

# **Course 2: Rendering and Transformations**

# Today

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- *Game loop*
- *2D Transformations*
- *Rendering*

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# **The game loop**

**Can you imagine a game without?**

# A game is a simulator

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**1. *AI and user input***

**2. *Environment reaction***

**3. *Equations of Motion***

- sum forces & torques, solve for accelerations:  $\vec{F} = m\vec{a}$

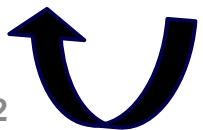
**4. *Numerical integration***

- update positions, velocities

**5. *Collision detection***

**6. *Collision resolution***

*We will have a separate  
lecture on physics  
simulation!*



# A game loop

```
// Set all states to default
world.restart();
auto t = Clock::now();
// Variable timestep loop
while (!world.is_over())
{
    // Processes system messages, if this wasn't present the window would become unresponsive
    glfwPollEvents();

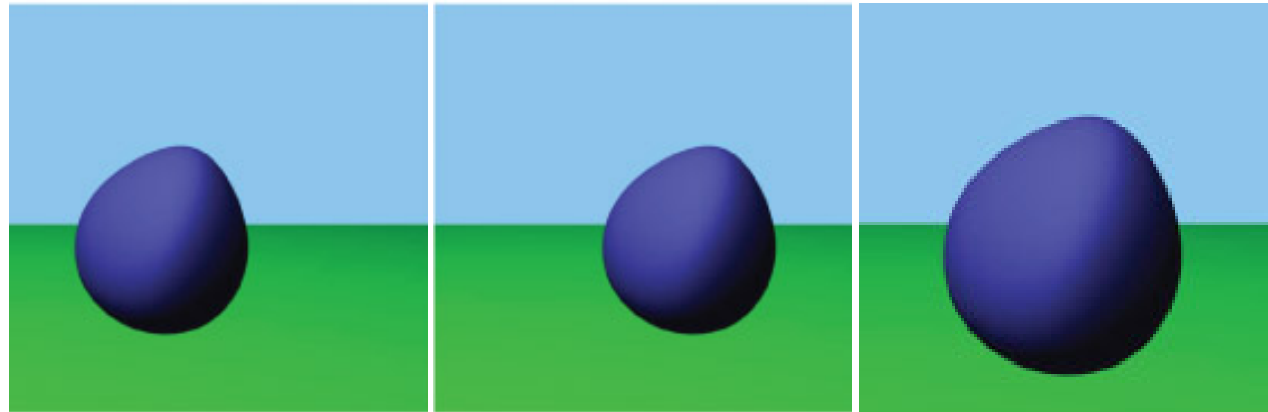
    // Calculating elapsed times in milliseconds from the previous iteration
    auto now = Clock::now();
    float elapsed_ms = static_cast<float>((std::chrono::duration_cast<std::chrono::microseconds>(now - t)).count()) / 1000.f;
    t = now;

    DebugSystem::clearDebugComponents();
    ai.step(elapsed_ms, window_size_in_game_units);
    world.step(elapsed_ms, window_size_in_game_units);
    physics.step(elapsed_ms, window_size_in_game_units);
    world.handle_collisions();

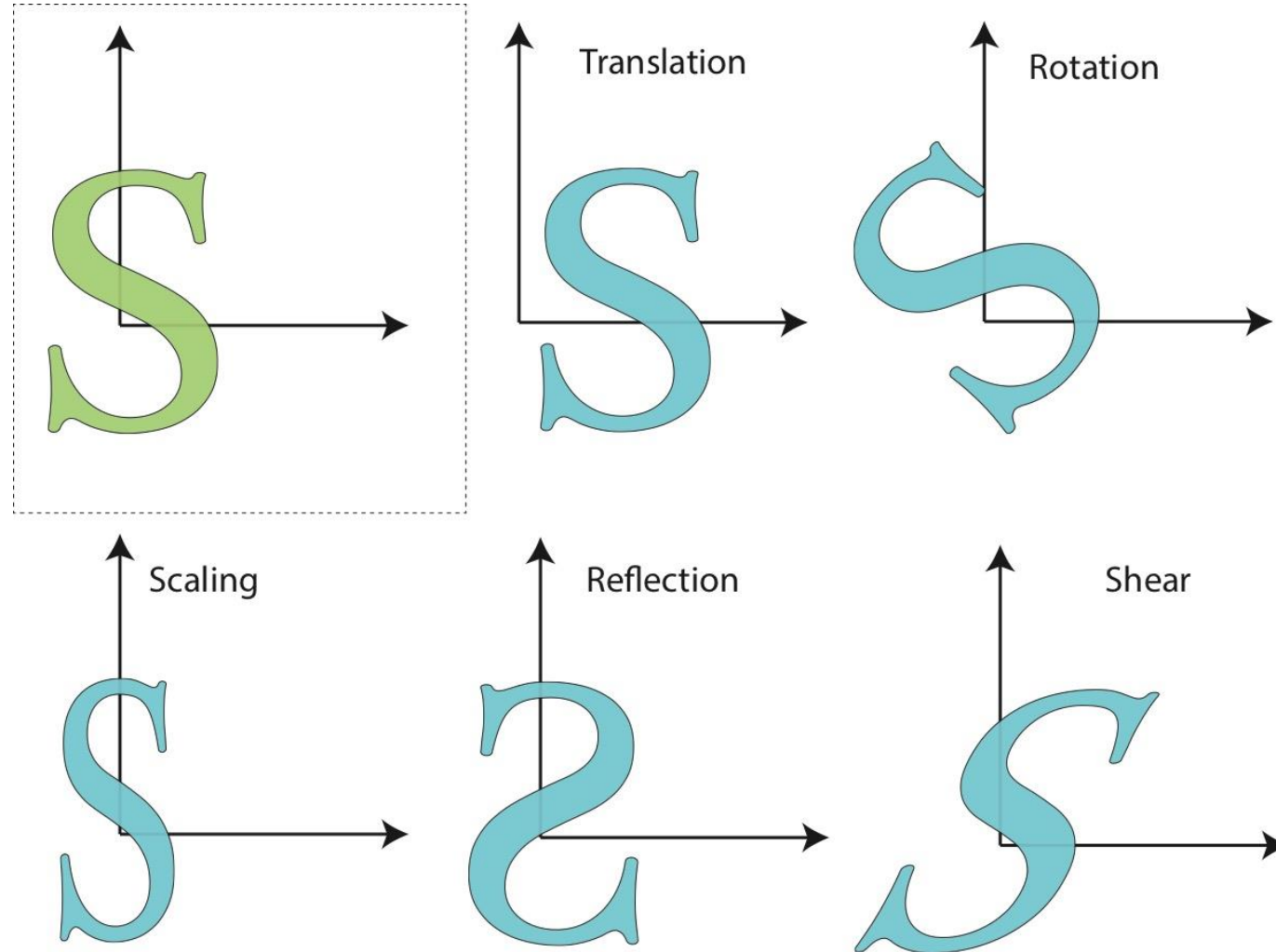
    renderer.draw(window_size_in_game_units);
}

return EXIT_SUCCESS;
```

