Rigid Body Dynamics of Transforming Spinning Tops

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Project Description

The focus of this project is on designing, analyzing, and building a transforming spinning top that will expand and deploy its arms as it reaches a certain speed of rotation; the expansion occurs via centrifugal force and there are no electronic components or actuators used in the process. This top is built entirely based upon passive dynamics; therefore, at lower speeds the arms of the top will be in a retracted position, locked in by a simple mechanical constraint (a spring loaded latch or a mass based trigger) and as the top's speed increases the radial forces developed by centrifugal force will exceed the restoring force developed by the mechanical constraint allowing the arms of the top to expand outward.

The basis of this design came from the behavior of commercially available spinning tops such as Beyblades that utilize gyroscopic stability and have moving parts. However, while most spinning tops use pre-programmed mechanisms and/or the force generated by an impact to cause a movement or transformation in their geometry, the purpose of this project was to create a spinning top that would transform solely because of the rotation of the top itself. Therefore, this design allows for the demonstration of pure centrifugal force and the effect that angular momentum has on both stabilizing and destabilizing a spinning object. When the arms of the spinning top are extended, the mass distribution of the spinning top is drastically changed, resulting in a new moment of inertia that also affects the spin of the top. As a result, the spinning top will exhibit several different dynamic behaviors such as an upright stable state, a wobble state, precession, and finally rolling behavior that are all associated with physical principles in the study of rigid-body dynamics.

Once the design had been created in Creo, the model was constrained for manufacture, cost, and assembly. The top was modeled as a single rigid body where the dimensions were carefully selected so that the strength and durability of the structure would match the required performance characteristics of the top. The deployable mechanism was integrated into the central portion of the top where sufficient clearance and tolerance were provided to ensure that the deployable mechanism would operate reliably when subjected to high-speed rotation. After the CAD design had been completed, the top was printed using a 3-D printer and then assembled to facilitate physical testing of the top. The ability to see the transformation occur in real-time and to assess how well the experimental results agreed with the predicted results allowed the team to gain insight into the mechanical design of the top.

Ultimately, the transforming spinning top represents a small, but powerful, example of how one can combine a basic understanding of classical dynamics with creative mechanical design to demonstrate many interesting physics concepts including variations in moments of inertia, gyroscopic motion, the conservation of angular impulse-momentum, and torque free precession.

Kinematic and Force Analysis Using Creo

Background

Centrifugal force is one of the most important forces to consider in engineering, not only because nearly every mechanical system has rotating parts, but centrifugal force also allows engineers to design systems that use this outward force for separation, stability, and efficient mechanical operation. These applications of centrifugal force can be seen in our daily lives; one major example is the laundry machine. In a laundry machine, the spin cycle uses centrifugal force to force water out of your clothes to help with the drying process. With these ideas in mind, group 7 set out to design a project to demonstrate the stabilizing ability of centrifugal force. The idea behind this project started with a popular toy that is prominent in many childhood toy boxes, the Bayblade. This toy uses centrifugal force to not only stay upright but also to battle other Bayblades, as the toys make contact with each other. The goal of these toys is to destabilize the other, causing them to stop spinning through contact with each other. Centrifugal is the outward force experienced when an object is rotating about a central point or axis.

Scope

By applying a classical mechanical lens for analysis, approximated predictions can be made by neglecting parameters that would otherwise exasperate depth needed to solve the problem. The first assumption being made here is at the core of the classical mechanical scope and is interpreted as Newton's Laws. The analysis is then made even simpler by treating the spinning top as a nondeformable and non-destructable object, especially since a 3D printed spinning top is not meant to carry any weight other than its own which can be regarded in the form of mass moment of inertia.

When analyzing mechanisms, it is usually best to develop the understanding from multiple two dimensional perspectives, starting with the simplest explanation first which is often rooted in statics. Setting up the FBD so that the spinning top sits perfectly upright provides a possible simplification for an opening analysis, but with one inherent flaw: it's unstable. The instant the angle between the spinning top's axis and the absolute vertical axis becomes nonzero, it becomes a dynamics problem. However, the most critical observation is actually the fact that spinning tops wobble without falling over but only while spinning. The moment that seems to exist solely to keep the spinning top upright is that of nutation, related to the rate of precession, both being time derivatives of particular Euler angles.

Since the mathematics describing unsteady precession are more advanced, an assumption for steady precession can be made and treated as a rough design constraint. By using a bicycle wheel suspended by a string as an example and the fact that the nutation angle's max value is reached when the spinning top completely falls over, we can define the scope for the analysis of the spinning top as having the following time states:

Charge-up State:
- The spinning top is connected to the launcher prongs as a gear-meshed cord is being pulled.
- Defines initial dynamic parameters.

Upright State:
- An ideal stable state enabling static analysis without regarding precession.
- This state is practically nonexistent and if it does occur, only lasts for a negligible amount of time.
- Initial vertical plane FBD's are drawn this way for simplification.

Wobble State:
- The main state for analysis but also the most complex.
- Inherently possesses unsteady precession.
- This project aims to approximate unsteady precession as existing as various states with steady or no precession.
- Steady precession is achieved at certain equilibrium angle values when momentum is no longer transferring between the Euler angles and is conserved. Steady precession can be treated as an approximation that acts to ignore nutation angle changes by smoothing the fluctuations out over time.

