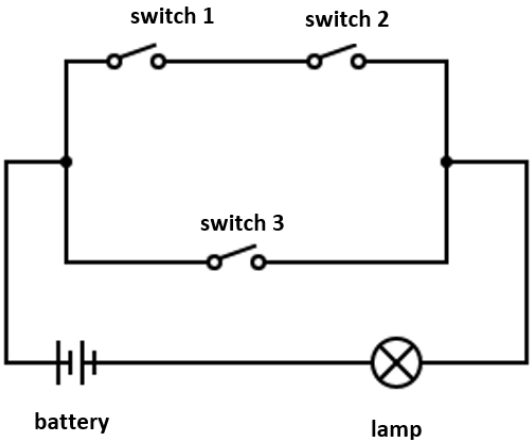
Seminar 4

1. Four electronic devices have the property that, for every $i \in \{1, 2, 3, 4\}$, the probability that any i fixed devices are all functional is $\frac{1}{4^i}$. Using the inclusion-exclusion principle, compute the probability of the event A:"none of the devices is functional".

2. Four antivirus programs are tested by scanning independently an infected file. They detect the virus with corresponding probabilities: $\frac{3}{4}, \frac{1}{4}, \frac{2}{4}, \frac{1}{4}$. Compute the probabilities of the following events:
A:"All programs detect the virus."
B:"Exactly one program detects the virus."
C:"Exactly three programs detect the virus."
D:"At most one program detects the virus."
E:"At least one program detects the virus."

3. In the diagram below the three switches are either ON or OFF, independently, with probability $\frac{1}{2}$ for each state. Compute the probability that the circuit operates.



4. The owner of three shops decides to give a bonus to the salary of a randomly chosen employee. The first shop has 50 employees and 50% of them are men, the second shop has 75 employees and 60% of them are men and the third shop has 100 employees and 70% are men.
a) Find the probability that the lucky employee works in the third shop, given that the lucky employee is a woman.
b) Find the probability that the lucky employee is a woman, given that the lucky employee works in the third shop.

5. Three dice are rolled. Let N_k be number that showed on the kth die, $k \in \{1, 2, 3\}$. Find:
a) $P(N_1 = 1, N_2 = 2, N_3 = 3)$.
b) $P(N_1 = N_2 = N_3)$.
c) $P(N_1 + N_2 + N_3 \geq 5)$.
d) $P(N_1 + N_2 + N_3 \geq 5 | N_1 < N_2 < N_3)$.
e) $P(N_1 < N_2 < N_3 | N_1 < N_2)$.
f) $P(N_1 > N_2 < N_3 | N_1 = N_3)$.
g) $P(N_1 = N_2, N_2 > 2 | N_3 > 2)$.

6. A fair coin is tossed infinitely many times. Compute the probability of the events:
a) A:"All tosses show heads."
b) B:"At least one toss shows head."