# Transformations

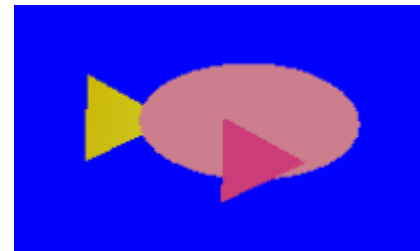
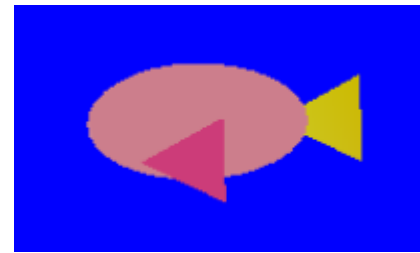
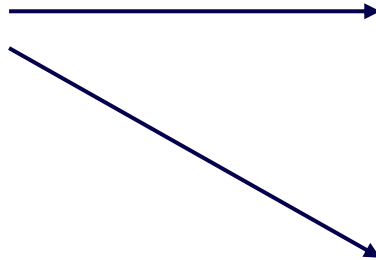
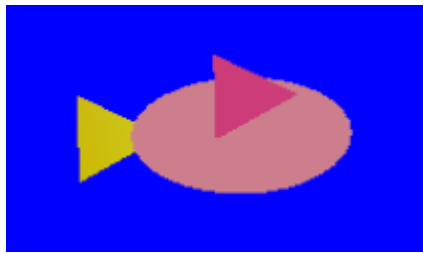


# Modeling Transformations



# How to turn the fish?

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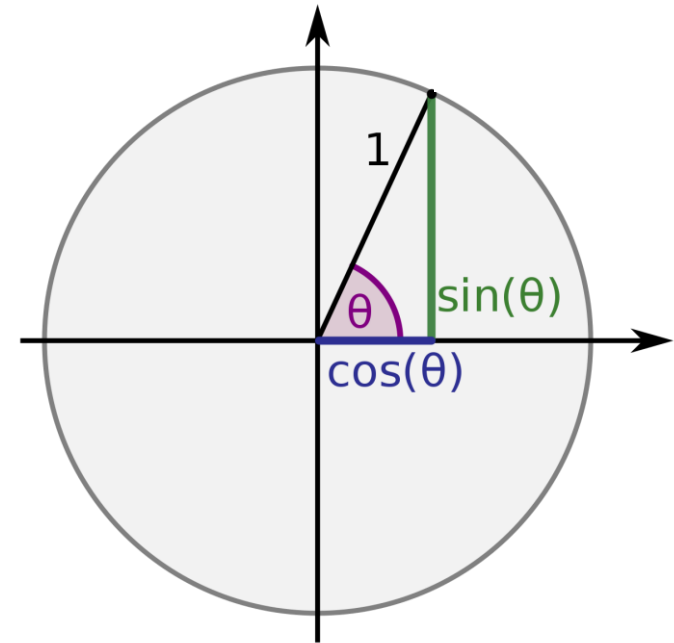
# Linear transformations

- Rotations, scaling, shearing
- Can be expressed as 2x2 matrix (for 2D points)
- E.g.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- or a rotation

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$



Rotation angle  $\theta$ ,  $\cos$ , and  $\sin$

[https://en.wikipedia.org/wiki/Trigonometric\\_functions](https://en.wikipedia.org/wiki/Trigonometric_functions)

# Affine transformations

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- Linear transformations + translations
- Can be expressed as 2x2 matrix + 2 vector
- E.g. scale+ translation:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

# Modeling Transformation

## *Adding a third coordinate*

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 0 & t_x \\ 0 & 2 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Affine transformations are now linear

- one 3x3 matrix can express: 2D rotation, scale, shear, and translation

# Combination of Transformations?

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- ***How can we combine***
  - translation
  - rotation
  - scaling
- ***... into one matrix?***

## Self study: Homogeneous coordinates

- Homogeneous coordinates are defined as vectors, with equivalence

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x/z \\ y/z \\ 1 \end{pmatrix} = \begin{pmatrix} x\lambda \\ y\lambda \\ z\lambda \end{pmatrix}$$

- Can also represent projective equations
- homogeneous matrix becomes 4x4

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & t_x \\ 0 & 2 & 0 & t_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Rendering basics

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# What is rendering?

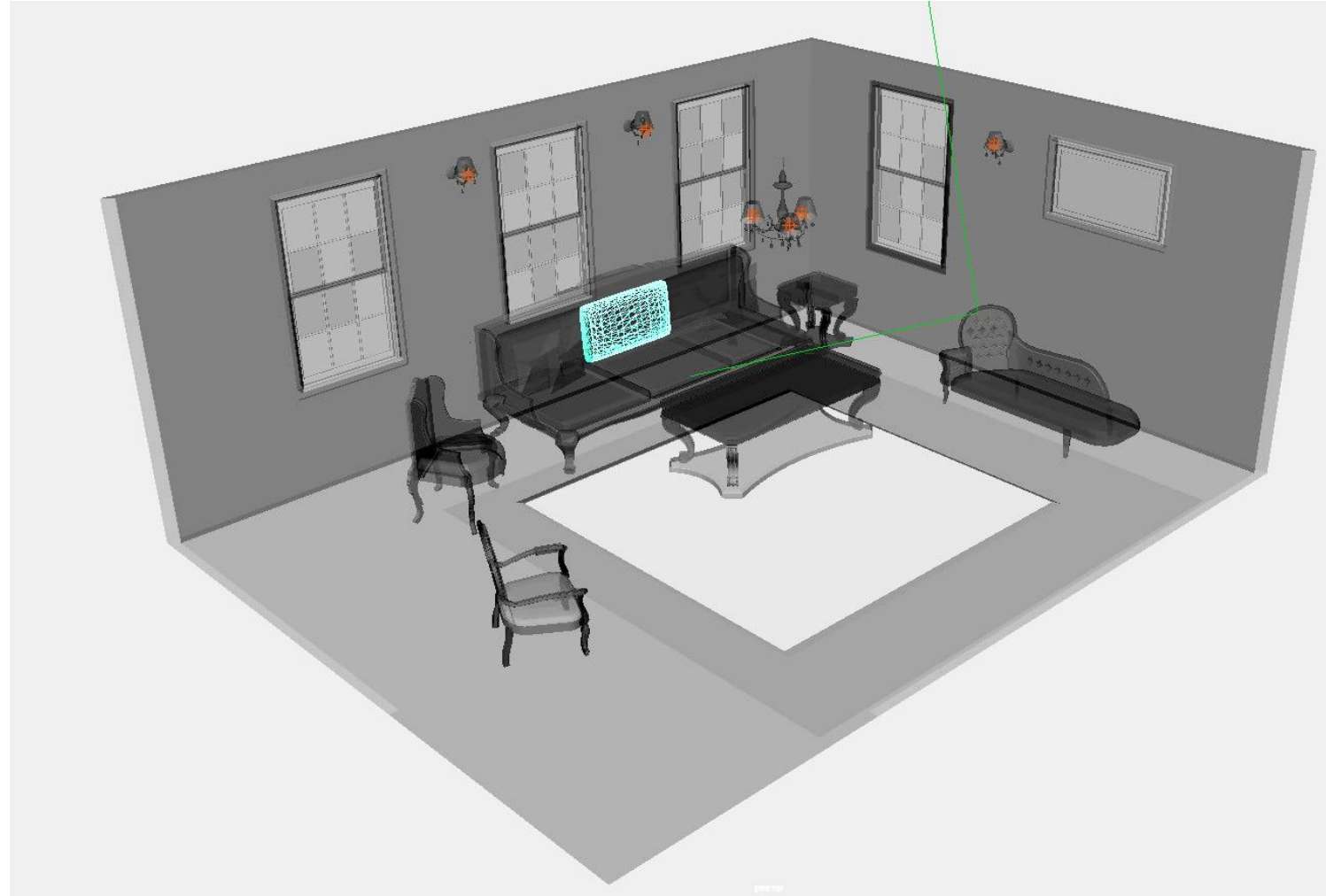
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*Generating an image from a (3D) scene*