Horizontal State:
- An ideal state for analysis simplification that does not actually exist.
- Relates the mathematics to the bicycle wheel example.
- Assumed to have steady precession if momentum is conserved.

Roll State:
- Occurs when the maximum nutation angle is reached.
- A slip state may be considered too if slipping cannot be neglected.
- Assumed to have steady precession

Resting State:
- The spinning top is not moving
- There are two resting states, one before charge-up, and one after roll.

A freefall state is not needed if there is negligible energy loss during fall and apex to ground collision.

Theory

The theoretical mathematical analysis for the project primarily revolves around the principles of linear and angular impulse-momentum and conservation of momentum. This can be treated as two different superposing coordinate systems: rectilinear and cylindrical. Equations (#s) describe how the two coordinate systems relate to each other for describing the same points in space:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \qquad \text{[0.1 Eq. (1)]}$$

$$r = \sqrt{x^2 + y^2 + z^2} \qquad \text{[0.2 Eq. (1)]}$$

Euler Angles:

1. **Spin angle:** ψ
   Measures the angle of the shaft or initial rotational axis.
2. **Nutation angle:** θ
   Measures the angle that the spinning top leans at from the absolute z-axis.
3. **Precession angle:** ϕ
   Measures the angle for the circular position for the spin axis about the absolute z-axis.

Table 1: Kinetic Parameters:

	Linear	**Angular**
Force or Moment (F & M)	$\Sigma\vec{F} = m\vec{a}$ [1.1a, Eq. (1)]	$\Sigma\vec{M}_O = I\vec{\alpha}_O$ [1.1b, Eq. (1)]
Mass or Inertia (m & I)	$\Sigma m = m_{ABS} + m_{ball\_bearings}$ [1.2a, Eq. (1)]	$\Sigma I = I_{constant} + I_{variable}$ [1.2b, Eq. (1)]
Momentum (L & H)	$\dot{\vec{L}} = \Sigma\vec{F}$ [1.3a, Eq. (1)]	$\dot{\vec{H}}_O = \Sigma\vec{M}_O$ [1.3b, Eq. (1)]

Uniform Circular Trajectory Kinematics

As a particle moves at a constant velocity along a uniform circular trajectory, it must move with an acceleration toward the center of this circle equal to the tangential velocity squared divided by the radius of curvature. This centripetal acceleration a_r, which can also be extrapolated from the centrifugal force, can also be stated in terms of the period T, as the time needed to complete one revolution.

$$a_r = \frac{v^2}{R}$$ [2.1, Eq. (2)]

$$T = \frac{2\pi R}{v}$$ [2.2, Eq. (2)]

$$a_r = \frac{4\pi^2 R}{v}$$ [2.3, Eq. (2)]

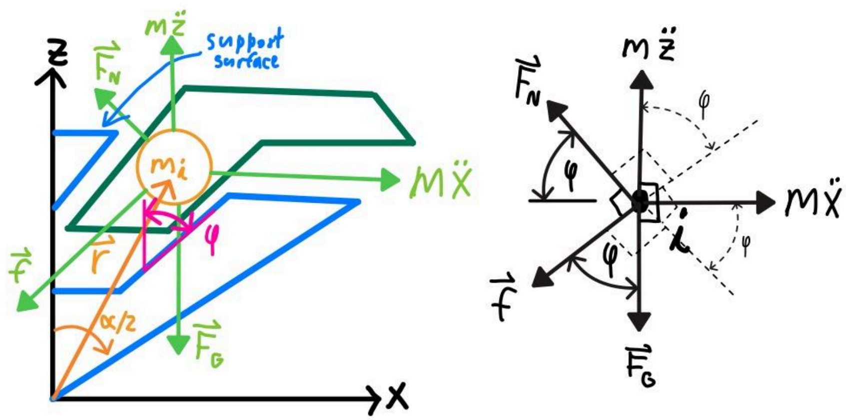
$$R = \sqrt{x^2 + y^2}$$ [2.4, Eq. (1)]

Turntable Kinematics (Local XY or R plane)

By comparing a spinning top with a turntable, it is quickly realized that as any point along the surface of a turntable (or horizontal plane within spinning top as it is assumed to be standing perfectly upright) rotates about a fixed axis in a uniform circular trajectory, a particle adjacent or just above an arbitrary point at rest will be moving at the same basic linear kinematic rates relative to said arbitrary point. This would mean that, while neglecting friction, a particle resting on a turntable will appear to have the tendency to move in the opposite direction of that of the arbitrary surface point's motion as it moves along the uniform circular trajectory. Basic relative kinematic analysis while neglecting friction provides that this particle would move outward equal to the centripetal acceleration.

Arm Extension/Retraction (Local RZ Plane)

When analyzing the RZ plane as it aligns with one of the three arms to provide the following free body diagram, the summation of forces can also be used to prove the authenticity of equations (2.1-2.3) by comparing equations of equilibrium of a nonspinning top standing perfectly upright with one that is kept from falling over due to a moment due to momentum transfer about angle θ as it transfers from the initial applied torque about angle ψ that set it in motion.



