List In Array



Application: Sparse Matrices

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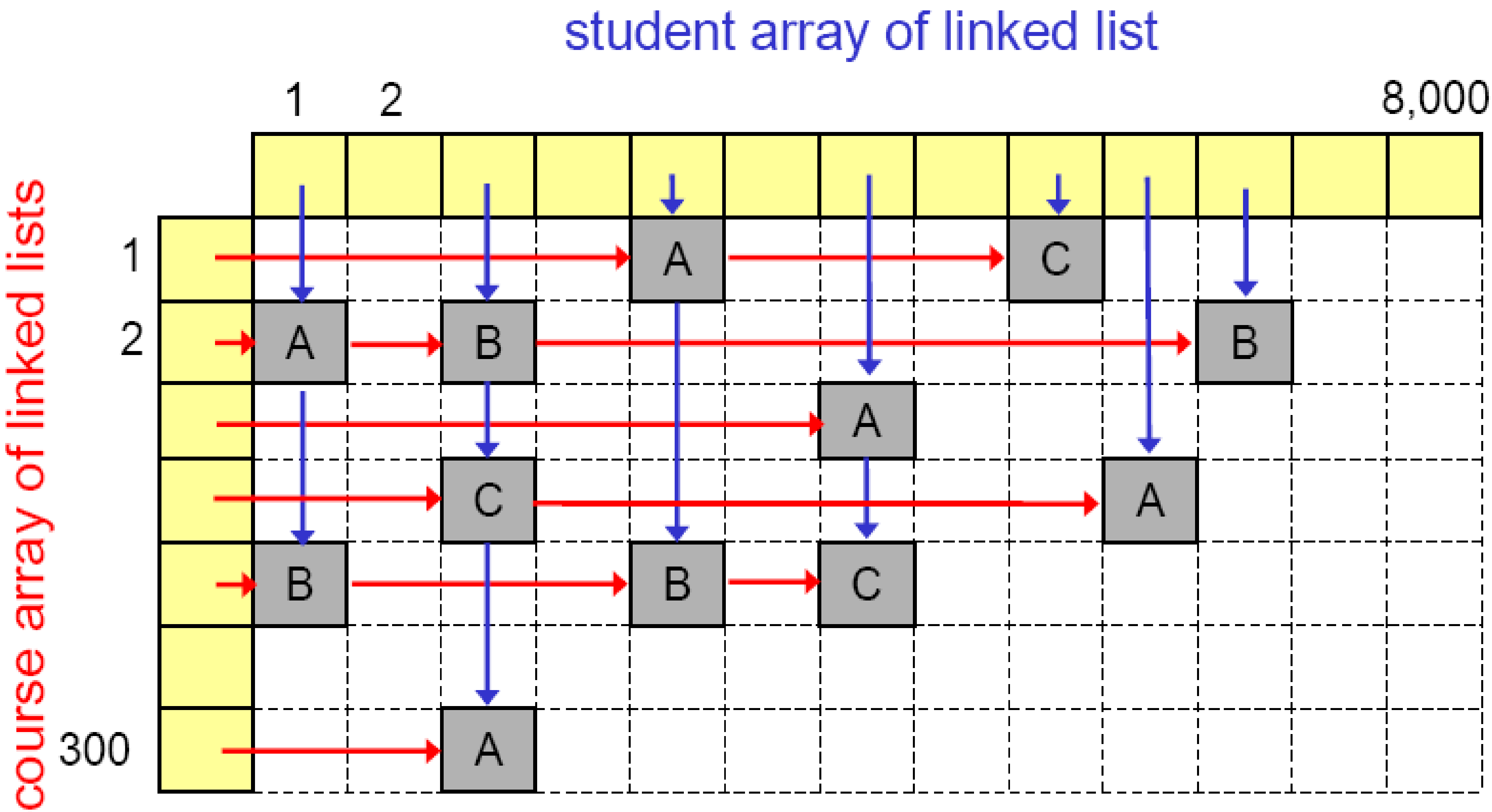
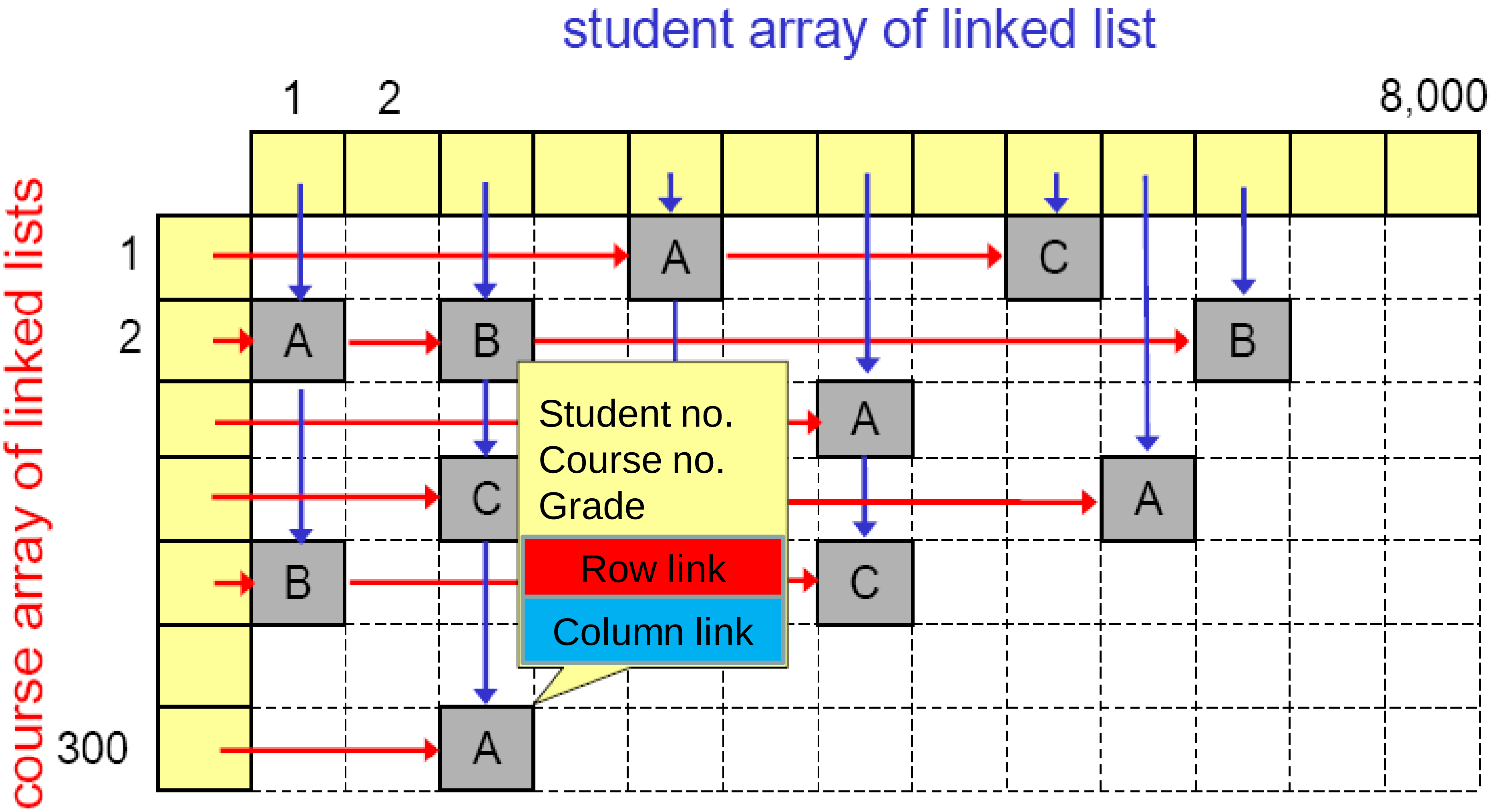
							A						
			C							A			
	B				B		C						
300			A										

Sparse Matrices



Two one-dimensional arrays of Linked List are used

Sparse Matrices



Sparce Matrices

- Why two arrays of linked lists?
- How about two linked lists of linked lists?
- How about 3-D sparse matrices?

Variants of List are used for Graph and Hash Table, we will see later.