*Let's think how!*

# Scene

- A coordinate frame
- Objects
- Their materials
- (Lights)
- (Camera)





# Object

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***Most common:***

- ***surface representation***

# GEOMETRY

## Triangle meshes

- Set of vertices
- Connectivity defined by indices

- `uint16_t indices[] = {vertex_index1, vertex_index2, vertex_index3, ...}`

three indices make one triangle

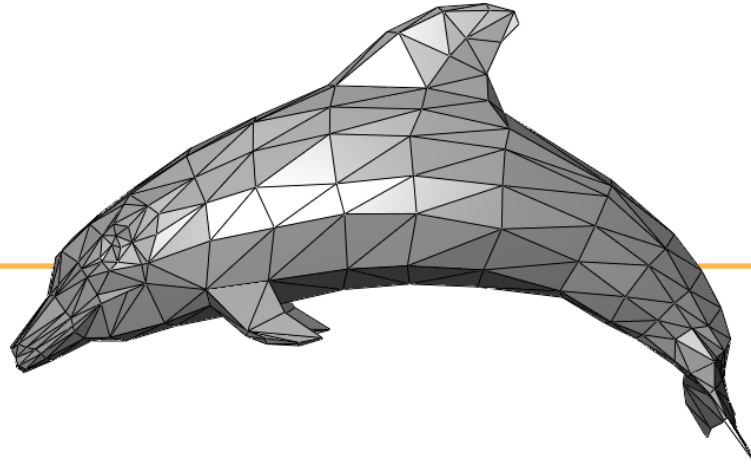
### OpenGL resources

- vertex buffer
- index buffer

### Creation

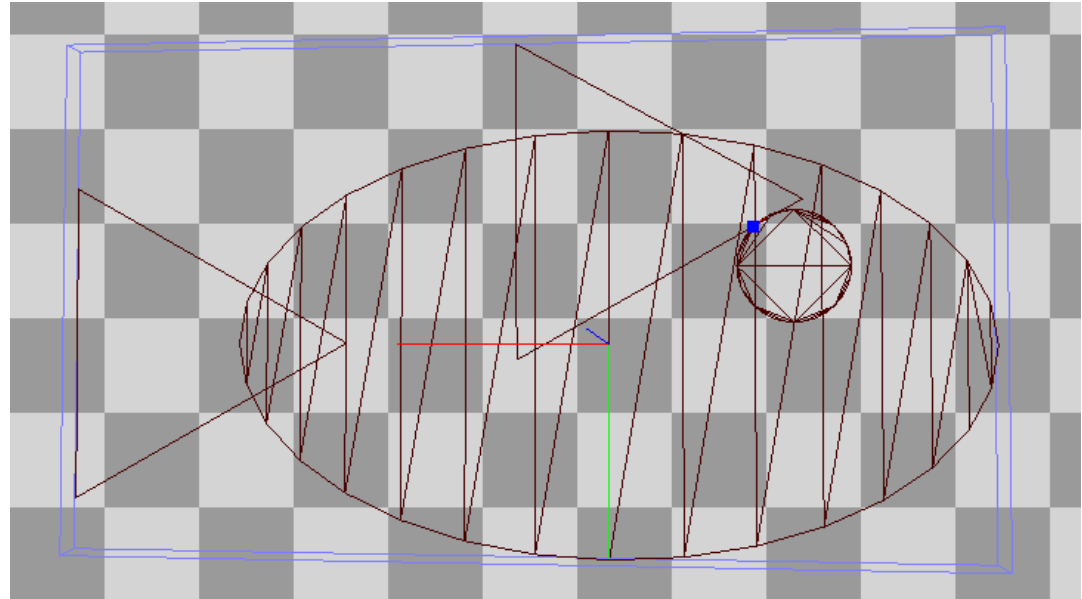
```
GluInt vbo;  
glGenBuffers (vbo) ;
```

```
GluInt ibo;  
glGenBuffers (ibo) ;
```



# Programmatic geometry definition

```
vec3 vertices[153];  
vertices[0].position = { -0.54, +1.34, -0.01 };  
vertices[1].position = { +0.75, +1.21, -0.01 };  
...  
vertices[152].position = { -1.22, +3.59, -0.01 };  
  
uint16_t indices[] = { 0,3,1, 0,4,1,... , 151,152,150 };  
  
Gluint vbo;  
glGenBuffers(vbo);  
glBindBuffer(vbo);  
glBufferData(vbo, vertices);  
  
Gluint ibo;  
glGenBuffers(ibo);  
glBindBuffer(ibo);  
glBufferData(ibo, indices);
```