[5, Fig. 1]: RZ=RX Plane FBD

The summation of forces while treating the vertical arm midplane as direction x while also considering turntable dynamics for the XY Plane can output the following equations:

$$\Sigma F_x = m_i \ddot{x}_i = F_{centrifugal} - F_N cos\varphi - f sin\varphi$$ [3.0, Eq. (5)]

$$F_{centrifugal} = m\omega^2 x$$ [3.1, Eq. (1)]

$$\Sigma F_x = 0 = -F_N cos\varphi + f sin\varphi$$ [3.2, Eq. (5)]

Using linear-angular momentum conservation, the following formulas can provide further details for analysis and control system development for spinning tops:

$$\vec{L}_1 + \Sigma \int_{t_1}^{t_2} \vec{F} dt = \vec{L}_2$$ [4.1, Eq. (1)]

$$(\vec{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \vec{M}_O dt = (\vec{H}_O)_2$$ [4.2, Eq. (1)]

$$\vec{H}_O = \vec{r} \times m\vec{v}$$ [4.3, Eq. (1)]

$$L = m v_G$$ [4.4, Eq. (1)]

$$H_G = I_G \omega_G$$ [4.5, Eq. (1)]

Creo Motion Study

In essence, the sole objective for this motion study is to prove [3.0, Eq. (5)] so as to have more precisely controlled geometric transformation triggered by the relative dynamics for masses moving about relative to the spinning top. We evaluated the kinematics and forces using Creo's mechanism constraints and mass-property extraction, then used analytical rigid-body dynamics to determine deployment conditions. A full time-domain motion study was not necessary because the deployment is governed by centrifugal force thresholds rather than prescribed input motion.

Prototype

The transformed Spinning Top prototype was created using a final Creo design with 3D printing technology in ABS plastic and tested the degree to which the developed system was consistent with its modeled behavior. The central body, hinge elements, and deployable parts were printed accurately, maintaining the fundamental geometric configuration of the mechanism. The 3D printing process did not allow the team to manufacture the deployable arms longer than they had been designed to be in the CAD concept. Since the deployable arms were shorter than designed, the physical properties of the spinning top's mass distribution and the centrifugal loading experienced by the spinning top, along with the visual effect of the transformation, would also be different from what the team had envisioned when designing the device.

Although the manufactured prototype preserved the functional mechanism of the spinning top, since the arms of the prototype were shorter than those in the CAD concept the degree of outward extension of the arms following deployment was less than that originally intended for the design.

Using the launcher to launch the prototype, the team found that it demonstrated consistent and predictable mechanical behavior at relatively low and moderate rotational velocities, with the deployable arms being retracted as expected. At higher rotational velocities, centrifugal forces acted upon the deployable arms causing them to release from their retracted position. The transformation of the arms from the retracted to deployed positions occurred every time, demonstrating that the mechanical constraint and the hinges functioned as the team had intended. However, since the arms were shorter than the team had originally anticipated, the centrifugal force acting on each arm was less than that estimated during the initial analysis. Therefore, the prototype had to be spun at a slightly greater rate than initially calculated for the deployable arms to extend and the moment of inertia to increase after the arms extended.

Following the deployment of the prototype, the team observed the dynamic characteristics of the prototype to be similar to that described in the analysis, consisting of an increased wobble and a transition to precession. However, the magnitude of the stability changes was less dramatic than that indicated by the team's Creo model for a prototype with arms of the same length as those used in the design study. The decrease in the amount of inertia caused by the short arms reduced the magnitude of the angular deceleration and the extent of the widening of the spin stance, indicating a more subtle change in the dynamics of the system. Some of the observed wobble and asymmetries may have been attributed to minor manufacturing imperfections associated with the 3D printing process, i.e., slight mass imbalances, wall thickness variations, and surface roughness. Regardless of the imperfections present in the 3D printed parts, the prototype remained structurally sound and functionally reliable through numerous test runs.

In general, the manufactured prototype supported the basic idea of the design by showing that it could deploy centrifugally and transition between states of motion. However, the limitations placed on the dimensions of the printed arms limited the magnitude of the transformation and changed the behavior of the prototype following deployment. These results demonstrate the importance of scale, mass distribution, and clearances in centrifugal mechanisms and will provide valuable information for the development of revised designs that can restore the desired mechanical and dynamic performance of the device.

Redesign Idea

While the prototype provided a successful demonstration of centrifugally deploying into a rotating state, there were several issues that have become obvious, and that clearly indicate where an updated version can be developed. The most significant deviation from the initial design was the reduced arm length in the last printed version of this model due to geometric and dimension restrictions established by the project's manufacturing guidelines. Thus, the deployed arms did not extend outward as originally designed, which decreased the change in moment of inertia once deployed. Both the visual change associated with the deployment and the dynamics of the system are impacted by this issue, because smaller arms create less centrifugal force and thus require greater angular velocities to break the latch mechanism free. Additionally, the limited deployment of the arms produced a different top stability and precession behavior then what was predicted using the larger-arm model.

A second aspect of the design to be improved is the accuracy and reliability of the deployment mechanism. Although the arms released as needed upon achieving sufficient centrifugal force, the shortening of the levers involved (as a result of the shortened components) created a much narrower operating window and caused inconsistent results from trial to trial. Several factors contributed to the variations seen in the deployment, including the varying degrees of hinge friction, slight misalignment from 3-D printing, and variation in contact between surfaces. To achieve a consistent release over multiple trials, one of the options available would be to increase the arm length within the allowable dimensions; another option would be to redistribute mass to the outer radial edge. Improving the design of the hinges (i.e., incorporating metal pins, reducing sliding contact, etc.) would improve the repeatability of the deployment.

