

Examen

Probabilitati și Statistică

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G. 234

i = 107

- 1) Fie: $D+$ = aveți boala $T+$ = testat pozitiv
 $D-$ = nu aveți boala $T-$ = testat negativ

$$P(D+) = \frac{107}{1000} = 0.107$$

$$P(D-) = 1 - 0.107 = 0.893$$

$$P(T+|D-) = 0.03 \text{ (fals pozitiv)}$$

$$P(T-|D+) = 0.03 \text{ (fals negativ)}$$

$$P(D+|T+) = \frac{P(T+|D+) \cdot P(D+)}{P(T+)} = \frac{0.97 \cdot 0.107}{0.13058} = 0.794$$

$$P(T+) = (0.03 \cdot 0.893) + (0.97 \cdot 0.107) = 0.13058$$

Răspuns: 0.794

$$2) X \sim \left(\begin{array}{ccc} 1 & 2 & 3 \\ \frac{107}{1000} & \frac{107}{500} & \frac{1000 - 3 \cdot 107}{1000} \end{array} \right). \quad E(X^2) = ?$$

Când R folosind librăria discrete RV

$$X \leftarrow RV(c(1, 2, 3), c(107/1000, 107/500, (1000 - 3 * 107)/1000))$$

$$E(X^2)$$

Rezultat: $E(X^2) = 7.074$

$$\text{Alternativ: } E(X^2) = 1 \cdot \frac{107}{1000} + 2^2 \cdot \frac{107}{500} + 3^2 \cdot \frac{1000 - 3 \cdot 107}{1000} = 7.074$$

$$3) X \sim U(0, 107)$$

$$Y = \sqrt{X}$$

$$F_X(x) = \frac{x-0}{107-0} = \frac{x}{107}$$

$$F_Y(y) = P(Y \leq y) = P(\sqrt{X} \leq y) = P(X \leq y^2) = F_X(y^2) = \frac{y^2}{107}$$

$$f_Y(y) = F'_Y(y) = \left(\frac{y^2}{107}\right)' = \frac{2y}{107}$$

$$Y = \sqrt{X} \rightarrow \text{domeniul de valori este } [0, \sqrt{107}]$$

$$\underline{F_Y(y) = \frac{y^2}{107}} \quad ; \quad \underline{f_Y(y) = \frac{2y}{107}}$$

$$4) P(107 \text{ averseuri}) = C_{200}^{107} p^{107} (1-p)^{200-107} = C_{200}^{107} p^{107} (1-p)^{93}$$

$$\frac{d}{dp} P(\text{date} | p) = C_{200}^{107} (107 p^{106} (1-p)^{93} - 93 p^{107} (1-p)^{92}) = 0$$

$$\Rightarrow 107 p^{106} (1-p)^{93} = 93 p^{107} (1-p)^{92}$$

$$107 (1-p) = 93 p$$

$$107 = (93 + 107) p$$

$$107 = 200 p$$

$$\Rightarrow \underline{MLE = 0.535}$$

$$5) P(D_H | A) = 0.5 \quad P(A) = \frac{107}{202} = 0.535$$

$$P(D_H | B) = 0.8 \quad P(B) = \frac{95}{202} = 0.465$$

Probabilitatea predictivă a priori:

$$P(D_H) = P(D_H | A) P(A) + P(D_H | B) P(B) = 0.5 \cdot 0.535 + 0.8 \cdot 0.465$$

$$\underline{P(D_H) = 0.6395}$$

Probabilitatea predictivă a posteriori:

$$\begin{aligned} \underline{P(D_H | D)} &= P(D_H | A) \cdot P(A | D) + P(D_H | B) \cdot P(B | D) \\ &= 0.5 \cdot 0.418 + 0.8 \cdot 0.581 \\ &= 0.209 + 0.4648 = \underline{\underline{0.6738}} \end{aligned}$$

Tabel Bayes:

H	P(H)	P(D H)	P(D H) · P(H)	P(H D)
A	0.535	0.5	0.2675	$\frac{0.2675}{0.6395} = 0.418$
B	0.465	0.8	0.372	$\frac{0.372}{0.6395} = 0.581$
total	1		0.6395	1

Răspuns : $\underline{P(D_H) = 0.6395}$
 $\underline{P(D_H | D) = 0.6738}$

- 6) $107 \cdot 10 = 1070$ persoane preferă candidatul A
 $1600 - 1070 = 530$ persoane preferă candidatul B

$$\text{Interval de încredere: } \bar{x} \pm z_{\alpha/2} \cdot \frac{1}{\sqrt{25n}}$$

$$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05$$

$$\bar{x} = \frac{1070}{1600} = 0.6687$$

$$z_{\alpha/2} = z_{0.025} \approx 1.96$$

$$n = 1600$$

\Rightarrow Intervalul de încredere de 95% va fi:

$$I \approx 0.6687 \pm 1.96 \cdot \frac{1}{\sqrt{25 \cdot 1600}} = 0.6687 \pm 0.0245$$

$$\underline{I = [0.644, 0.692]}$$