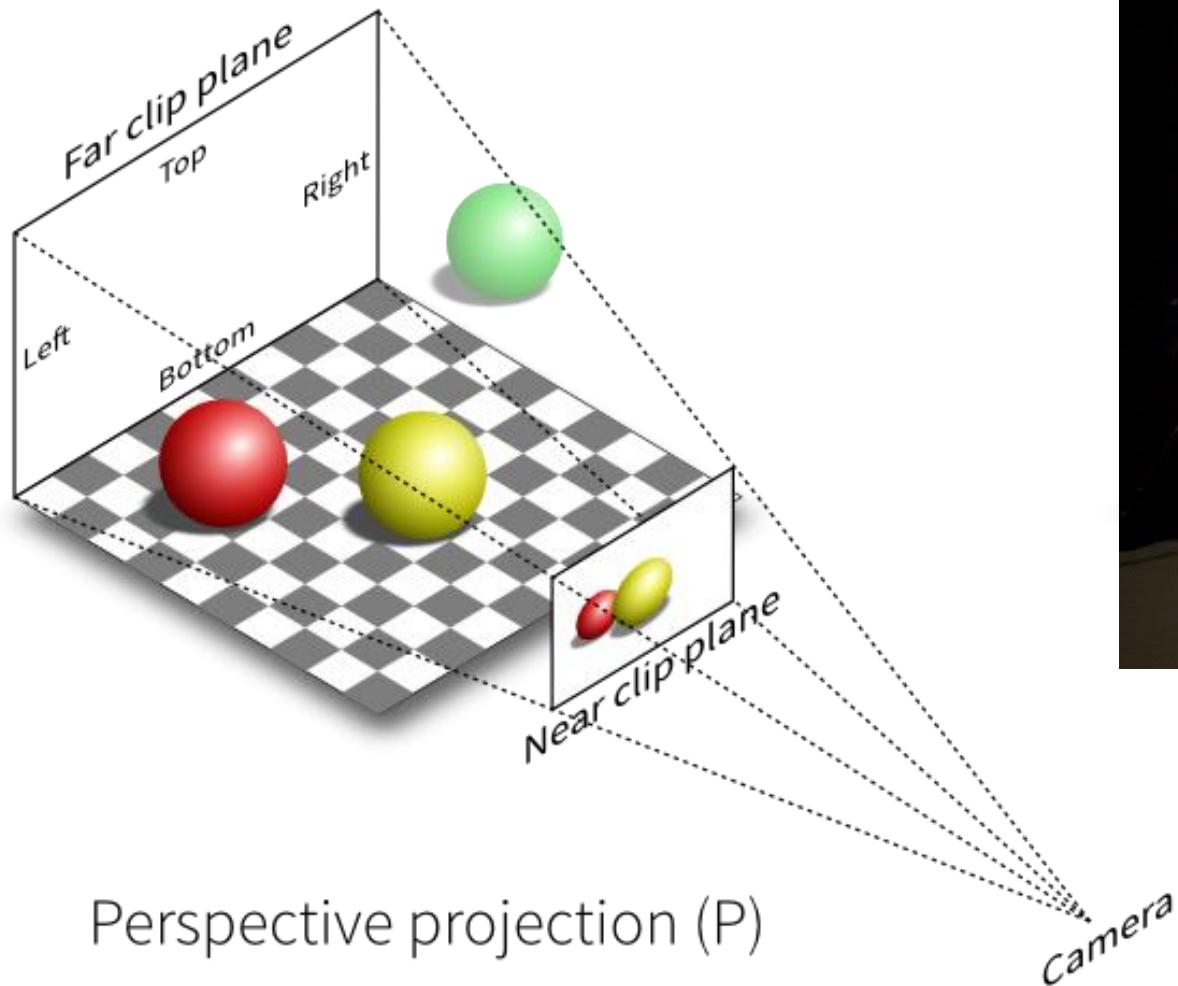


# Image

## *A grid of color values*



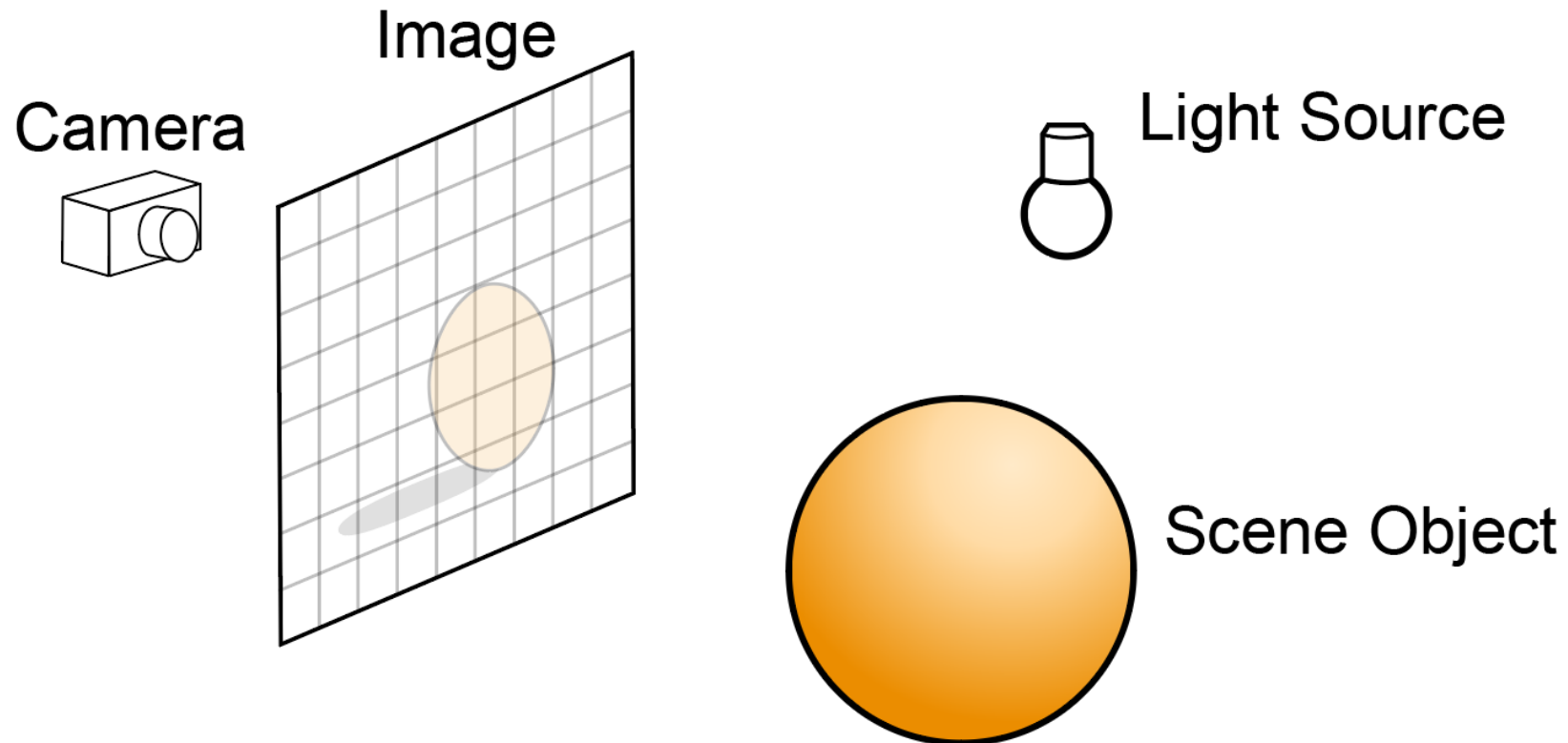
# Virtual Camera



Virtual camera registered in the real world  
(using marker-based motion capture)

# Rendering?

- ***Simulating light transport***
  - How to simulate light efficiently?





# Rendering – Photon Tracing

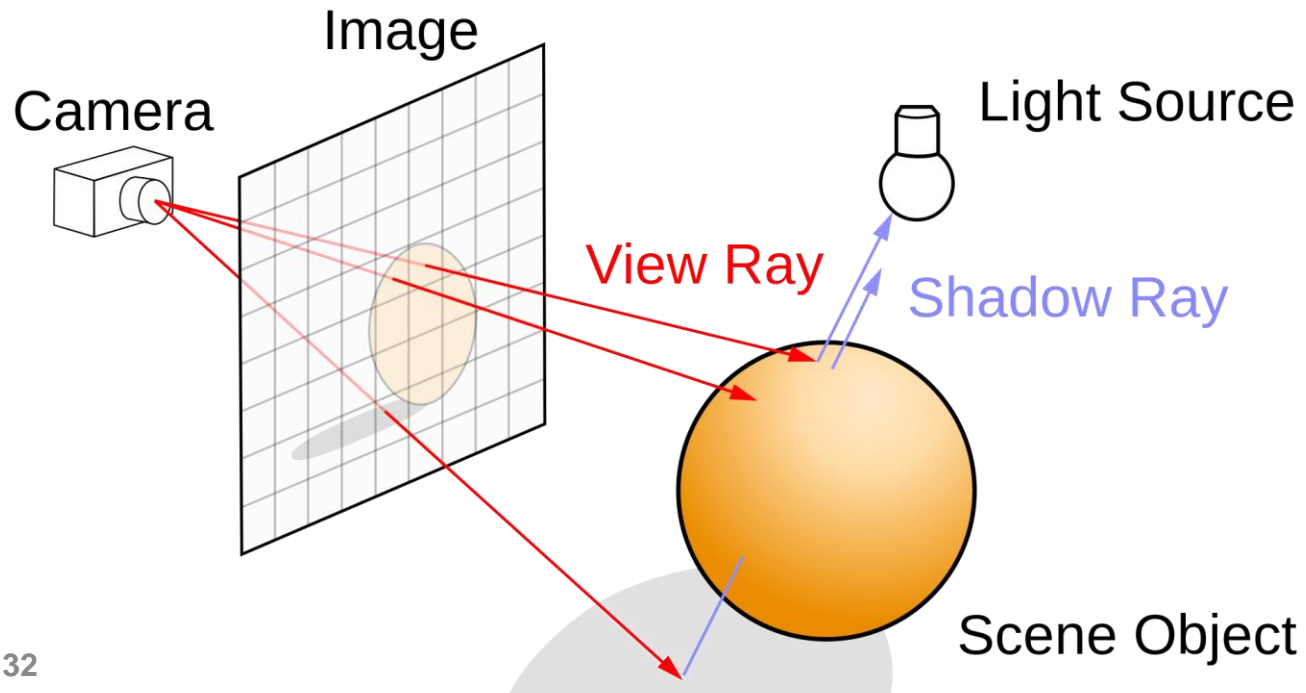
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- ***simulate physical light transport from a source to the camera***
  - *the paths of photons*
- ***shoot rays from the light source***
  - *random direction*
- ***compute first intersection***
  - *continue towards the camera*
- used for indirect illumination: ‘photon mapping’