Additionally, the sudden transition from rotational behavior prior to deployment to that after deployment has also shown possible avenues of design improvements. Since the arms were shorter than originally designed, the moment of inertia was not significantly changed and the resultant decrease in angular velocity was also not as large as anticipated. Therefore, while the prototype still showed evidence of wobble, precession, and eventually roll, the transition between each of these behaviors was not as distinct as the analytical model suggested. To again show a distinct difference in rotational behavior between the pre-deployment and post-deployment conditions, one could either add a slightly longer or heavier deployable element(s), or design the arms so they will deploy outward in a more linear fashion than the current design.

Lastly, structural integrity may present an additional design opportunity. In addition to the arms being shorter than desired, the stress levels on the small hinge regions and the load bearing features were increased, and some of these locations displayed wear after repeated testing. By increasing the thickness of the critical regions, creating fillets, or changing the material from PLA to a stronger material (such as nylon or a reinforced composite filament), the longevity of the prototype can be increased without compromising its functionality.

In summary, the primary goal of the redesign should be to restore the desired arm length or deployment radius, to improve the design of the hinges, and to refine the tolerances so that the transformation occurs in a smoother and more predictable manner. With these changes, the actual performance of the prototype should more closely match the analytical predictions and the centrifugal effects that the project sought to illustrate.

Rendered Images of Design

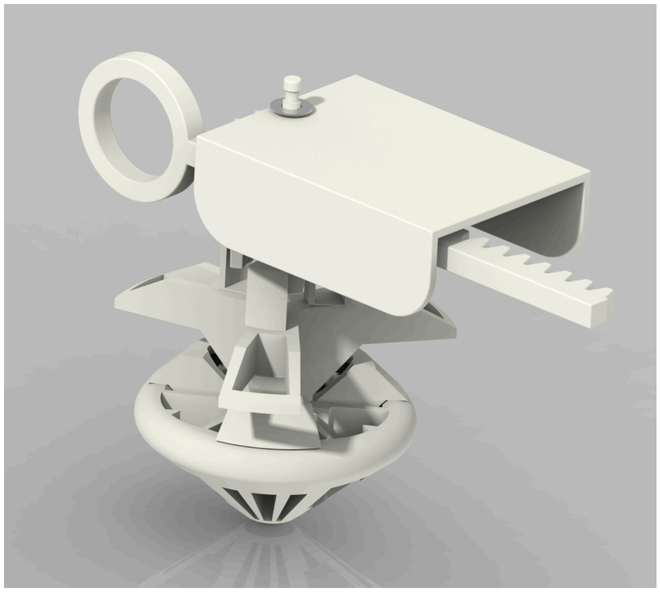


Fig. 2: Full Design Assembly

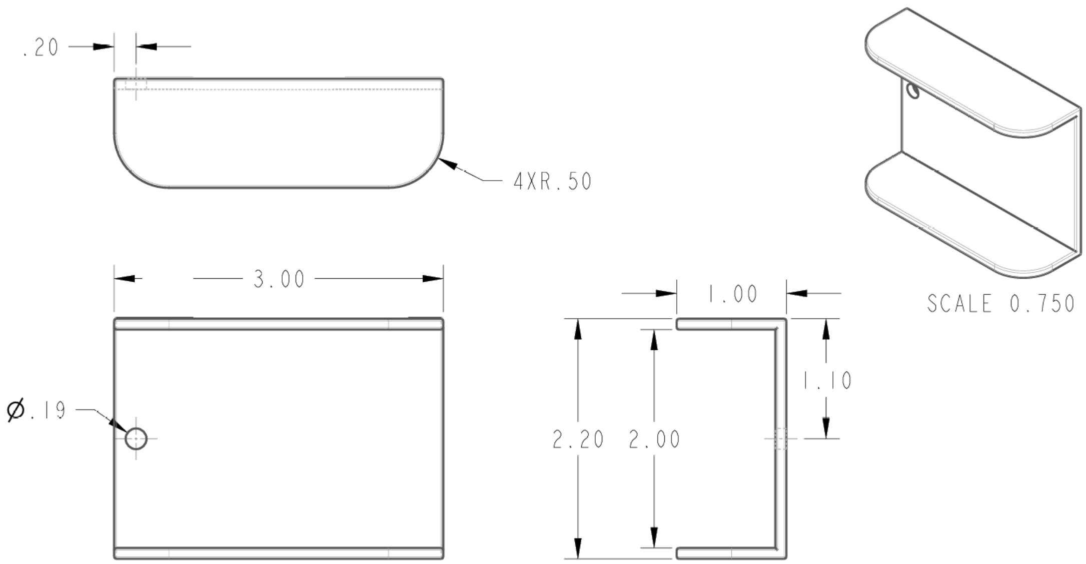
Fig. 3: Exploded View

Component Cost (Bill of Materials)

Table 2:

