Differentiable Functions

Exercise 1: Determine the n-th derivative of the following functions:

a) $f : (-1, \infty) \to \mathbb{R}$ defined by $f(x) = (1 + x)^r$, where $r \in \mathbb{R}$;

b) $f : (-1, \infty) \to \mathbb{R}$ defined by $f(x) = x \cdot \ln(1 + x)$;

c) $f : (-\infty, -1) \to \mathbb{R}$ defined by $f(x) = x \cdot \ln(1 - x)$;

d) $f : (-1, 1) \to \mathbb{R}$ defined by $f(x) = \sqrt{3x + 4}$;

e) $f : (-\frac{1}{2}, \infty) \to \mathbb{R}$ defined by $f(x) = \frac{1}{\sqrt{2x+1}}$.

Exercise 2: Determine the n-th derivative of the following functions:

a) $f : \mathbb{R} \setminus -\frac{b}{a} \to \mathbb{R}$ defined by $f(x) = \frac{1}{ax+b}$;

b) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(ax + b)$;

c) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \cos(ax + b)$;

d) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^{ax+b}$.

Exercise 3: Compute the derivatives of the following functions

a) $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = x^x$;

b) $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = x^{\frac{1}{x}}$;

c) $f : (0, \pi) \to \mathbb{R}$ defined by $f(x) = \sin x^x$;

d) $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = x^{\sin x}$;

Excercise 4:

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = x + |x - 1|$$

for all $x \in \mathbb{R}$.

a) Prove that f has side derivatives at $x_0 = 1$;

b) Compute the side derivates of f at $x_0 = 1$;

c) Is f differentiable on the left at $x_0 = 1$? What about on the right?

d) Does f have a derivative at $x_0 = 1$?

e) Is f differentiable at $x_0 = 1$?