# Rendering – Ray Tracing

*Start rays from the camera (opposes physics, an optimization)*

- *View rays: trace **from every pixel** to the first occlude*
- *Shadow ray: test light visibility*



Nvidia RTX does ray tracing



# Problems of ray tracing

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- ***the collision detection is costly***
  - ray-object intersection
    - *n objects*
    - *k rays*
    - *naïve:  $O(n*k)$  complexity*

# Rendering – Splatting

*Approximate scene with spheres*

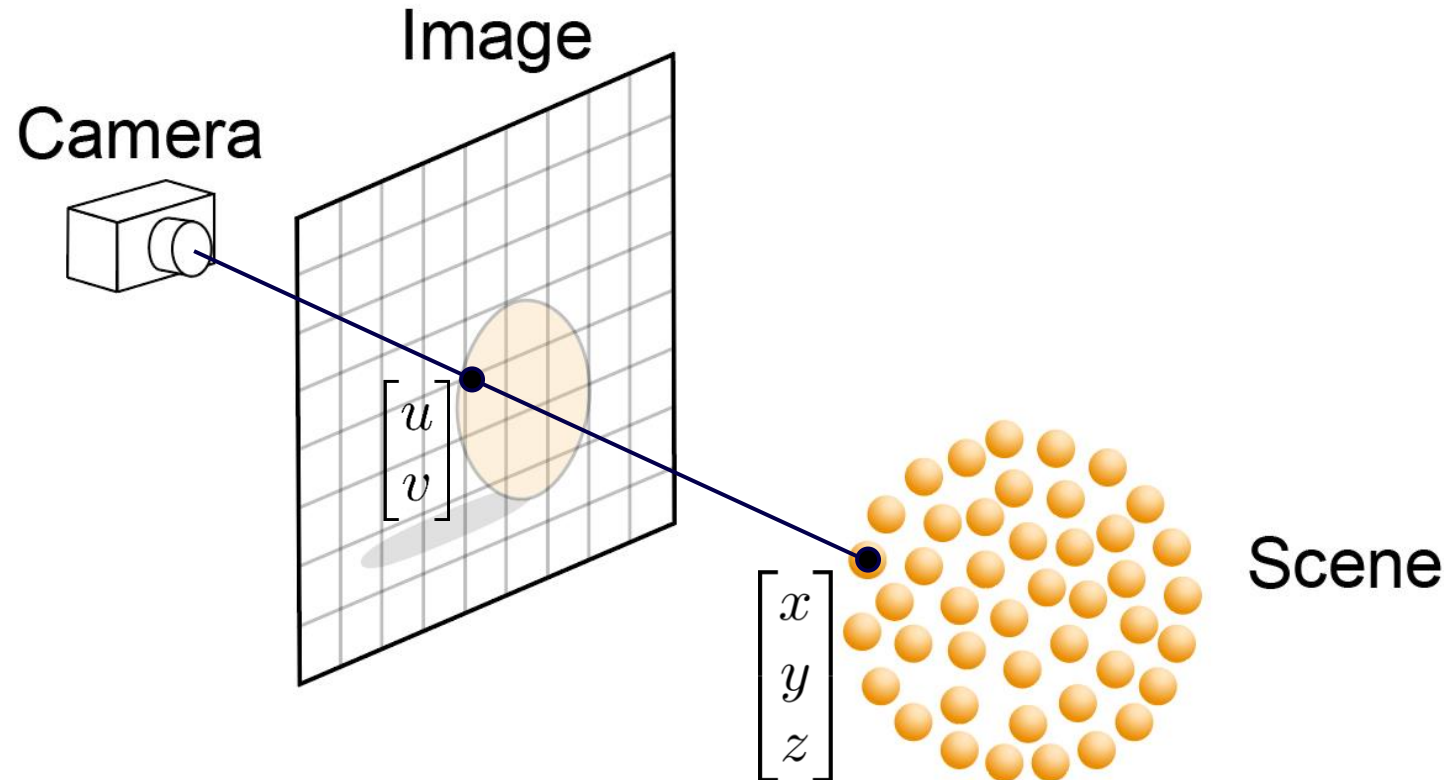
- *sort spheres back-to front*
- *project each sphere*
  - simple equation

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z} \begin{bmatrix} x \\ y \end{bmatrix}$$

- $O(n)$  for  $n$  spheres

*Many spheres needed!*

*Shadows?*



# Rendering – Rasterization

*Approximate objects with triangles*

## 1. *Project each corner/vertex*

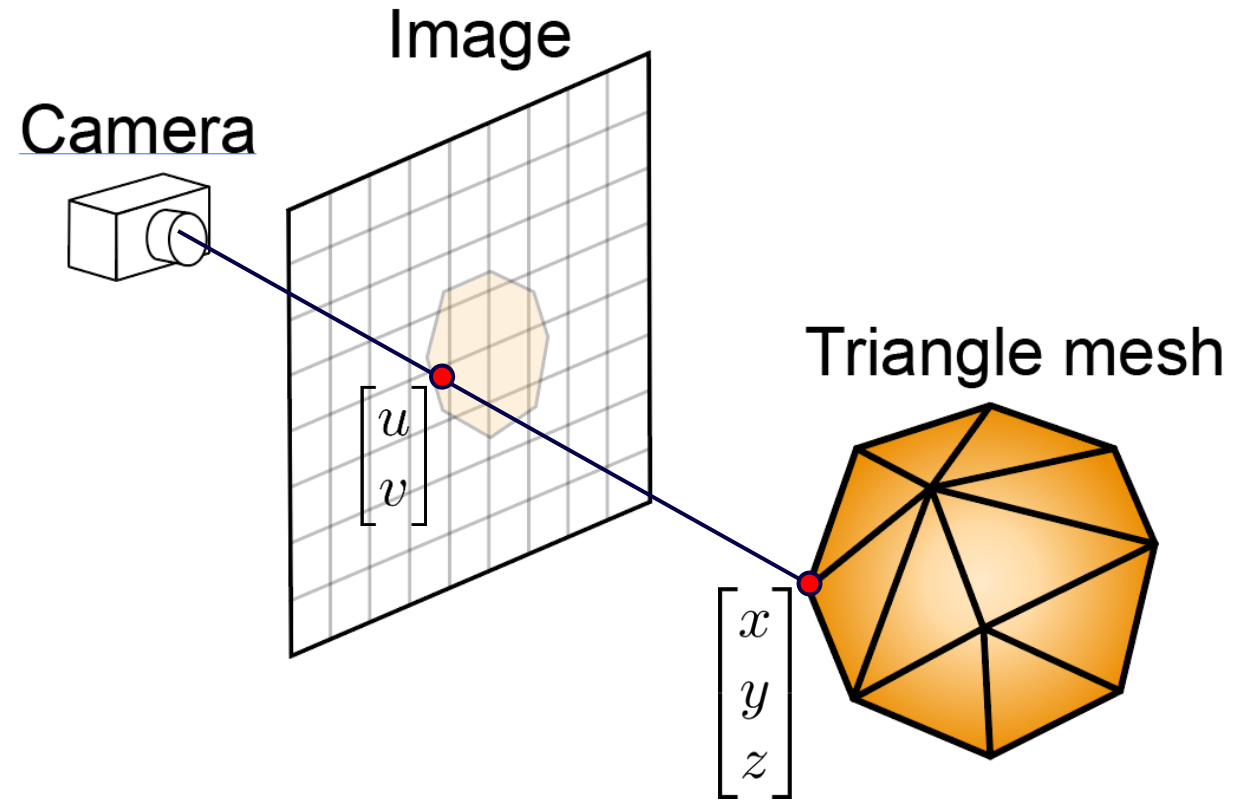
- projection of triangle stays a triangle