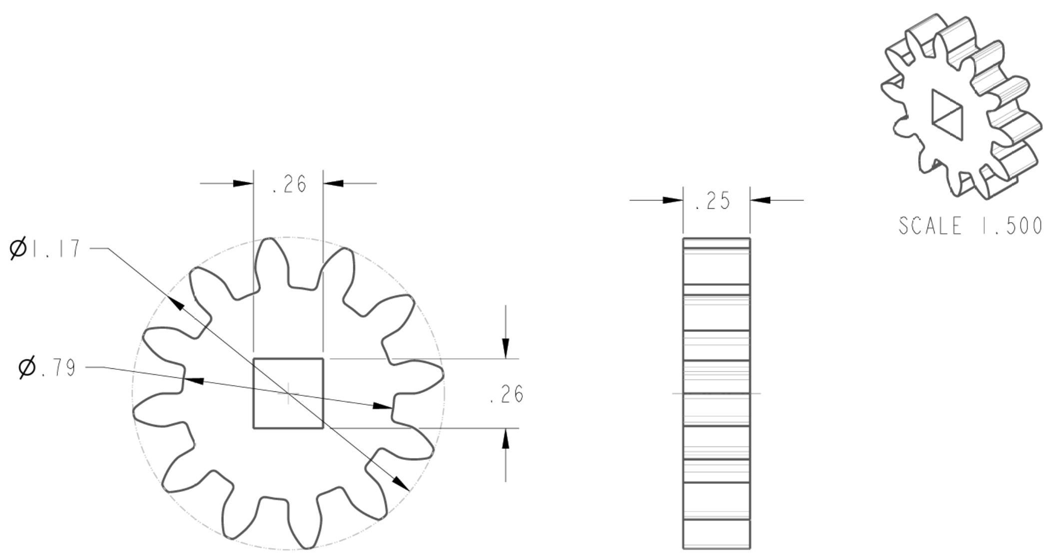
Printed Parts			
Name	**Volume (in^3)**	**Quantity**	**Cost**
Holder	1.173	1	$7.62
Rack	0.633	1	$4.11
Gear	0.163	1	$1.06
Shaft & Connector	0.303	1	$1.97
Mount	0.943	1	$6.13
Arm	0.149	3	$2.91
Cone	1.583	1	$10.29
		Total for Printed parts	$34.09
Non-Printed Parts			
Name	**Individual Cost Estimation**	**Quantity**	**Cost**
Washer	$0.25	1	$0.25
Bearings	$0.04	4	$0.16
		Total for Non-printed parts	$0.41
Hot Glue Estimation			
	Volume Estimate (in^3)	**Cost per in^3**	**Cost**
Hot Glue	0.05	$0.89	$0.04
		Total for Hot Glue Used	$0.04
		Total Cost of Prototype	$34.55

Appendix



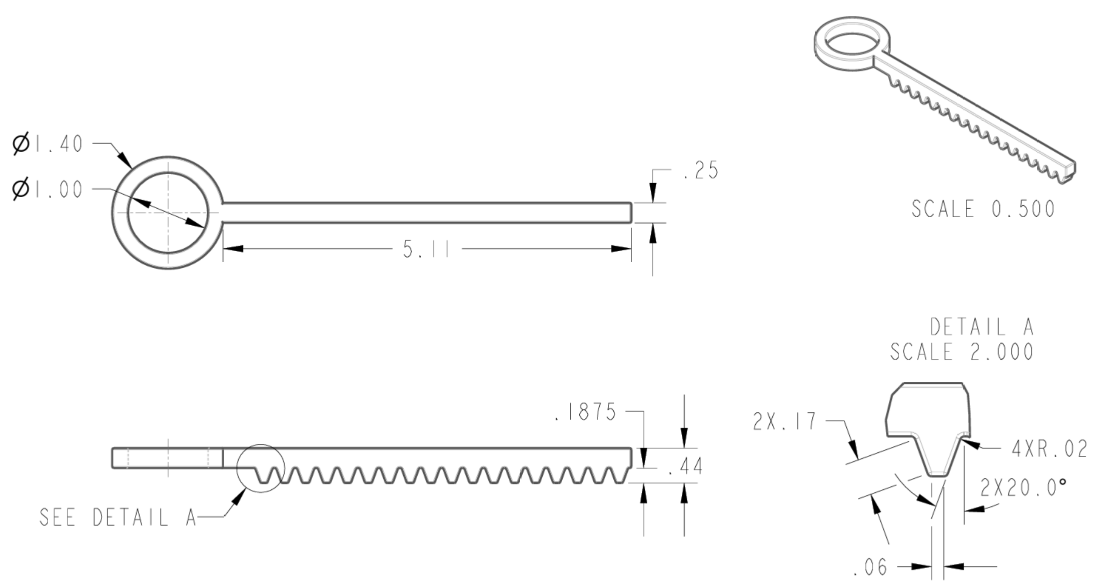
Holder has a uniform thickness of 0.10 and 0.02 radius rounds on all edges

