

INTEGRALE EULERIENE

1) Integrala gamma

$$\Gamma(a) = \int_0^{\infty} x^{a-1} \cdot e^{-x} dx, \quad \underline{a > 0}$$

Proprietati

$$1) \Gamma(1) = 1$$

$$2) \Gamma(a) = (a-1) \cdot \Gamma(a-1) \quad \forall a > 1$$

$$3) \Gamma(n) = (n-1)! \quad \forall n \in \mathbb{N}^*$$

$$4) \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

3) Integrala Euler-Poisson

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

2) Integrala beta

$$\beta(a, b) = \int_0^1 x^{a-1} \cdot (1-x)^{b-1} dx, \quad \underline{a > 0, b > 0}$$

Proprietati

$$1) \beta(a, b) = \beta(b, a) \quad \forall a, b > 0$$

$$2) \beta(a, b) = \frac{\Gamma(a) \cdot \Gamma(b)}{\Gamma(a+b)} \quad \forall a, b > 0$$

$$3) \beta(a, b) = \int_0^{\infty} \frac{x^{a-1}}{(1+x)^{a+b}} dx, \quad \forall a, b > 0$$

$$4) \text{ Dacă } \underline{a+b=1} \text{ atunci: } \beta(a, b) = \frac{\pi}{\sin(a\pi)}$$