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z} \begin{bmatrix} x \\ y \end{bmatrix}$$

- $O(n)$  for  $n$  vertices

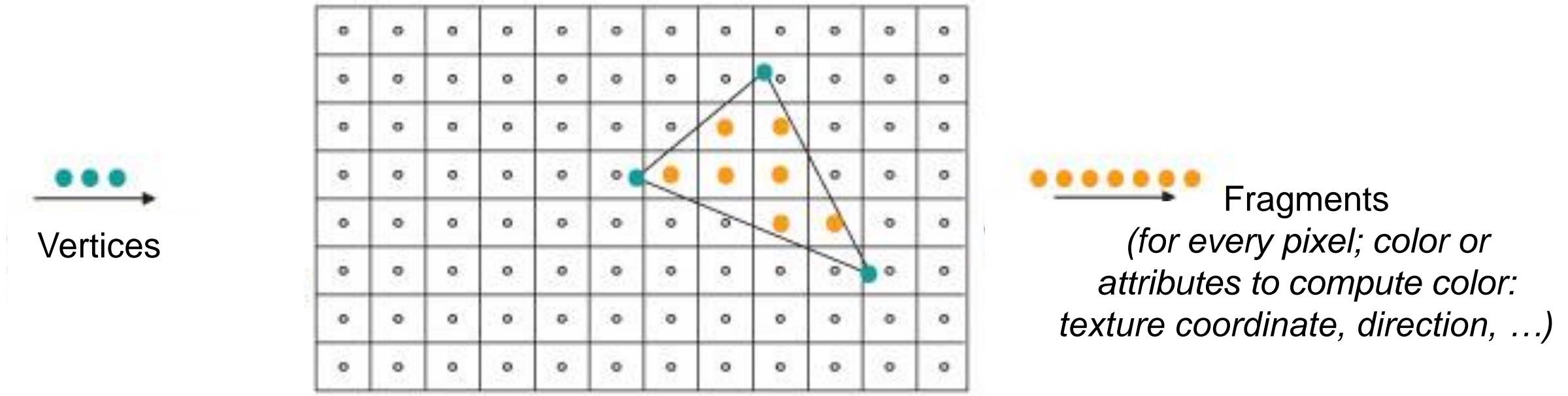
## 2. *Fill pixels enclosed by triangle*

- e.g., scan-line algorithm



# Rasterizing a Triangle

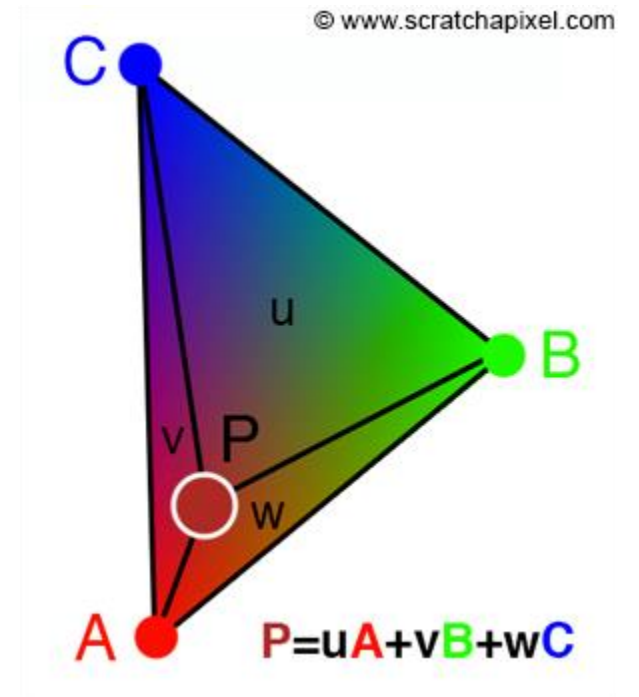
- *Determine pixels enclosed by the triangle*
- *Interpolate vertex properties linearly*



# Self study:

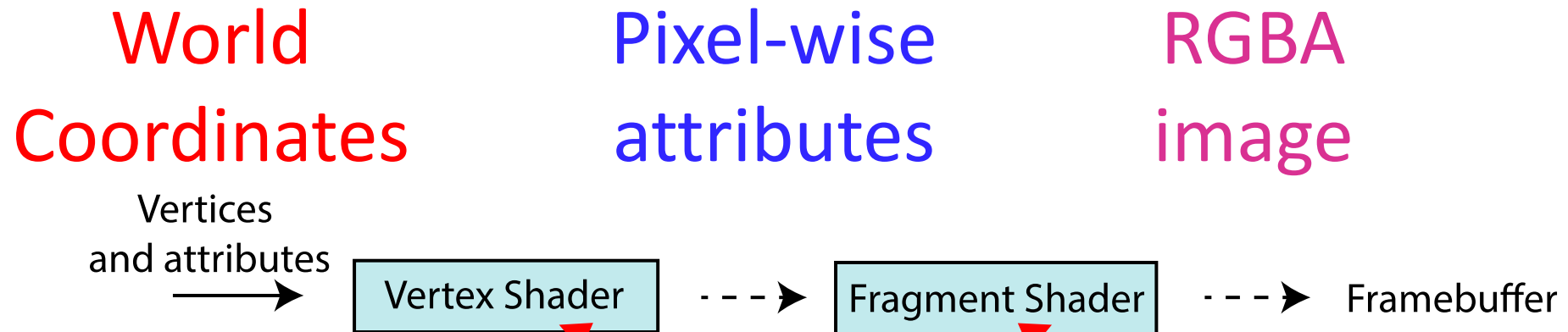
## Interpolation with barycentric coordinates

- *linear combination of vertex properties*
  - e.g., color, texture coordinate, surface normal/direction
- *weights are proportional to the areas spanned by the sides to query point  $P$*



# OpenGL Rendering Pipeline (simplified)

- 1. Vertex shader: geometric transformations*
- 2. Fragment shader: pixel-wise color computation*

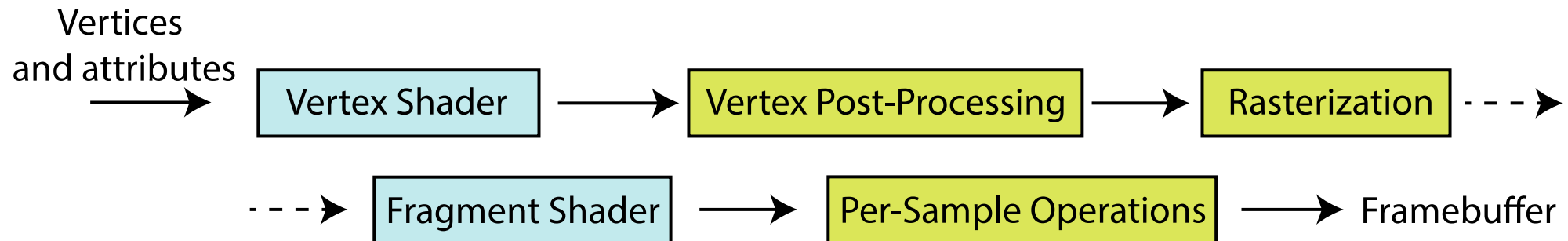


Shader: Programmable functions  
to define object appearance locally  
(vertex wise or fragment wise)

