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# Strategic Manipulation of Bids in Auction-Based Transport Collaborations

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University:

University of Vienna

# Agenda

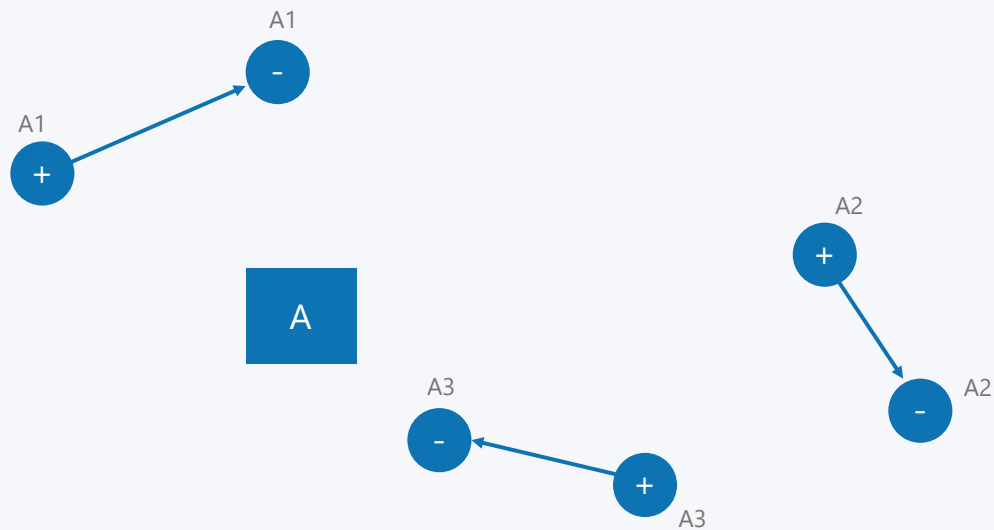
- 1 Introduction to Transport Collaborations
- 2 Implementation of Auction-based Transport Collaborations
- 3 Payment Calculation and Profit Sharing Methods in Auction-based Transport Collaborations
- 4 Strategic Manipulation of Bids in Auction-based Transport Collaborations
  - 4.1 Bidding Strategies for Egalitarian Profit Sharing*
  - 4.2 Bidding Strategies for Modified Egalitarian Profit Sharing*
  - 4.3 Bidding Strategies for Purchase/Sale Weight Profit Sharing*
  - 4.4 Bidding Strategies for Shapley Value Profit Sharing*
  - 4.5 Bidding Strategies for Critical Weight Profit Sharing*
- 5 Comparison of the analysed Profit Sharing Methods
- 6 Outlook

## 1

# Introduction to Transport Collaborations

Introduction to Transport Collaborations

## Initial Situation

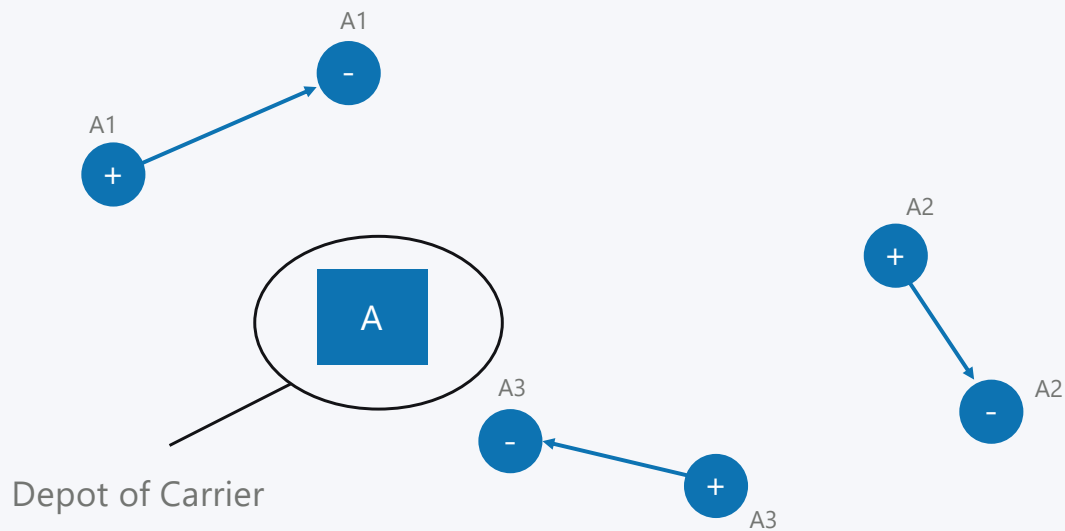


Legend:  Depot  Pickup  Delivery

Strategic Manipulation of Bids in Auction-based Transport Collaborations

Introduction to Transport Collaborations

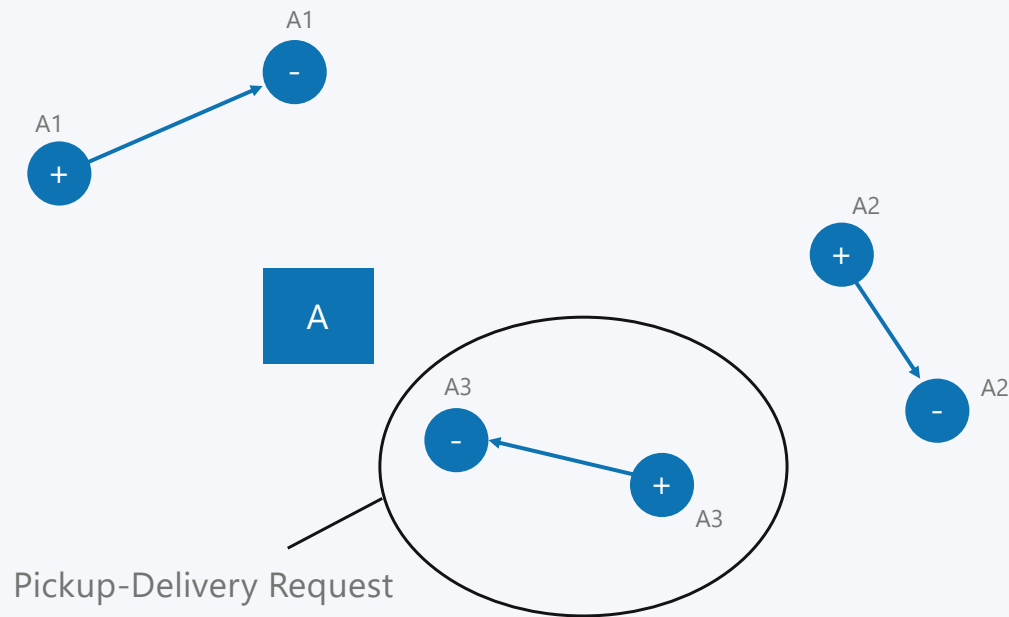
## Initial Situation– Depot



Legend:  Depot  Pickup  Delivery

Strategic Manipulation of Bids in Auction-based Transport Collaborations

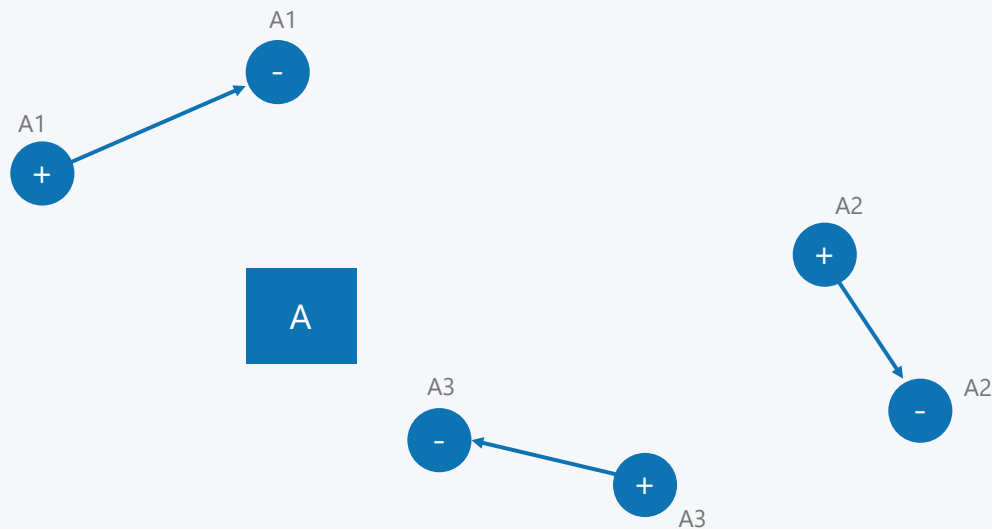
## Initial Situation– Pickup-Delivery Requests



Legend:  Depot  Pickup  Delivery

Introduction to Transport Collaborations

## Initial Situation - Revenue Calculation



### Revenue

*Request Revenue*

(+) Fixed Revenue

(+) Variable Revenue \* Direct Length

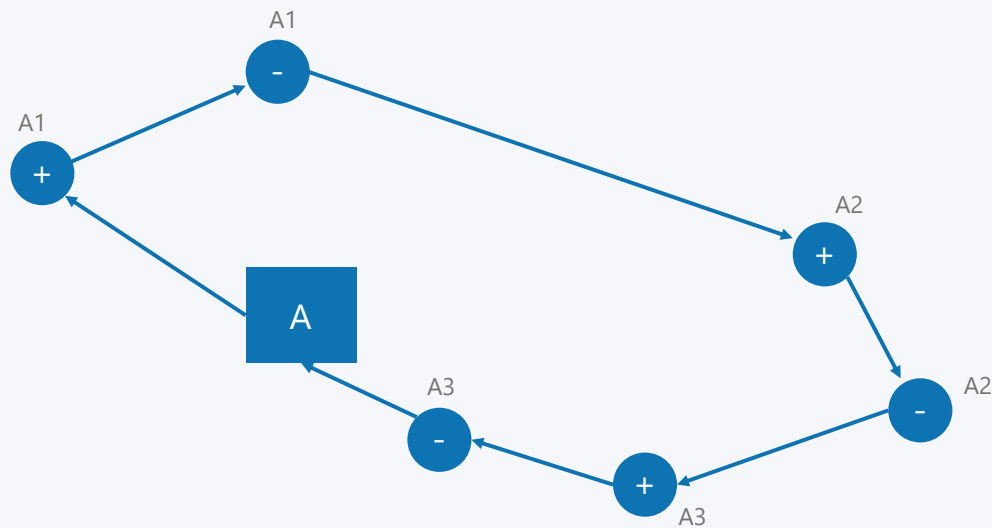
*Total Revenue*

(+) Sum[Request Revenues]

Legend:  Depot  Pickup  Delivery

Introduction to Transport Collaborations

## Initial Situation - Cost Calculation



### Costs

*Total Costs*

(-) Variable Cost \* Routing Distance

*Marginal Cost Request*

(-) Variable Cost \* Marginal Routing Distance

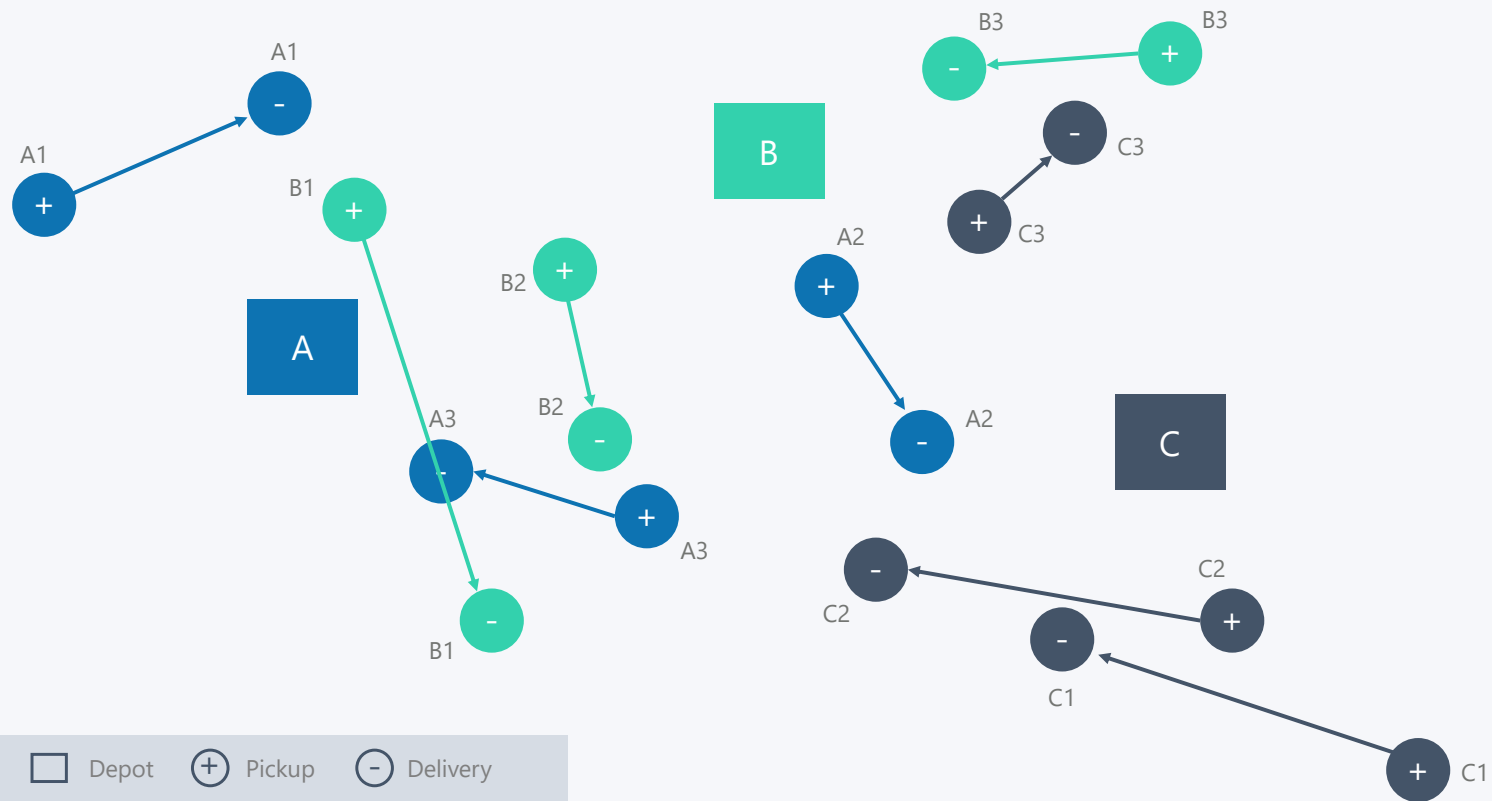
→ Maximize Profit by optimizing the route

Legend:  Depot  Pickup  Delivery



Introduction to Transport Collaborations

## Introduction of Multiple Carriers



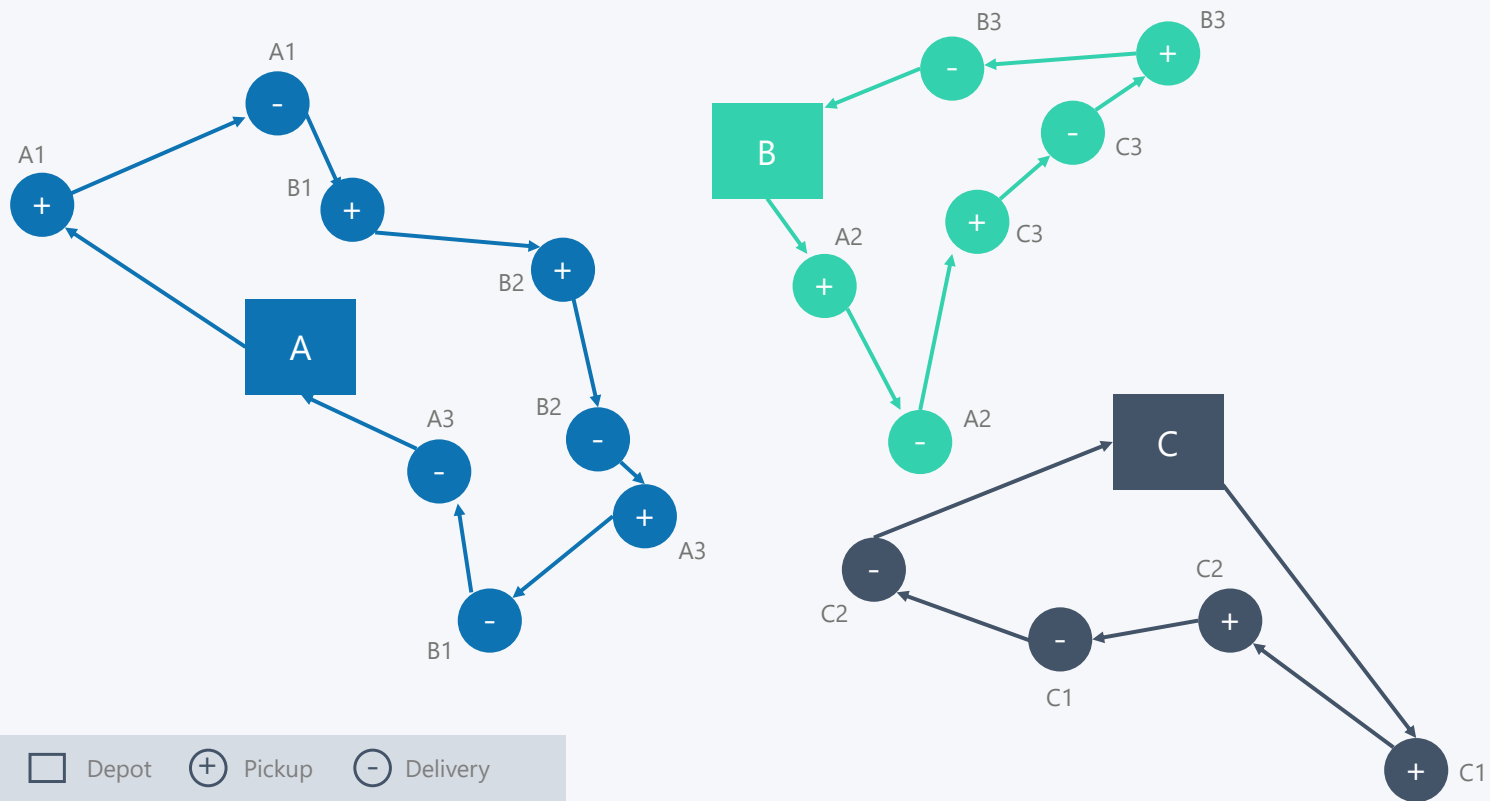
Legend:  Depot  Pickup  Delivery

Strategic Manipulation of Bids in Auction-based Transport Collaborations



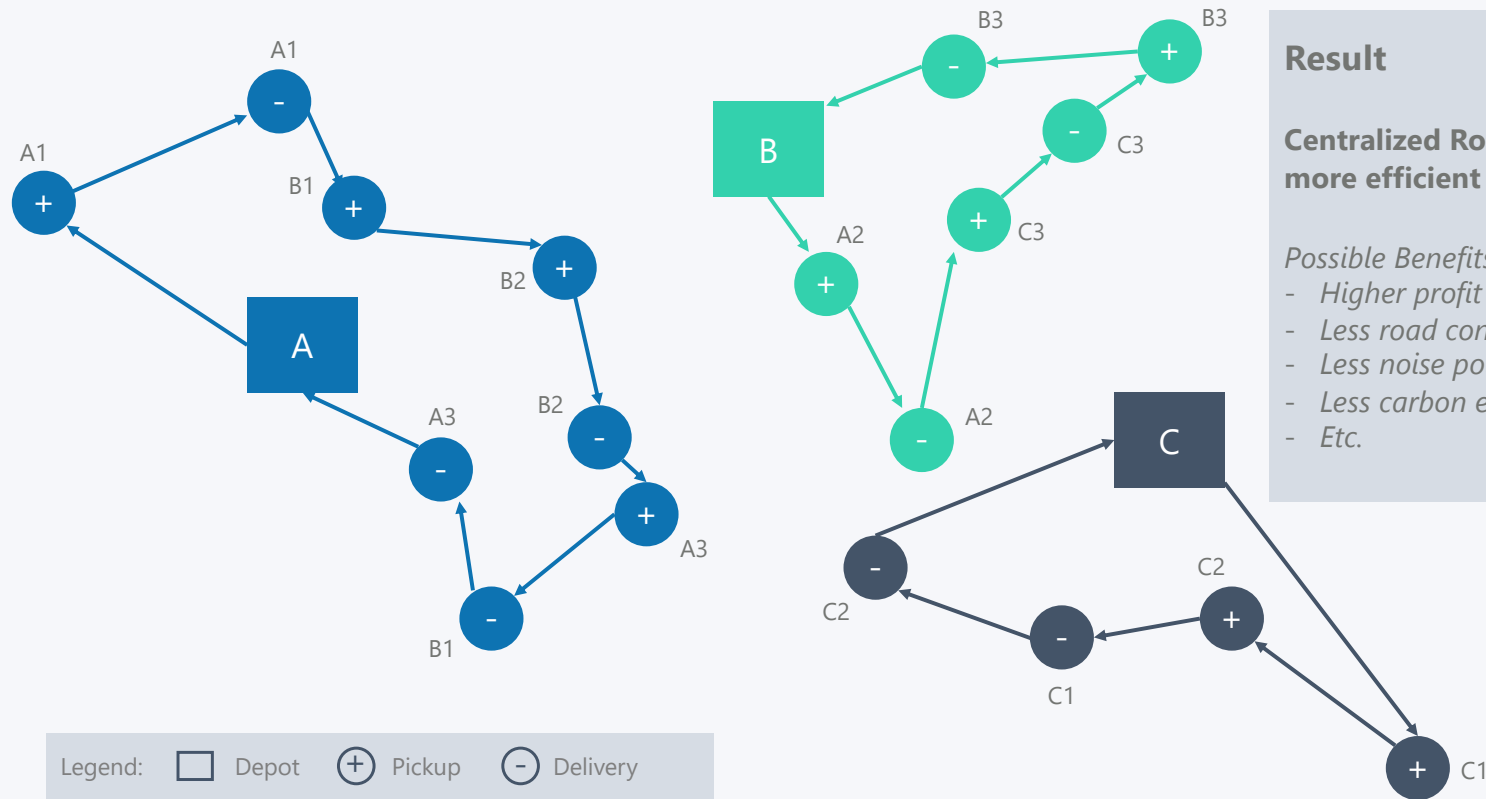
Introduction to Transport Collaborations

## Centralized Routing Solution



## Introduction to Transport Collaborations

# Centralized Routing Solution



### Result

**Centralized Routing Solution is more efficient**

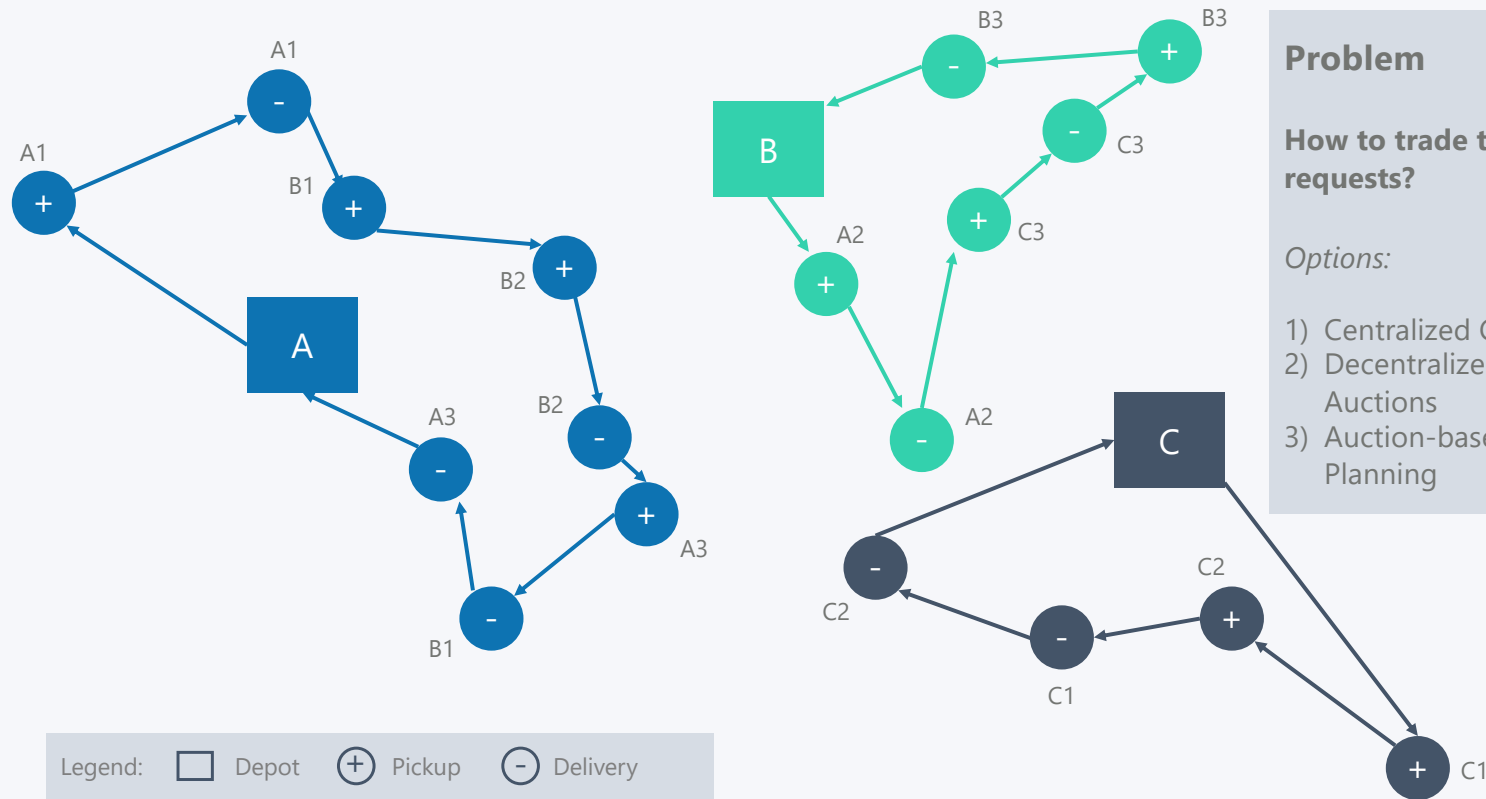
*Possible Benefits:*

- Higher profit margins
- Less road congestion
- Less noise pollution
- Less carbon emissions
- Etc.

See [4]

# Introduction to Transport Collaborations

## Centralized Routing Solution



### Problem

How to trade the Pickup-Delivery requests?

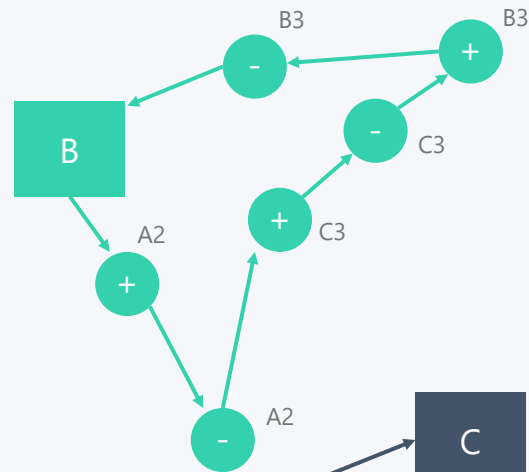
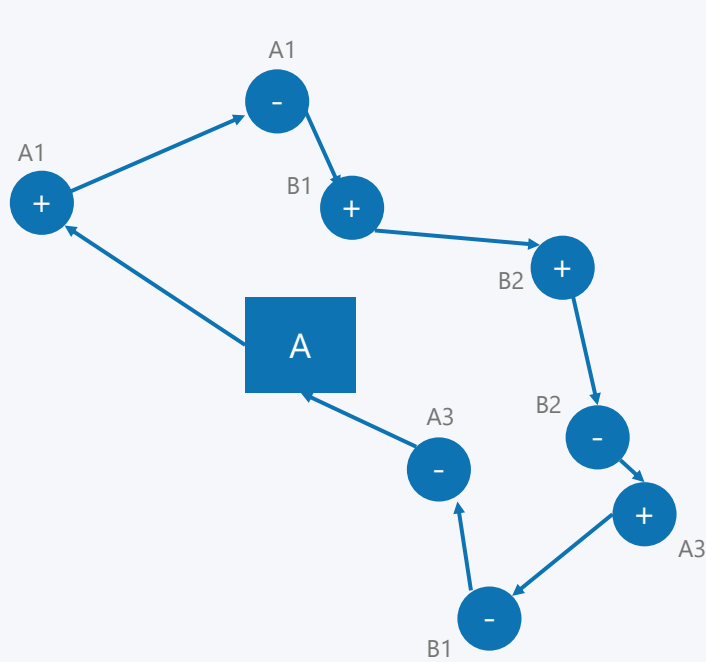
Options:

- 1) Centralized Collaborative Planning
- 2) Decentralized Planning without Auctions
- 3) Auction-based Decentralized Planning

See [4]

# Introduction to Transport Collaborations

## Centralized Routing Solution



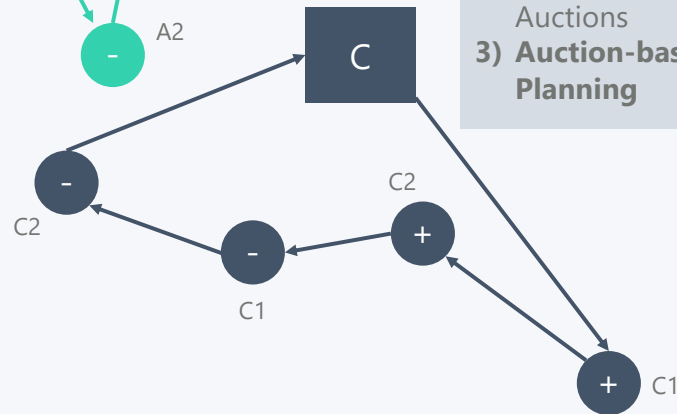
### Problem

How to trade the Pickup-Delivery requests?