Part Name: Holder
Designer: Phillip C. Leach
Units: Inches
Scale: 1.000



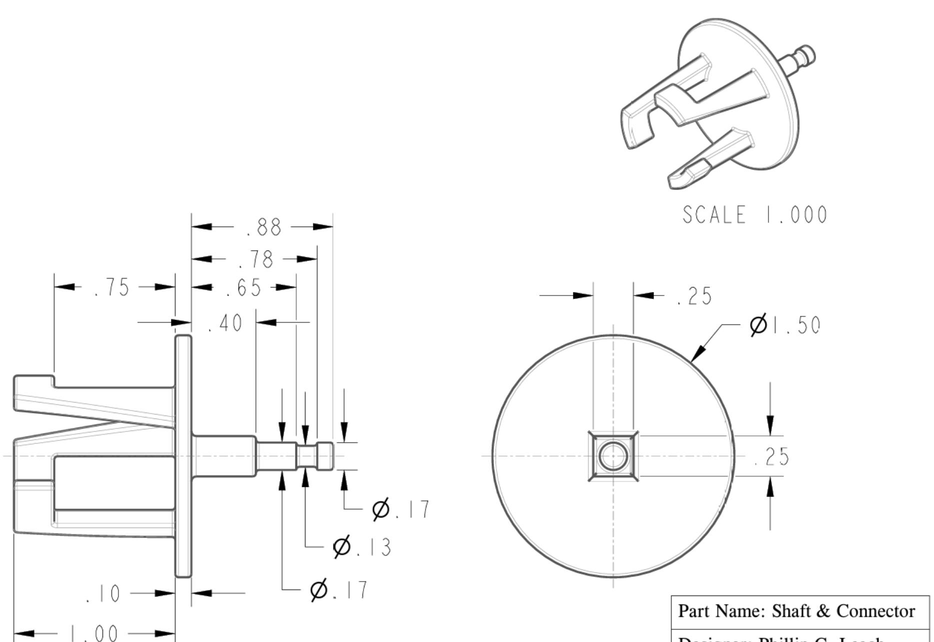
Pitch Diameter: 1 in
Presssure angle: 20°
No. of teeth: 12

Part Name: Gear
Designer: Phillip C. Leach
Units: Inches
Scale: 2.500



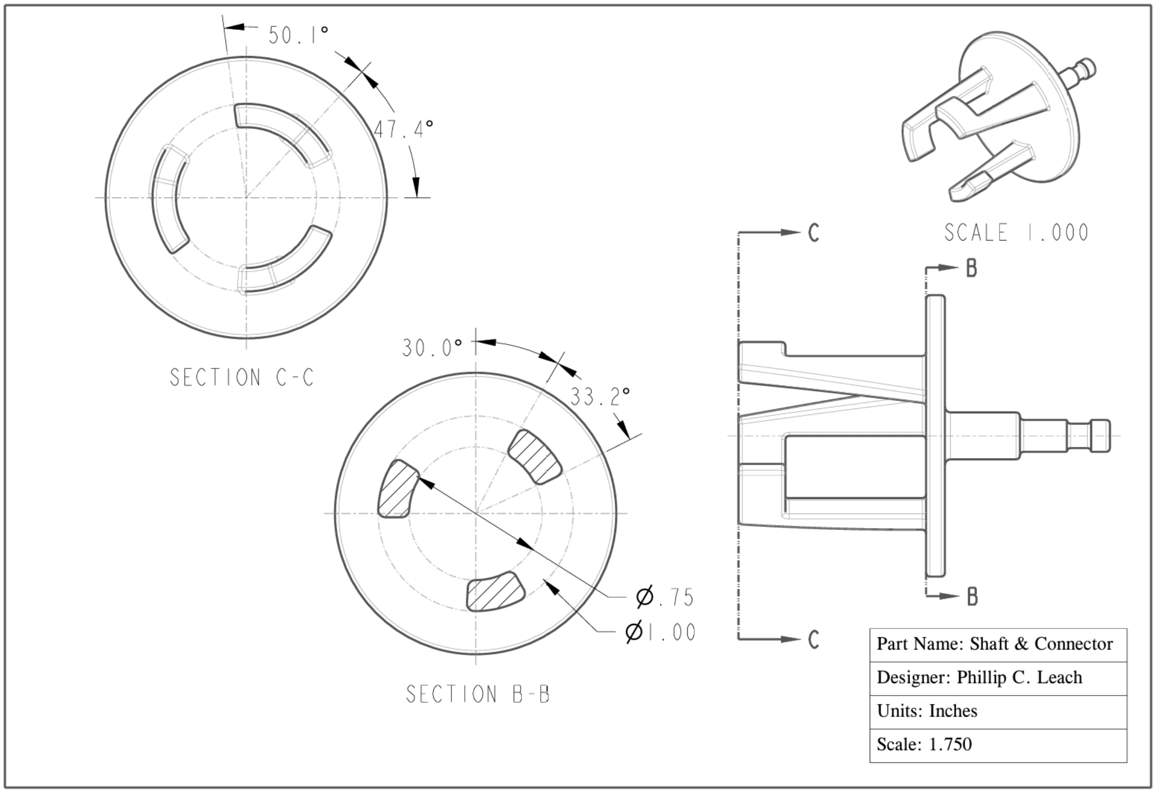
A round of 0.02 was applied to all edges

