

## Series of real numbers- series with random terms

**Exercise 1:** Study both the absolute convergence and the convergence of the following series of real numbers:

a)

$$\sum_{n \geq 1} (-1)^n \frac{(n+1)^{n+1}}{n^{n+2}}.$$

b)

$$\sum_{n \geq 1} (-1)^n \frac{2n+1}{3^n},$$

c)

$$\sum_{n \geq 1} (-1)^n \frac{1}{\ln n}$$

d)

$$\sum_{n \geq 1} (-1)^n \frac{1}{\sqrt{n(n+1)}}$$

e)

$$\sum_{n \geq 1} (-1)^n \frac{(2n-1)!!}{(2n)!!}$$

where  $!!$  means double factorial, i.e. either all the even number multiplied, or the odd ones ( for example  $8!! = 2 \cdot 4 \cdot 6 \cdot 8$ )

f)

$$\sum_{n \geq 1} (-1)^{\frac{n(n+1)}{2}} \frac{n^{100}}{2^n};$$

g)

$$\sum_{n \geq 1} (-1)^{\frac{n(n+1)}{2}} \sin \frac{\pi}{n\sqrt{n+1}}.$$

**Exercise 2:** Consider  $a, b > 0$ . Study the convergence or divergence of the following series of real numbers:

a)

$$\sum_{n \geq 1} \frac{a(2a+1)(3a+1) \cdot \dots \cdot (na+1)}{b(2b+1)(3b+1) \cdot \dots \cdot (nb+1)};$$

b)

$$\sum_{n \geq 1} \frac{a(a+1)\dots(a+n)}{n!} \cdot \frac{1}{n^b};$$

c)

$$\sum_{n \geq 1} \frac{2^n}{a^n + b^n}$$

d)

$$\sum_{n \geq 1} \frac{a^n b^n}{a^n + b^n}.$$