

4.

Parity function: $p(x) = 1$ iff there is an odd number of 1's in x
 else
 $p(x) = 0$

for $n=3$, $\theta \in \mathbb{R}$

$$\begin{aligned} x = (1, 0, 0) & \quad w \cdot x = w_1 \Rightarrow w_1 \geq \theta \\ x = (1, 1, 0) & \quad w \cdot x = w_1 + w_2 \Rightarrow w_1 + w_2 < \theta \\ x = (0, 0, 0) & \quad w \cdot x = 0 \\ x = (0, 1, 1) & \quad w \cdot x = w_2 + w_3 \\ x = (0, 1, 0) & \quad w \cdot x = w_2 \Rightarrow w_2 \geq \theta \end{aligned}$$

given $w_1 \geq \theta$ then, $w_1 + w_2 \geq \theta$
 $w_2 \geq \theta$

therefor $w_1 + w_2 < \theta$ cannot be true

→ Perceptrons cannot compute the parity function.

for $n \in \mathbb{Z}$,
 do the same as with $n=3$, but replace all values of x_i for $i \geq 3$ with 0
 for test cases.