Options:

- 1) Centralized Collaborative Planning
- 2) Decentralized Planning without Auctions
- 3) **Auction-based Decentralized Planning**

See [4]



Legend: Depot Pickup Delivery

Strategic Manipulation of Bids in Auction-based Transport Collaborations

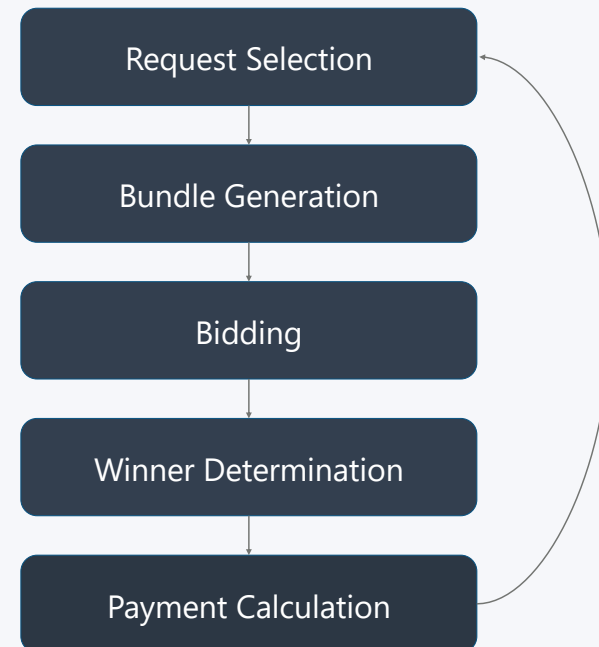
## 2

## Implementation of Auction-based Transport Collaborations

### Auction-based mechanism

*Coordinated by a mechanism manager*

- 1) Each carrier should **select pickup-delivery requests** that she/he is willing to trade and state a price (**Input Bid**)
- 2) The mechanism manager has to **bundle the requests** to attractive packages
- 3) Each carrier has to **select a price** that she/he is willing to pay for the offered **bundles (Bid)**
- 4) The mechanism manager has to **determine the optimal bids** and allocate the requests (**Winning Bids**)
- 5) The mechanism manager has to **determine the payments** for each carrier
- ( 6) Mechanism terminates or restarts at 1) )

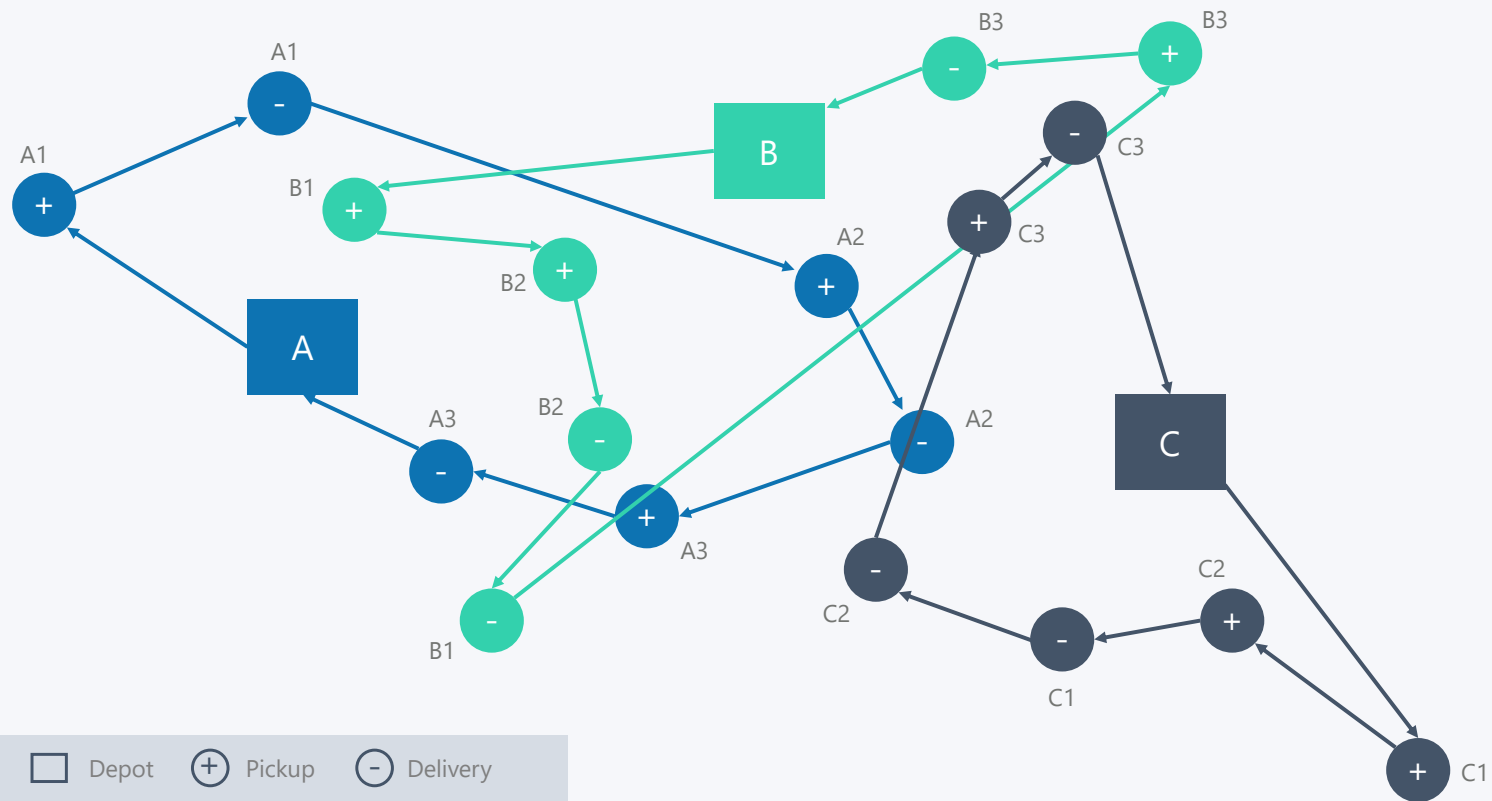


See [1]



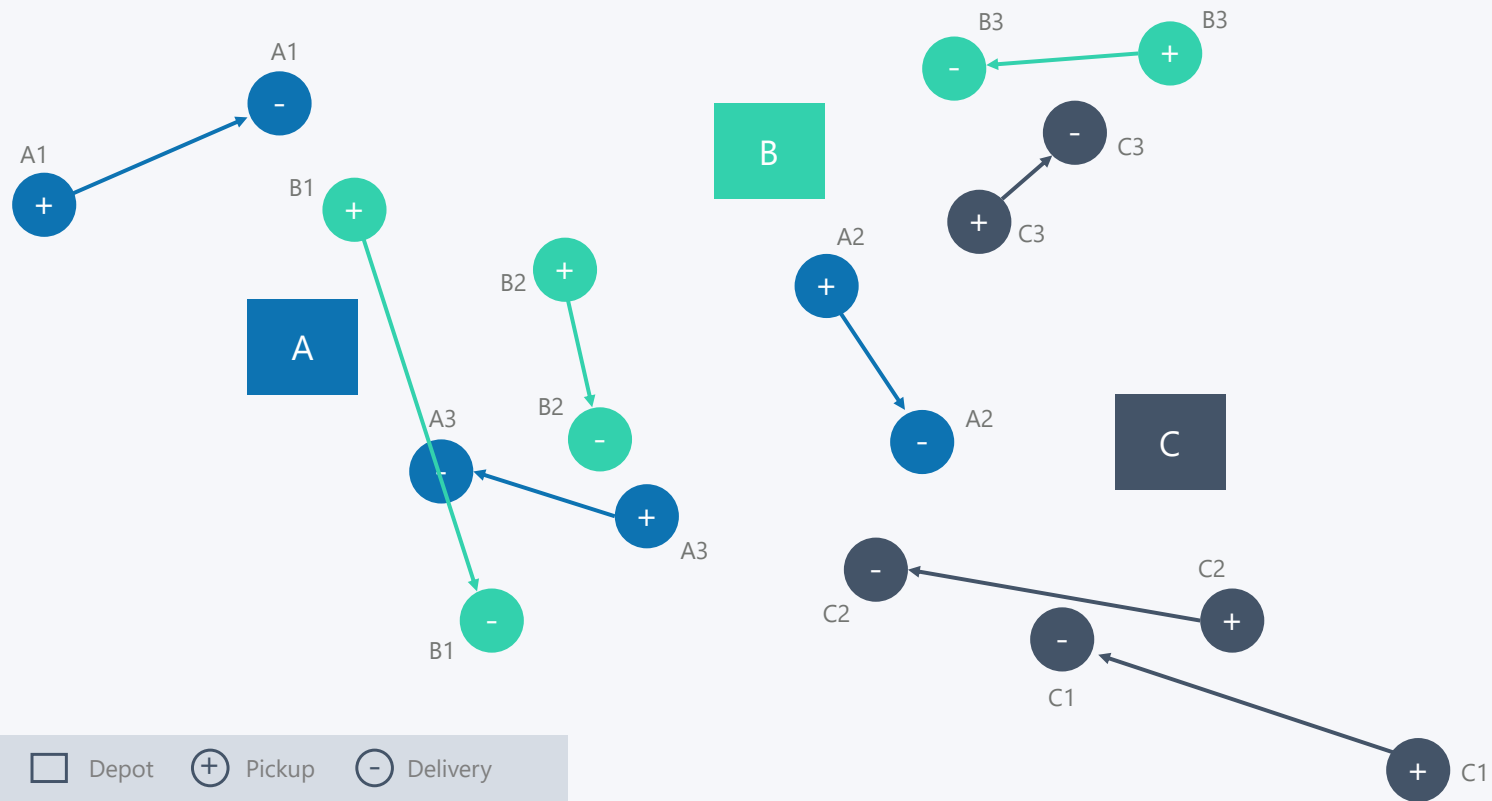
Implementation of Auction-based Transport Collaborations

## Starting Point – Decentralized Routing Solution



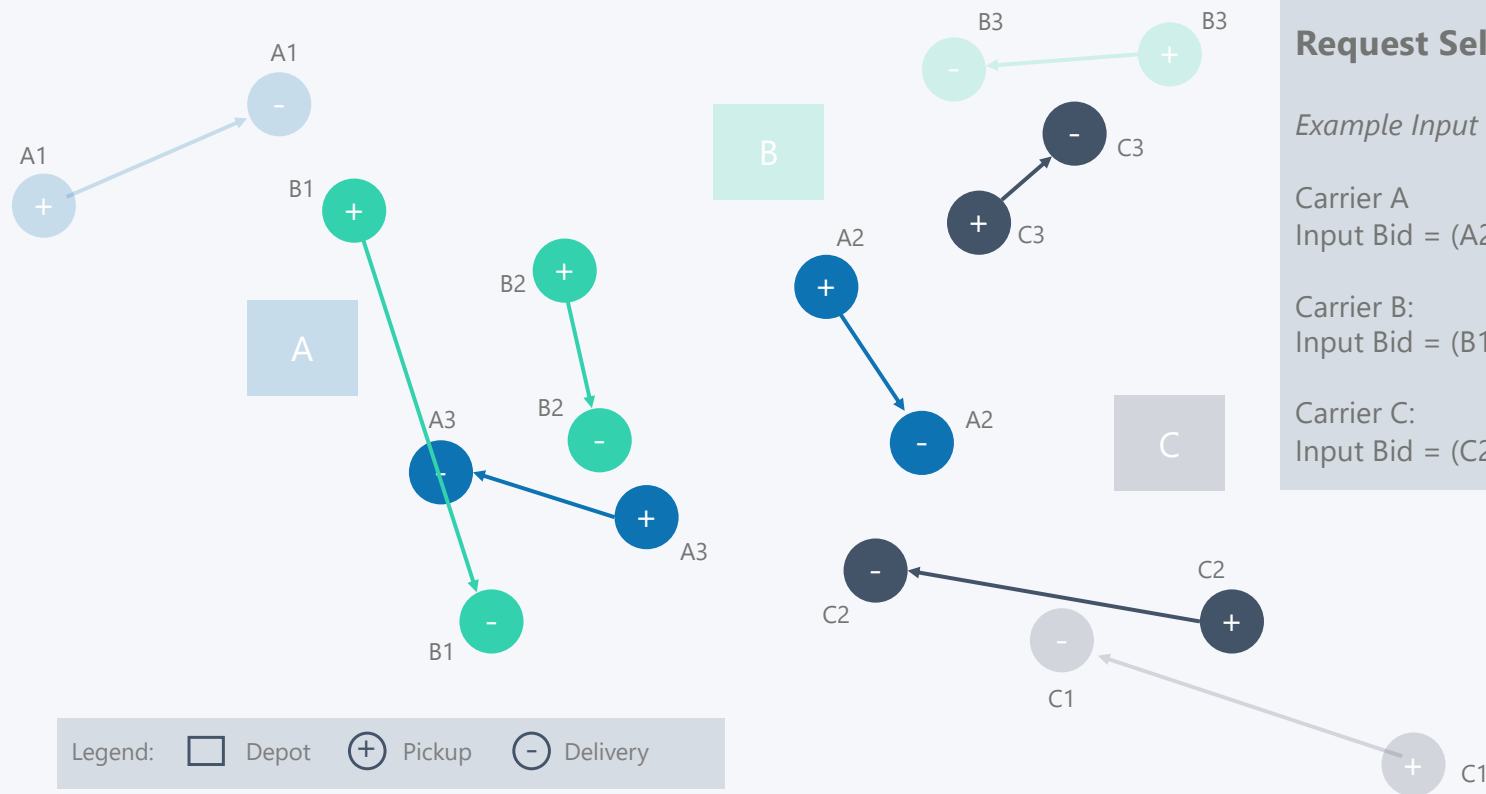
Implementation of Auction-based Transport Collaborations

## Initial Situation – Pickup-Delivery Requests



## Implementation of Auction-based Transport Collaborations

### Request Selection - Carriers



### Request Selection

*Example Input Bids:*

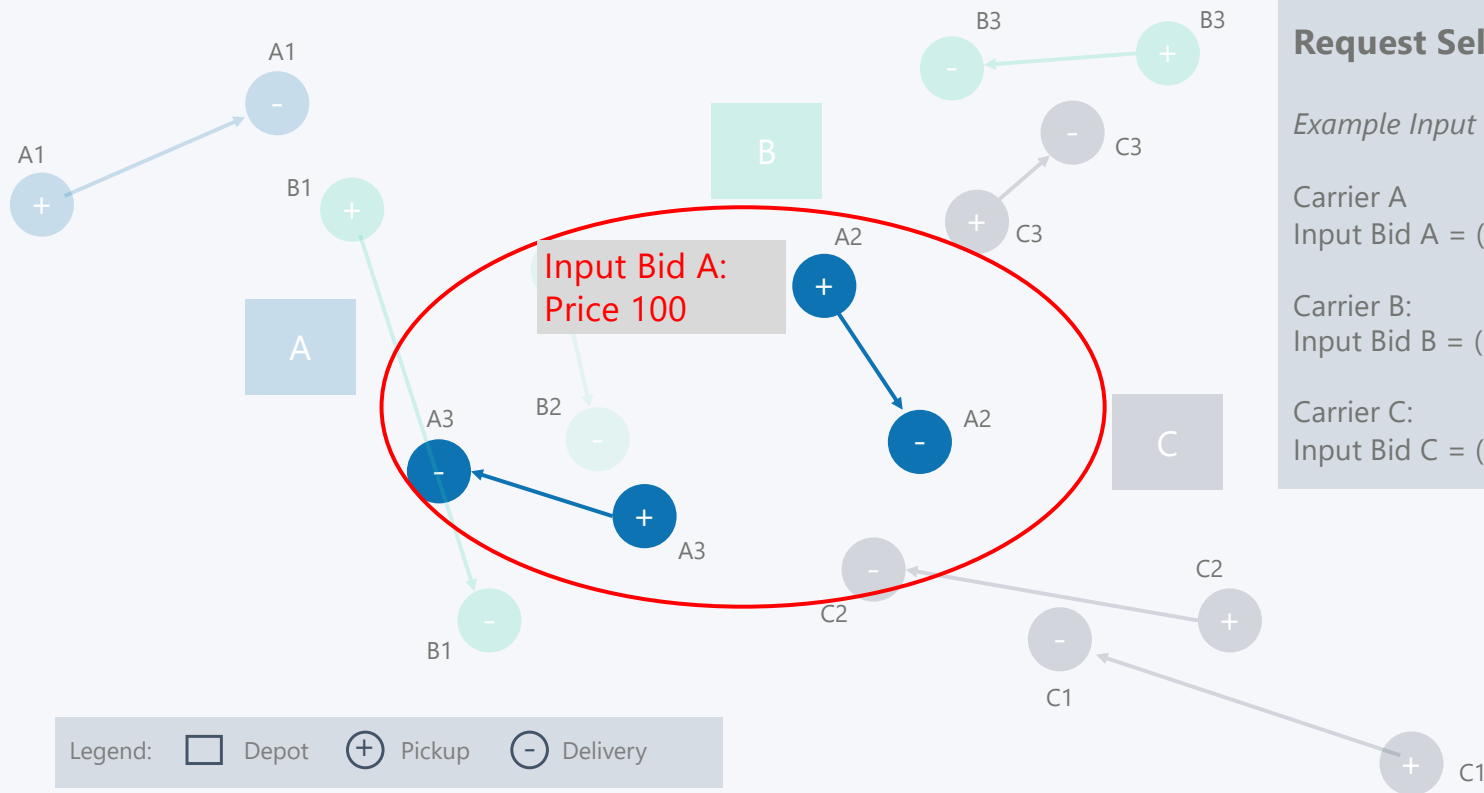
Carrier A  
Input Bid = (A2, A3) with price 100

Carrier B:  
Input Bid = (B1, B2) with price 130

Carrier C:  
Input Bid = (C2, C3) with price 90

## Implementation of Auction-based Transport Collaborations

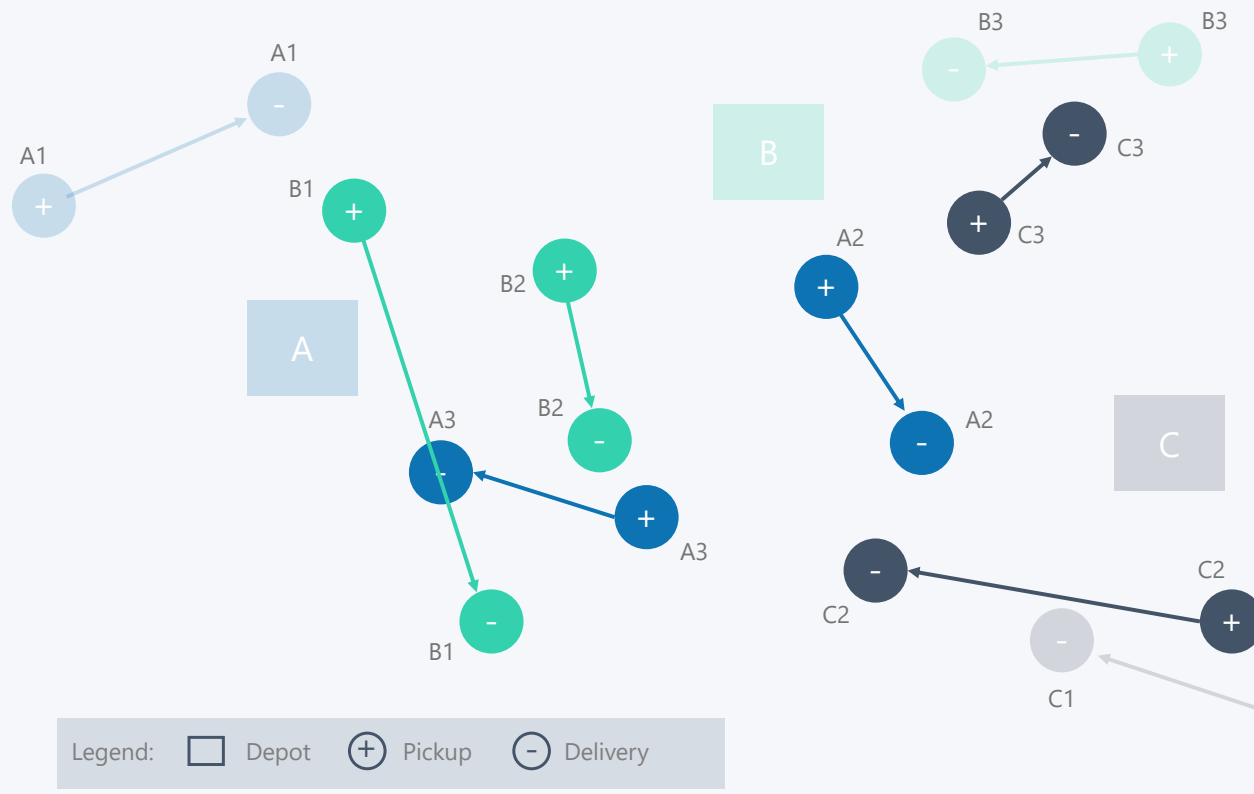
### Request Selection - Carriers



Strategic Manipulation of Bids in Auction-based Transport Collaborations

# Implementation of Auction-based Transport Collaborations

## Bundle Generation – Mechanism Manager



### Bundle Generation

*Example Bundles:*

Bundle 1: (B1, B2, A3)

Bundle 2: (A2, C3)

Bundle 3: (B1, A3)

Bundle 4: (C2)

Etc.

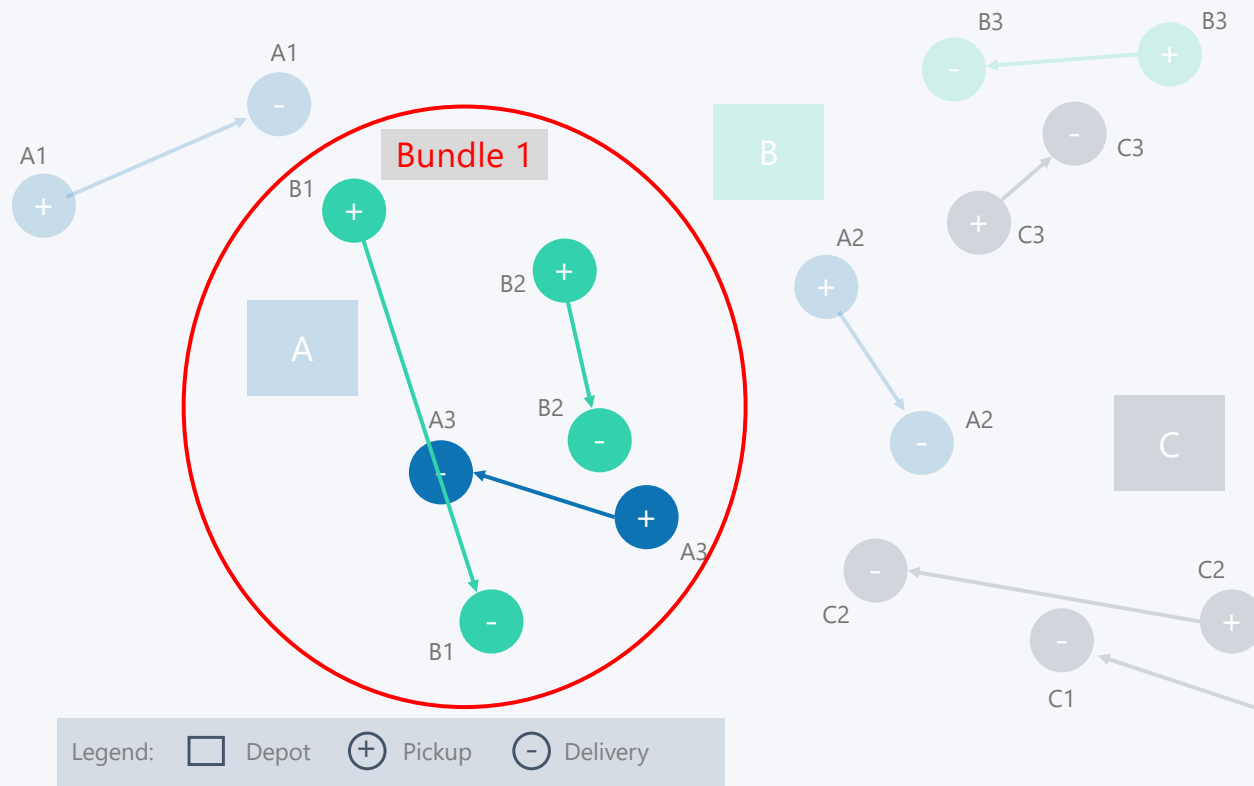
*Challenge*

$2^{(\text{Number of Requests})} - 1$  possible bundles

**Generating bundles is necessary because of synergy effects!**

# Implementation of Auction-based Transport Collaborations

## Bundle Generation – Mechanism Manager



Strategic Manipulation of Bids in Auction-based Transport Collaborations

### Bundle Generation

*Example Bundles:*

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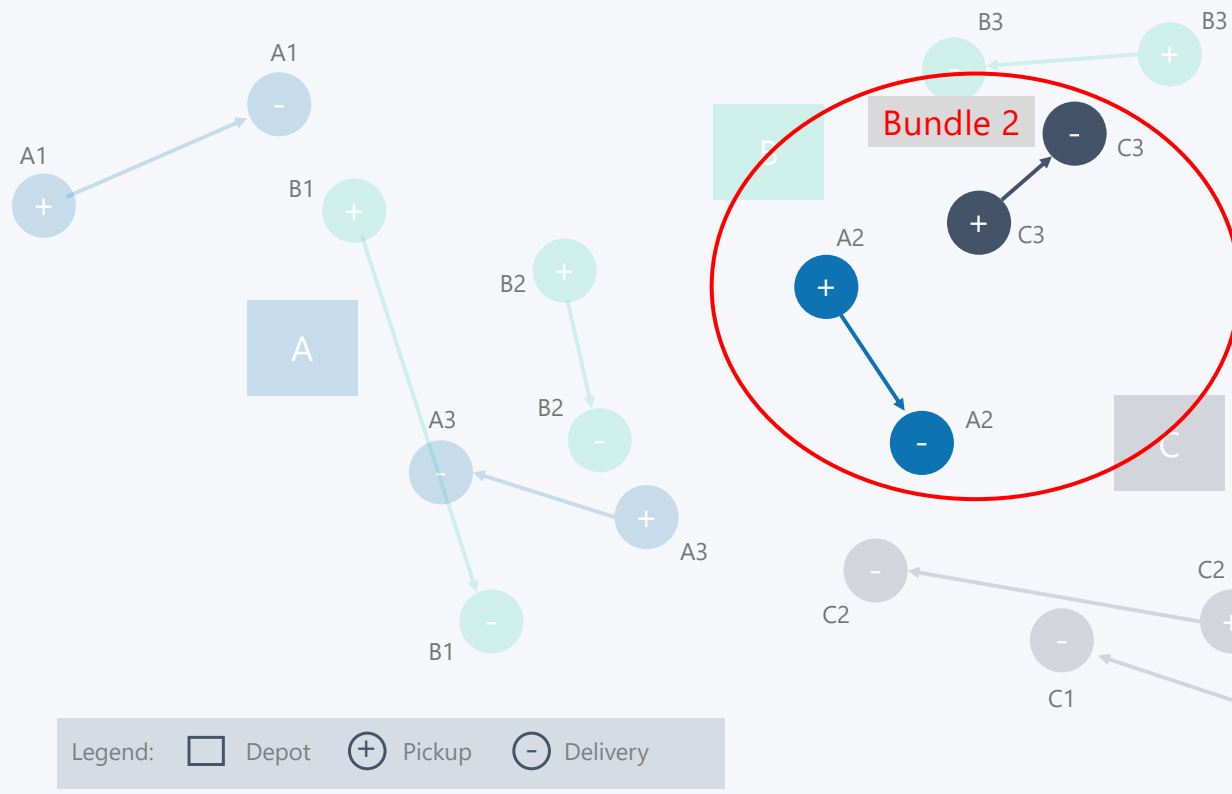
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## Bundle Generation – Mechanism Manager



### Bundle Generation

*Example Bundles:*

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Etc.

*Challenge*

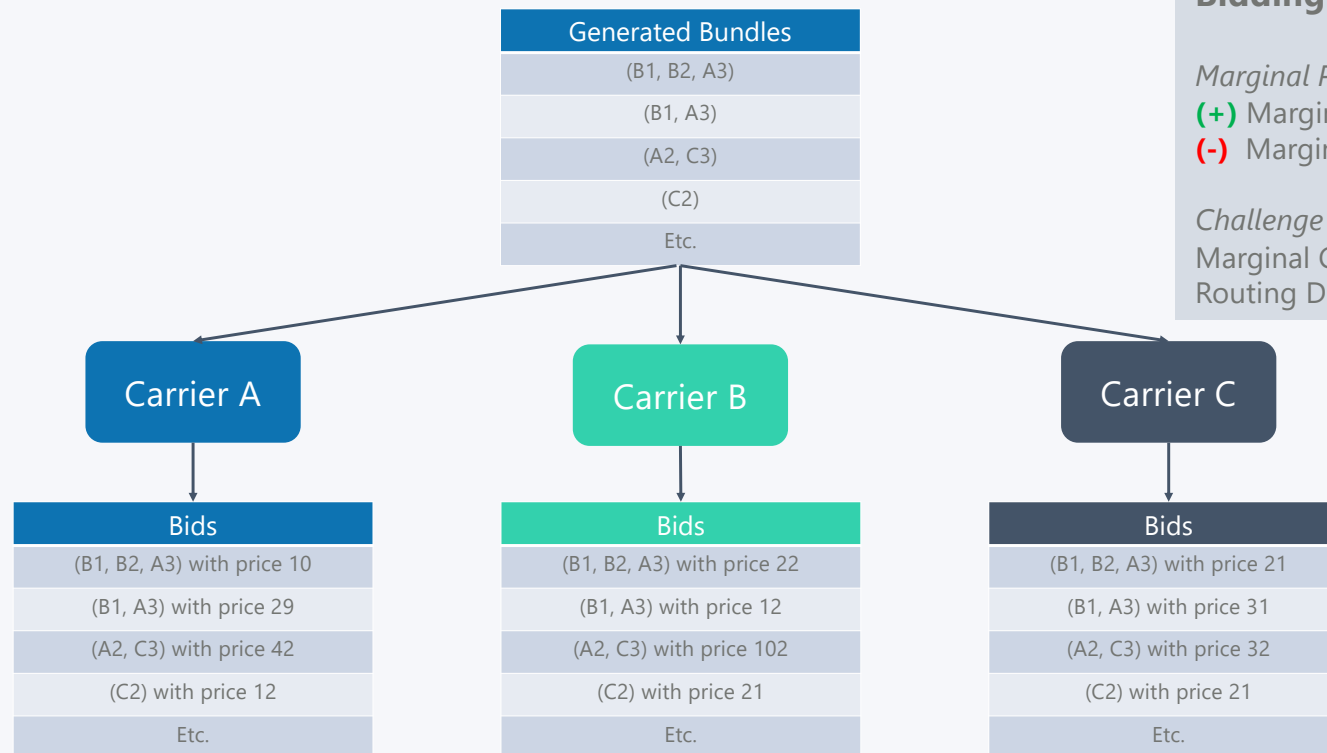
$2^{(\text{Number of Requests})} - 1$  possible bundles

**Generating bundles is necessary because of synergy effects!**

Legend: Depot Pickup Delivery

Strategic Manipulation of Bids in Auction-based Transport Collaborations

## Bidding - Carriers



### Bidding

*Marginal Profit of Bundle*

(+) Marginal Revenue

(-) Marginal Cost

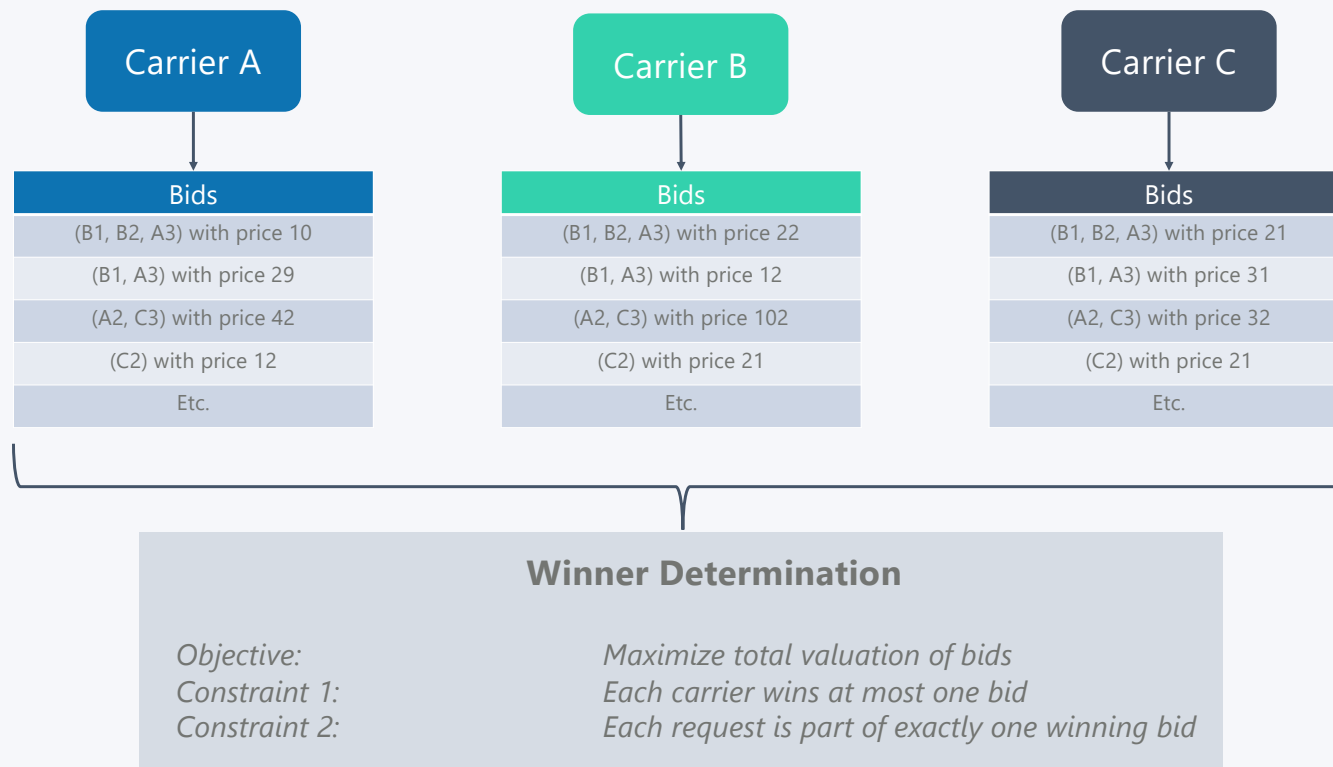
*Challenge*

Marginal Cost depend on Marginal Routing Distance (hard to compute!)



Implementation of Auction-based Transport Collaborations

## Winner Determination – Mechanism Manager



Implementation of Auction-based Transport Collaborations

## Payment Calculation – Mechanism Manager

### Winner Determination

*Objective:*

*Maximize total valuation of bids*

*Constraint 1:*

*Each carrier wins at most one bid*

*Constraint 2:*

*Each request is part of exactly one winning bid*

### Request Allocation

If (total valuation of winning bids > total valuation of input bids):

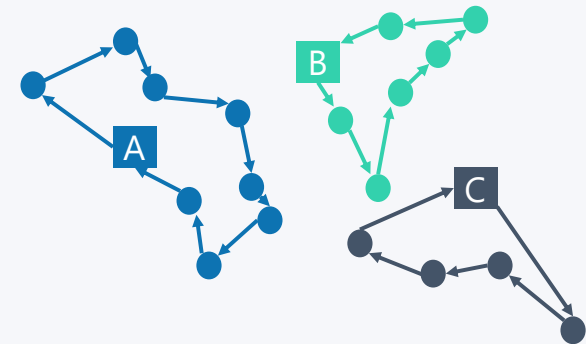
→ allocate requests according to the winning bids

Else:

→ Stop mechanism or go back to Request Selection

### Payment Calculation

Final Question: Who has to pay/gets paid (subject of next chapter)



?