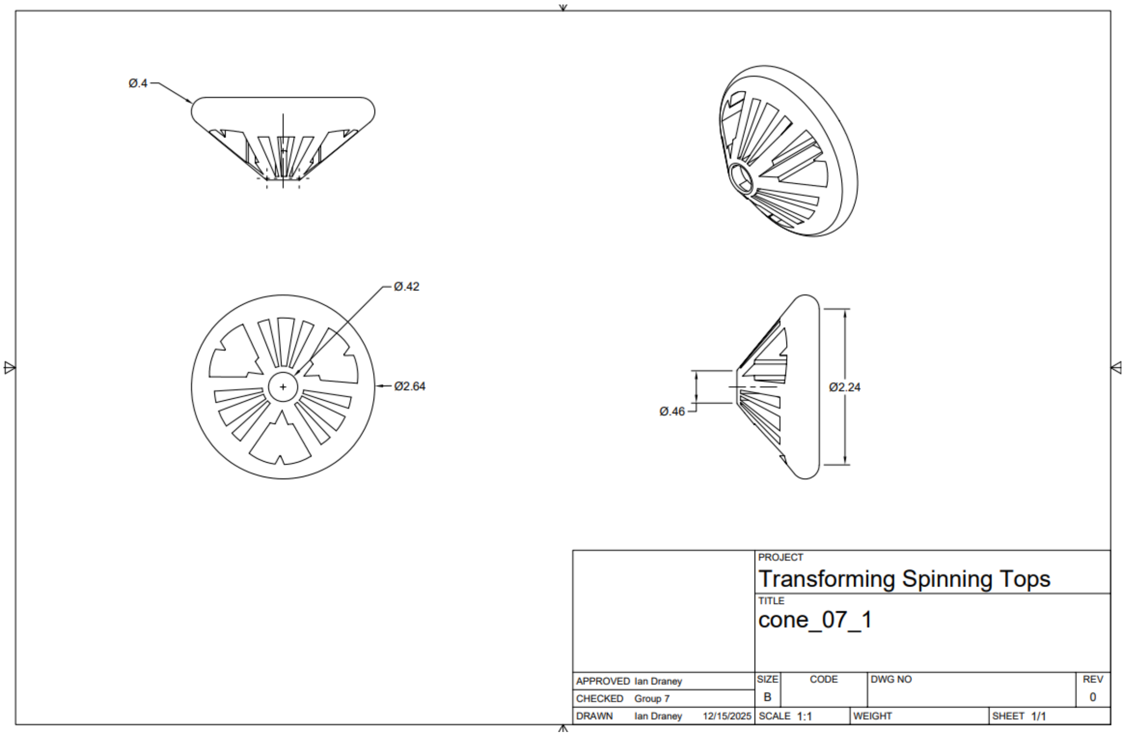
Part Name: Rack
Designer: Phillip C. Leach
Units: Inches
Scale: 0.750



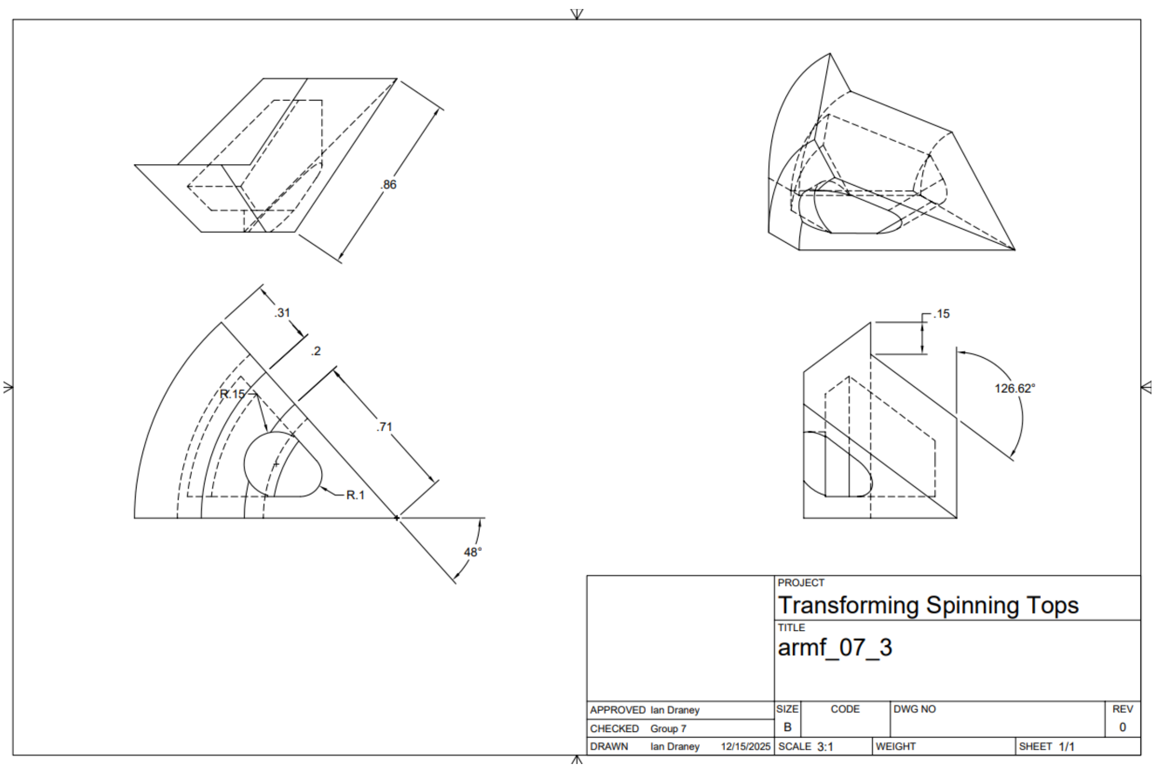
Round from square shaft to base plate is 0.03 in. All other edges are rounded at 0.02 in.