# OpenGL Rendering Pipeline

## *Input:*

- *3D vertex position*
- *Optional vertex attributes: color, texture coordinates, ...*



## *Output:*

- **Frame Buffer** : GPU video memory, holds image for display
- **RGBA pixel color** (*Red*, *Green*, *Blue*, *Alpha* / opacity)

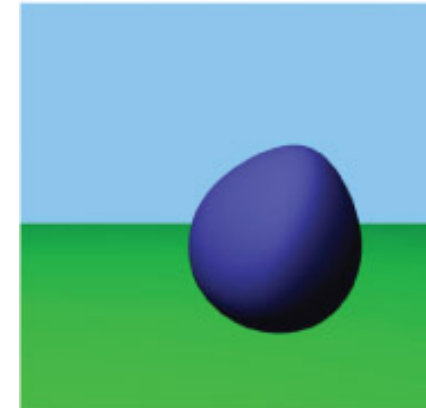
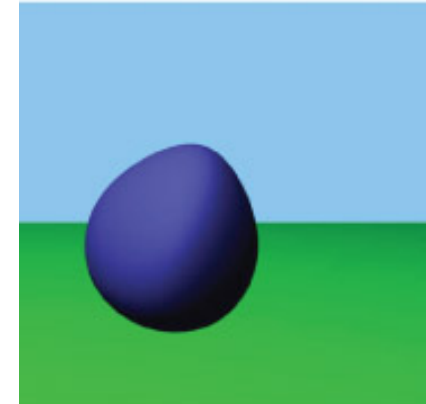
# Vertex shader examples

## ***Object motion & transformation***

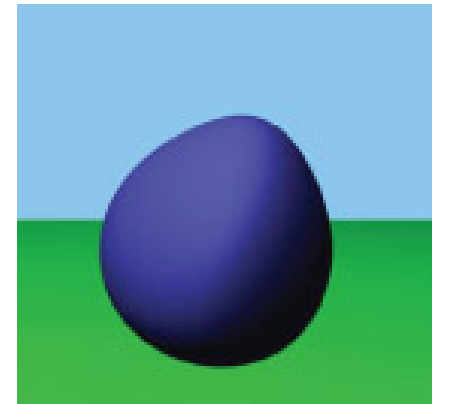
- translation
- rotation
- scaling

## ***Projection***

- Orthographic
  - *simple, without perspective effects*
- Perspective
  - *pinhole projection model*



Translation



Scaling



# GLSL Vertex shader

## The OpenGL Shading Language (GLSL)

- Syntax similar to the C programming language
- Build-in vector operations
  - functionality as the GLM library our assignment template uses

```
void main()
```

```
{
```

```
    // Transforming The Vertex
```

```
    vec3 out_pos = projection * transform * vec3(in_pos.xy, 1.0);
```

```
    gl_Position = vec4(out_pos.xy, in_pos.z, 1.0);
```

```
}
```

world  
-> camera

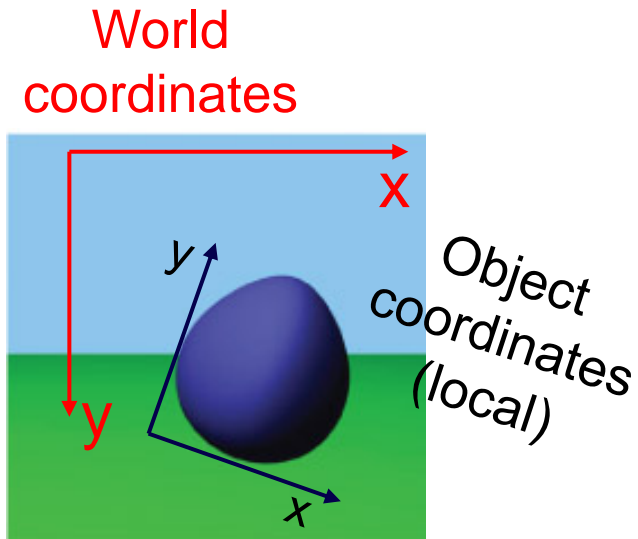
object  
-> world

x and y coordinates  
of a vec2, vec3 or vec4

vector of 3 (vec3) and 4 (vec4) floats

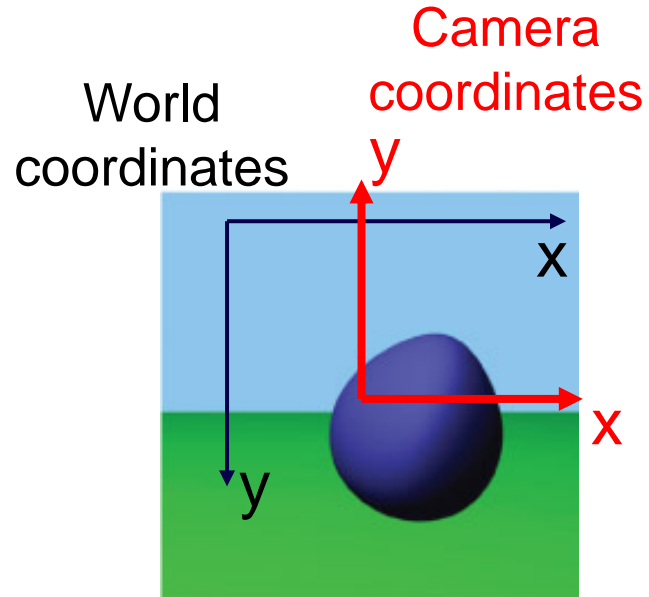
float  
(32 bit)