## Implementation of Auction-based Transport Collaborations

### Setup – Auction-based Mechanism

Request Selection	Bundle Generation	Bidding	Winner Determination	Payment Calculation
<p>Requests selected based on:</p> <ul style="list-style-type: none"> <li>- Marginal profit</li> <li>- Distance to one's own depot</li> <li>- Distance to another carrier's depot</li> <li>- Closeness between each other</li> </ul> <p>→ Input Bid</p> <p>See [2]</p>	<p>All possible bundles of requests are offered</p> <p><i>Alternative:</i> Genetic Algorithm which selects the most attractive bundles (not used for tests)</p> <p><i>Challenge:</i></p> <ul style="list-style-type: none"> <li>- High synergy effects</li> <li>- Many possible bundles</li> </ul> <p>See [3]</p>	<p>Requires the carriers to calculate their marginal profit for each bundle</p> <p><i>Marginal Profit:</i> (+) Revenues of requests in the bundle (-) Marginal cost of including the bundle in route</p> <p><i>Routing Calculation:</i> Double Insertion with 3-opt (initial) or 2-opt improvement</p> <p><i>Strategies:</i> Truthful, Conspiring, Strategic</p> <p>See [11]</p>	<p>A candidate is a set of bids whereas each request has to be allocated to exactly one carrier</p> <p>Winning Candidate = Most valuable Candidate</p> <p>Optimization program: Set partitioning problem (solved optimally with Google OR-Tools)</p> <p>See [1]</p>	<p>Different Payment Approaches:</p> <ul style="list-style-type: none"> <li>- Egalitarian</li> <li>- Purchase/Sale Weights</li> <li>- Shapley Value</li> <li>- Critical Weight (explained later)</li> </ul>

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Strategic Manipulation of Bids in Auction-based Transport Collaborations

See [2]

## Implementation of Auction-based Transport Collaborations

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Strategic Manipulation of Bids in Auction-based Transport Collaborations

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## Implementation of Auction-based Transport Collaborations

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Strategic Manipulation of Bids in Auction-based Transport Collaborations

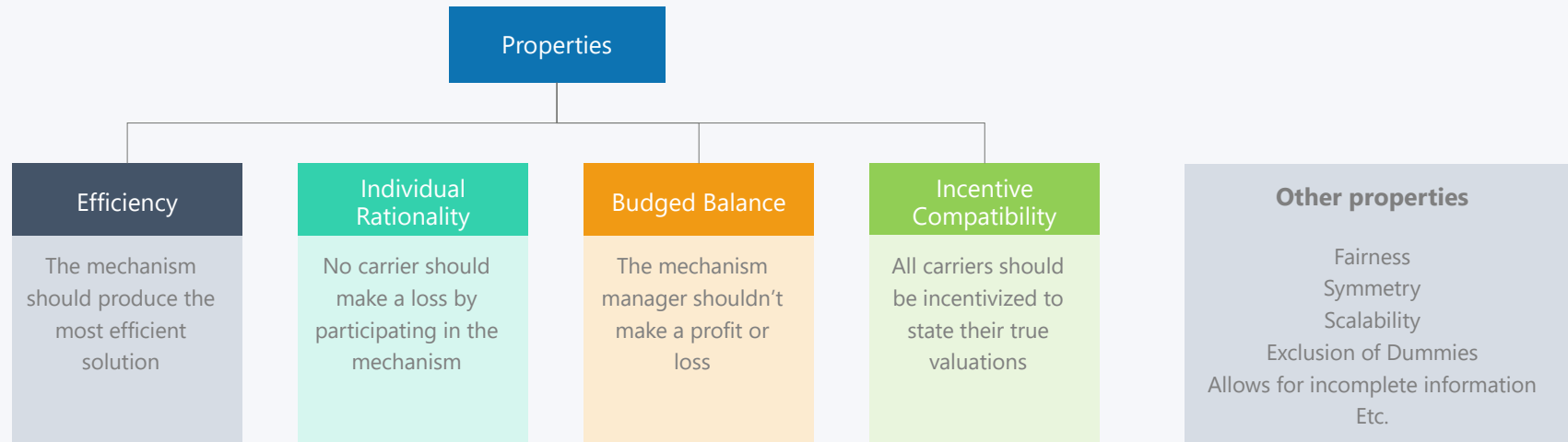
See [2]



## 4.2

## Bidding Strategies for Modified Egalitarian Profit Sharing

## Desirable Properties



See [6]

e.g., see [9], [11]

**Problem:** Not all properties can be achieved simultaneously

## 3

## Payment Calculation and Profit Sharing Methods in Auction-based Transport Collaborations

## Generic Payment Method

1

**Collect** the money from all carriers who won a bid (on a bundle of requests)

2

**Pay** every carrier her/his valuation of her/his offered requests (Input Bid)

3

**Share** the remaining collaboration gain between the carriers

Sum of Winning Bid Prices

Sum of Input Bid Prices

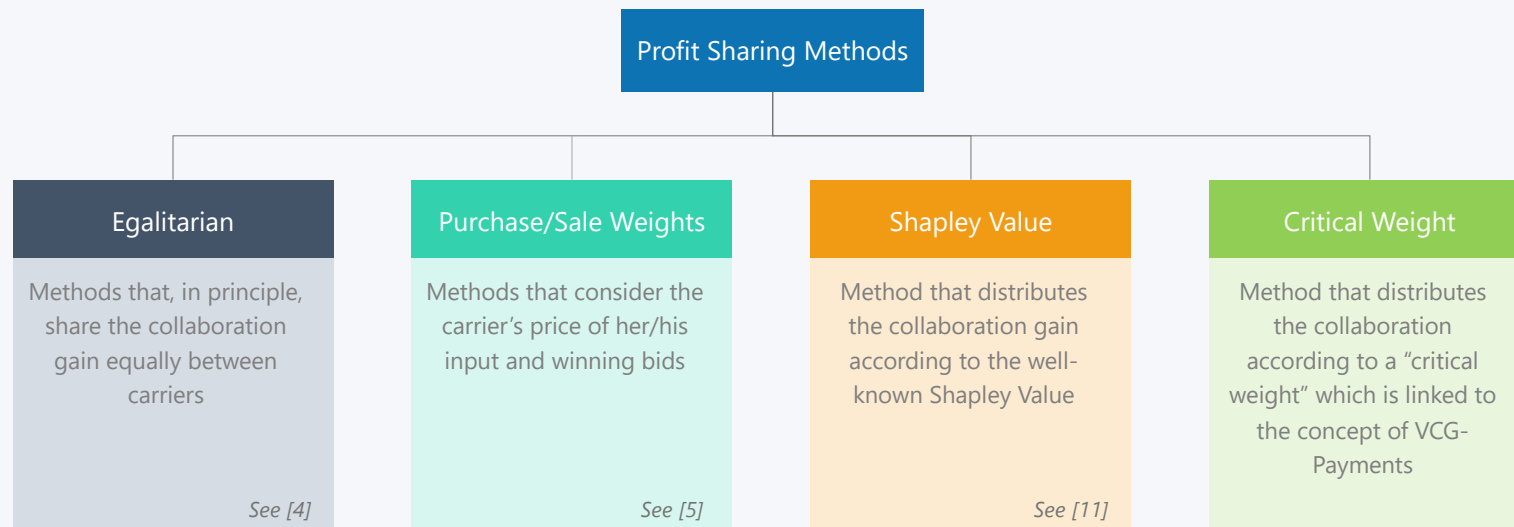
Collaboration Gain

Challenge: How to distribute?

**Guaranteed Properties:** Individual Rationality; Budget Balance

**Challenge:** Incentive Compatibility or rather the mitigation of strategic manipulation

## Analyzed Profit Sharing Methods



## 4

## Strategic Manipulation of Bids in Auction-based Transport Collaborations

## Different Bidders for Evaluation

### Truthful Bidders

Always bid truthfully their valuations

→ Used to **evaluate** the **truthful outcome** of the mechanism

→ Help to **evaluate** the strategic potential of a **single strategic/conspiring carrier**

### Conspiring Bidders

Receive information about all bid prices  
+  
Use the information to manipulate their bid prices

→ Used to **evaluate** the **upside of strategic manipulation**

→ Help to get **insight** about the construction of **successful strategies**

### Strategic Bidders

Manipulate their bid prices

→ Used to **evaluate** the **potential of realistic strategic behavior**

→ Help to **estimate** the **likelihood** that **carriers will act strategically**

## Tests Configuration

Property	Value
Number of carriers	3
Initial number of requests per carrier	9
Competition Level	Medium (see [2], "02")
Number of traded requests per carrier (per mechanism round)	3
Number of instances per test	100
Max capacity of carrier	1,3x distance of initial routing solution
Min number of maintained requests per carrier	4
Number of retries of request selection (if no improvement)	2
Default bidding strategy	Truthful
Profitability	All Equal



## 4.1

## Bidding Strategies for Egalitarian Profit Sharing

**Profit Sharing Rule:**

Share the collaboration gain equally  
between the carriers

*See [4]*



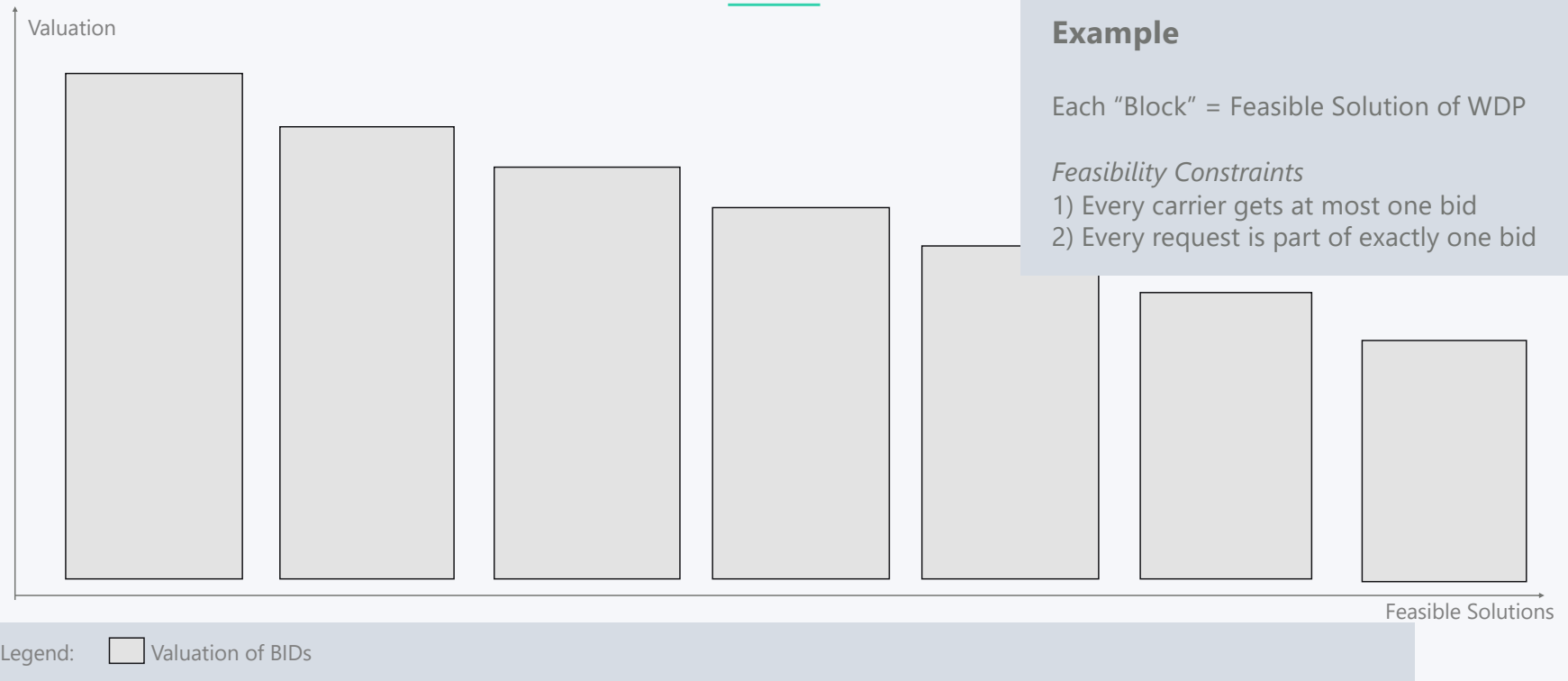
Bidding Strategies for Egalitarian Profit Sharing

## Perspective of Conspiring Bidder



Bidding Strategies for Egalitarian Profit Sharing

## Perspective of Conspiring Bidder



Bidding Strategies for Egalitarian Profit Sharing


## Perspective of Conspiring Bidder



### Example

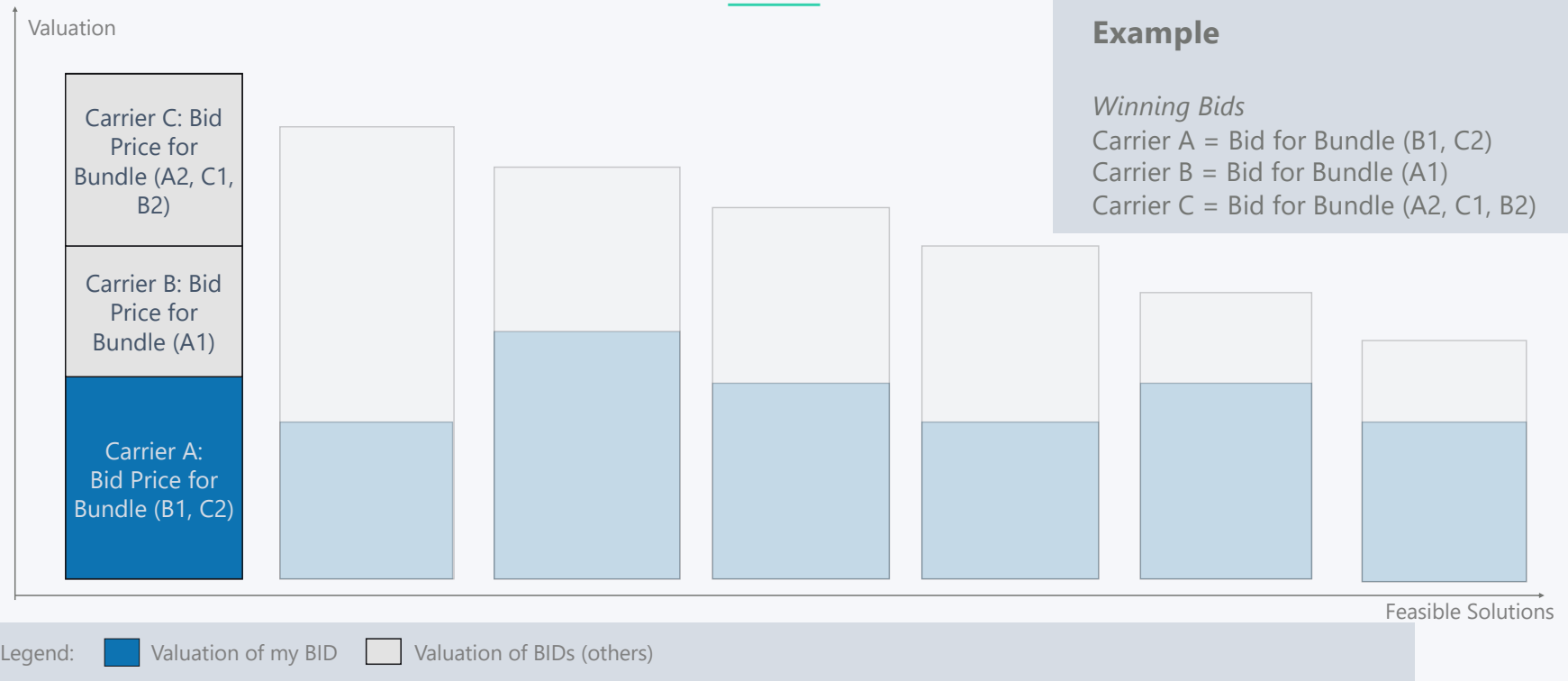
Each "Block" = Feasible Solution of WDP

→ Ranking of Feasible Solutions

Legend:  Valuation of BIDs

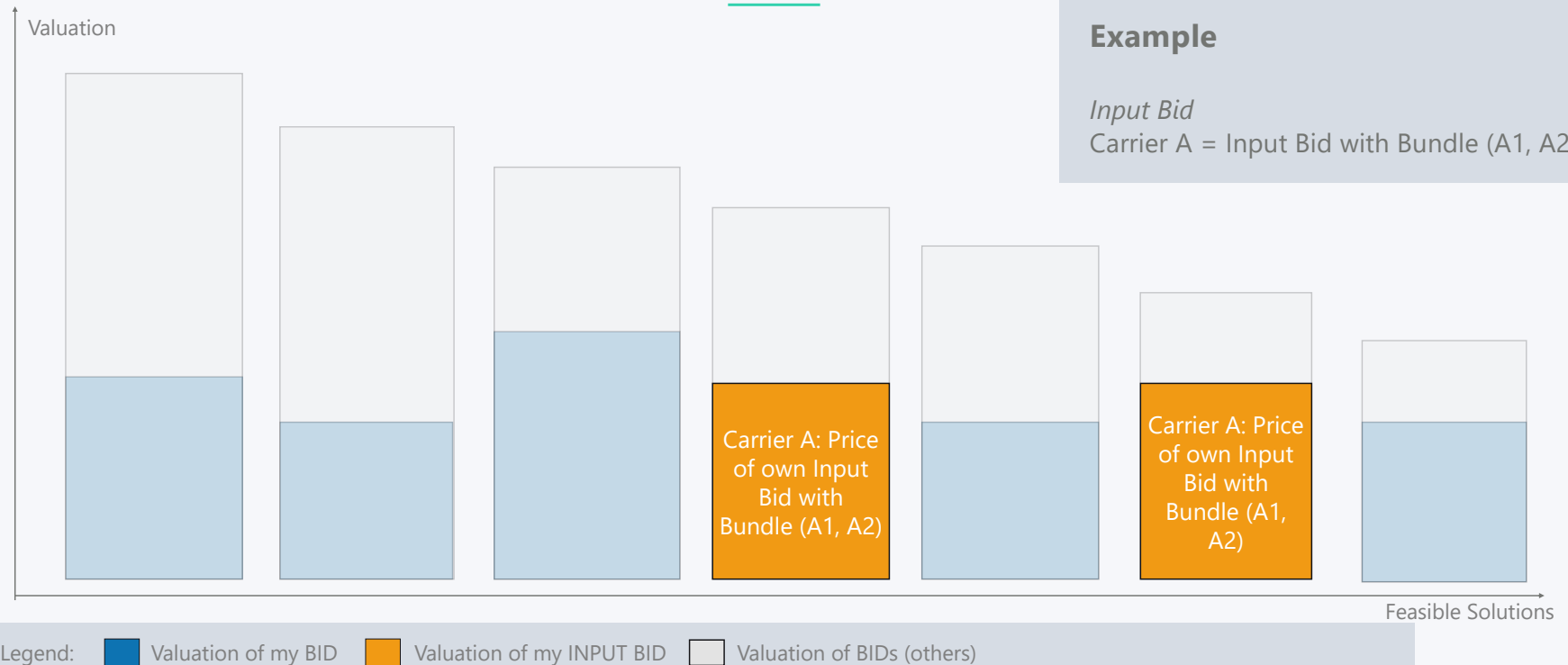
## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



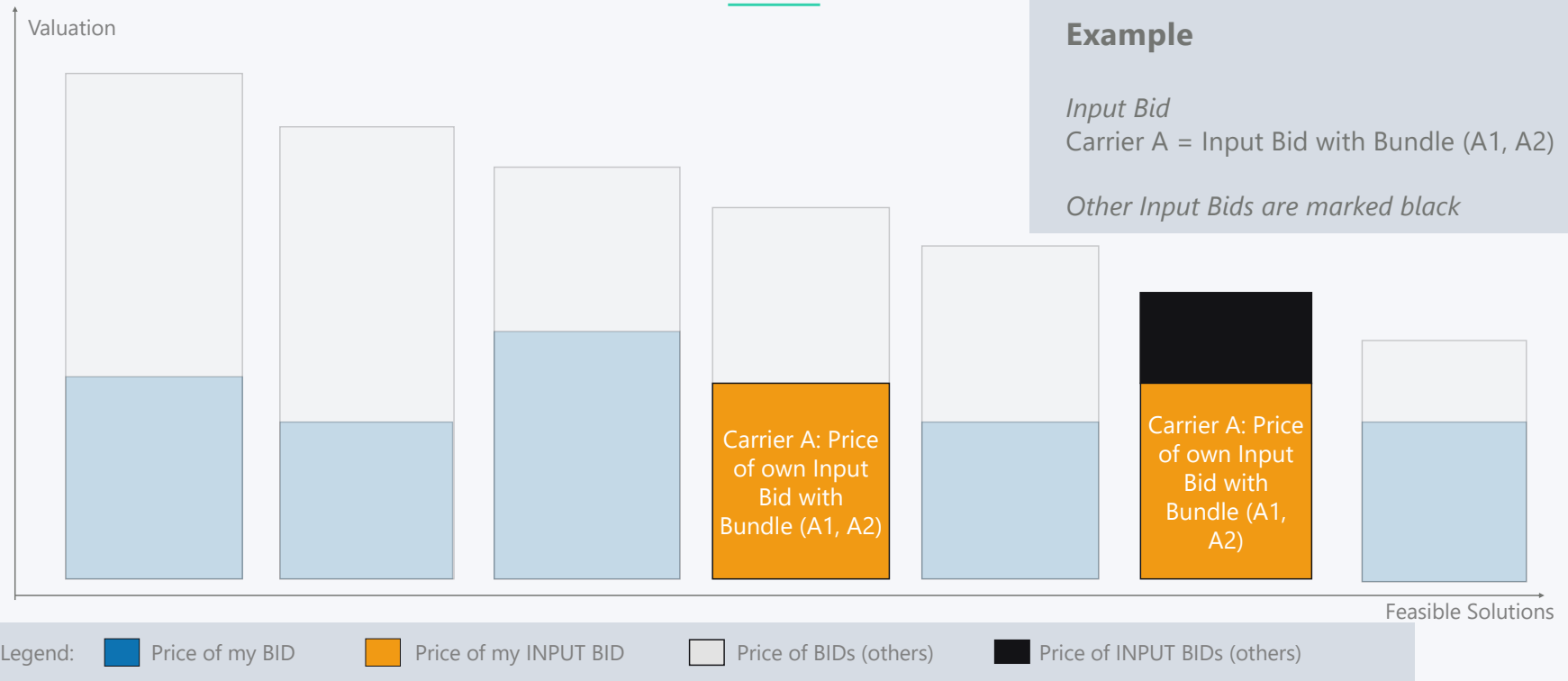
## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



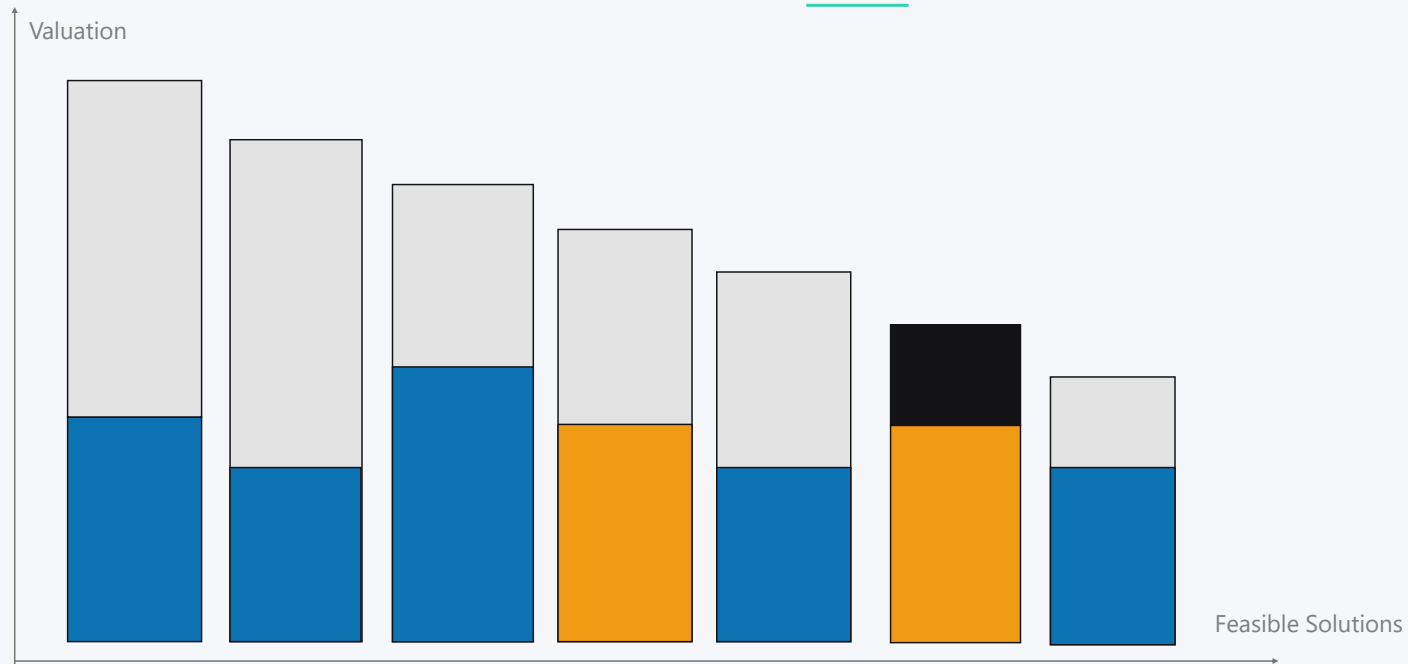
## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



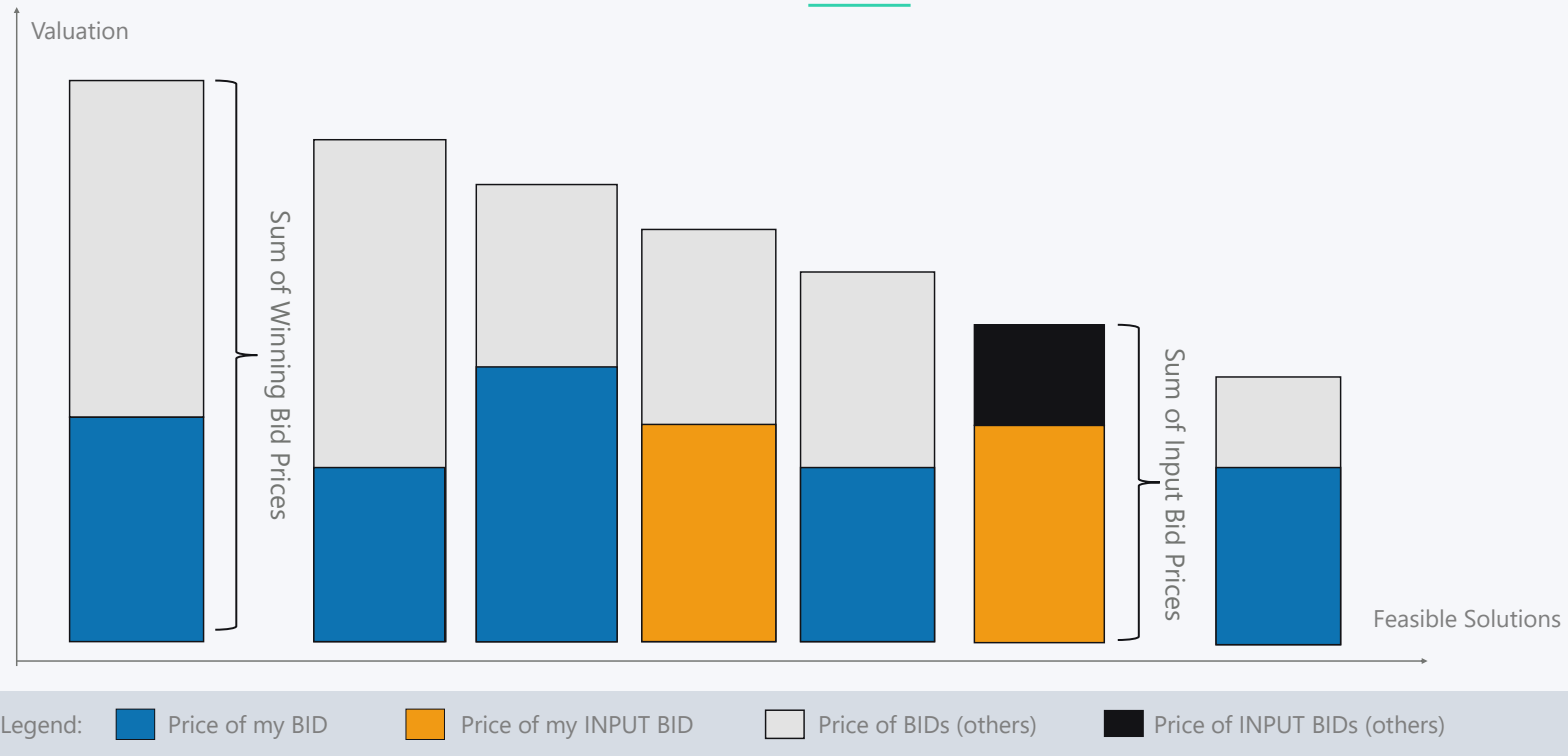


## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder

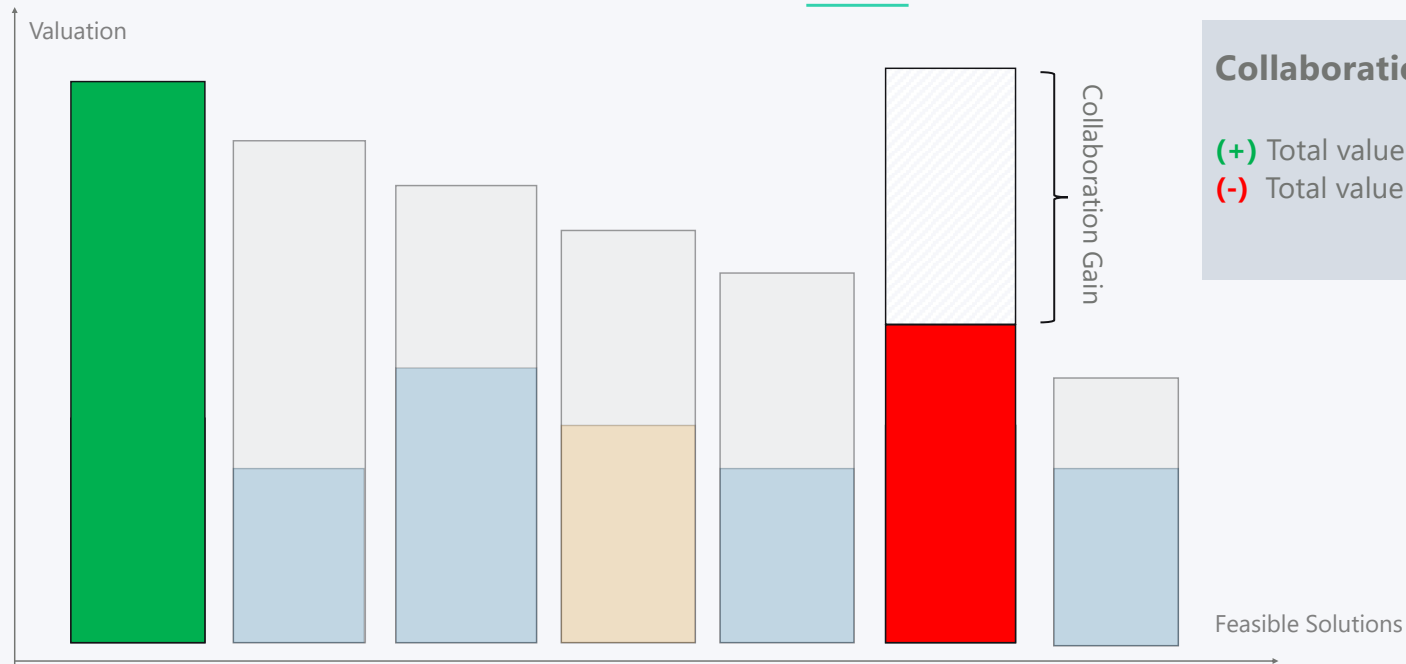


Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)

## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder



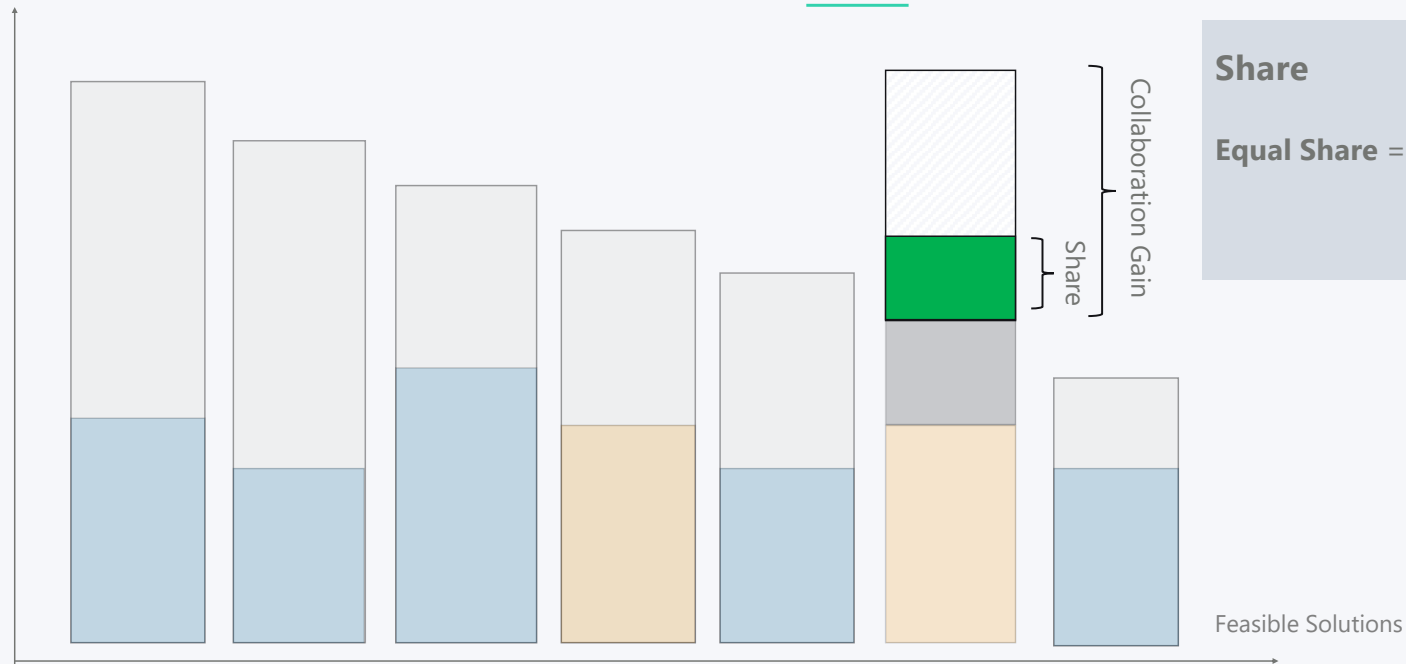
## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder



### Collaboration Gain

- (+) Total value of Winning Bid prices
- (-) Total value of Input Bid prices

## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder



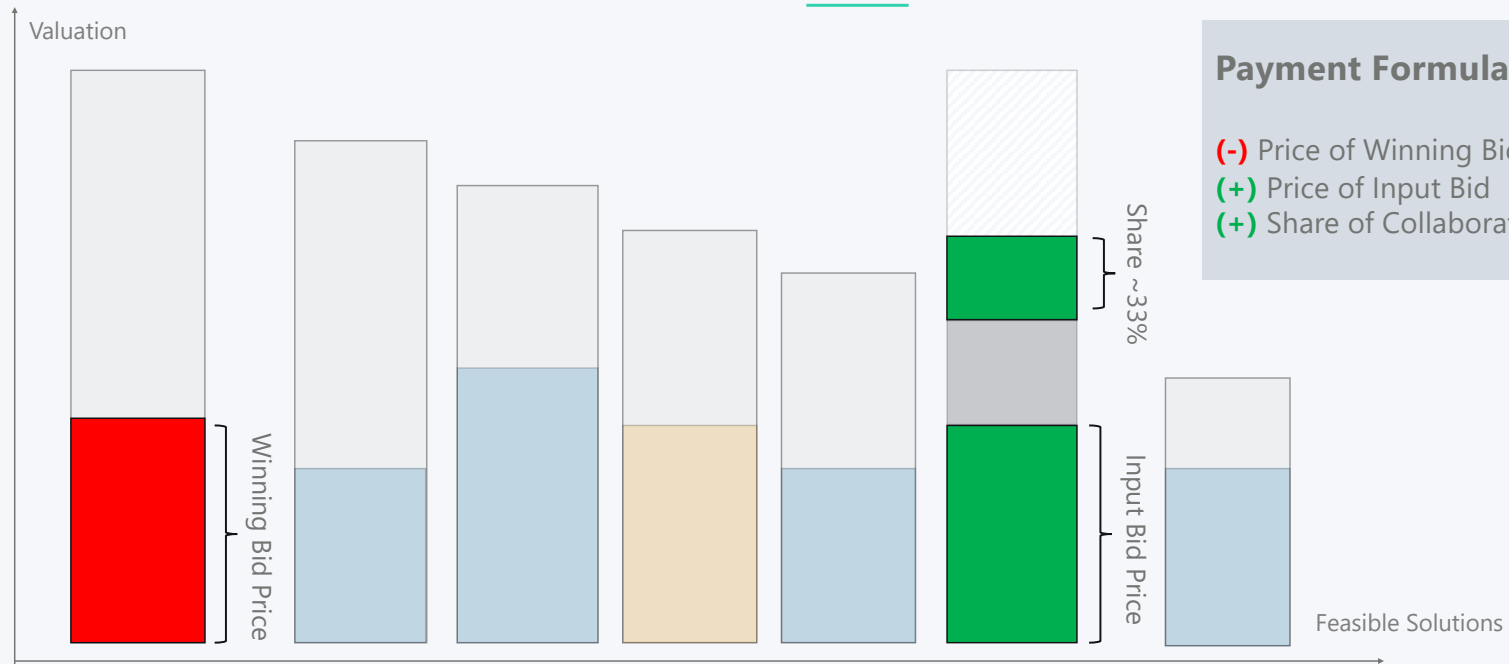
**Share**

**Equal Share** =  $1 / (\text{number of carriers})$

Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)  
■ Gain ■ Loss

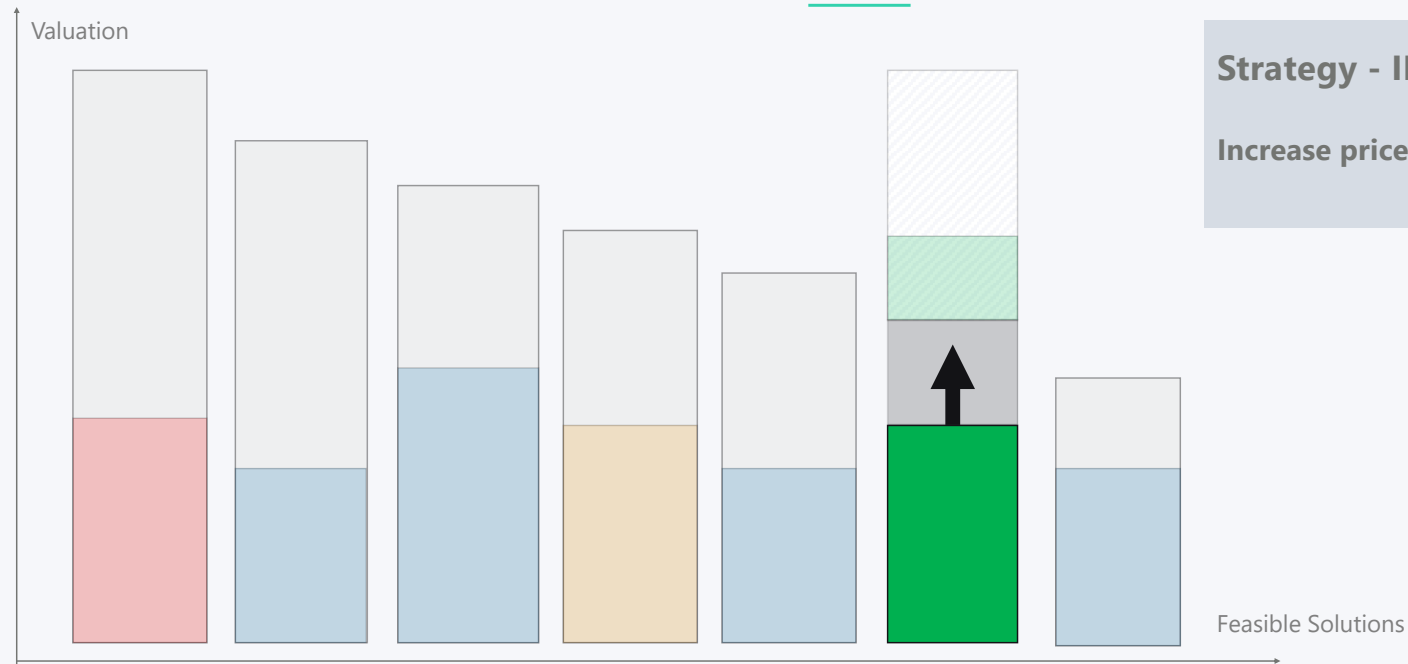
Strategic Manipulation of Bids in Auction-based Transport Collaborations

## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder



## Bidding Strategies for Egalitarian Profit Sharing

### Perspective of Conspiring Bidder

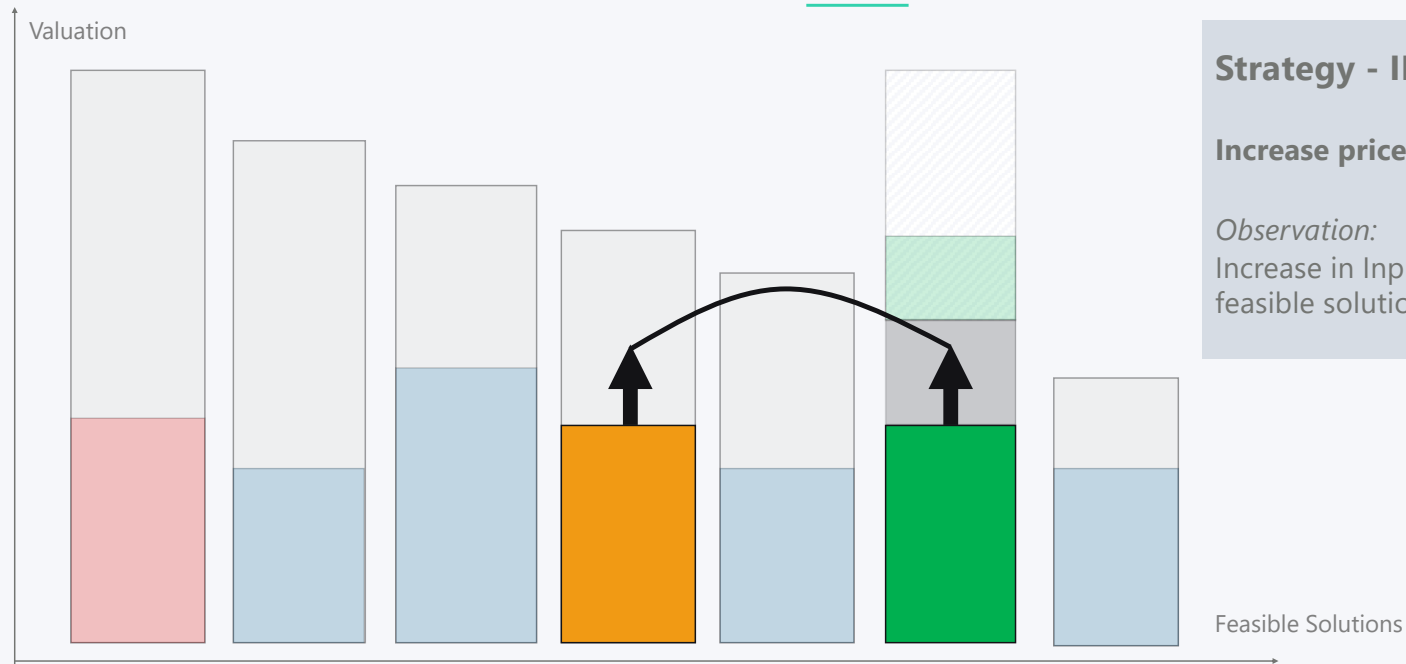


Legend:

- Price of my BID
- Price of my INPUT BID
- Price of BIDs (others)
- Price of INPUT BIDs (others)
- Gain
- Loss

## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



### Strategy - INPUT\_MAX

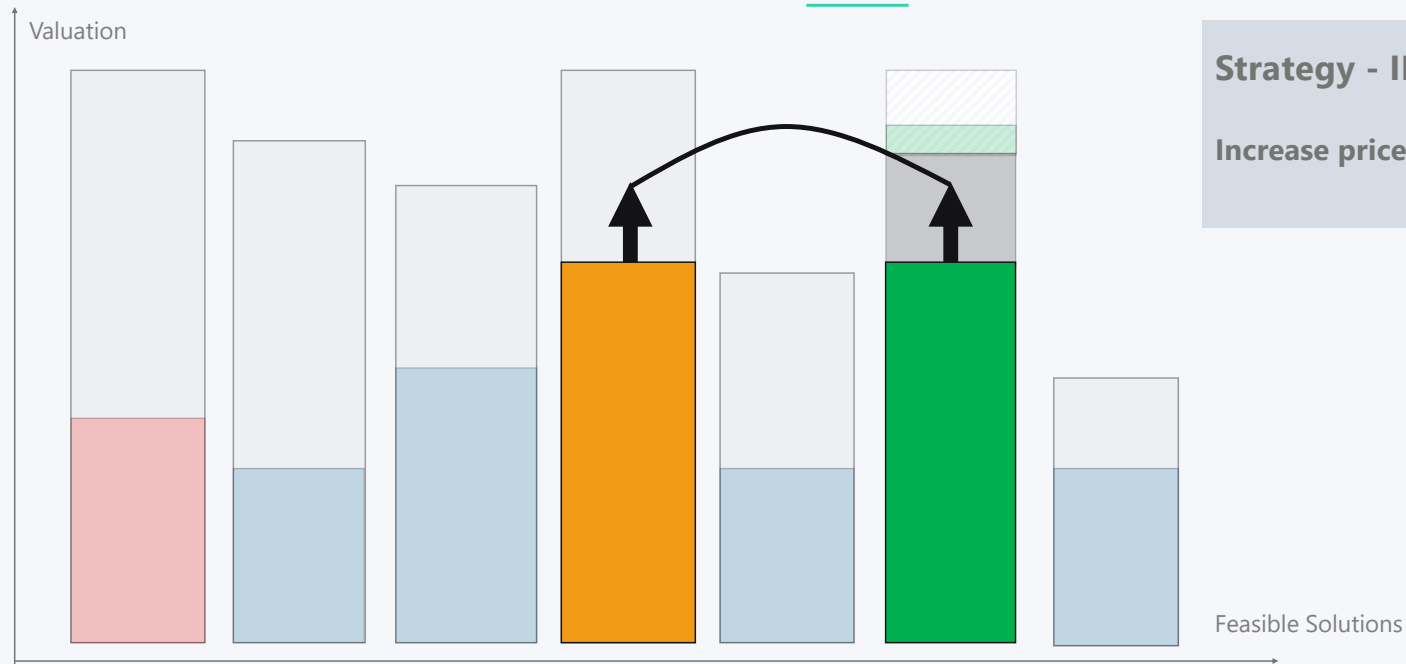
**Increase price of Input Bid**

*Observation:*

Increase in Input Bid price effects multiple feasible solutions

Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)  
■ Gain ■ Loss

## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder



**Strategy - INPUT\_MAX**

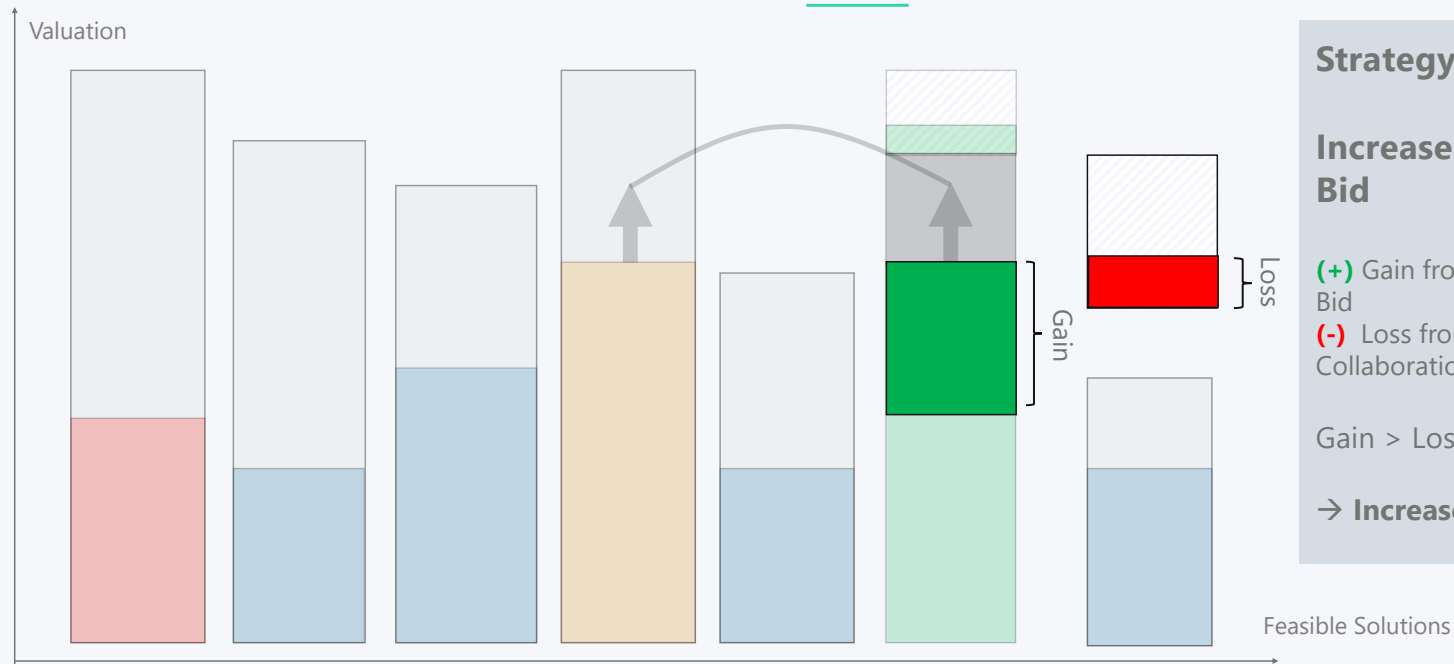
**Increase price of Input Bid**

Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)  
■ Gain ■ Loss



## Bidding Strategies for Egalitarian Profit Sharing

### Perspective of Conspiring Bidder



#### Strategy - INPUT\_MAX

#### Increase price of Input Bid

(+) Gain from overbidding on Input Bid

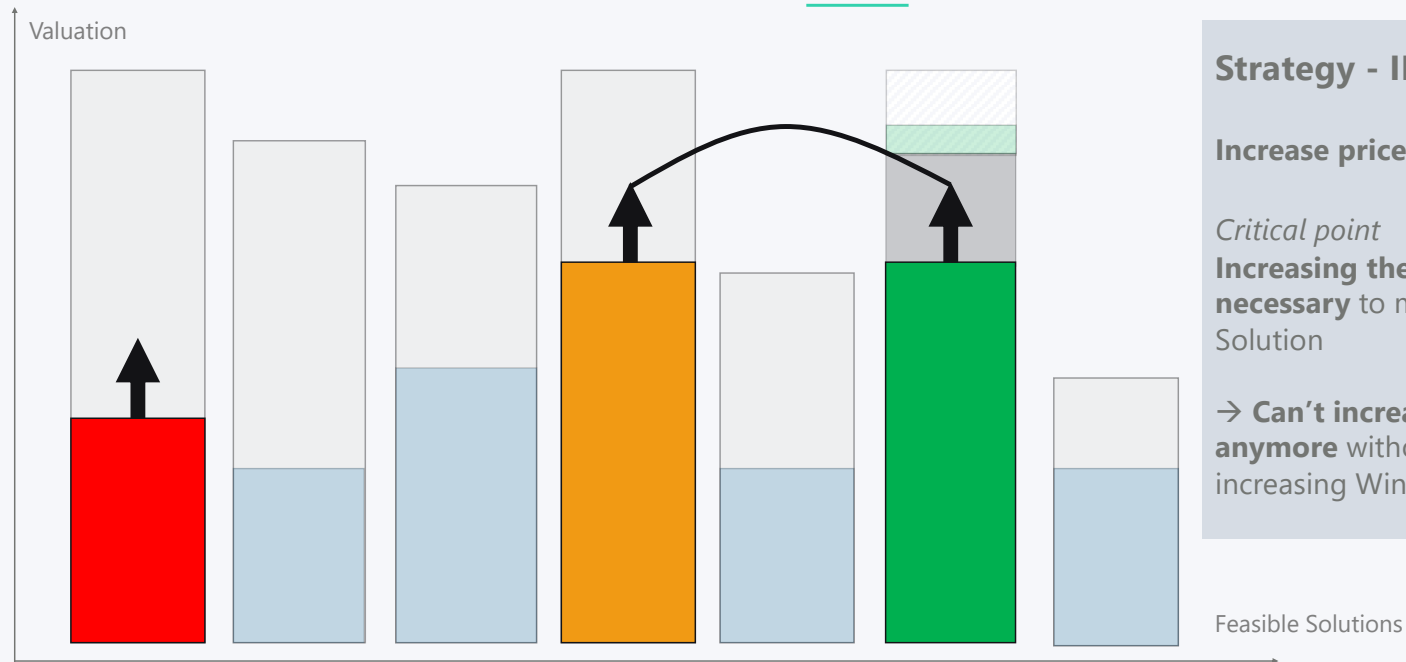
(-) Loss from decreasing Collaboration Gain

Gain > Loss

→ Increase of Profit

Legend:   
■ Price of my BID   
■ Price of my INPUT BID   
■ Price of BIDs (others)   
■ Price of INPUT BIDs (others)   
■ Gain   
■ Loss

## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder



### Strategy - INPUT\_MAX

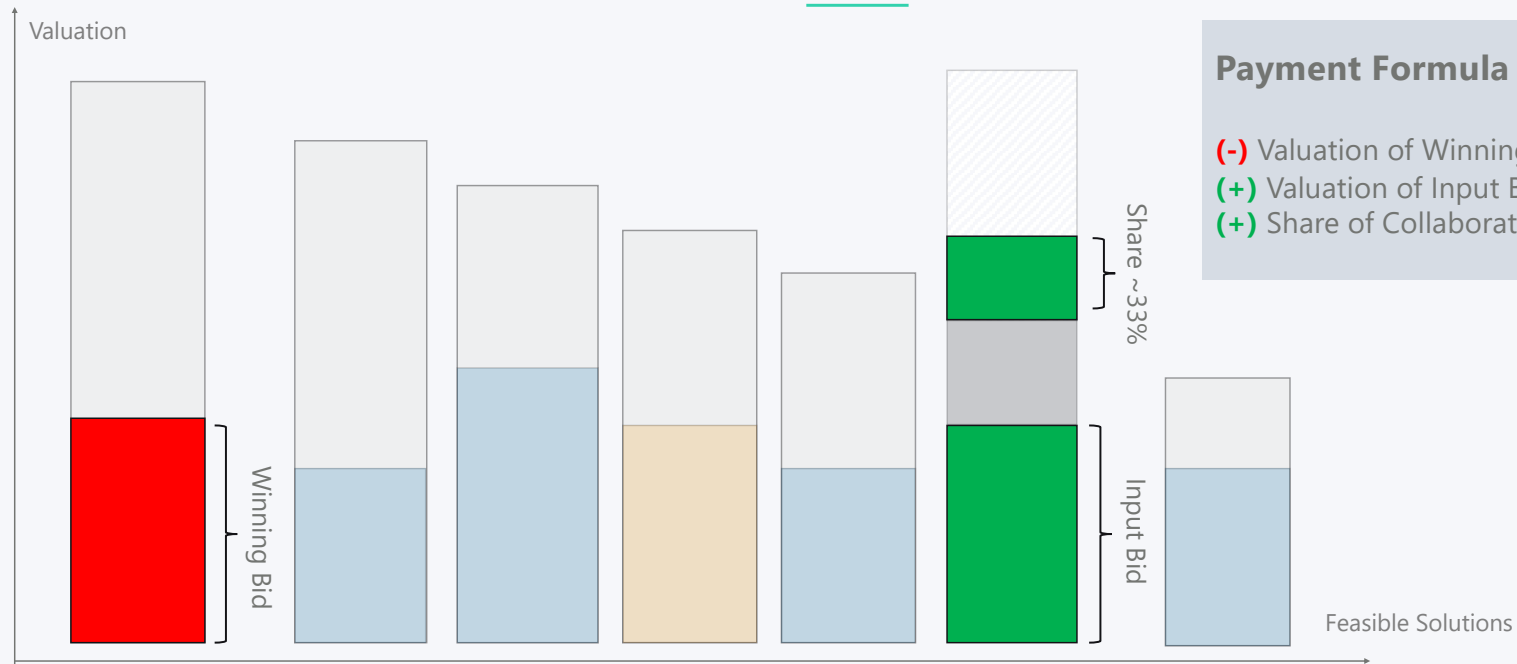
Increase price of Input Bid

*Critical point*

**Increasing the Winning Bid price is necessary** to maintain the Winning Bid Solution

→ **Can't increase** the Input But price **anymore** without simultaneously increasing Winning Bid price

## Bidding Strategies for Egalitarian Profit Sharing Perspective of Conspiring Bidder



### Payment Formula

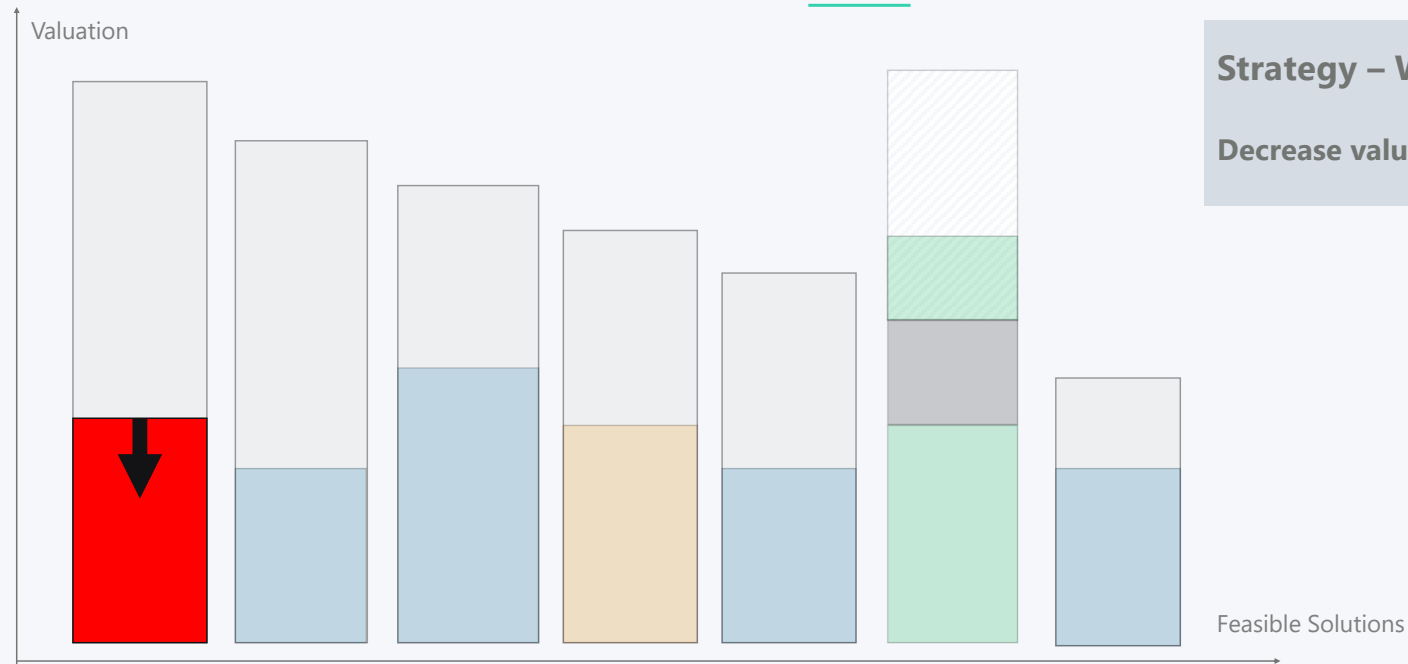
- (-) Valuation of Winning Bid
- (+) Valuation of Input Bid
- (+) Share of Collaboration Gain

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)

■ Gain ■ Loss

## Bidding Strategies for Egalitarian Profit Sharing

### Perspective of Conspiring Bidder



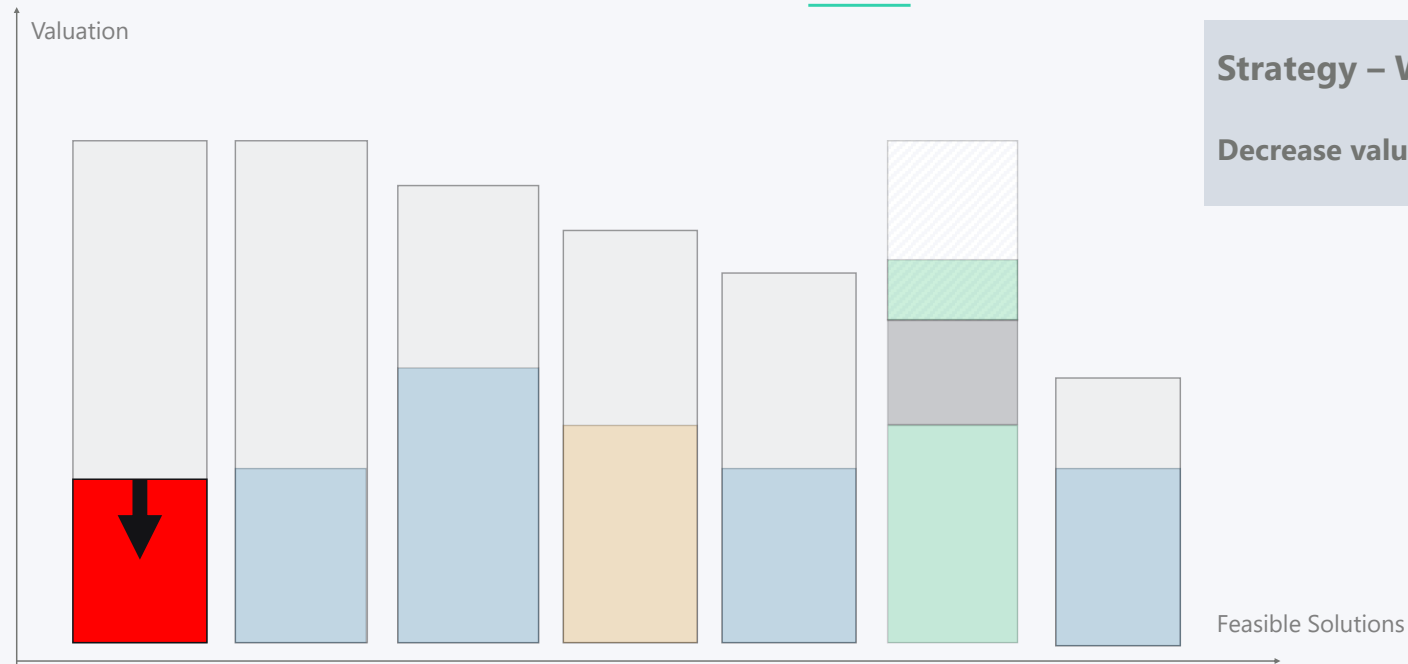
**Strategy – WIN\_LOW**

**Decrease valuation of Winning Bid**

Legend:   
 ■ Valuation of my BID   ■ Valuation of my INPUT BID   ■ Valuation of BIDs (others)   ■ Valuation of INPUT BIDs (others)   
 ■ Gain   ■ Loss

## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



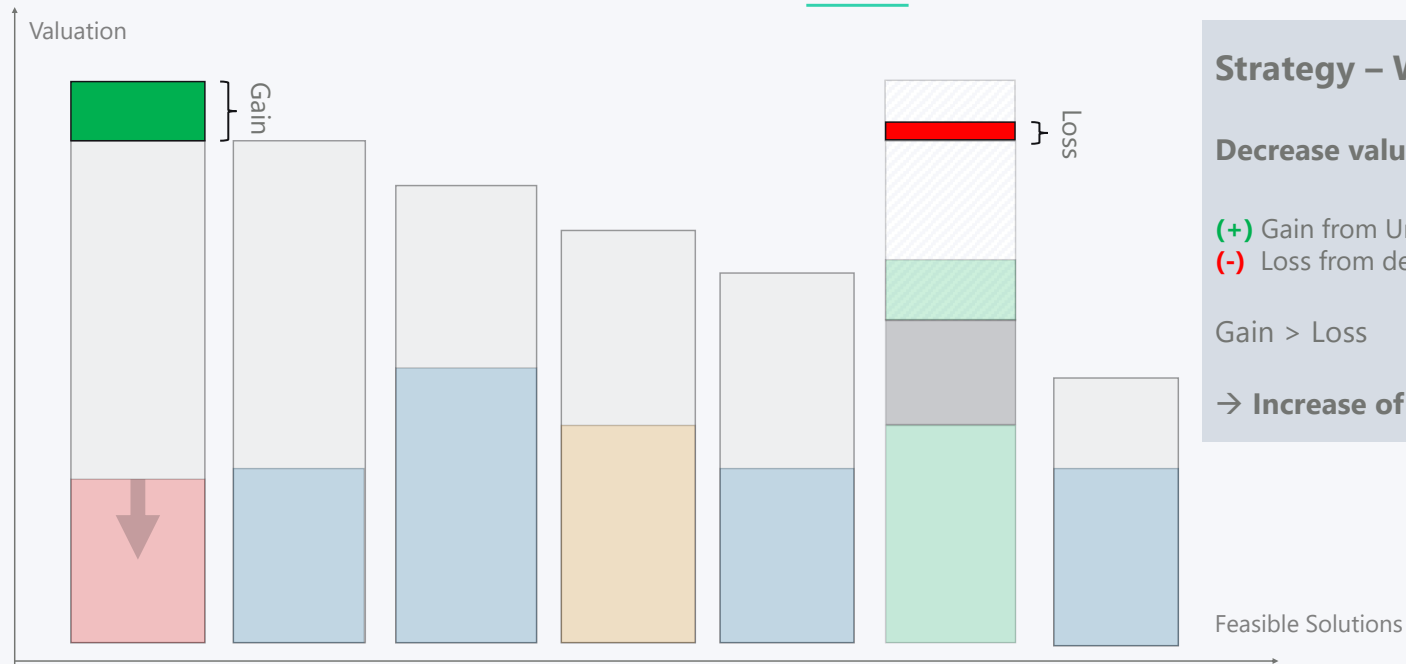
**Strategy – WIN\_LOW**

**Decrease valuation of Winning Bid**

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)  
■ Gain ■ Loss

## Bidding Strategies for Egalitarian Profit Sharing

### Perspective of Conspiring Bidder



#### Strategy – WIN\_LOW

Decrease valuation of Winning Bid

(+) Gain from Underbidding

(-) Loss from decreasing Collaboration Gain

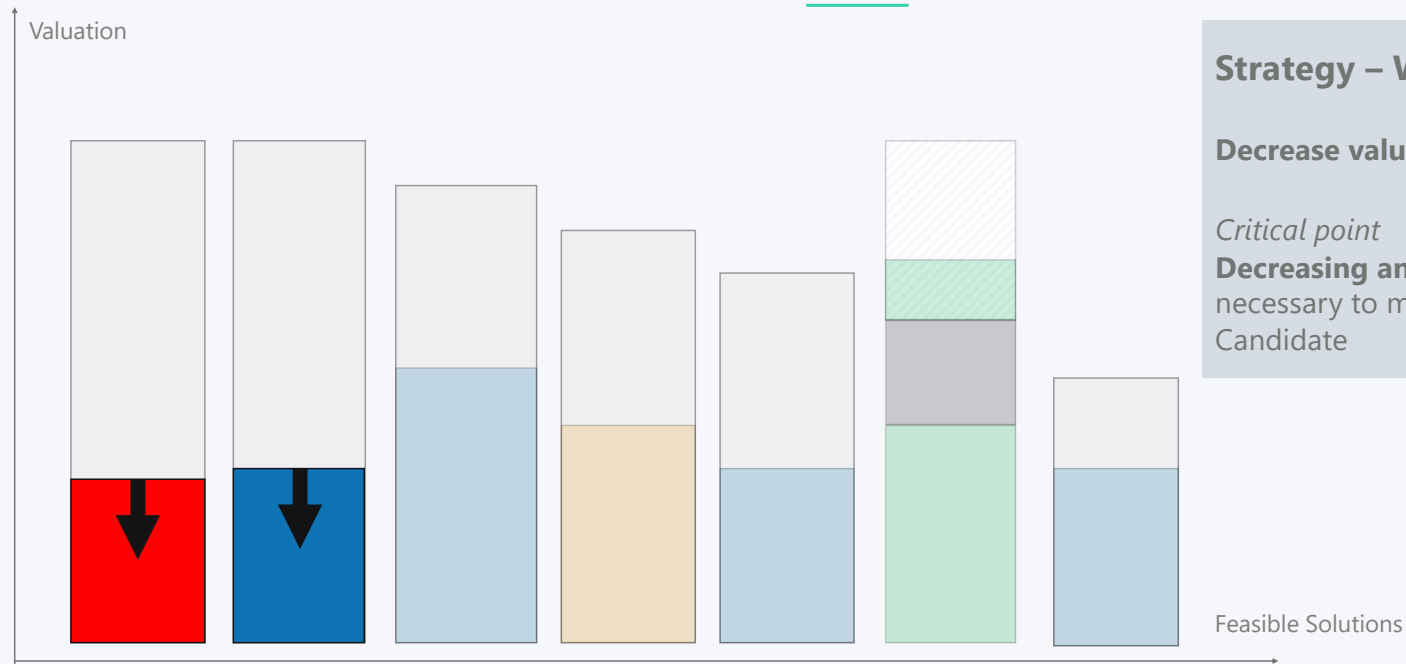
Gain > Loss

→ Increase of Profit

Legend:   
■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)   
■ Gain ■ Loss

## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



### Strategy – WIN\_LOW

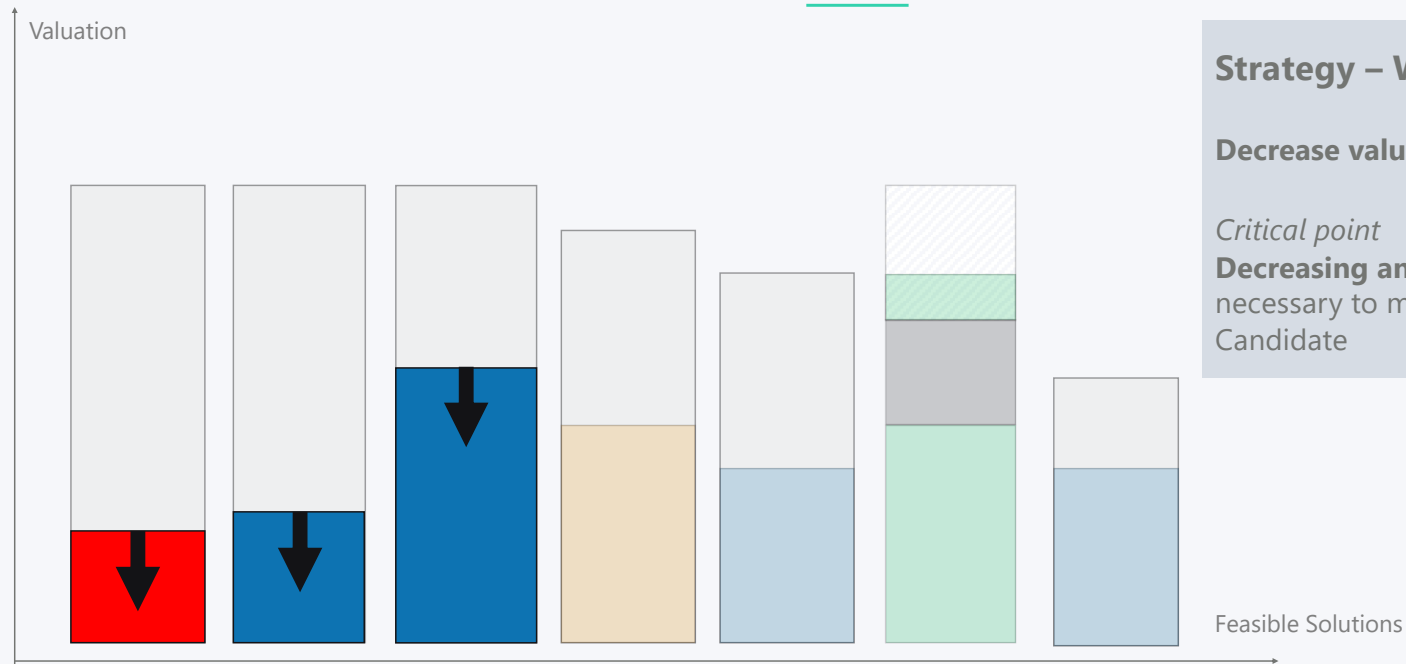
Decrease valuation of Winning Bid

*Critical point*

**Decreasing another Bid Valuation** is necessary to maintain the same Winning Candidate

## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



### Strategy – WIN\_LOW

Decrease valuation of Winning Bid

*Critical point*

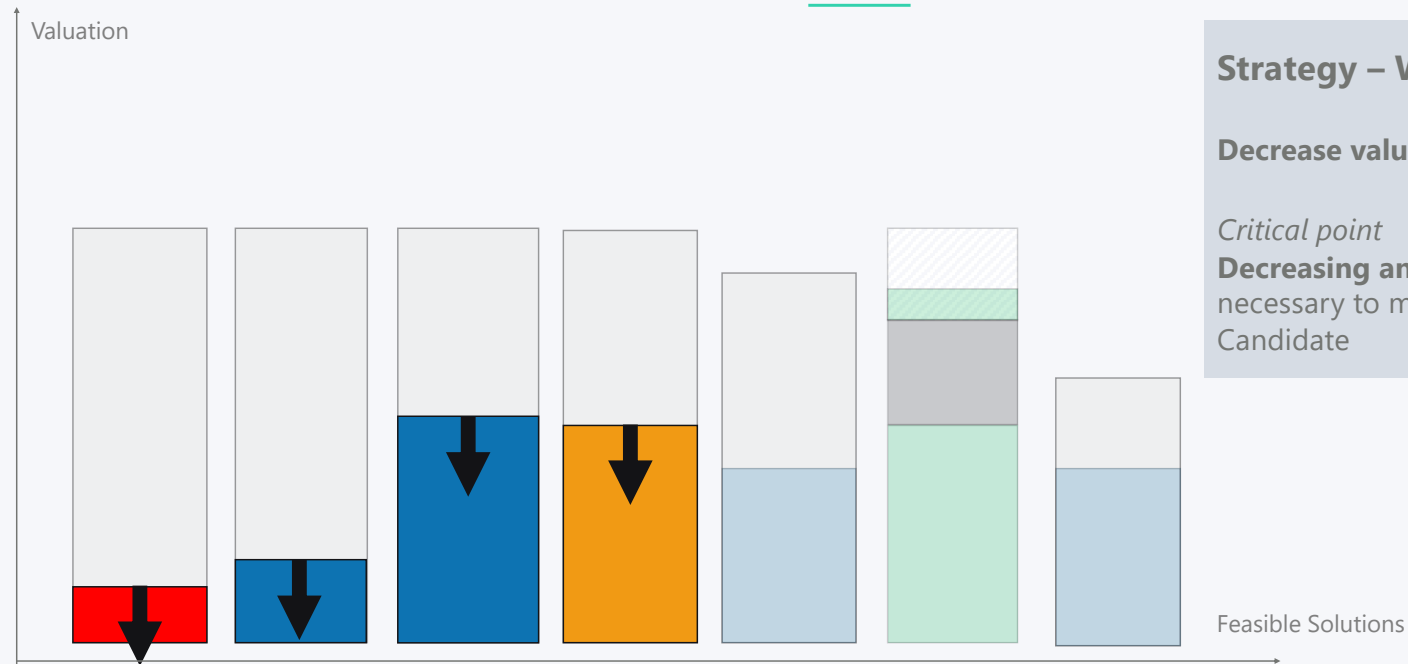
**Decreasing another Bid Valuation** is necessary to maintain the same Winning Candidate

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)  
■ Gain ■ Loss



## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



### Strategy – WIN\_LOW

Decrease valuation of Winning Bid

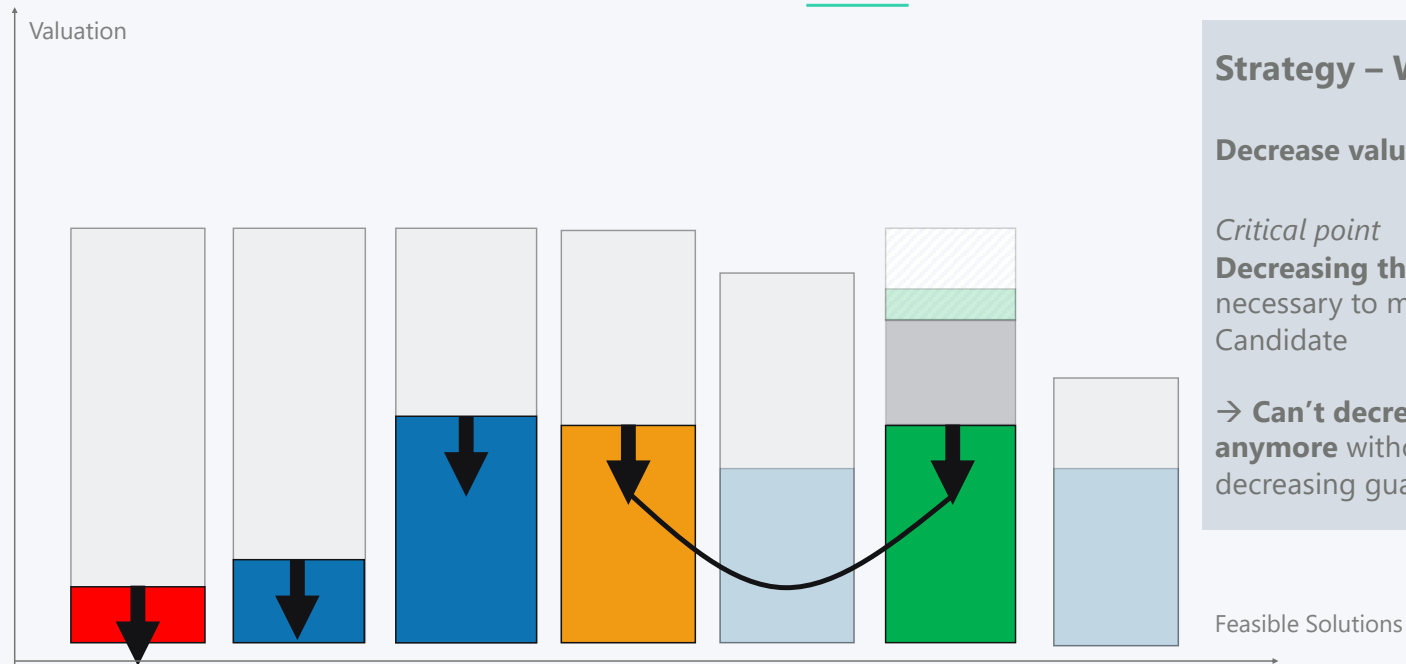
*Critical point*

**Decreasing another Bid Valuation** is necessary to maintain the same Winning Candidate

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)  
■ Gain ■ Loss

## Bidding Strategies for Egalitarian Profit Sharing

# Perspective of Conspiring Bidder



### Strategy – WIN\_LOW

Decrease valuation of Winning Bid

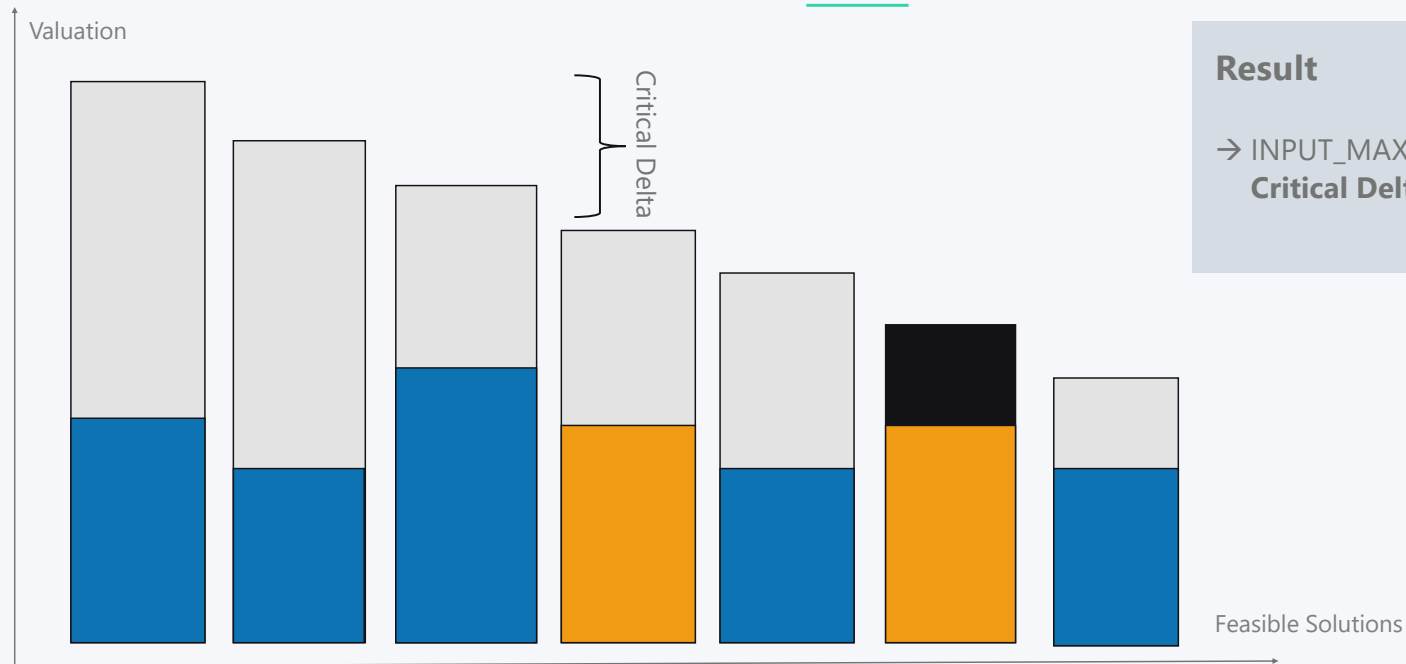
*Critical point*

**Decreasing the Input Bid Valuation** is necessary to maintain the Winning Candidate

→ **Can't decrease** bid valuations **anymore** without simultaneously decreasing guaranteed profit

## Bidding Strategies for Egalitarian Profit Sharing

### Perspective of Conspiring Bidder



#### Result

→ INPUT\_MAX and WIN\_LOW exploit the **Critical Delta**

Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)

## Conspiring Bidder Strategies

### INPUT\_MAX

Increase price of Input Bid

### WIN\_LOW

Decrease price of Winning Bid



## Strategic Bidder Strategies

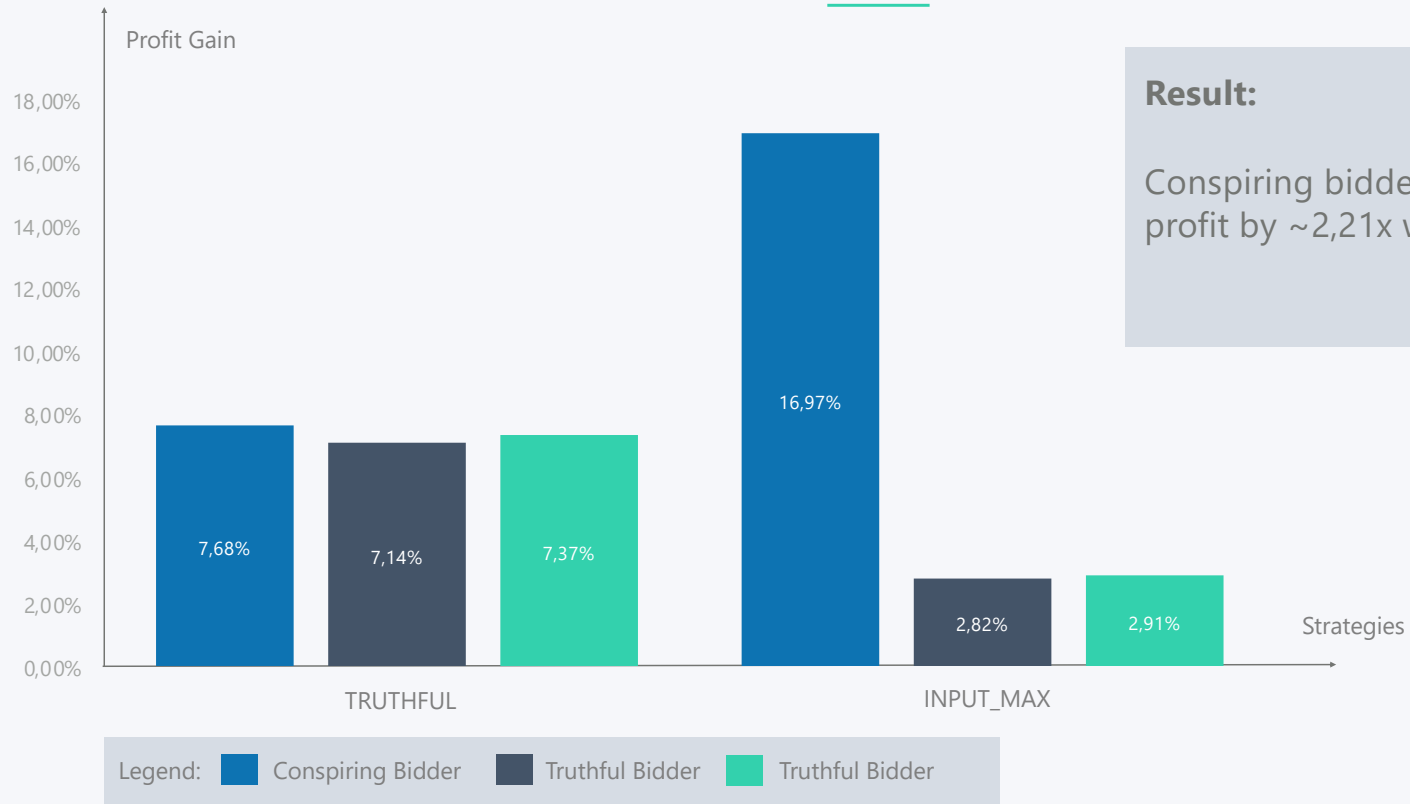
### INPUT\_MANIPULATION

Overbid/Underbid on the Input Bid



## Bidding Strategies for Egalitarian Profit Sharing

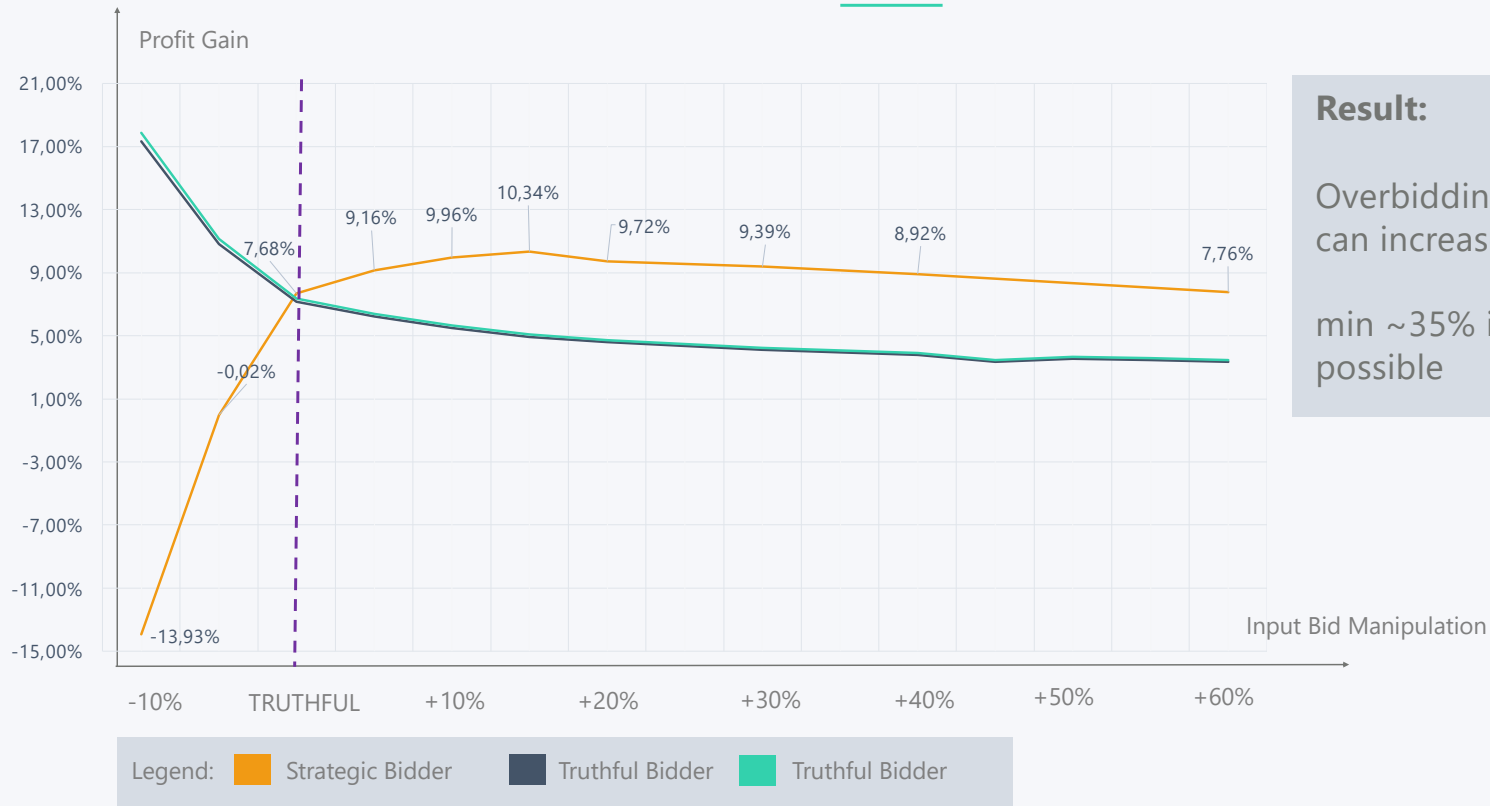
### Test Results for Conspiring Bidder



#### Result:

Conspiring bidder can increase her profit by ~2,21x with INPUT\_MAX

## Bidding Strategies for Egalitarian Profit Sharing Test Results for Strategic Bidder



### Result:

Overbidding on the Input Bid can increase profit

min ~35% increase of profit possible

## 4.2

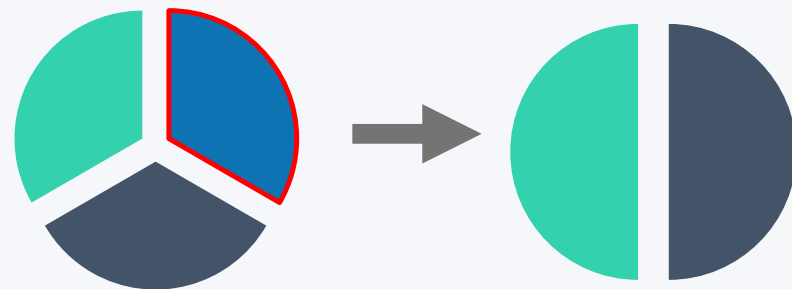
## Bidding Strategies for Modified Egalitarian Profit Sharing

**Profit Sharing Rule:**

Share the collaboration gain equally  
between the carriers

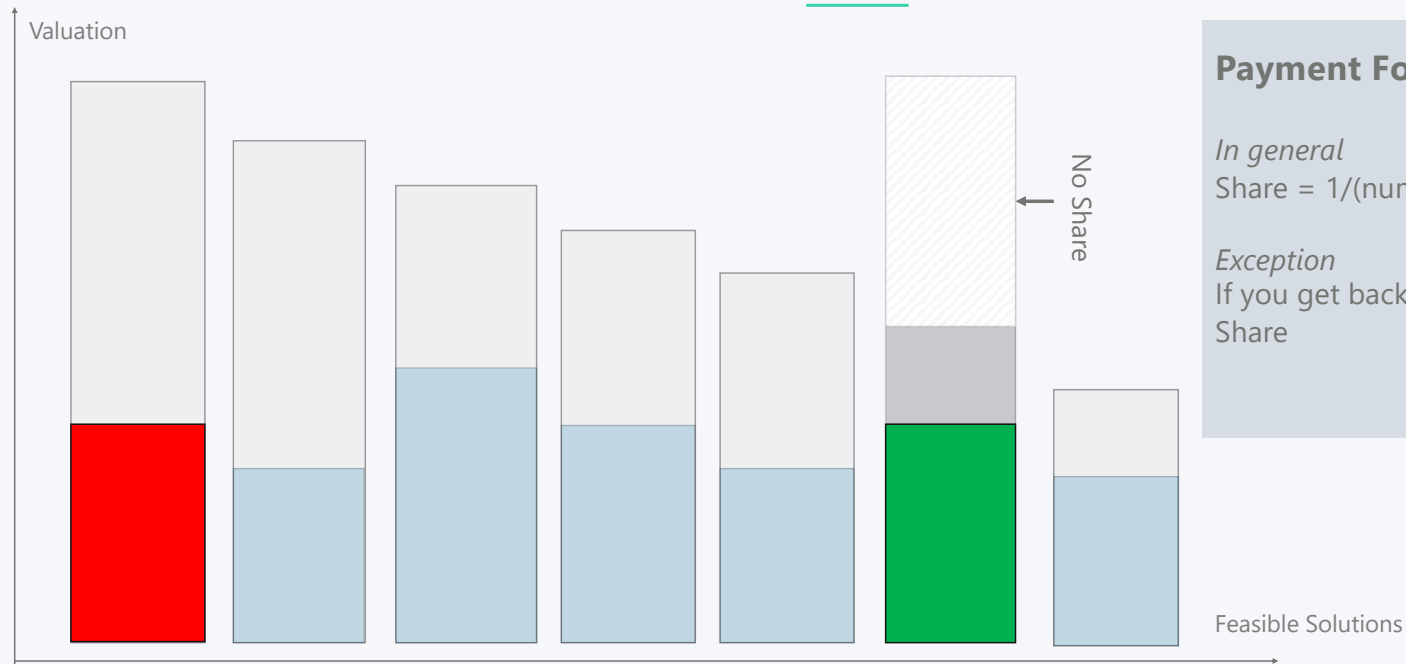
*Modification:*

If a carrier wins her own Input Bid then  
she is excluded from the profit sharing





## Bidding Strategies for Modified Egalitarian Profit Sharing Perspective of Conspiring Bidder



### Payment Formula

*In general*

Share =  $1/(\text{number of carriers})$

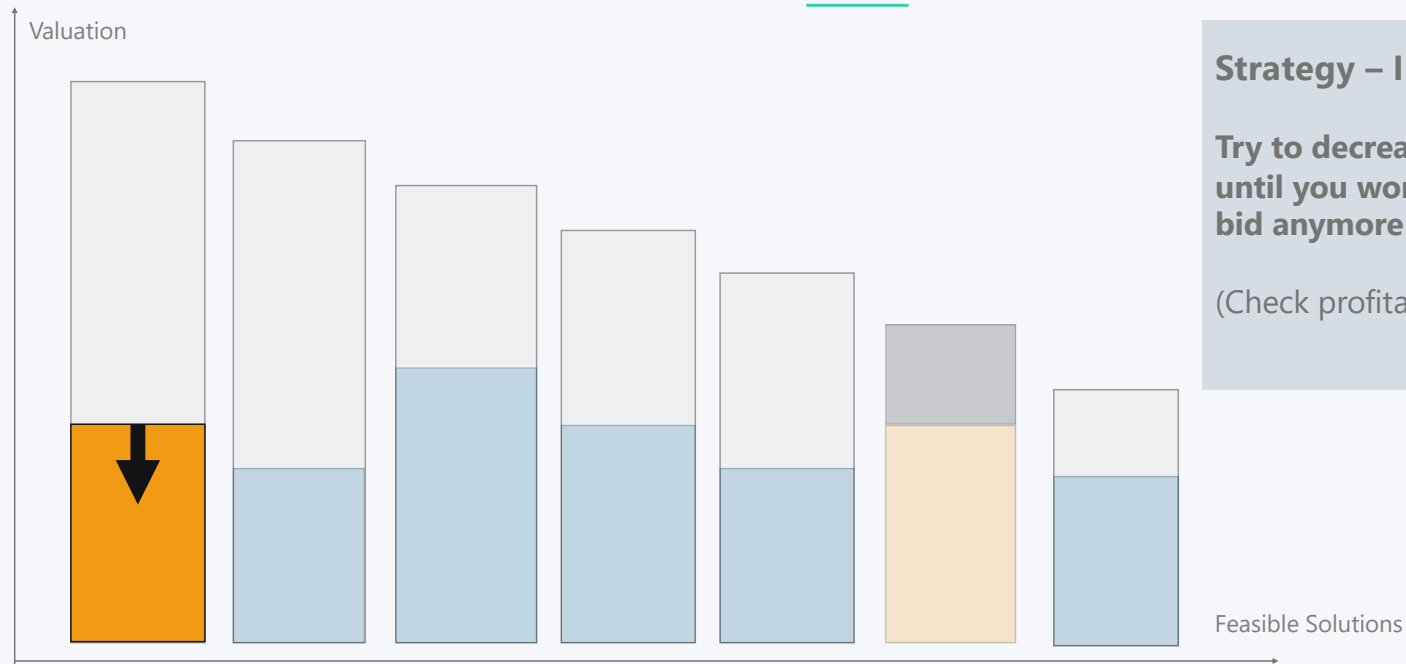
*Exception*

If you get back your own input bit → No Share

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)  
■ Gain ■ Loss

Bidding Strategies for Modified Egalitarian Profit Sharing




## Perspective of Conspiring Bidder



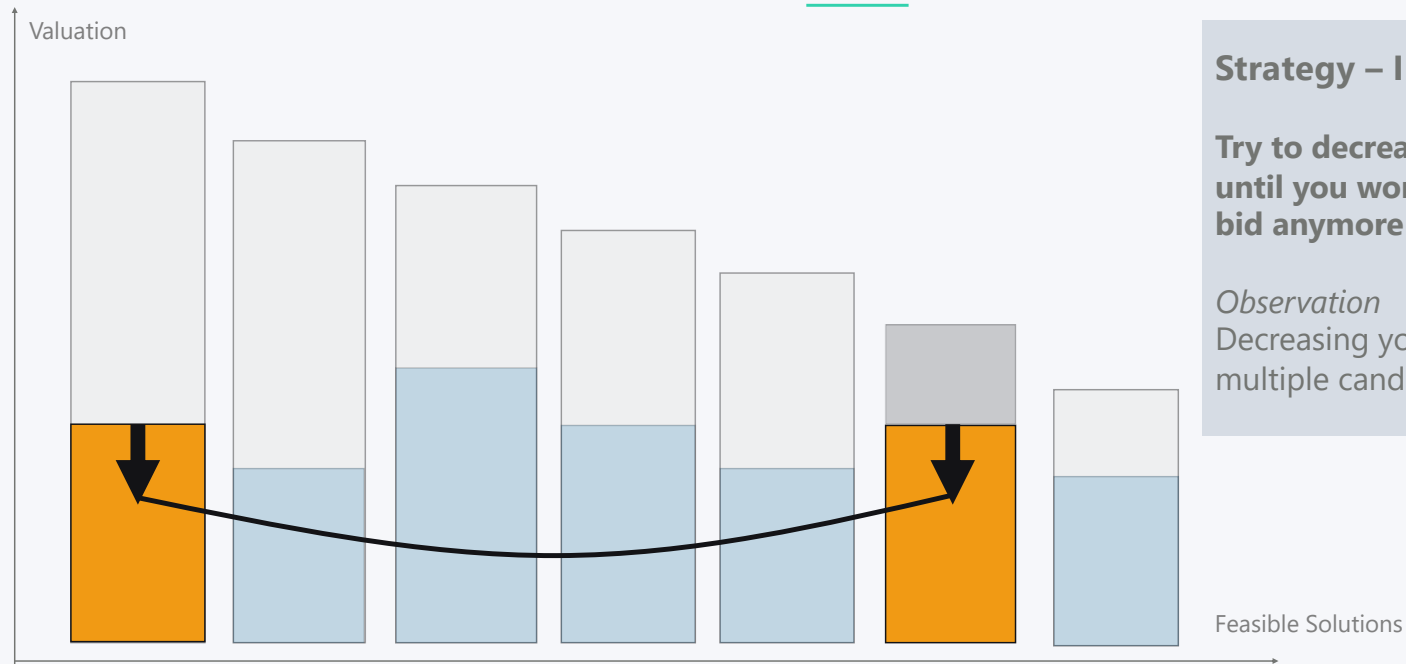
### Strategy – INPUT\_ENTER

**Try to decrease your input valuation until you won't win your own input bid anymore**

(Check profitability afterwards)

Legend:  Valuation of my BID  Valuation of my INPUT BID  Valuation of BIDs (others)  Valuation of INPUT BIDs (others)

## Bidding Strategies for Modified Egalitarian Profit Sharing Perspective of Conspiring Bidder



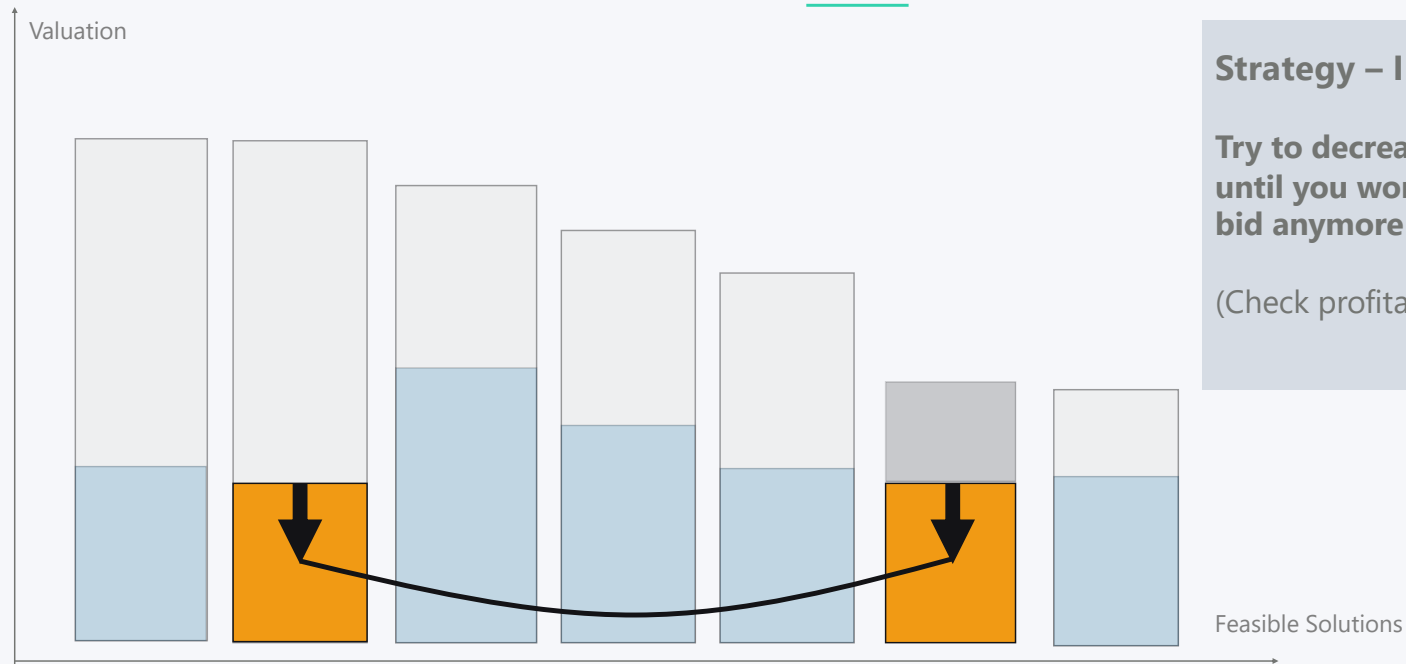
### Strategy – INPUT\_ENTER

**Try to decrease your input valuation until you won't win your own input bid anymore**

*Observation*  
Decreasing your input bid impacts multiple candidates

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)

