Analysis - Homework 1

1. a) $A = (-\infty; -1] \cup (2; +\infty)$

- $LB(A) = \emptyset$
- $UB(A) = \emptyset$
- $\inf A = -\infty$
- ~~inf~~
- $\sup A = +\infty$
- $\min A = -$
- $\max A = -$

b) $A = (-1; 9] \cup [10; 20)$

- $LB(A) = (-\infty; -1]$
- $UB(A) = [20; +\infty)$
- $\inf A = -1$
- $\sup A = 20$
- $\min A = -$, $\inf A = -1 \notin A$
- $\max A = -$, $\sup A = 20 \notin A$

c) $A = ((-1; 9] \cup [10; 20)) \cap \mathbb{N}$

- $LB(A) = (-\infty; 1]$
- $UB(A) = [20; +\infty)$
- $\inf A = 1$
- $\sup A = 20$
- $\min A = 1$, $\inf A = 1 \in A$
- $\max A = -$, $\sup A = 20 \notin A$

d) $A = \{1; 2; 3\}$

- $LB(A) = (-\infty; 1]$
- $UB(A) = [3; +\infty)$
- $\inf A = 1$
- $\sup A = 3$
- $\min A = 1$, $\inf A = 1 \in A$
- $\max A = 3$, $\sup A = 3 \in A$

e) $A = \mathbb{N}$

- $LB(A) = (-\infty; 1]$
- $UB(A) =$
- $\inf A = 1$
- $\sup A = +\infty \rightarrow \nexists \max A$
- ~~max~~ min $A = 1$, $\inf A = 1 \in A$
- $\max A = -$

f) $A = \mathbb{R} \setminus \{1, 2, 3\}$

- $LB(A) = \emptyset$
- $UB(A) = \emptyset$
- $\inf A = -\infty \rightarrow \nexists \min A$
- $\sup A = +\infty \rightarrow \nexists \max A$
- $\min A = -$
- $\max A = -$

g) $A = \mathbb{R} / \mathbb{N}$

- $LB(A) = \emptyset$
- $UB(A) = \emptyset$
- $\inf A = -\infty \rightarrow \nexists \min A$
- $\sup A = +\infty \rightarrow \nexists \max A$
- ~~min A = ∄~~

h) $A = \mathbb{Z}$

- $LB(A) = \emptyset$
- $UB(A) = \emptyset$
- $\inf A = -\infty \rightarrow \nexists \min A$
- $\sup A = +\infty \rightarrow \nexists \max A$

i) $A = \mathbb{R} \setminus \mathbb{Z}$

- $LB(A) = \emptyset \to \inf A = -\infty \to \nexists \min A$
- $UB(A) = \emptyset \to \sup A = +\infty \to \nexists \max A$

j) $A = \mathbb{Q}$

- $LB(A) = \emptyset \to \inf A = -\infty \to \nexists \min A$
- $UB(A) = \emptyset \to \sup A = +\infty \to \nexists \max A$

k) $A = \mathbb{R} \setminus \mathbb{Q}$

- $LB(A) = \emptyset \to \inf A = -\infty \to \nexists \min A$
- $UB(A) = \emptyset \to \sup A = +\infty \to \nexists \max A$

l) $A = \mathbb{R}$

- $LB(A) = \emptyset \to \inf A = -\infty \to \nexists \min A$
- $UB(A) = \emptyset \to \sup A = +\infty \to \nexists \max A$

2. a) $A = \bigcup_{n \in \mathbb{N} \setminus \{1\}} \left(-1+\frac{1}{n} ; 1-\frac{1}{n}\right)$

$$A = \left(-1+\frac{1}{2} ; 1-\frac{1}{2}\right) \cup \left(-1+\frac{1}{3} ; 1-\frac{1}{3}\right) \cup \dots \cup \left(-1+\underbrace{\frac{1}{n}}_{\xrightarrow{n\to\infty} 0} ; 1-\underbrace{\frac{1}{n}}_{\xrightarrow{n\to\infty} 0}\right) =$$

$$= \left(-\frac{1}{2} ; \frac{1}{2}\right) \cup \left(-\frac{2}{3} ; \frac{2}{3}\right) \cup \dots \cup (-1 ; 1) = (-1;1)$$

$LB(A) = (-\infty ; -1] \to \inf A = -1 \notin A \to \nexists \min A$

$UB(A) = [1 ; +\infty) \to \sup A = 1 \notin A \to \nexists \max A$

b) $B = \bigcup_{n \in \mathbb{N}} \left[-1+\frac{1}{n} ; 1-\frac{1}{n}\right]$

$$B = \left[-1+\frac{1}{1} ; 1-\frac{1}{1}\right] \cup \left[-1+\frac{1}{2} ; 1-\frac{1}{2}\right] \cup \left[-1+\frac{1}{3} ; 1-\frac{1}{3}\right] \cup \dots \cup$$

$$\cup \left[-1+\underbrace{\frac{1}{n}}_{\xrightarrow{n\to\infty} 0} ; 1-\underbrace{\frac{1}{n}}_{\xrightarrow{n\to\infty} 0}\right] = \{0\} \cup \left[-\frac{1}{2} ; \frac{1}{2}\right] \cup \left[-\frac{2}{3} ; \frac{2}{3}\right] \cup \dots \cup [-1;1] =$$

$$= [-1;1]$$

$LB(B) = (-\infty ; -1] \to \inf B = -1 \in B \to \min B = -1$

$UB(B) = [1 ; +\infty) \to \sup B = 1 \in B \to \max B = 1$

c) $C = \bigcap_{n \in \mathbb{N} \setminus \{1\}} \left(-1+\frac{1}{n} ; 1-\frac{1}{n}\right)$

