CSC 212: Data Structures and Abstractions

Quick Sort

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Fall 2020

Quick Notes

- Assignment 4 is out (recursion / stacks)
  - 1-2 students
  - time-consuming, start early
- Problem Set
  - grading 60% completed
  - 2 submissions can't be graded properly
- Midterm Exam (timed)
  - past exams available
  - perhaps a Saturday / Monday review session?

Quick Sort

- **Divide** the array into **two** partitions (subarrays)
  - need to pick a *pivot* and rearrange the elements into two partitions

- Conquer **Recursively** each half
  - call Quick Sort on each partition (i.e. solve 2 smaller problems)

- **Combine** Solutions
  - there is no need to combine the solutions

Quick Sort: pseudocode

```
if (hi <= lo) return;

int p = partition(A, lo, hi);

quicksort(A, lo, p-1);

quicksort(A, p+1, hi);
```

Partition

10	12	3	7	4	13	11	9

10

← pick a **pivot** (it can be the first element)

4	9	3	7	10	13	11	12

<= **pivot**

>= **pivot**

Partition: algorithm

lo									hi
10	1	31	20	3	4	22	15	2	35
	i →							← j	

```
while (true)
    scan i left-to-right (while a[i] < a[lo])
    scan j right-to-left (while a[j] > a[lo])
    if i and j crossed then
        break
    swap a[i] with a[j]
swap a[lo] with a[j]
```

Partition: do it yourself

12	1	31	20	10	11	8	2	23	1

```
while (true)
    scan i left-to-right (while a[i] < a[lo])
    scan j right-to-left (while a[j] > a[lo])
    if i and j crossed then
        break
    swap a[i] with a[j]
swap a[lo] with a[j]
```

Partition: implementation

```
int partition(int *A, int lo, int hi) {
    int i = lo;
    int j = hi + 1;
    while (1) {
        // while A[i] < pivot, increase i
        while (A[++i] < A[lo]) if (i == hi) break;
        // while A[i] > pivot, decrease j
        while (A[lo] < A[--j]) if (j == lo) break;
        // if i and j cross exit the loop
        if (i >= j) break;
        // swap A[i] and A[j]
        std::swap(A[i], A[j]);
    }
    // swap the pivot with A[j]
    std::swap(A[lo], A[j]);
    // return pivot's position
    return j;
}
```

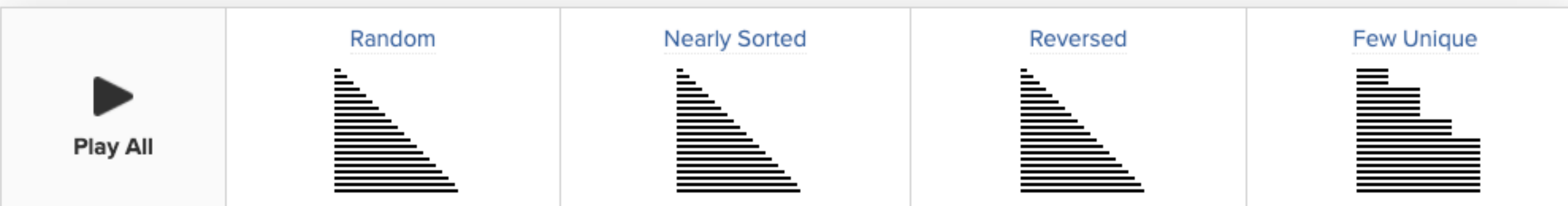
Quick Sort: implementation

```
void r_quicksort(int *A, int lo, int hi) {
    if (hi <= lo) return;
    int p = partition(A, lo, hi);
    r_quicksort(A, lo, p-1);
    r_quicksort(A, p+1, hi);
}
```

```
void quicksort(int *A, int n, int m) {
    // shuffle the array
    std::random_shuffle(A, A+n);
    // call recursive quicksort
    r_quicksort(A, 0, n-1);
}
```

Animation

https://www.toptal.com/developers/sorting-algorithms/quick-sort



Analysis of Quick Sort

- **Best-case**
  - pivot partitions array evenly (almost never happens)

$$
\begin{aligned}
T(n) &= 2T(n/2) + \Theta(n) \\
&= \ldots \\
&= \Theta(n \log n)
\end{aligned}
$$

Analysis of Quick Sort

- **Worst-case**
  - input sorted, reverse order, equal elements

$$
\begin{aligned}
T(n) &= T(n-1) + T(0) + \Theta(n) \\
&= T(n-1) + \Theta(1) + \Theta(n) \\
&= T(n-1) + \Theta(n) \\
&= \dots \\
&= \Theta(n^2)
\end{aligned}
$$

`can shuffle the array (to avoid the worst-case)`

Analysis of Quick Sort

- **Average-case**
  - analysis is more complex (assumes distinct elements)

> - Consider a 9-to-1 proportional split
> - Even a 99-to-1 split yields same running time
> - Faster than merge sort in practice (less data movement)

$$
\begin{aligned}
T(n) &= T(9n/10) + T(n/10) + \Theta(n) \\
&= \ldots \\
&= \Theta(n \log n)
\end{aligned}
$$

Comments on Quick Sort

- Properties
  - it is **in-place** but **not stable**
  - benefits substantially from **code tuning**
- Improvements
  - use insertion sort for small arrays
    - avoid overhead on small instances (~10 elements)
  - median of 3 elements
    - estimate true median by inspecting 3 random elements
  - three-way partitioning
    - create three partitions < pivot, == pivot, > pivot

Sorting Algorithms

	Best-Case	Average-Case	Worst-Case	Stable?	In-place?
Selection Sort	$\theta(n^2)$	$\theta(n^2)$	$\theta(n^2)$	No	Yes
Insertion Sort	$\theta(n)$	$\theta(n^2)$	$\theta(n^2)$	Yes	Yes
Merge Sort	$\theta(n\log n)$	$\theta(n\log n)$	$\theta(n\log n)$	Yes	No
Quick Sort	$\theta(n\log n)$	$\theta(n\log n)$	$\theta(n^2)$	No	Yes

Empirical Analysis

Running time estimates:

- Home PC executes 10^8 compares/second.
- Supercomputer executes 10^{12} compares/second.

	insertion sort (N^2)			mergesort (N log N)			quicksort (N log N)		
computer	thousand	million	billion	thousand	million	billion	thousand	million	billion
home	instant	2.8 hours	317 years	instant	1 second	18 min	instant	0.6 sec	12 min
super	instant	1 second	1 week	instant	instant	instant	instant	instant	instant

Lesson 1. Good algorithms are better than supercomputers.

Lesson 2. Great algorithms are better than good ones.

http://www.cs.princeton.edu/courses/archive/spring18/cos226/lectures/23Quicksort.pdf