

## Seminar 5

1. Let  $(S, \mathcal{K}, P)$  be a probability space and  $A, B \in \mathcal{K}$ .

(a) Prove that

$$P(A) + P(B) - 1 \leq P(A \cap B) \leq \min\{P(A), P(B)\} \\ \max\{P(A), P(B)\} \leq P(A \cup B) \leq P(A) + P(B).$$

(b) It is known that  $P(A) = \frac{2}{5}$ ,  $P(B) = \frac{9}{10}$ . Indicate four concrete values  $a, b, c, d \in (0, 1]$  such that

$$a \leq P(A \cap B) \leq b < c \leq P(A \cup B) \leq d.$$

Explain why your choice is correct!

(c) It is known that  $P(A) = \frac{2}{5}$ ,  $P(B) = \frac{9}{10}$ ,  $P(A|B) = \frac{10}{27}$ .

(c1) Compute the probabilities:  $P(A \cap B)$ ,  $P(A \cup B)$ ,  $P(B \cap \bar{A})$ ,  $P(B|A)$ .

(c2) Are  $A$  and  $B$  independent events?

2. John has in his pocket 2 red dice and 3 blue dice. He takes randomly a die. If the chosen die is red, he rolls it 3 times. On the other hand, if the chosen die is blue, he rolls it 2 times. Compute the probability of the event  $E$ : "the sum of the numbers that show up after the rolls is 10."

3. Mary studies Probability Theory. She arrives late at the seminar with probability 0.2, if the day is rainy, and with probability 0.1, if the day is sunny. According to the weather forecast, the next day, when Mary has the seminar, is rainy with probability 0.8. Compute the probabilities of the events:

$A$ : "Mary arrives on time at the next seminar."

$B$ : "The next day is rainy, given that Mary arrives on time at the seminar."

4. A die is rolled. Let  $N$  be the number that is obtained. Next, the die is rolled  $N$  times. What is the probability that  $N = 3$ , given that  $N \geq 2$  and the numbers obtained after the  $N$  rolls are pairwise **a)** distinct? **b)** equal?

5. A pair of dice - one white die and one red die - is rolled two times. Compute the probability that the two pairs of numbers, obtained after the two rolls, are equal. (Example of favorable case: the white die shows number 2 and the red die shows number 4, both after the first roll and the second roll; example of unfavorable case: first roll "2 on white die, 4 on red die", second roll "4 on white die, 2 on red die".)

6. A computer center has three printers  $A$ ,  $B$ , and  $C$ , which print at different speeds. Programs are routed to the first available printer. The probability that a program is routed to printers  $A$ ,  $B$ , and  $C$  are 0.5, 0.3, and 0.2, respectively. Occasionally a printer will jam and destroy a printout. The probability that printers  $A$ ,  $B$ , and  $C$  will jam are 0.02, 0.06 and 0.1, respectively. Your program is destroyed when a printer jams. What is the probability that printer  $A$  is involved? Printer  $B$  is involved? Printer  $C$  is involved?

7. **a)** Let  $(S, \mathcal{K}, P)$  be a probability space and  $B \in \mathcal{K}$  such that  $P(B) > 0$ . Prove that  $(B, \mathcal{K}_B, P(\cdot|B))$  is a probability space, where  $\mathcal{K}_B := \{B \cap A : A \in \mathcal{K}\}$  and  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$ ,  $A \in \mathcal{K}_B$ .

**b)** Give examples, by considering a random experiment and its corresponding probability space  $(S, \mathcal{K}, P)$ , for the probability space  $(B, \mathcal{K}_B, P(\cdot|B))$  from **a)**.