## Bidding Strategies for Modified Egalitarian Profit Sharing Perspective of Conspiring Bidder



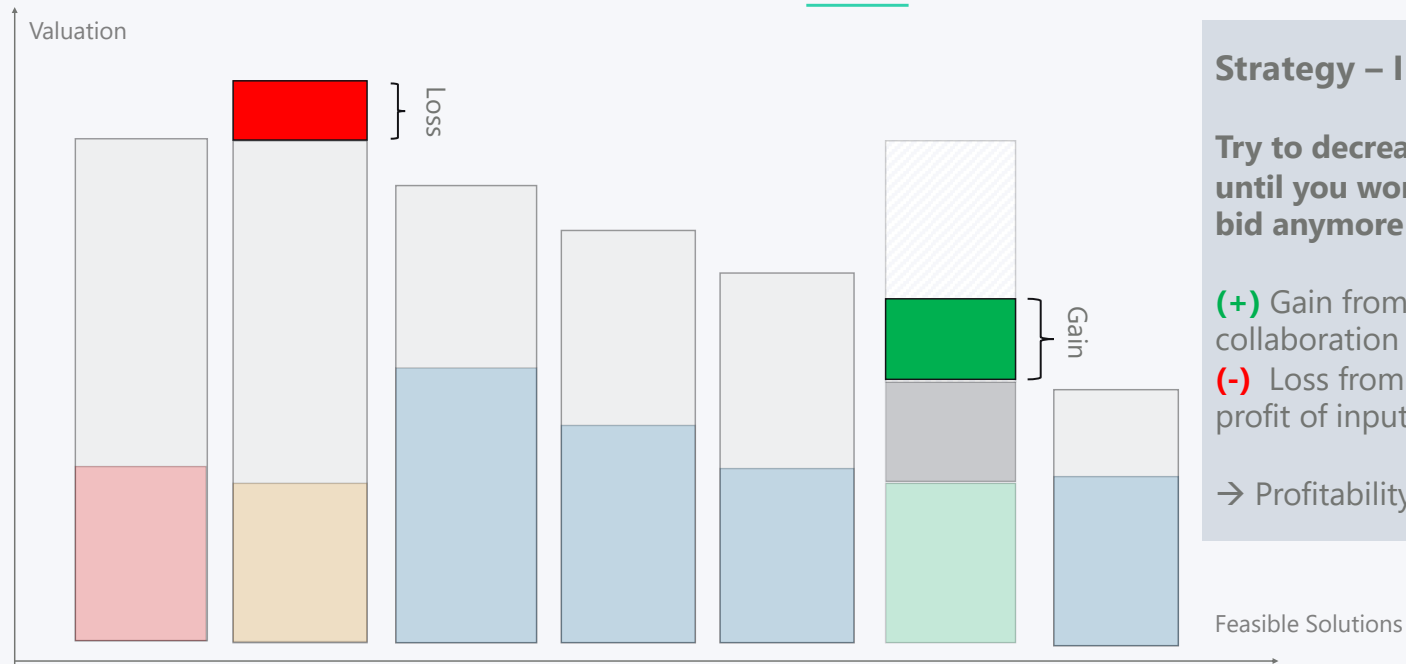
### Strategy – INPUT\_ENTER

**Try to decrease your input valuation until you won't win your own input bid anymore**

(Check profitability afterwards)

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)

## Bidding Strategies for Modified Egalitarian Profit Sharing Perspective of Conspiring Bidder



### Strategy – INPUT\_ENTER

**Try to decrease your input valuation until you won't win your own input bid anymore**

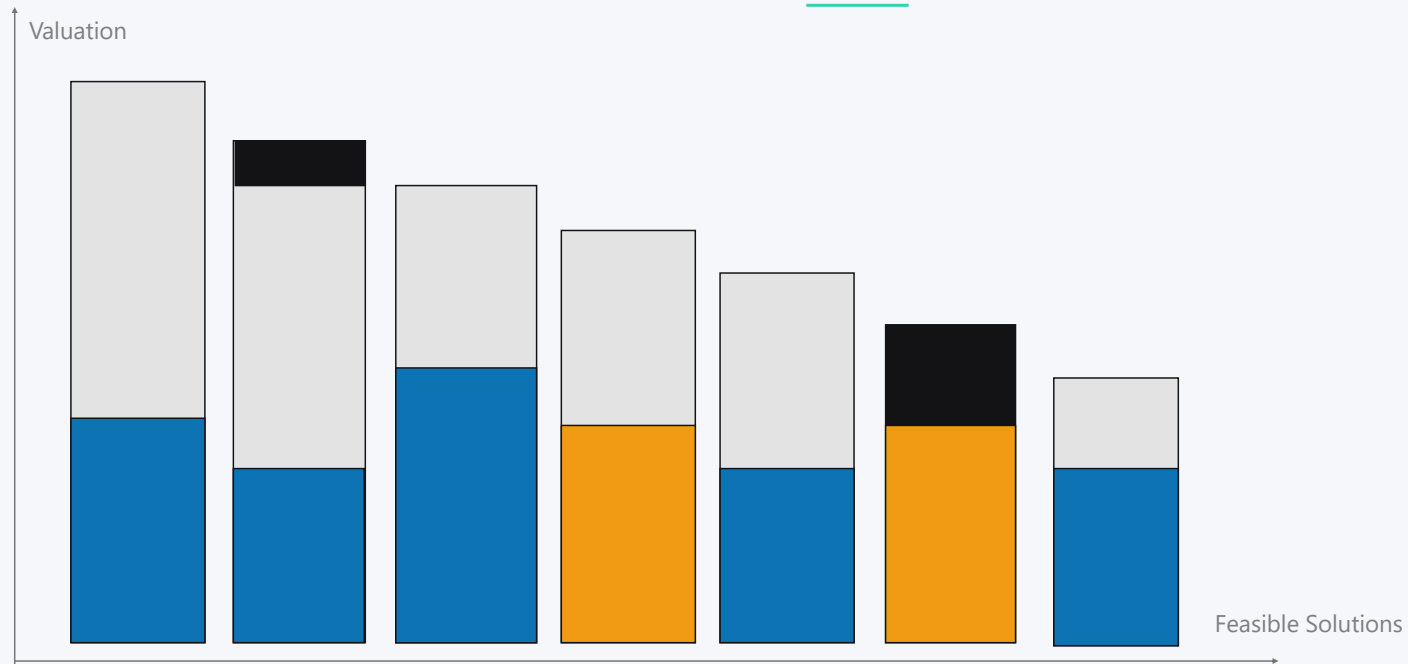
(+) Gain from getting part of the collaboration share

(-) Loss from decreasing guaranteed profit of input bid

→ Profitability check necessary

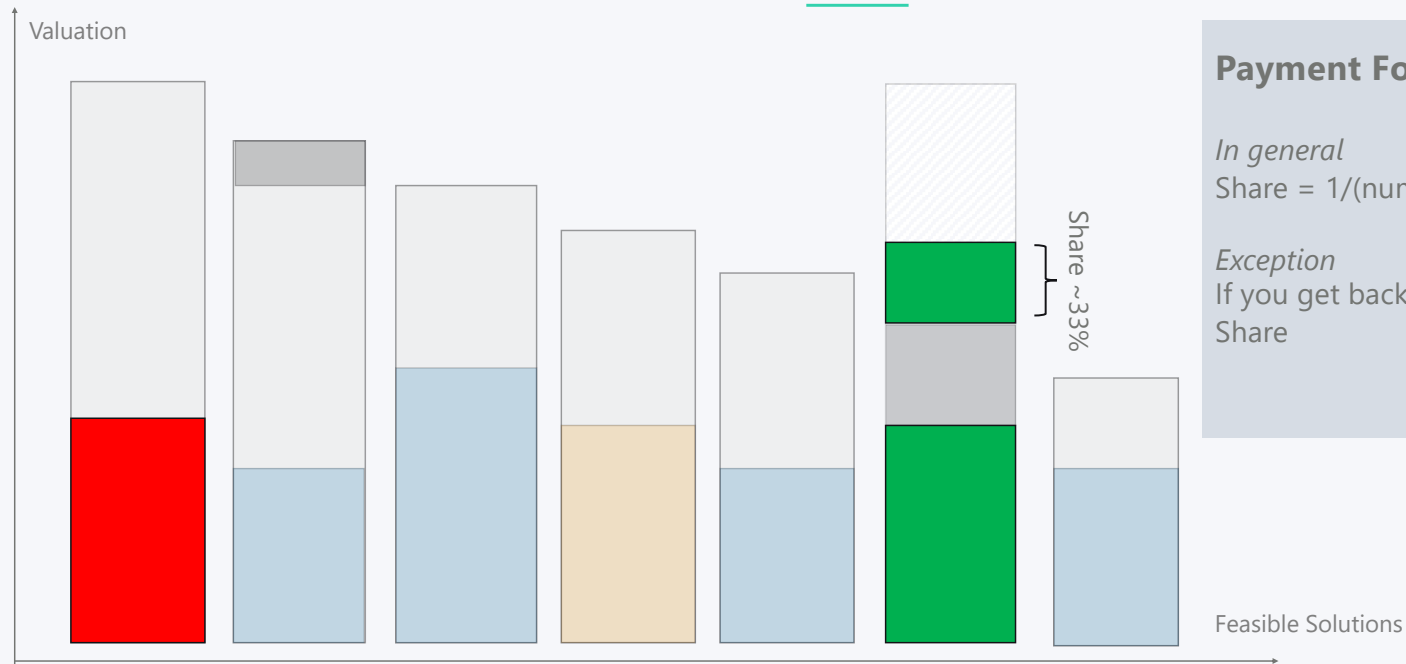
Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)  
■ Gain ■ Loss

## Bidding Strategies for Modified Egalitarian Profit Sharing Perspective of Conspiring Bidder



Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)

## Bidding Strategies for Modified Egalitarian Profit Sharing Perspective of Conspiring Bidder



### Payment Formula

*In general*

Share =  $1/(\text{number of carriers})$

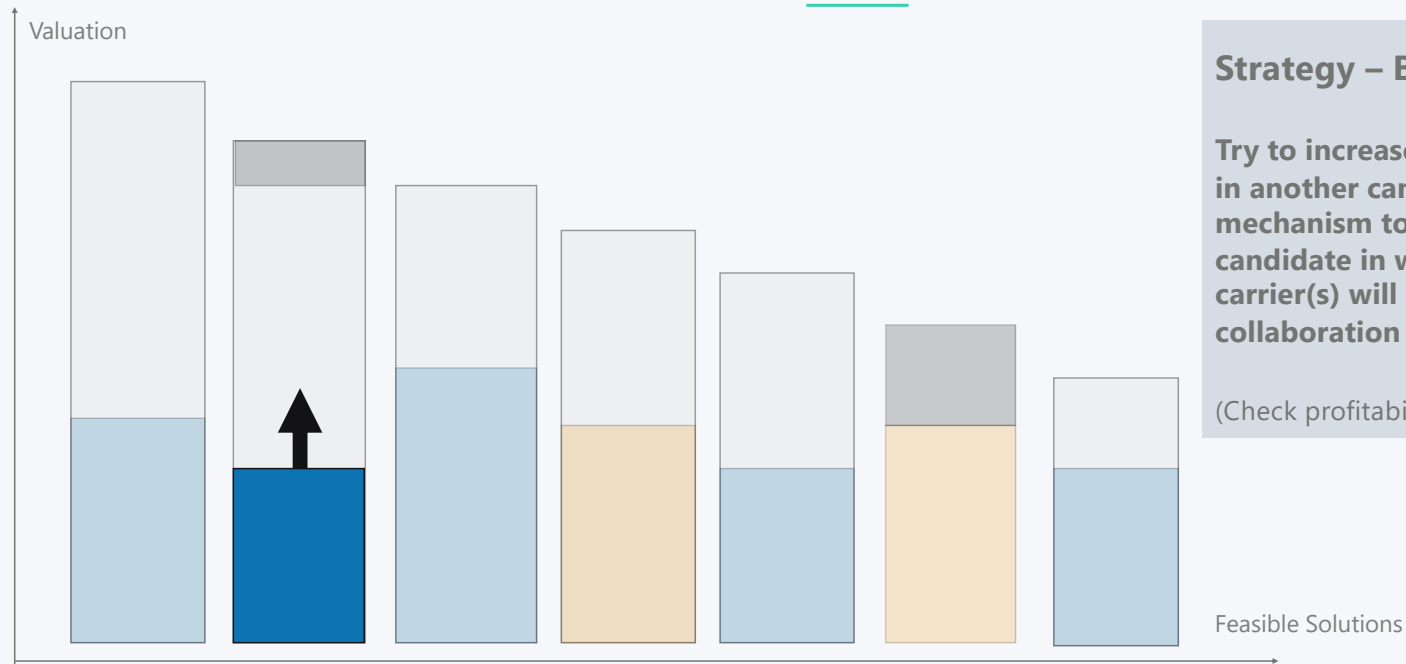
*Exception*

If you get back your own Input Bid → No Share

Legend:   
■ Valuation of my BID   
■ Valuation of my INPUT BID   
■ Valuation of BIDs (others)   
■ Valuation of INPUT BIDs (others)   
■ Gain   
■ Loss

## Bidding Strategies for Modified Egalitarian Profit Sharing

### Perspective of Conspiring Bidder



#### Strategy – BID\_KICKOUT

Try to increase your valuation of a bid in another candidate to force the mechanism to determine a winning candidate in which another/other carrier(s) will be excluded from the collaboration gain

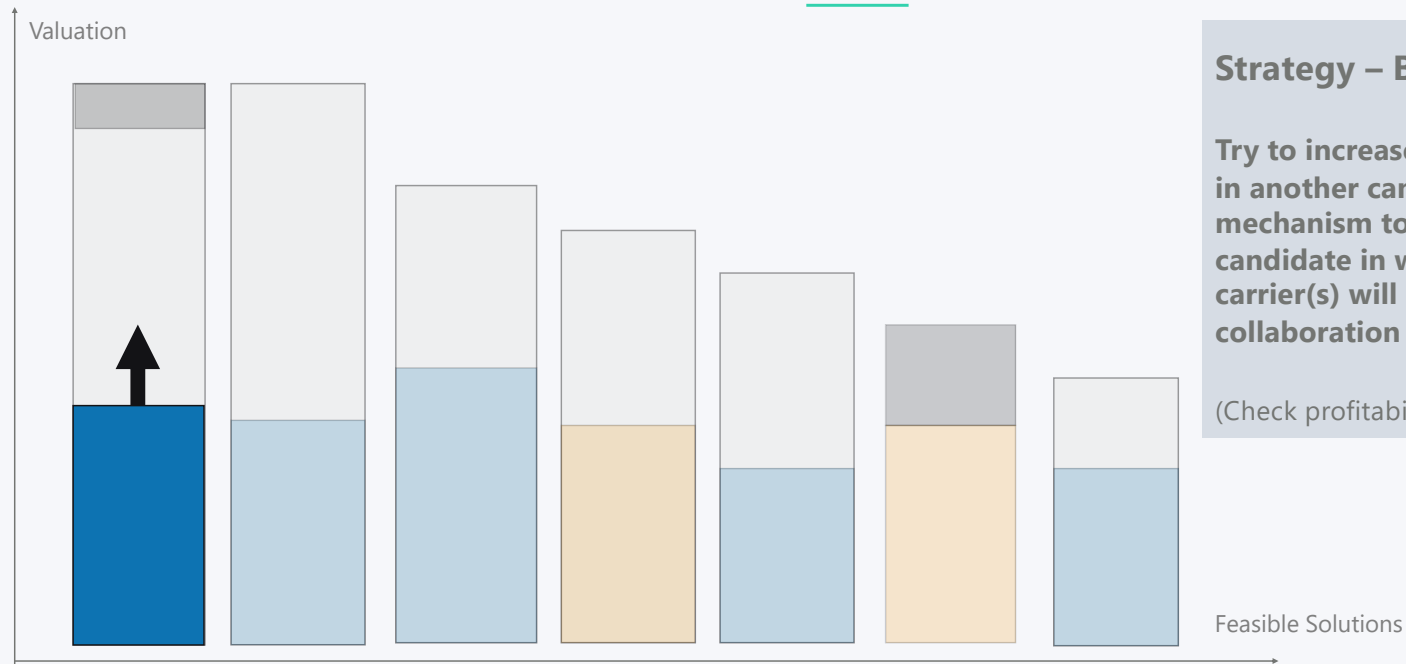
(Check profitability afterwards)

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of other BIDs ■ Valuation of other INPUT BIDs



## Bidding Strategies for Modified Egalitarian Profit Sharing

### Perspective of Conspiring Bidder



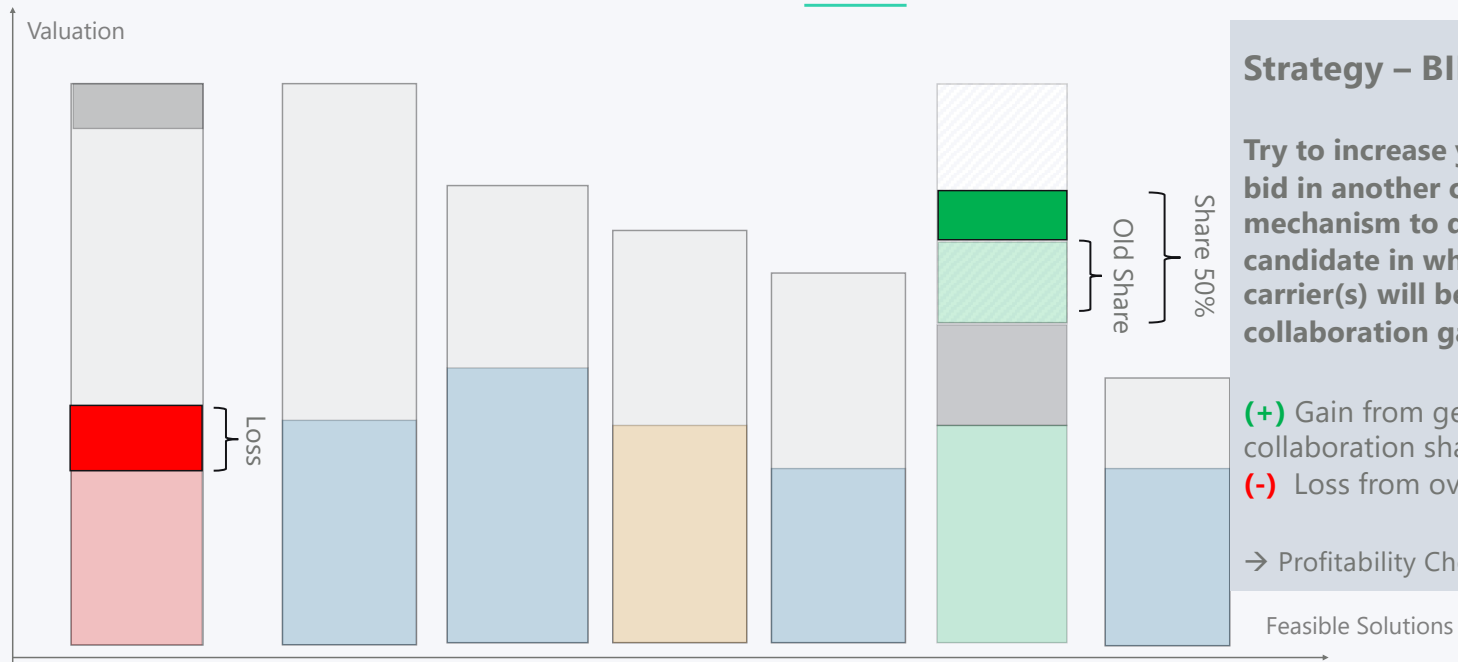
#### Strategy – BID\_KICKOUT

Try to increase your valuation of a bid in another candidate to force the mechanism to determine a winning candidate in which another/other carrier(s) will be excluded from the collaboration gain

(Check profitability afterwards)

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)

## Bidding Strategies for Modified Egalitarian Profit Sharing Perspective of Conspiring Bidder



### Strategy – BID\_KICKOUT

Try to increase your valuation of a bid in another candidate to force the mechanism to determine a winning candidate in which another/other carrier(s) will be excluded from the collaboration gain

(+) Gain from getting more of the collaboration share

(-) Loss from overbidding

→ Profitability Check necessary

Legend:   
■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)   
■ Gain ■ Loss

## Conspiring Bidder Strategies

### INPUT\_MAX

Increase valuation of Input Bid

### INPUT\_ENTER

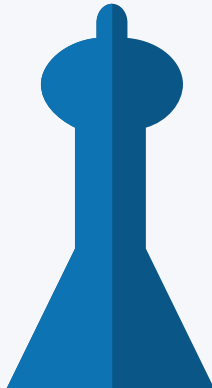
Try to decrease your input valuation until you won't win your own input bid anymore

### BID\_KICKOUT

Try to increase your valuation of a bid in another candidate to force the mechanism to determine a winning candidate in which another/other carrier(s) will be excluded from the collaboration gain

### LOW\_WIN

Decrease valuation of Winning Bid



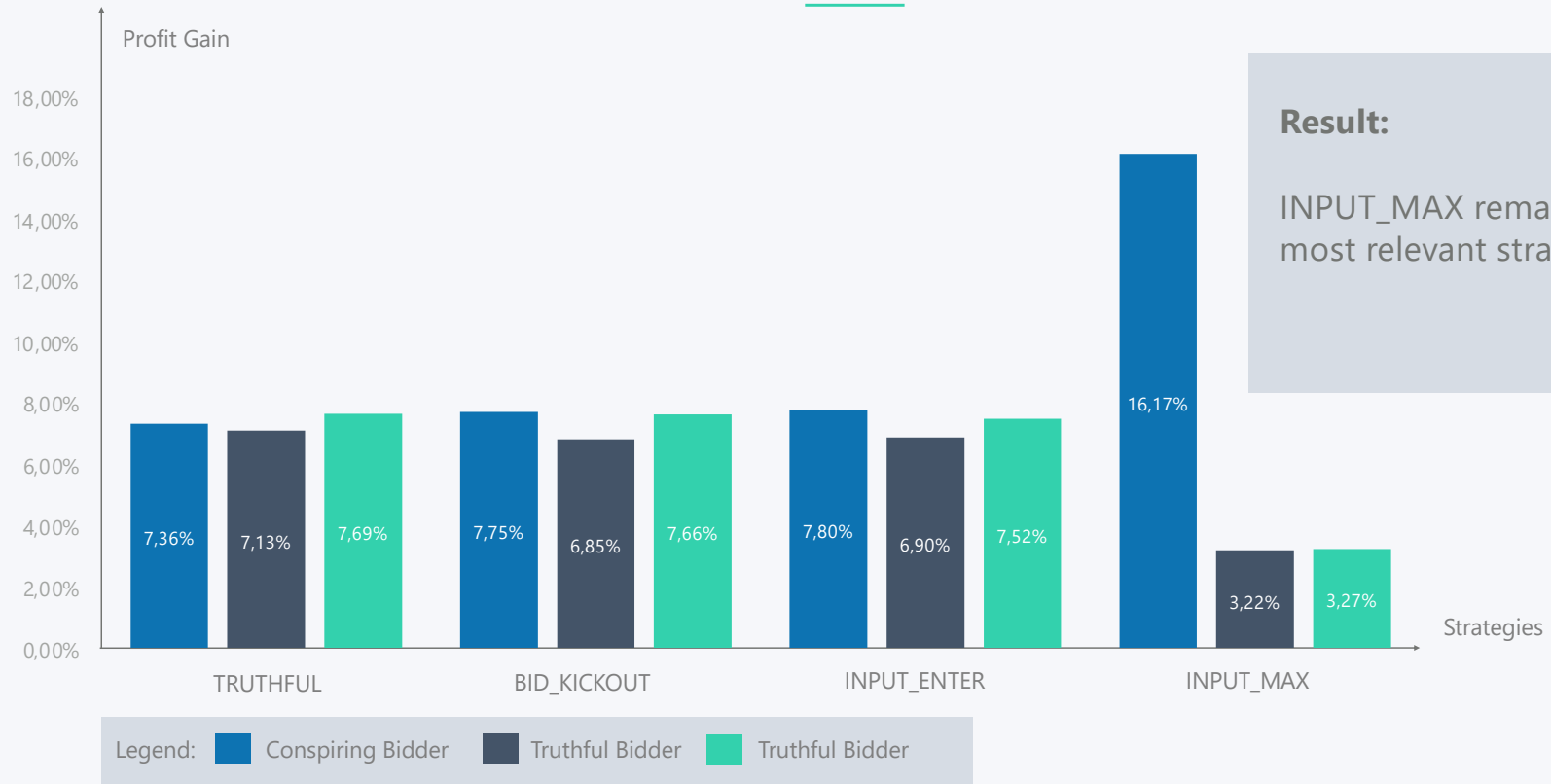
## Strategic Bidder Strategies

### INPUT\_MANIPULATION

Overbid/Underbid on the Input Bid

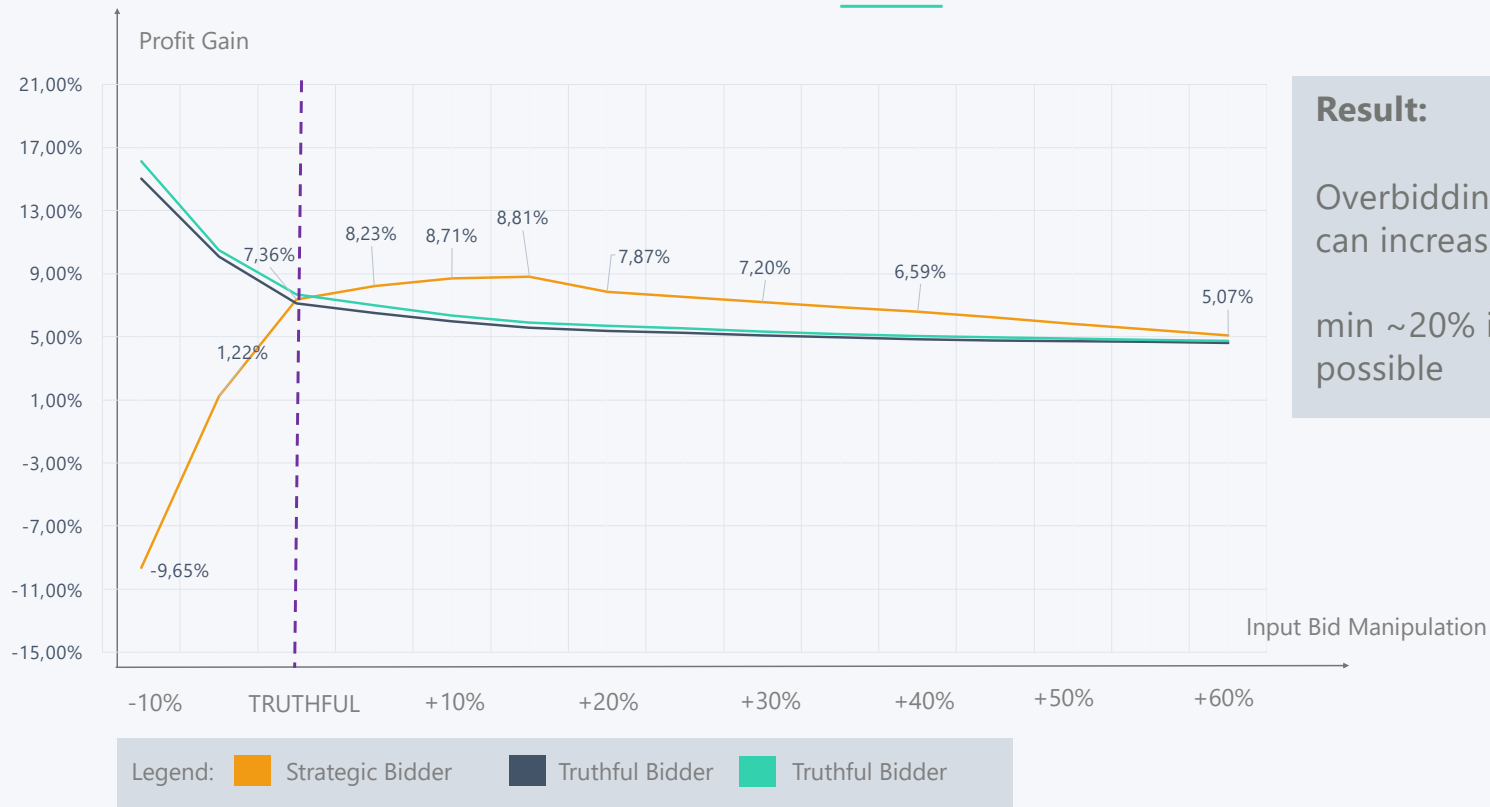


## Bidding Strategies for Modified Egalitarian Profit Sharing Simulation Results for Conspiring Bidder



## Bidding Strategies for Modified Egalitarian Profit Sharing

### Simulation Results for Strategic Bidder



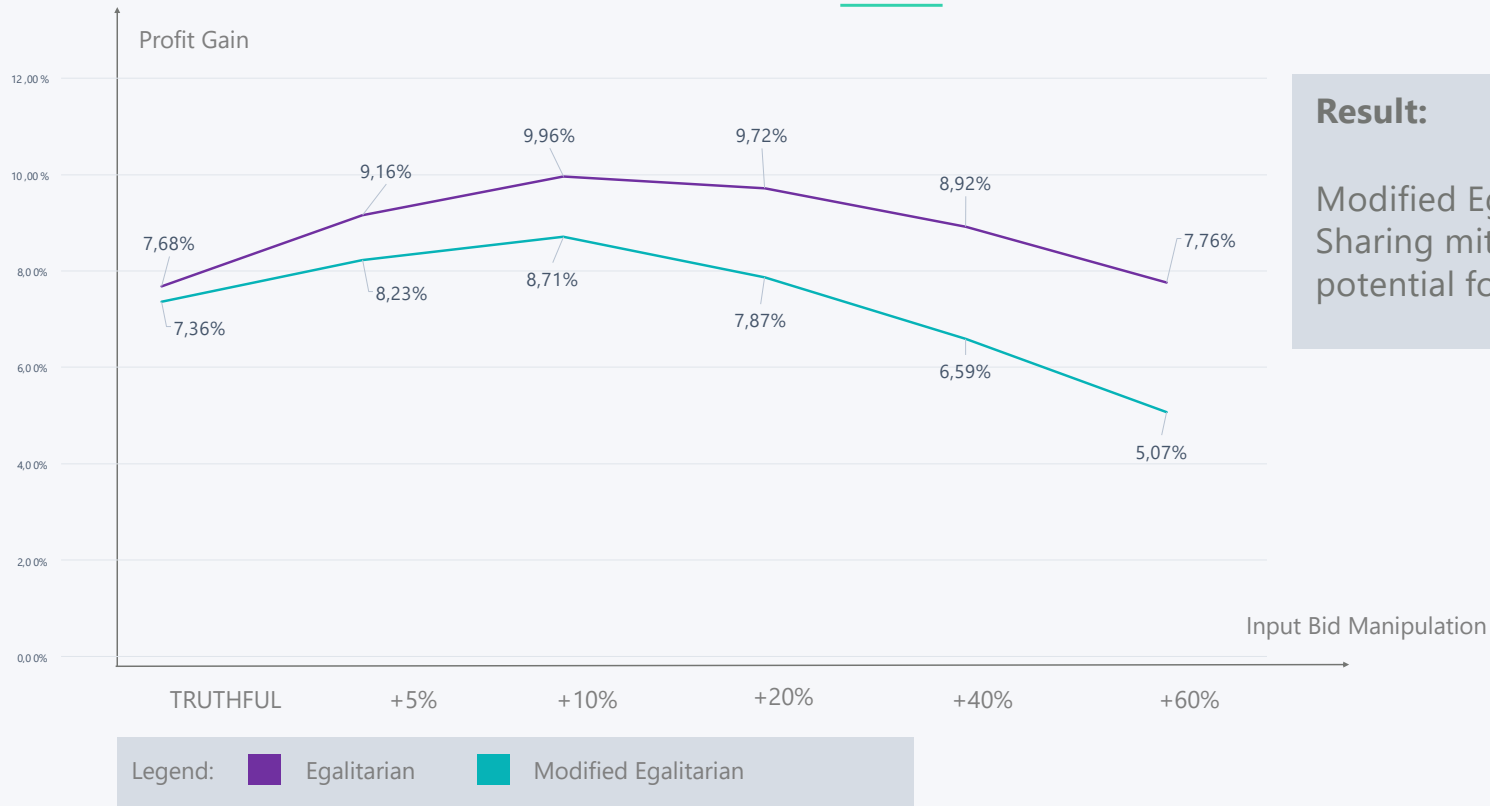
#### Result:

Overbidding on the Input Bid can increase profit

min ~20% increase of profit possible

## Bidding Strategies for Modified Egalitarian Profit Sharing

### Egalitarian vs. Modified Egalitarian Profit Sharing



#### Result:

Modified Egalitarian Profit Sharing mitigates the potential for overbidding

## 4.3

## Bidding Strategies for Purchase/Sale Weight Profit Sharing

Bidding Strategies for Purchase/Sale Weight Profit Sharing

## Purchase/Sale Weights Profit Sharing

### Profit Sharing Rule:

*Sale Weight*

$(\text{Your Input Bid price}) / (\text{All Input Bid prices})$

*Purchase Weight*

$(\text{Your Winning Bid price}) / (\text{All Winning Bid prices})$

*Purchase/Sale Weight*

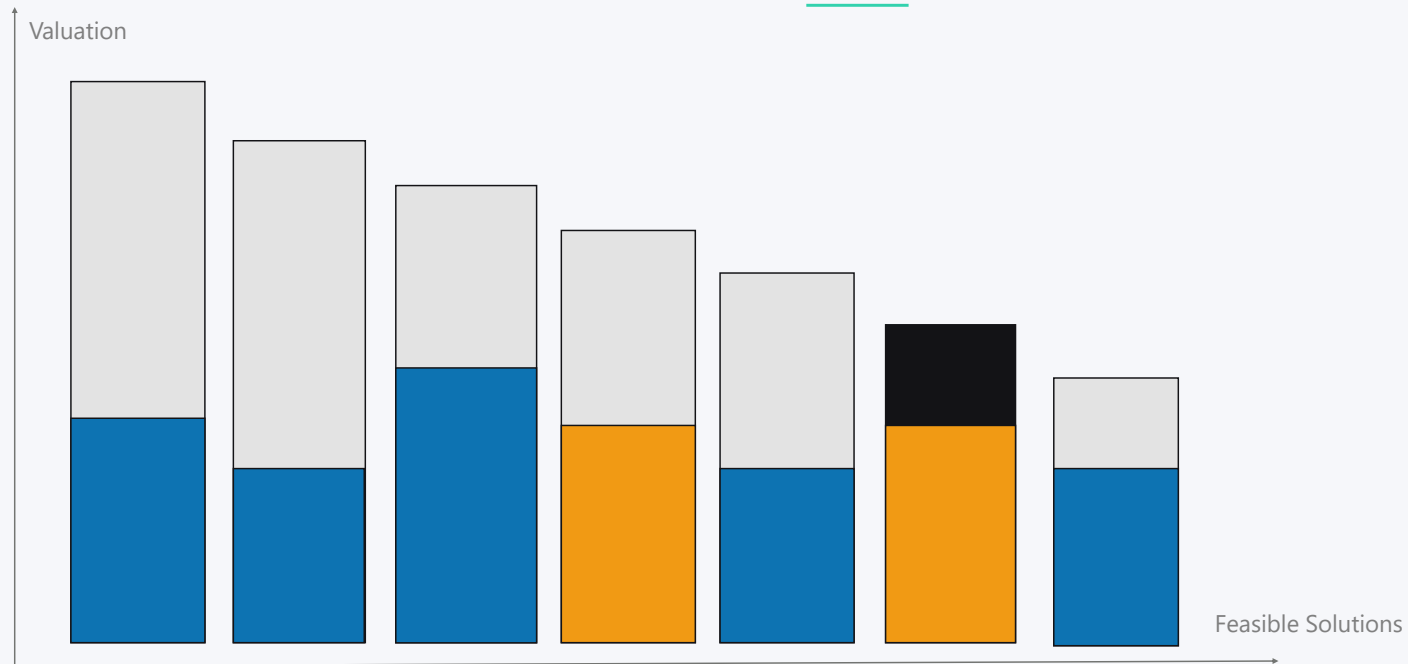
$0.5 * (\text{Sales Weight} + \text{Purchase Weight})$

See [5]



Bidding Strategies for Purchase/Sale Weight Profit Sharing

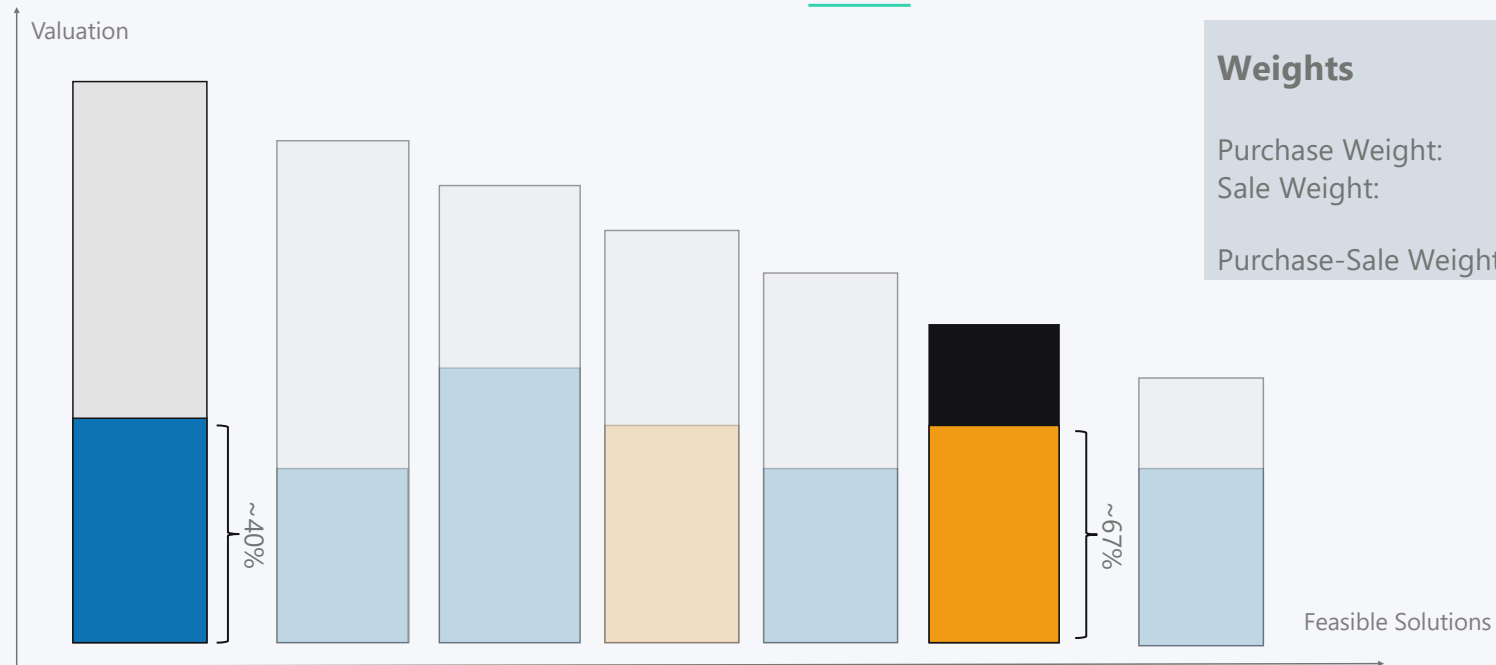
## Perspective of Conspiring Bidder



Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)

## Bidding Strategies for Purchase/Sale Weight Profit Sharing

### Perspective of Conspiring Bidder



#### Weights

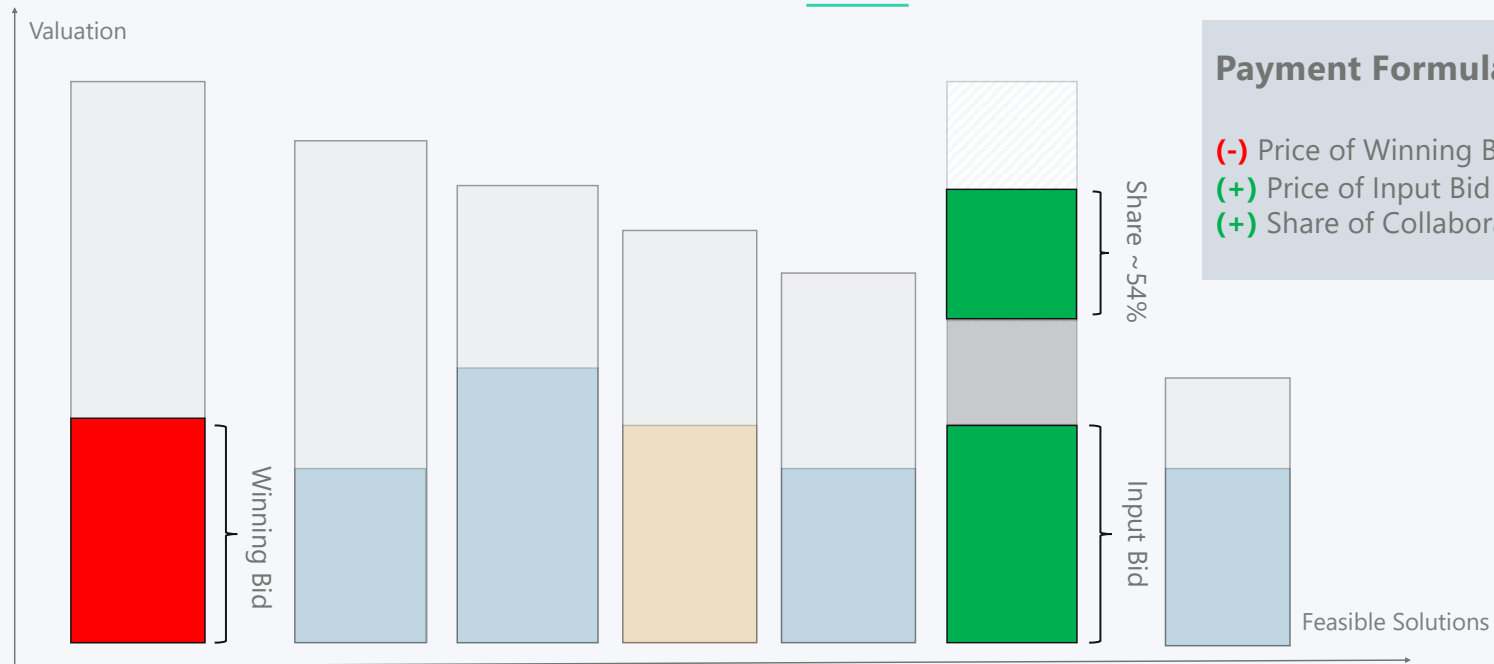
Purchase Weight: ~40%

Sale Weight: ~67%

Purchase-Sale Weight: ~54%

Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)

## Bidding Strategies for Purchase/Sale Weight Profit Sharing Perspective of Conspiring Bidder



### Payment Formula

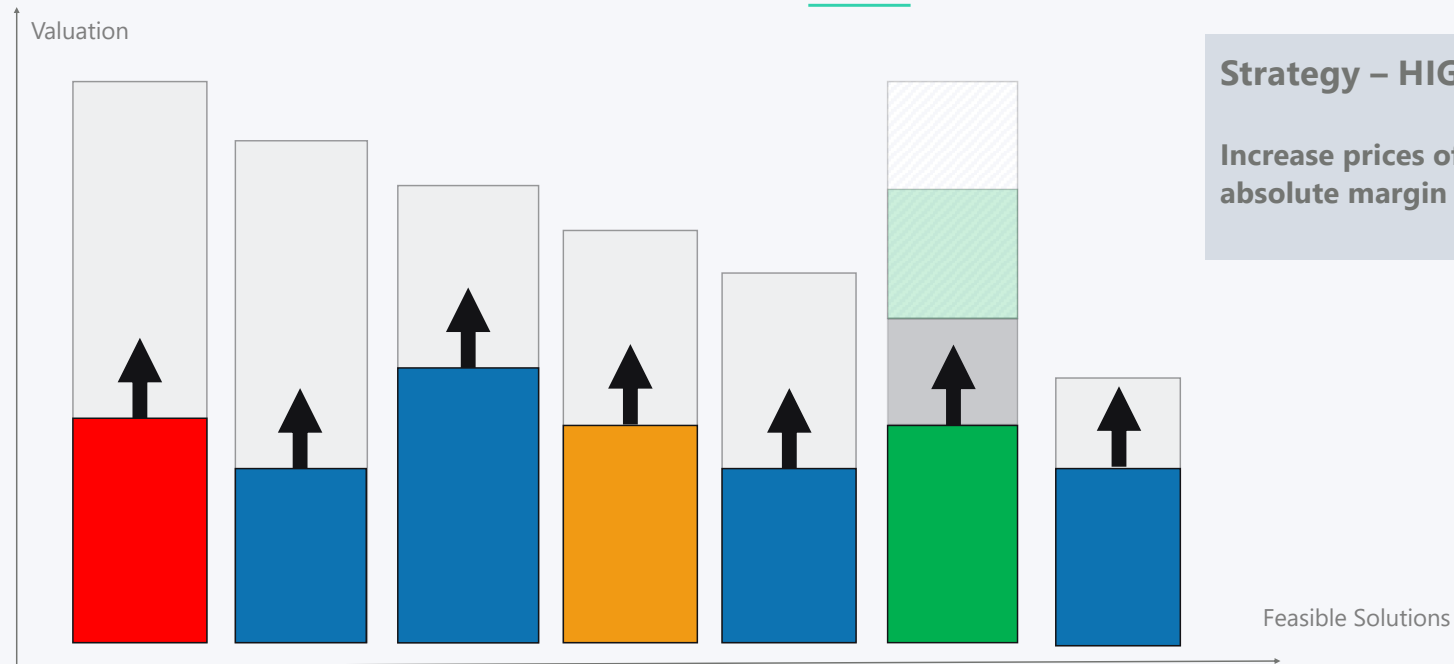
- (-) Price of Winning Bid
- (+) Price of Input Bid
- (+) Share of Collaboration Gain

Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)

■ Gain ■ Loss

Bidding Strategies for Purchase/Sale Weight Profit Sharing

## Perspective of Conspiring Bidder



### Strategy – HIGH\_ABS

Increase prices of all bids by the same absolute margin

Legend:

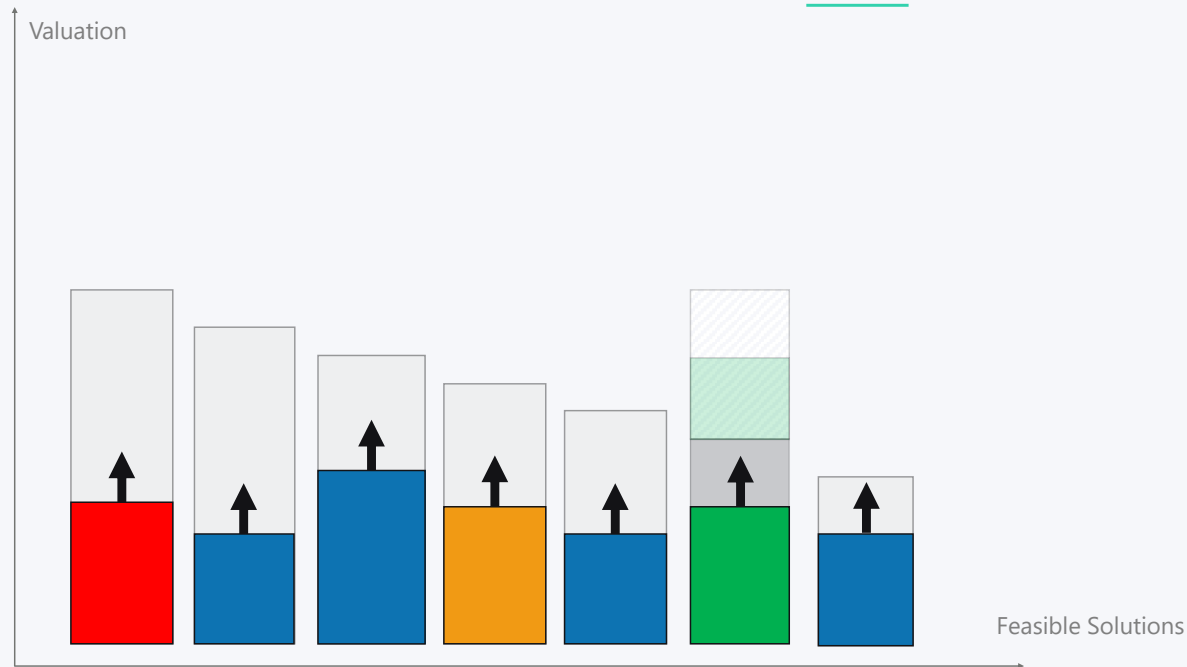
- Price of my BID
- Price of my INPUT BID
- Price of BIDs (others)
- Price of INPUT BIDs (others)
- Gain
- Loss

Bidding Strategies for Purchase/Sale Weight Profit Sharing

## Perspective of Conspiring Bidder

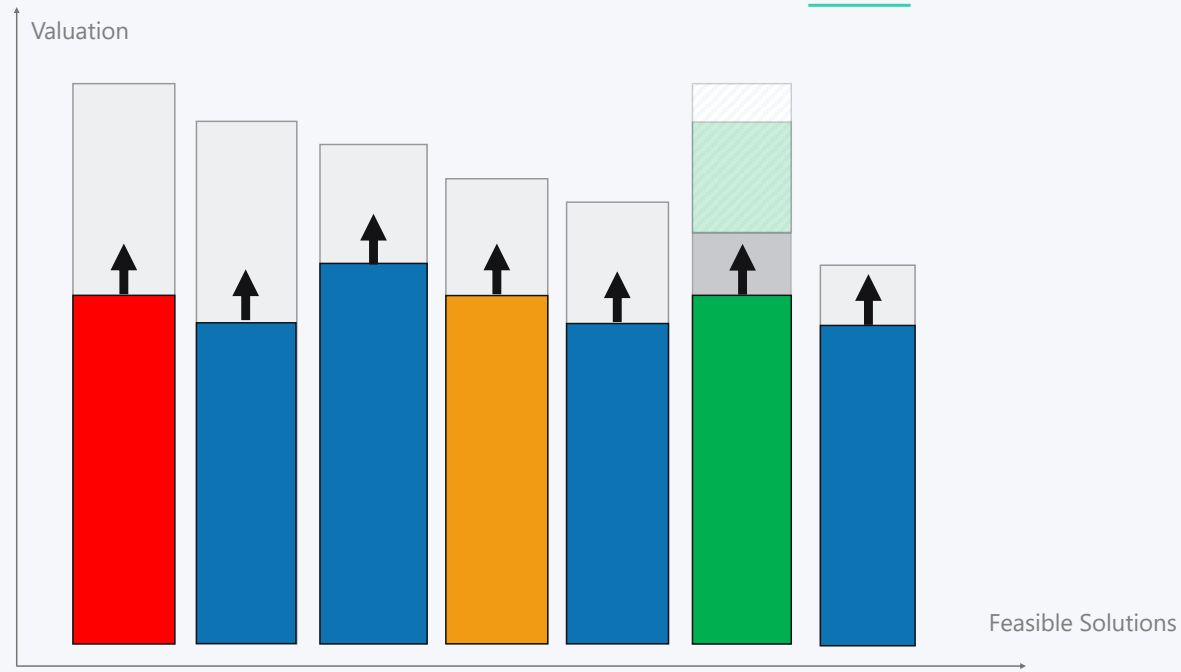
### Strategy – HIGH\_ABS

Increase prices of all bids by the same absolute margin



Bidding Strategies for Purchase/Sale Weight Profit Sharing

## Perspective of Conspiring Bidder

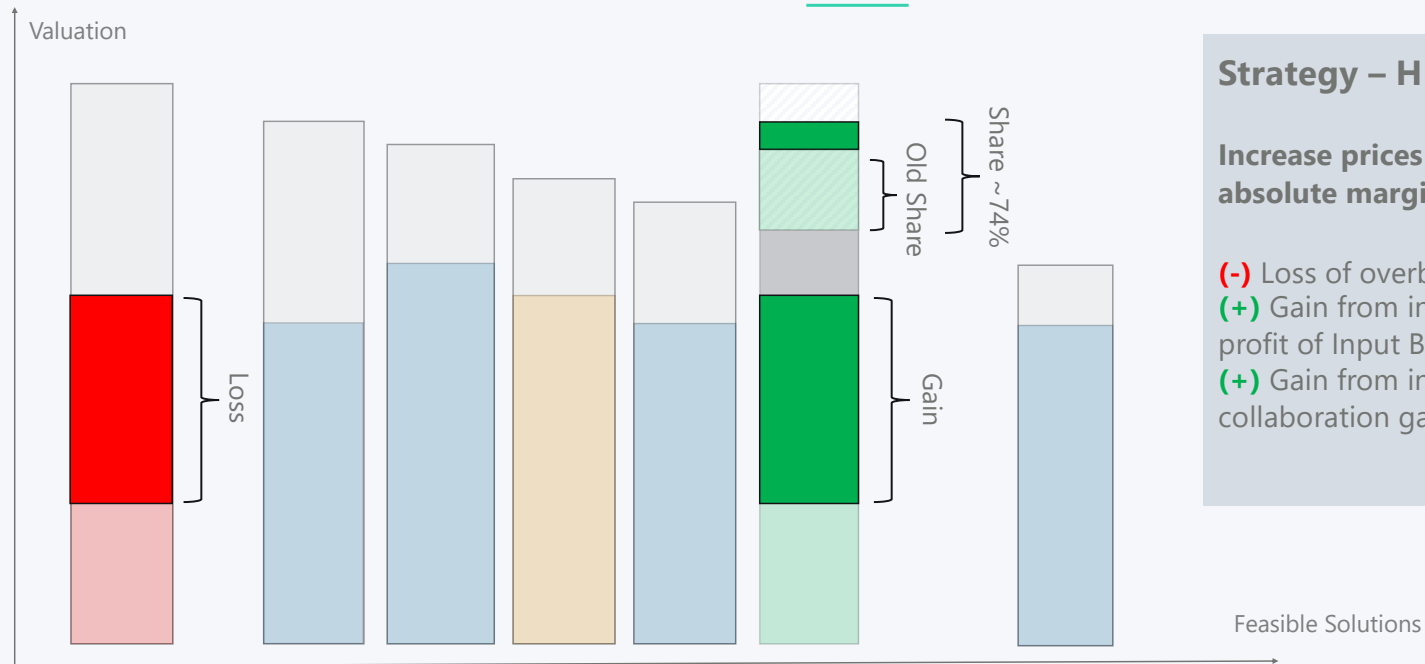


### Strategy – HIGH\_ABS

Increase prices of all bids by the same absolute margin

## Bidding Strategies for Purchase/Sale Weight Profit Sharing

# Perspective of Conspiring Bidder



### Strategy – HIGH\_ABS

**Increase prices of all bids by the same absolute margin**

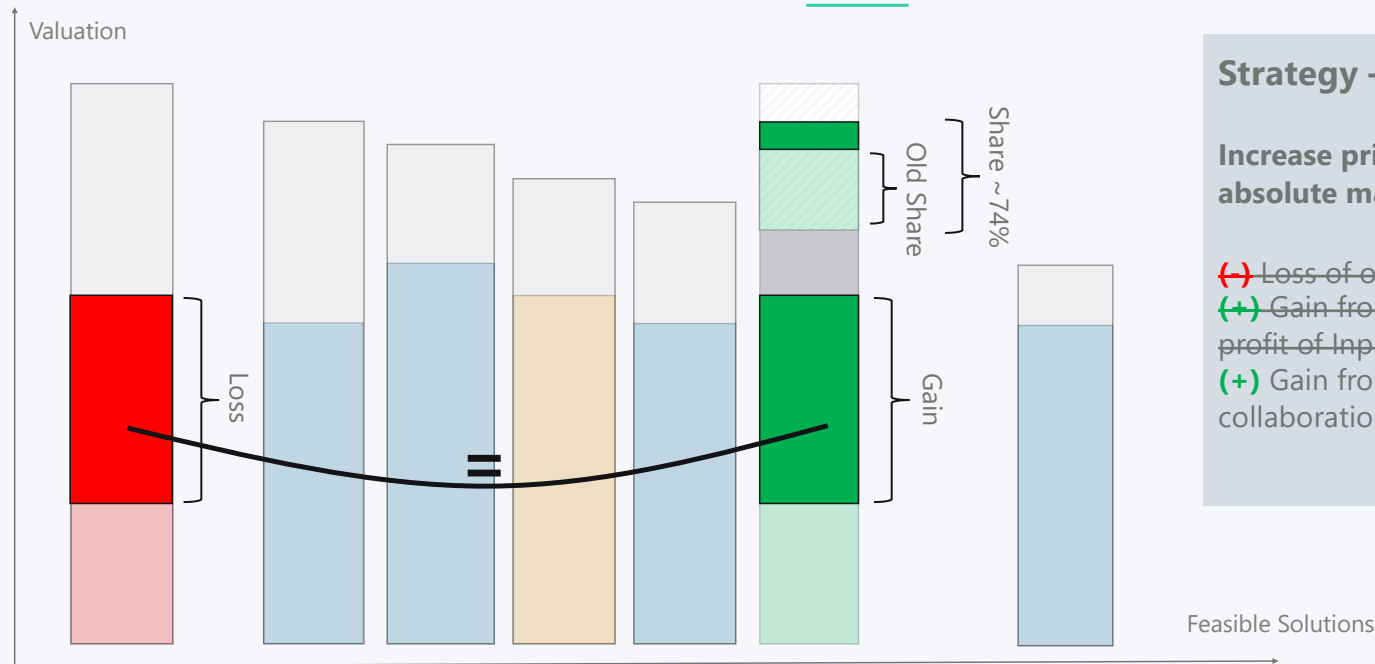
- (-) Loss of overbidding
- (+) Gain from increasing guaranteed profit of Input Bid
- (+) Gain from increasing share of collaboration gain

Legend: ■ Price of my BID ■ Price of my INPUT BID ■ Price of BIDs (others) ■ Price of INPUT BIDs (others)

■ Gain ■ Loss

## Bidding Strategies for Purchase/Sale Weight Profit Sharing

# Perspective of Conspiring Bidder



### Strategy – HIGH\_ABS

Increase prices of all bids by the same absolute margin

- (-) Loss of overbidding
- (+) Gain from increasing guaranteed profit of Input Bid
- (+) Gain from increasing share of collaboration gain

Legend:

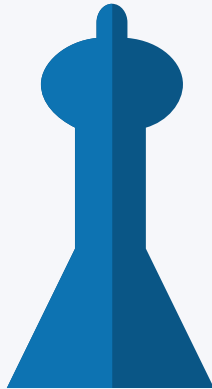
- Price of my BID
- Price of my INPUT BID
- Price of BIDs (others)
- Price of INPUT BIDs (others)
- Gain
- Loss



### Conspiring Bidder Strategies

#### **HIGH\_ABS**

Increase prices of all bids by the same absolute margin



### Strategic Bidder Strategies

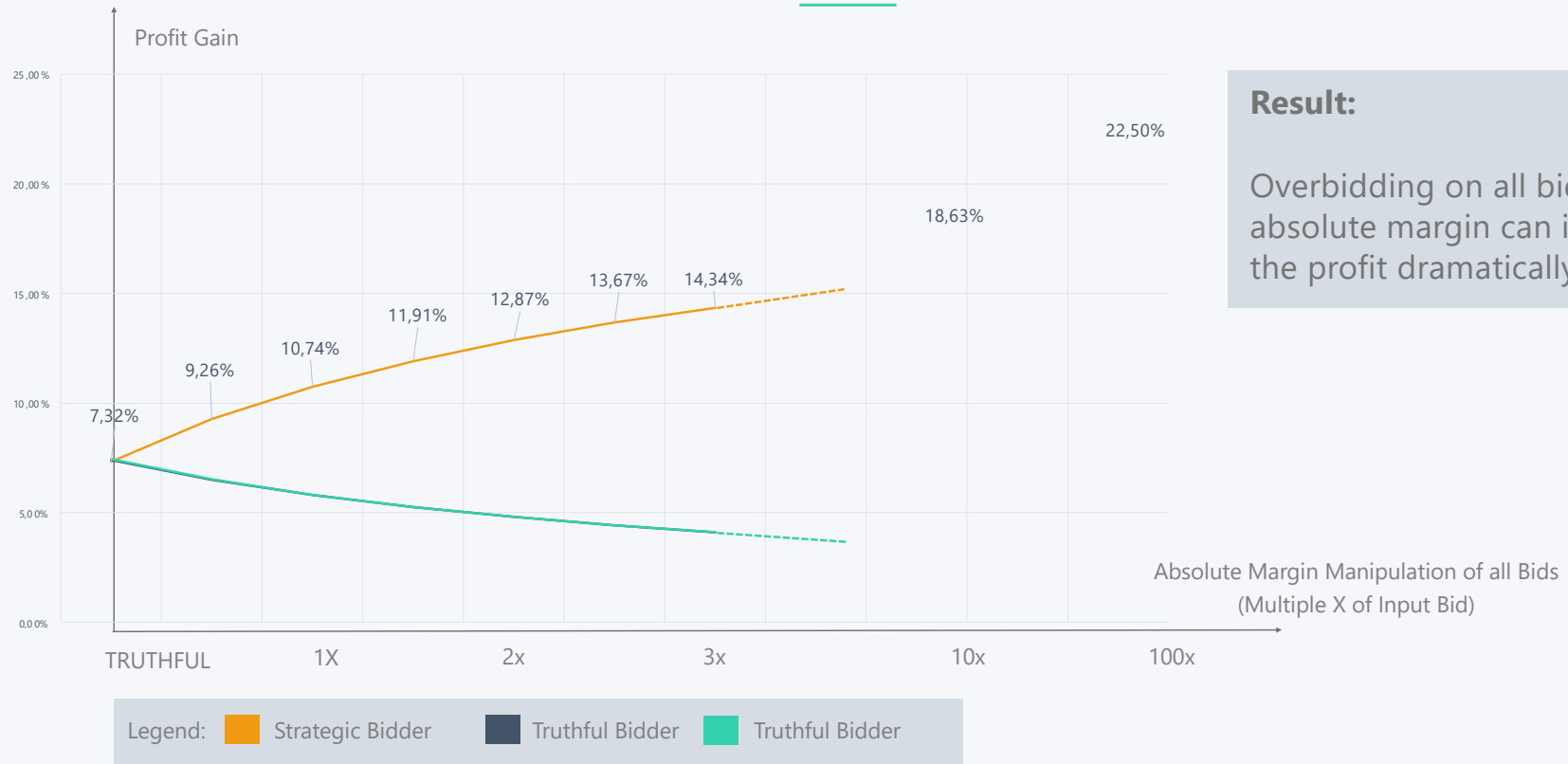
#### **HIGH\_ABS**

Increase prices of all bids by the same absolute margin



## Bidding Strategies for Purchase/Sale Weight Profit Sharing

### Test Results for Strategic Bidder



#### Result:

Overbidding on all bids by an absolute margin can increase the profit dramatically

## 4.4

## Bidding Strategies for Shapley Value Profit Sharing

**Profit Sharing Rule:**

*Calculating the Shapley Value*

$$\text{Shapley Value}_i = \sum_{S, i \in S} \frac{(|S| - 1)! * (|N| - |S|)!}{|N|!} * [g(S) - g(S \setminus i)]$$