Part Name: Shaft & Connector
Designer: Phillip C. Leach
Units: Inches
Scale: 1.500

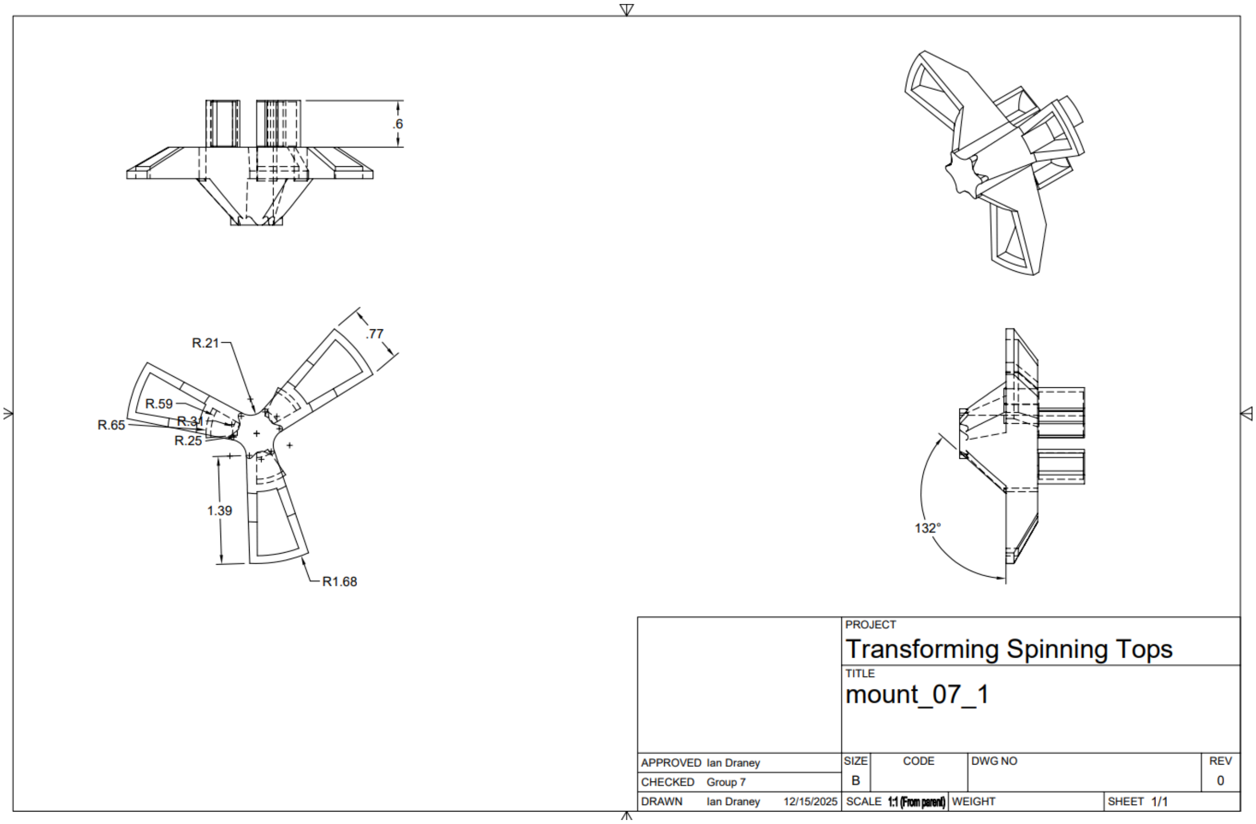

50.1°
47.4°
SECTION C-C
30.0°
33.2°
∅.75
∅1.00
SECTION B-B
SCALE 1.000
C
C
B
B
Part Name: Shaft & Connector
Designer: Phillip C. Leach
Units: Inches
Scale: 1.750



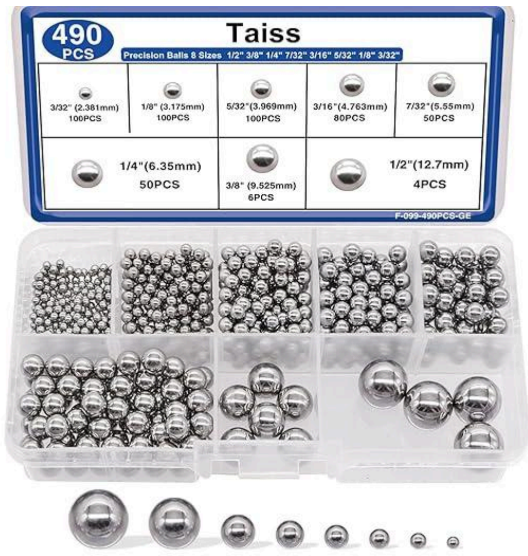
Designed by Ian Draney



Designed by Owen McLucas-Lopez



Designed by Ian Draney



Purchased Ball Bearings (4)

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