$C = \left(-1+\frac{1}{2}; 1-\frac{1}{2}\right) \cap \left(-1+\frac{1}{3}; 1-\frac{1}{3}\right) \cap \dots \cap \left(-1+\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}; 1-\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}\right) =$

$= \left(-\frac{1}{2}; \frac{1}{2}\right) \cap \left(-\frac{2}{3}; \frac{2}{3}\right) \cap \dots \cap (-1; 1) = \left(-\frac{1}{2}; \frac{1}{2}\right)$

$LB(C) = \left(-\infty; -\frac{1}{2}\right] \to \inf C = -\frac{1}{2} \notin C \to \nexists \min C$

$UB(C) = \left[\frac{1}{2}; +\infty\right) \to \sup C = \frac{1}{2} \notin C \to \nexists \max C$

d) $D = \bigcap_{n \in \mathbb{N}} \left[-1+\frac{1}{n}; 1-\frac{1}{n}\right]$

$D = \left[-1+\frac{1}{1}; 1-\frac{1}{1}\right] \cap \left[-1+\frac{1}{2}; 1-\frac{1}{2}\right] \cap \left[-1+\frac{1}{3}; 1-\frac{1}{3}\right] \cap \dots \cap$

$\cap \left[-1+\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}; 1-\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}\right] = \{0\} \cap \left[-\frac{1}{2}; \frac{1}{2}\right] \cap \left[-\frac{2}{3}; \frac{2}{3}\right] \cap \dots \cap [-1; 1] =$

$= \{0\}$

$LB(D) = (-\infty; 0] \to \inf D = 0 \in D \to \min D = 0$

$UB(D) = [0; +\infty) \to \sup D = 0 \in D \to \max D = 0$

e) $E = \bigcup_{n \in \mathbb{N}} \left[-1-\frac{1}{n}; 1+\frac{1}{n}\right]$

$\cancel{E = \left[-1-\frac{1}{2}; 1+\frac{1}{2}\right] \cup \left[-1-\frac{1}{3}; 1+\frac{1}{3}\right] \cup \dots \cup \left[-1-\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}; 1+\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}\right]}$

$\cancel{= \left[-\frac{3}{2}; \frac{3}{2}\right] \cup \left[-\frac{4}{3}; \frac{4}{3}\right] \cup \dots \cup [-1; 1]}$

$E = \left[-1-\frac{1}{1}; 1+\frac{1}{1}\right] \cup \left[-1-\frac{1}{2}; 1+\frac{1}{2}\right] \cup \dots \cup \left[-1-\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}; 1+\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}\right] =$

$= [-2; 2] \cup \left[-\frac{3}{2}; \frac{3}{2}\right] \cup \dots \cup [-1; 1] = [-2; 2]$

$LB(E) = (-\infty; -2] \to \inf E = -2 \in E \to \min E = -2$

$UB(E) = [2; +\infty) \to \sup E = 2 \in E \to \max E = 2$

f) $F = \bigcap_{n \in \mathbb{N}} \left(-1-\frac{1}{n}; 1+\frac{1}{n}\right)$

$F = \left(-1-\frac{1}{1}; 1+\frac{1}{1}\right) \cap \left(-1-\frac{1}{2}; 1+\frac{1}{2}\right) \cap \dots \cap \left(-1-\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}, 1+\underset{\substack{\downarrow n\to\infty \\ 0}}{\frac{1}{n}}\right) =$

$= (-2; 2) \cap \left(-\frac{3}{2}; \frac{3}{2}\right) \cap \dots \cap (-1; 1) = (-1; 1)$

$LB(F) = (-\infty; -1] \to \inf F = -1 \notin F \to \nexists \min F$

$UB(F) = [1; +\infty) \to \sup F = 1 \notin F \to \nexists \max F$

3.

$(-1;2]$	$(-2;1)$	$[-1;1]$	$\mathbb{R}\setminus\{1\}$	$\mathbb{Z}$	$\mathbb{R}\setminus(-1;0)$	$\mathbb{Q}$
✗	✓	✗	✗	✗	✗	✗

$x = -1$

- $(-1;2]$ - isn't a neighborhood of -1 because ~~th~~ $\nexists\ \varepsilon > 0$ s.t. $-1-\varepsilon \in (-1;2]$
- $(-2;1)$ - is a neighborhood of -1 because $\exists\ \varepsilon > 0$ s.t. $-1-\varepsilon \in (-2;1)$ and $-1+\varepsilon \in (-2;1)$
- $[-1;1]$ - isn't a neighborhood of -1 because $\nexists\ \varepsilon > 0$ s.t. $-1-\varepsilon \in [-1;1]$
- $\mathbb{R}\setminus\{1\}$ - isn't a neighborhood of -1 because is a discontinuous set
- $\mathbb{Z}$ - isn't a neighborhood of -1 because there aren't any elements in the proximity of -1
- $\mathbb{R}\setminus(-1;0)$ - is a discontinuous set