*Where:*

$N$  = Grand Coalition of Carriers

$S$  = Subset of Grand Coalition

$g(S)$  = Collaboration Gain of Coalition  $S$

*See [11]*

## Conspiring Bidder Strategies

### INPUT\_MAX

Increase price of Input Bid



## Strategic Bidder Strategies

### INPUT\_MANIPULATION

Overbid/Underbid on the Input Bid

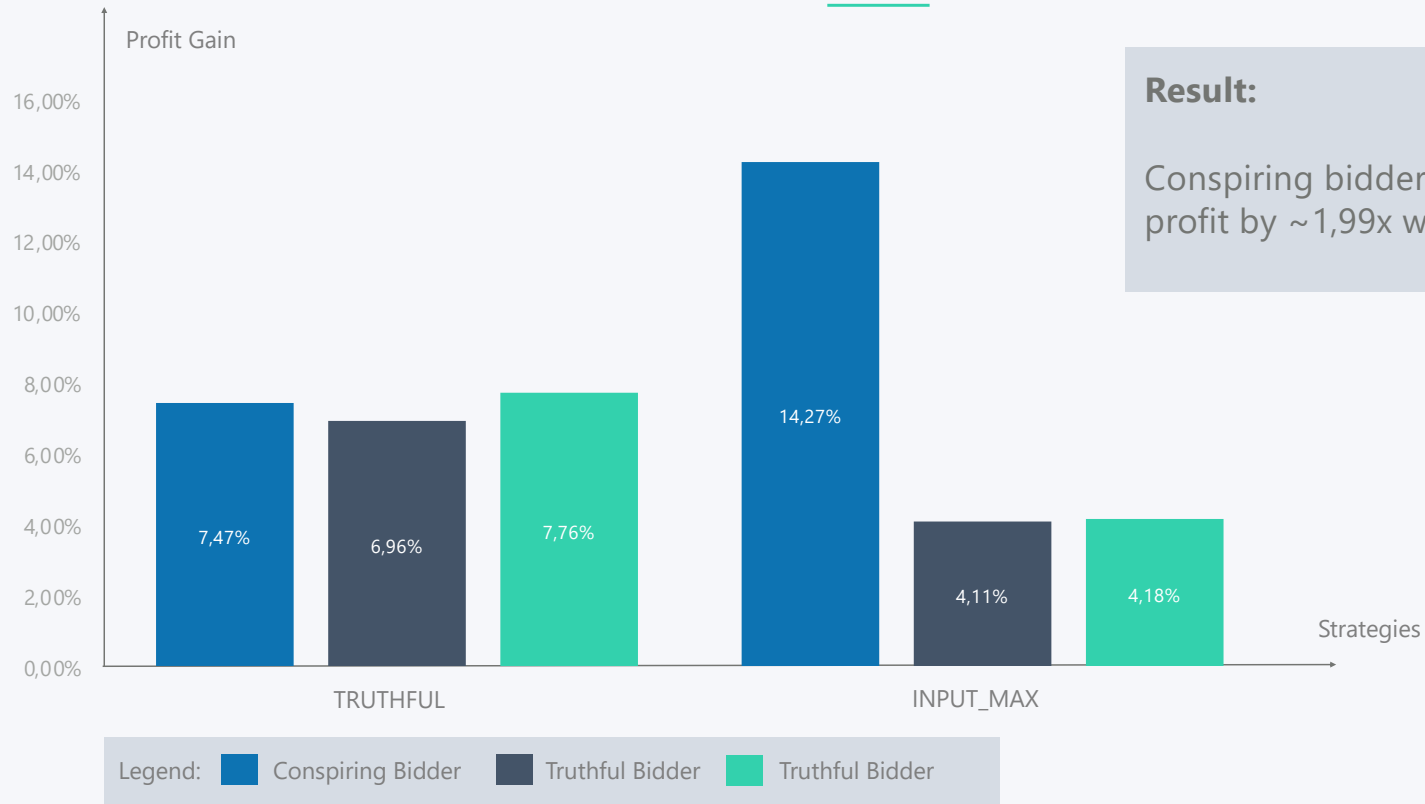
### BID\_MANIPULATION\_REL

Overbid/Underbid on all bids with a relative margin



## Bidding Strategies for Shapley Value Profit Sharing

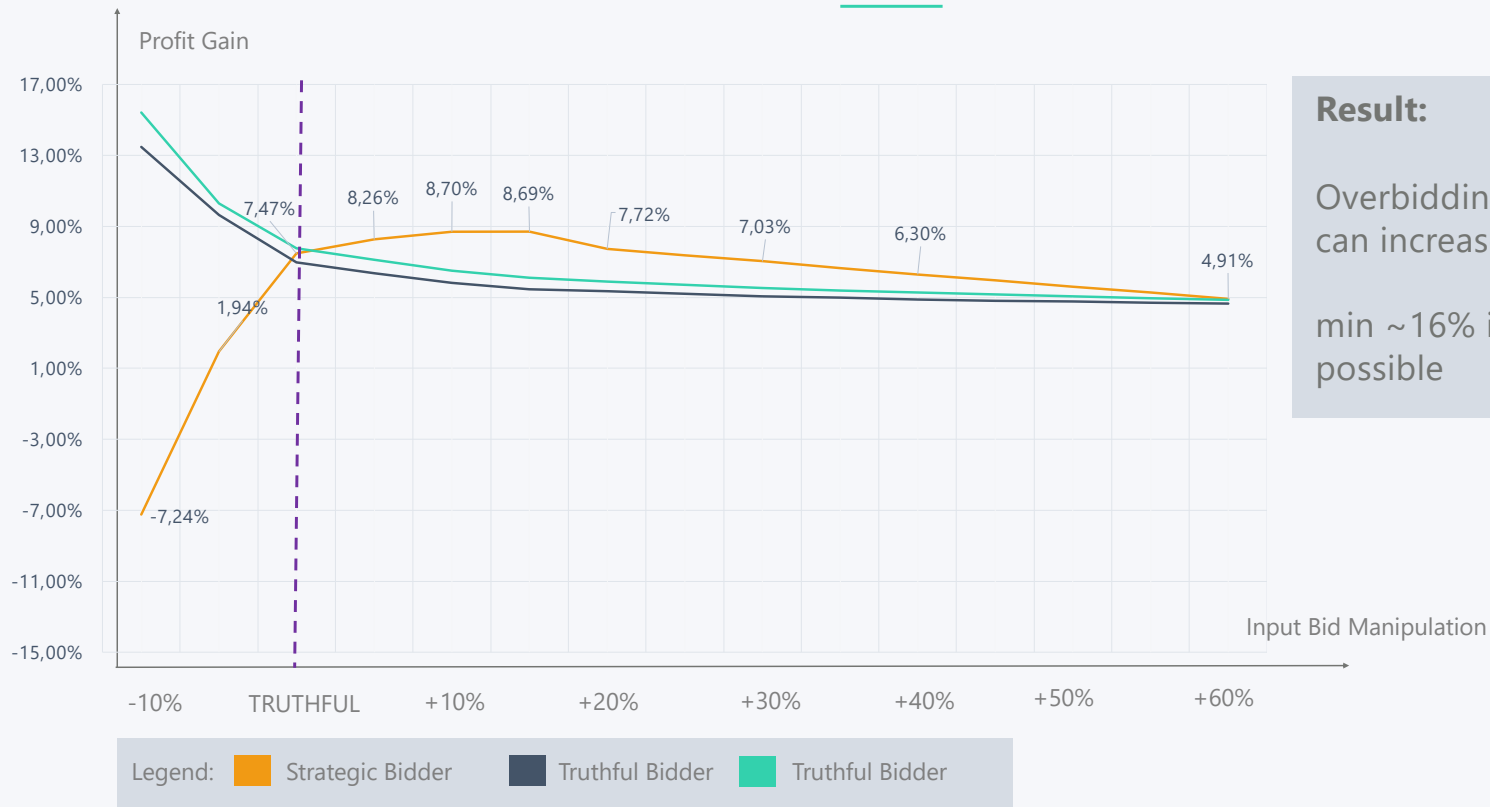
### Test Results for Conspiring Bidder



#### Result:

Conspiring bidder can increase her profit by ~1,99x with INPUT\_MAX

## Bidding Strategies for Shapley Value Profit Sharing Test Results for Strategic Bidder



### Result:

Overbidding on the Input Bid  
can increase profit

min ~16% increase of profit  
possible

## Bidding Strategies for Shapley Value Profit Sharing Test Results for Strategic Bidder



### Result:

Over- or underbidding, in general, is not a good idea



## 4.5

## Bidding Strategies for Critical Weight Profit Sharing

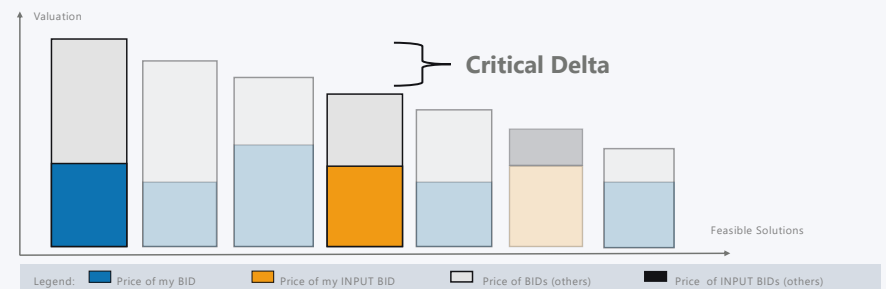
### Profit Sharing Rule:

Use the **Critical Delta** for the calculation of the profit share

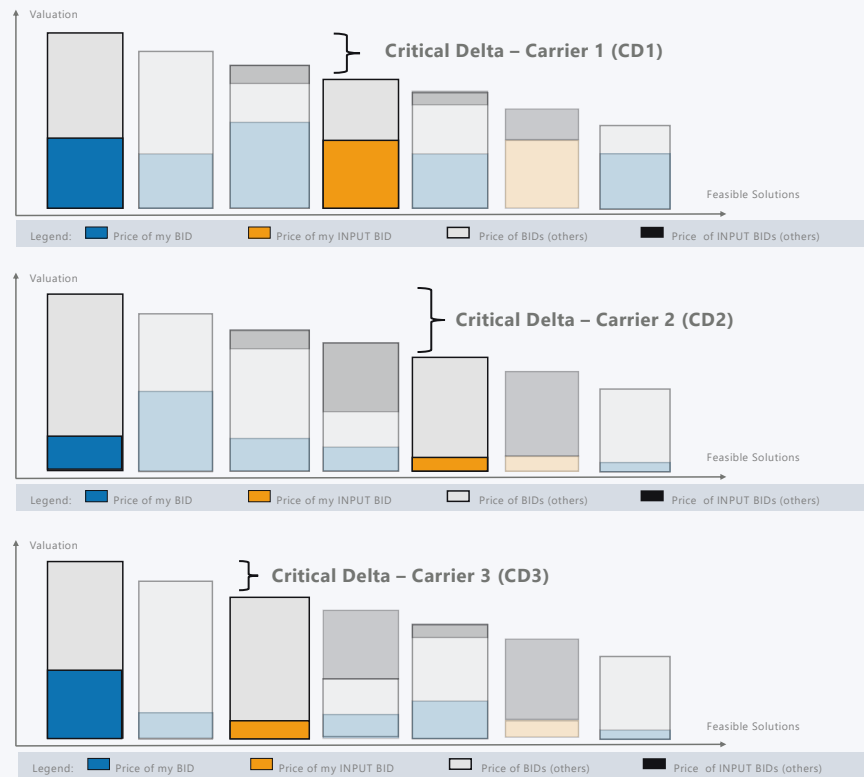
*Note*

**Paying** the **Critical Delta** to bidders would be equivalent to **paying** the **Vickrey-Clarke-Groves Payment** which creates an incentive compatible mechanisms (not budget balanced)

See [9]



## Bidding Strategies for Critical Weight Profit Sharing Perspective of Conspiring Bidder



### Critical Weight

#### For Carrier 1:

$$CD1 / (CD1 + CD2 + CD3) \sim 33\%$$

#### For Carrier 2:

$$CD2 / (CD1 + CD2 + CD3) \sim 43\%$$

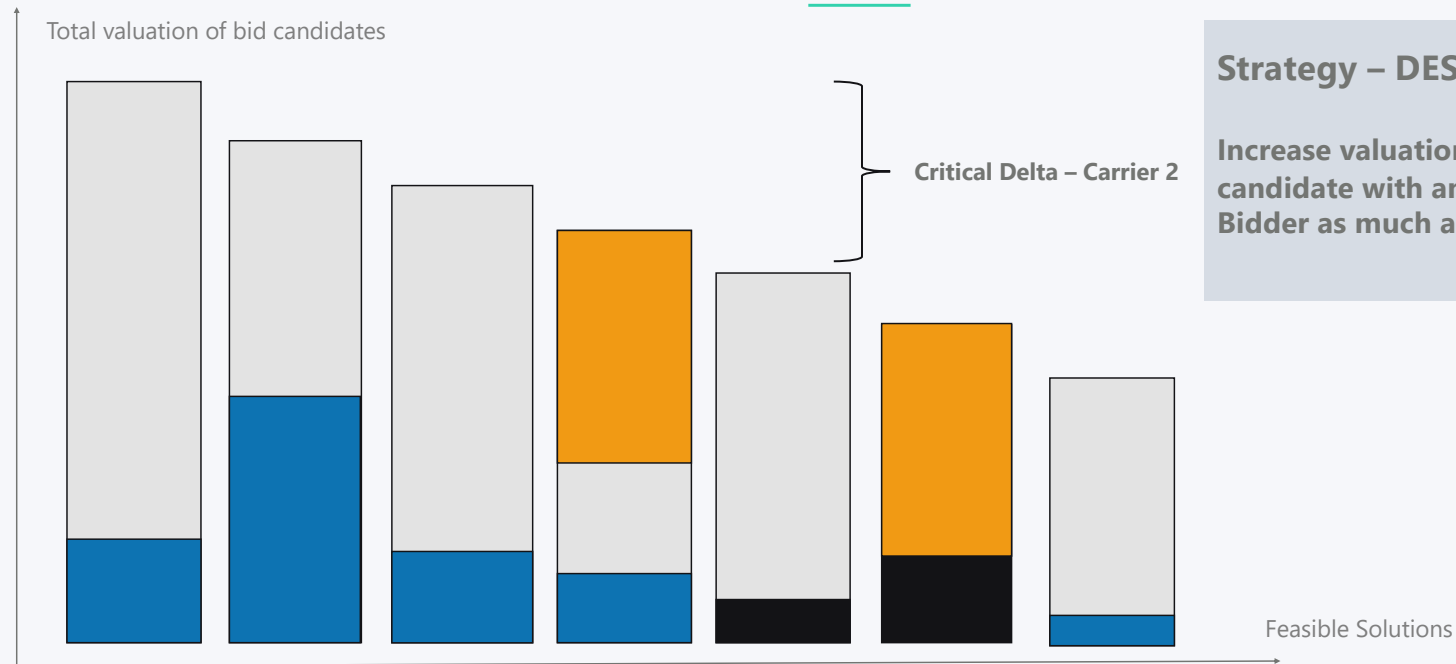
#### For Carrier 3:

$$CD3 / (CD1 + CD2 + CD3) \sim 24\%$$

#### Interpretation

**Marginal contribution** of the carrier  
(however, less accurate than the Shapley Value  
because not considering all sub-coalitions)

## Bidding Strategies for Critical Weight Profit Sharing Perspective of Conspiring Bidder



### Strategy – DESTROY\_WEIGHT

Increase valuation of a Bid of a candidate with an Input Bid of another Bidder as much as possible

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others)

## Bidding Strategies for Critical Weight Profit Sharing Perspective of Conspiring Bidder



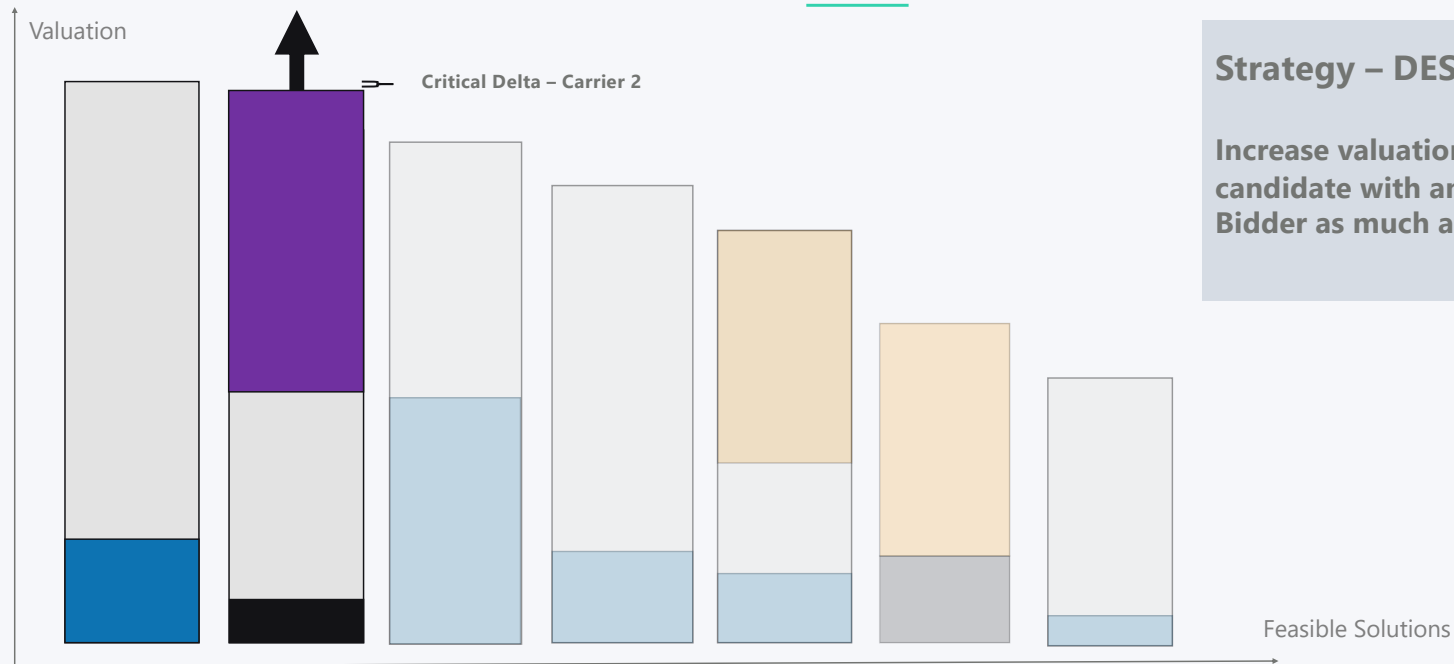
### Strategy - DESTROY\_WEIGHT

Increase valuation of a Bid of a candidate with an Input Bid of another Bidder as much as possible

Legend: ■ Valuation of my BID ■ Valuation of my INPUT BID ■ Valuation of BIDs (others) ■ Valuation of INPUT BIDs (others) ■ Valuation of INPUT BID of Carrier 2






Bidding Strategies for Critical Weight Profit Sharing

## Perspective of Conspiring Bidder

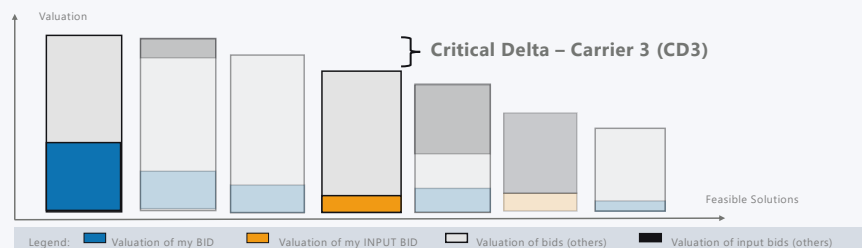
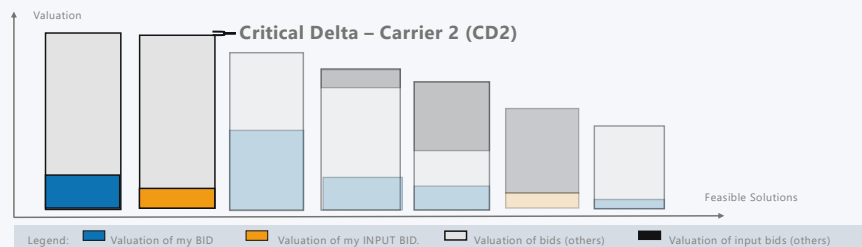
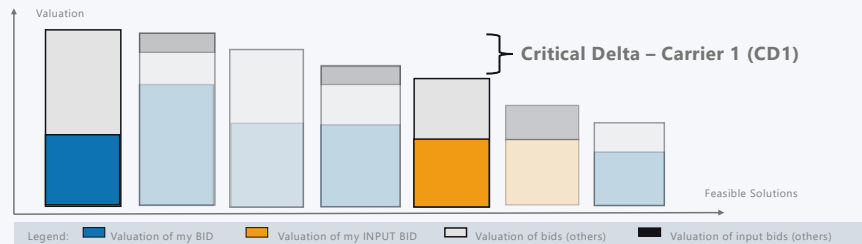


### Strategy – DESTROY\_WEIGHT

Increase valuation of a Bid of a candidate with an Input Bid of another Bidder as much as possible

Legend:  Valuation of my BID  Valuation of my INPUT BID  Valuation of BIDs (others)  Valuation of INPUT BIDs (others)  Valuation of INPUT BID of Carrier 2

## Bidding Strategies for Critical Weight Profit Sharing Perspective of Conspiring Bidder



### Strategy – DESTROY\_WEIGHT

**Increase valuation of a Bid of a candidate with an Input Bid of another Bidder as much as possible**

#### For Bidder 1:

$CD1 / (CD1 + CD2 + CD3) \sim 57\%$

→ (+) Increase of Collaboration Share

#### For Bidder 2:

$CD2 / (CD1 + CD2 + CD3) \sim 0\%$

→ (-) Decrease of Collaboration Share

#### For Bidder 3:

$CD3 / (CD1 + CD2 + CD3) \sim 43\%$

→ (+) Increase of Collaboration Share

## Conspiring Bidder Strategies

### **INPUT\_MAX**

Increase price of Input Bid

### **DESTROY\_WEIGHT**

Increase prices of bids in the feasible solutions with an Input Bid of other carrier(s) as much as possible



## Strategic Bidder Strategies

### **INPUT\_MANIPULATION**

Overbid/Underbid on the Input Bid

### **BID\_MANIPULATION\_REL**

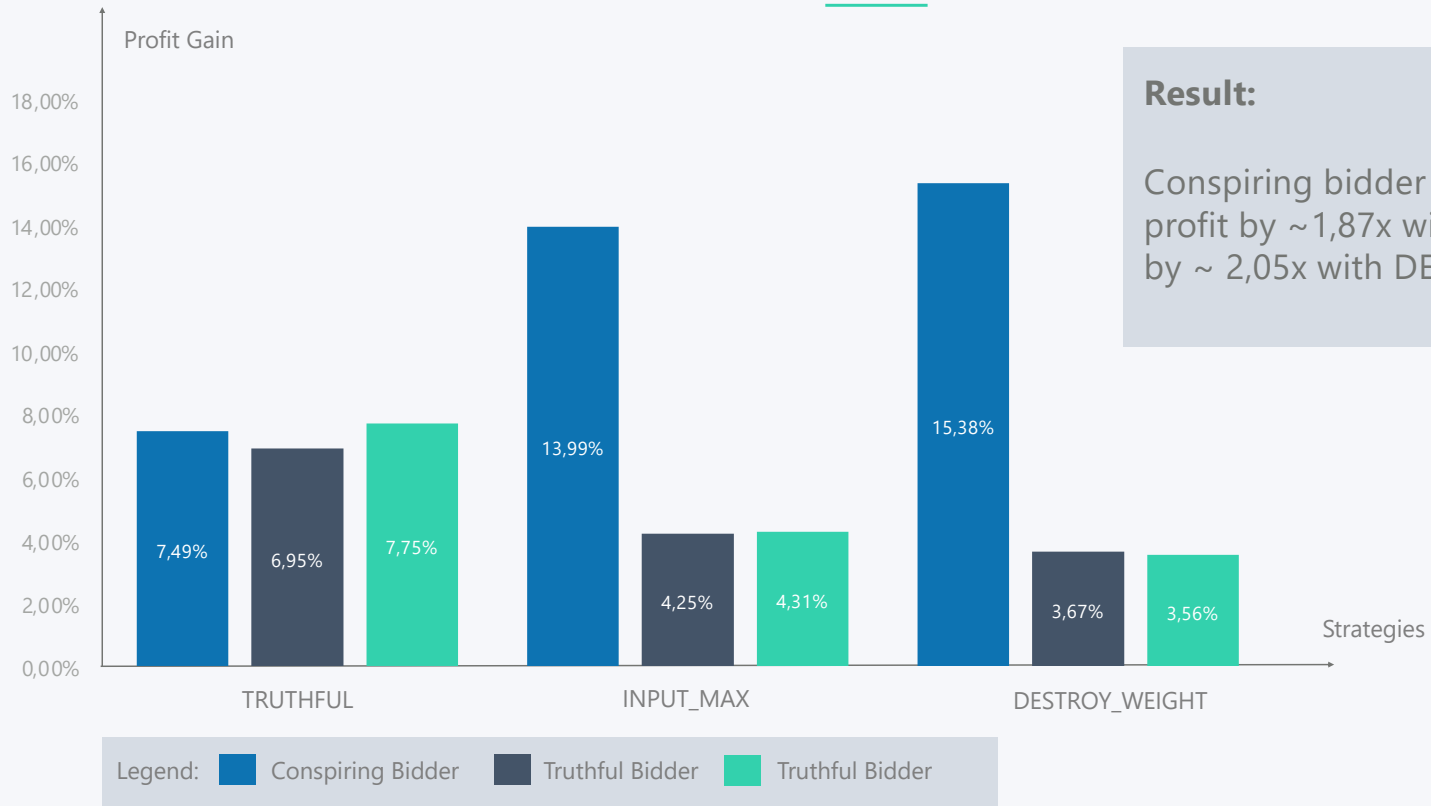
Overbid or Underbid on all bids with a relative margin



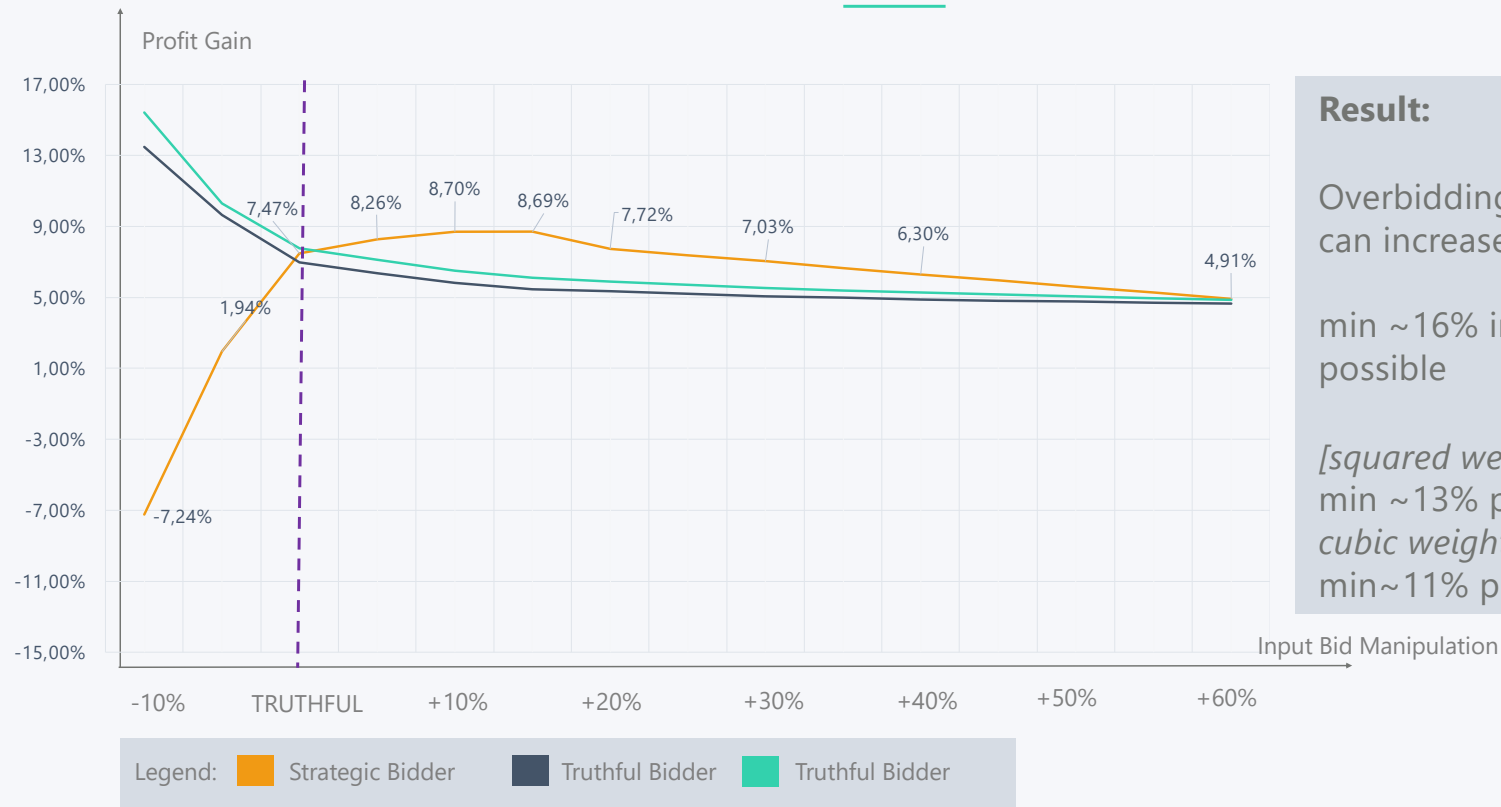


## Bidding Strategies for Critical Weight Profit Sharing

### Test Results for Conspiring Bidder



## Bidding Strategies for Critical Weight Profit Sharing Test Results for Strategic Bidder



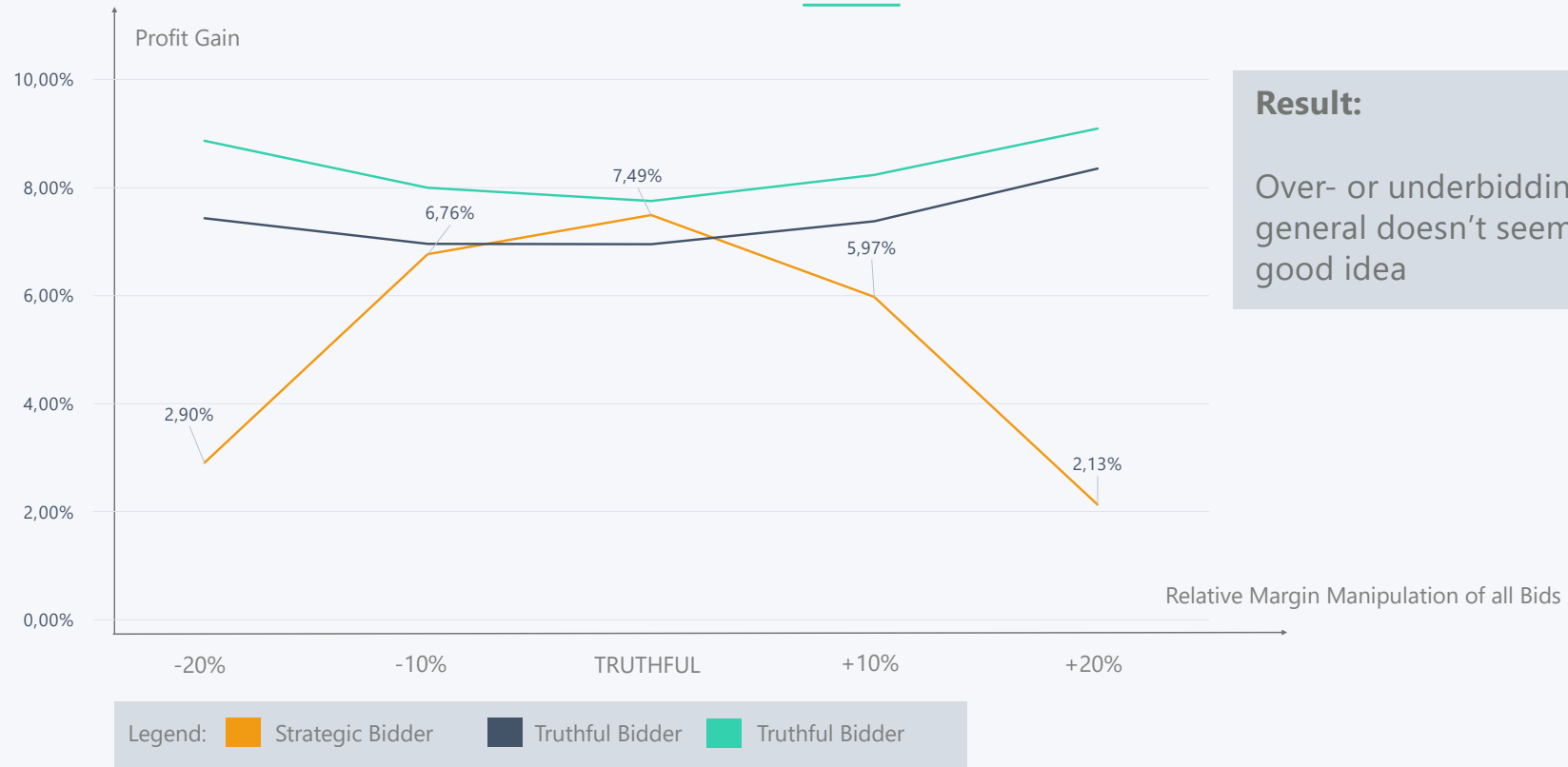
### Result:

Overbidding on the Input Bid can increase profit

min ~16% increase of profit possible

[squared weight  
min ~13% profit increase  
cubic weight  
min ~11% profit increase]

## Bidding Strategies for Critical Weight Profit Sharing Simulation Results for Strategic Bidder



### Result:

Over- or underbidding in general doesn't seem to be a good idea

## 5

## Comparison of the analysed Profit Sharing Methods

## Comparison

### Egalitarian

- ✓ computational efficient
- ✓ easy to understand
- ✗ could be considered unfair
- ✗ encourages overbidding the Input Bid

(Sidenote: Modified Egalitarian superior)

### Purchase/Sale Weights

- ✓ computational efficient
- ✓ incentivizes contribution
- ✗ manipulable through overbidding

### Shapley Value

- ✓ well-known economic formula
- ✓ desirable economic properties\*
- ✓ could be considered fair
- ✓ quite robust against strategic manipulation
- ✗ computational inefficient
- ✗ requires evaluation of all sub-coalitions

\* e.g., efficiency, symmetry, linearity, null player exclusion, anonymity etc. See [9]

### Critical Weights

- ✓ could be considered fair
- ✓ robust against simple strategic manipulation
- ✓ no need to evaluate all sub-coalitions
- ✗ less easy to understand
- ✗ potentially vulnerable to complex strategies

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## 6

## Outlook



## Outlook

# Overview of Research Topics

### *Potentially part of my Master Thesis*

- Comparison of the Shapley Value and Critical Weight Profit Sharing for more than 3 carriers
- Research/Development of complex strategies for manipulating the Shapley Value or Critical Weight Profit Sharing

### *Further Research*

- Evaluation of strategic behaviour during the request selection phase
- Evaluation/development of additional profit sharing methods
- Evaluation of various methods that approximate the Shapley Value
- Evaluation of equilibria and expected outcomes of a setting with multiple strategic carriers
- Strategic evaluation of payment methods that don't guarantee Individual Rationality
- Experimental analysis of strategic behaviour

## References

## References

- [1] **Berger, S. and Bierwirth, C., 2010.** Solutions to the request reassignment problem in collaborative carrier networks. *Transportation Research Part E: Logistics and Transportation Review*, 46(5), pp.627-638.
- [2] **Gansterer, M. and Hartl, R.F., 2016.** Request evaluation strategies for carriers in auction-based collaborations. *OR spectrum*, 38(1), pp.3-23.
- [3] **Gansterer, M. and Hartl, R.F., 2018.** Centralized bundle generation in auction-based collaborative transportation. *Or Spectrum*, 40(3), pp.613-635.
- [4] **Gansterer, M. and Hartl, R.F., 2018.** Collaborative vehicle routing: a survey. *European Journal of Operational Research*, 268(1), pp.1-12.
- [5] **Gansterer, M., Hartl, R.F. and Sörensen, K., 2020.** Pushing frontiers in auction-based transport collaborations. *Omega*, 94, p.102042.
- [6] **Gansterer, M., Hartl, R.F. and Vetschera, R., 2019.** The cost of incentive compatibility in auction-based mechanisms for carrier collaboration. *Networks*, 73(4), pp.490-514.
- [7] **Jacob, J. and Buer, T., 2018.** Impact of non-truthful bidding on transport coalition profits. In *Operations research proceedings 2016* (pp. 203-208). Springer, Cham.
- [8] **Krajewska, M.A. and Kopfer, H., 2006.** Collaborating freight forwarding enterprises. *OR spectrum*, 28(3), pp.301-317.
- [9] **Nisan, N., Roughgarden, T., Tardos, E. and Vazirani, V.V., 2007.** *Algorithmic Game Theory*. Cambridge University Press
- [10] **Renaud, J., Boctor, F.F. and Ouenniche, J., 2000.** A heuristic for the pickup and delivery traveling salesman problem. *Computers & Operations Research*, 27(9), pp.905-916.
- [11] **Shapley, L.S., 2016.** 17. A value for n-person games (pp. 307-318). Princeton University Press.