# From local object to camera coordinates



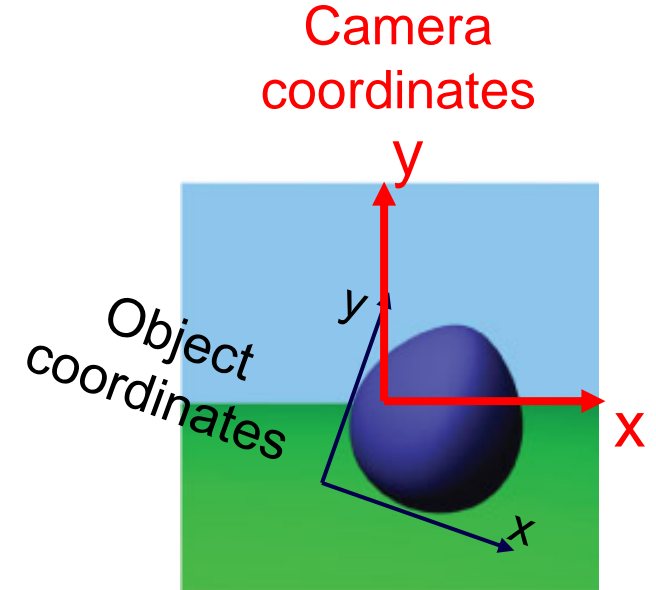
**object -> world**

**transform**



**world -> camera**

**projection**

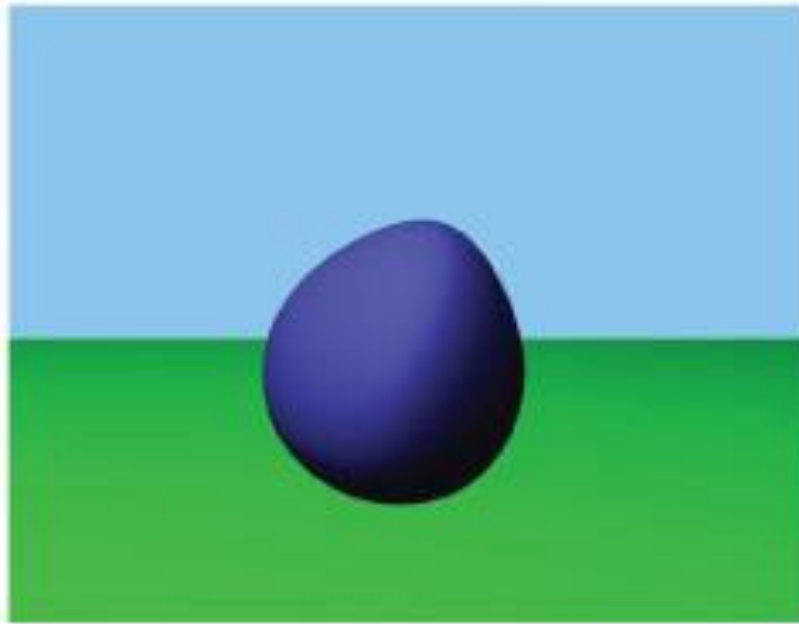


**object -> camera**

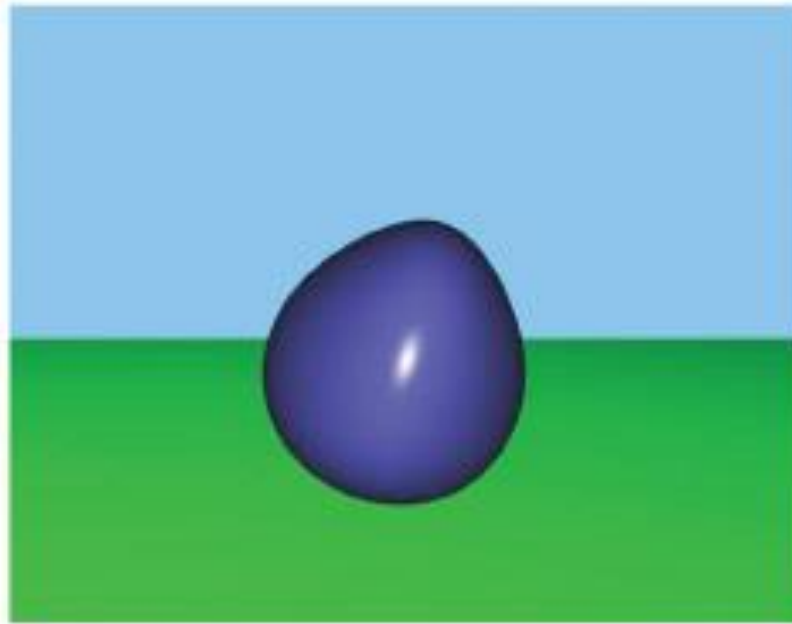
**projection \* transform**

# Fragment shader examples

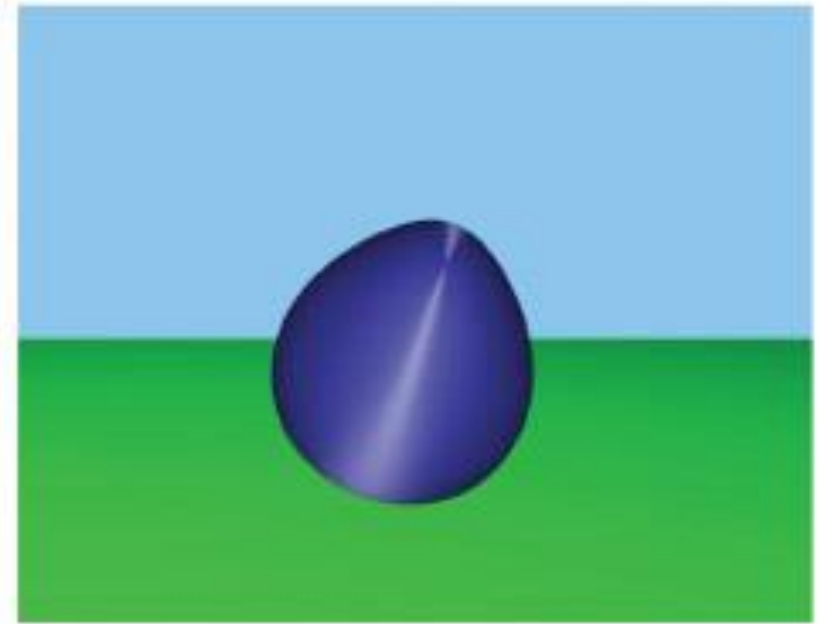
- *simulates materials and lights*
- *can read from textures*



Diffuse

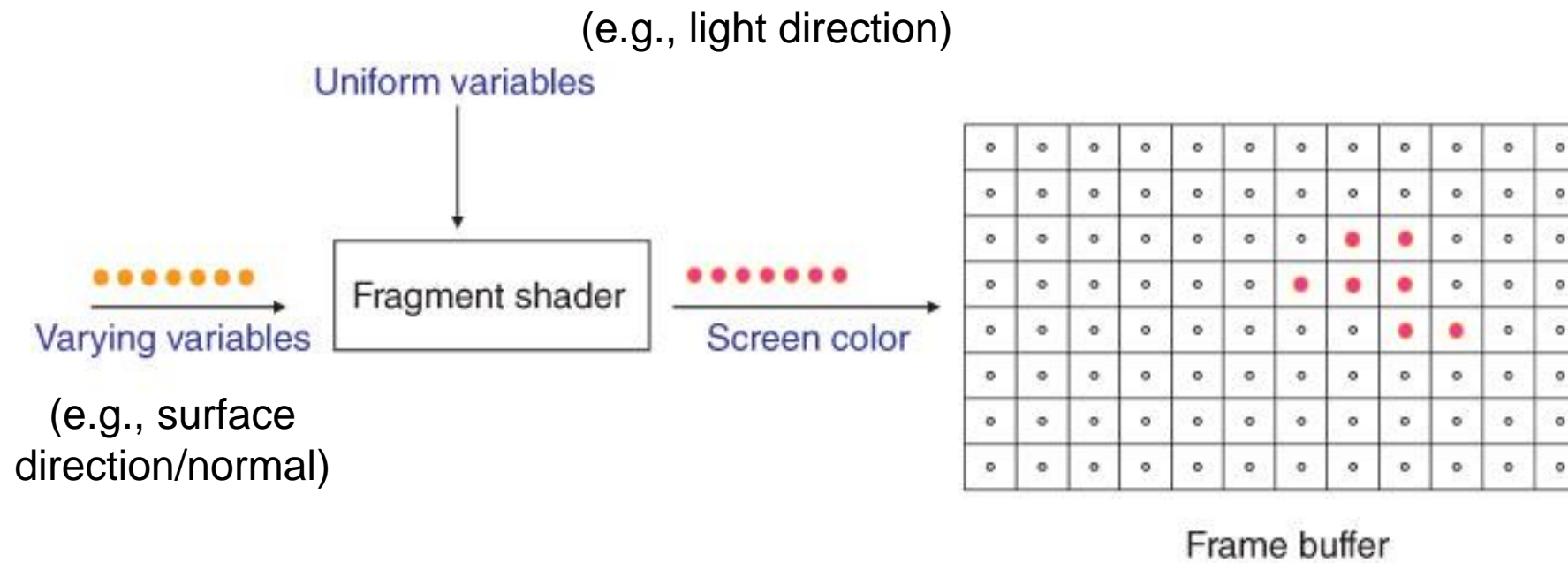


Specular



Directional

# Fragment shader overview



# GLSL fragment shader examples

## *Minimal:*

```
out vec4 out_color;
void main()
{
    // Setting Each Pixel To ???
    out_color = vec4(1.0, 0.0, 0.0, 1.0);
}
```

Specify color output

Red, Green, Blue, Alpha