Basic circuit elements — resistors

Electrical circuits can be modeled by a small number of "ideal" components. One of the simplest and most useful of these is the *resistor*. In some ways, electrical circuits can be modeled by fluid (hydraulic) systems, and this may provide a useful visual model for simple circuits.

The basic parameters for an electrical circuit are *current (I)* and *voltage (V)*.

Current is the rate of flow of *charge (Q)* — in good conductors (metals) the charge carriers are electrons.

$$I = dQ/dt$$

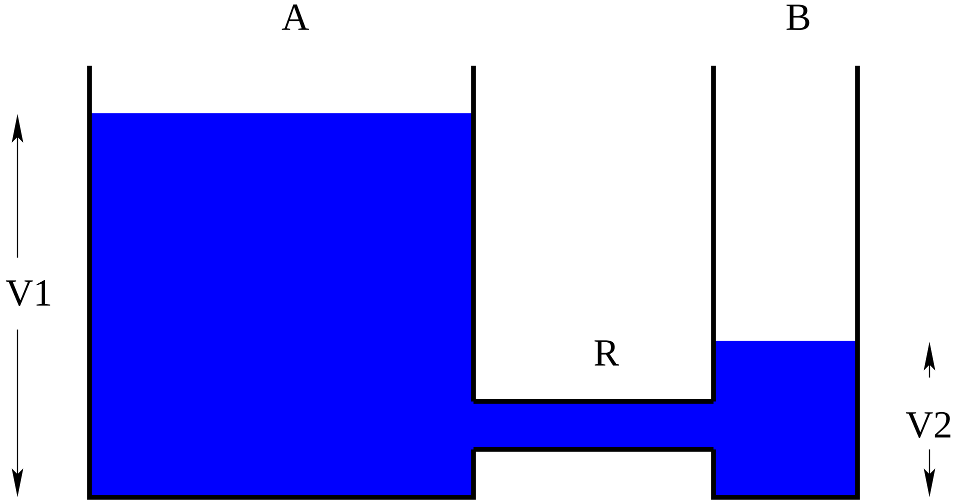
Voltage is a measure of potential energy.

A potential energy (voltage) *difference* causes current to flow from one point to another.

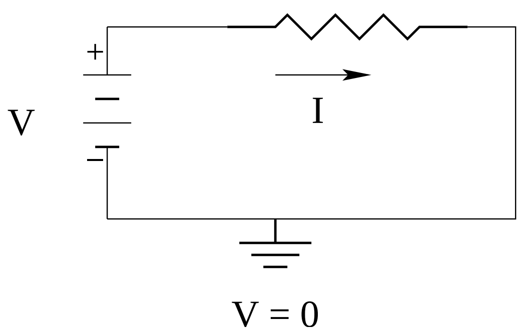
The rate at which the charge flows (current) is determined by the potential energy difference, and by the resistance in the circuit.

In a hydraulic model, consider two containers partly full of water. Water will flow from the reservoir with the higher potential (V1) to the lower potential (V2).

The rate at which the water will flow depends on the size (resistance) of the pipe connecting the two reservoirs.



Following is an equivalent electrical circuit:



The resistance, R, is defined as

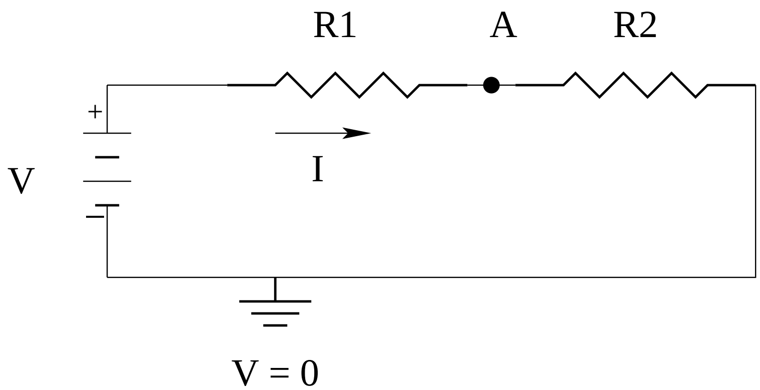
$$R = \frac{V}{I}$$

This relationship (written as $V = I \times R$) is called **Ohm's law**.

Note that v is a *potential difference*, and one terminal of a circuit is usually taken as having a potential of 0 volts.

Fun with resistors

Suppose we have a circuit with two resistors in series:



What is the total resistance here?

$$R_{total} = R1 + R2$$

In fact, for any resistors in series, the total resistance is the sum of the resistances in series.

The current flowing in each of the resistors in series is the same, and is found from Ohm's law as

$$I = \frac{V}{R_{series}}$$

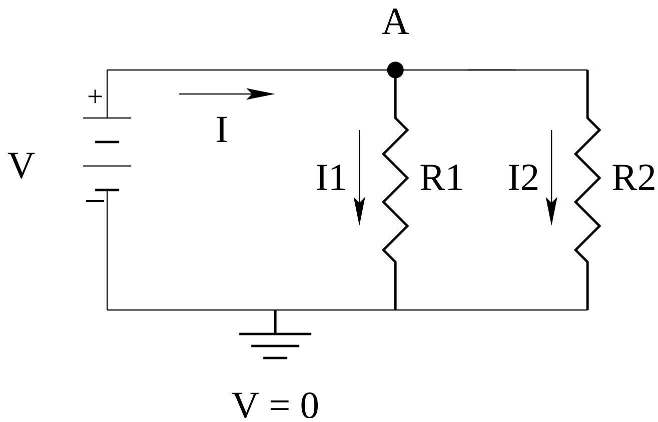
If R1 and R2 are identical, what is the voltage at node A?

How about if R2 = 2 × R1?

In general, how could the voltage at node A be determined?

$$V_A = I \times R2$$

Suppose the resistors are arranged in a parallel configuration:



What is the total resistance in the circuit now?

Suppose $R1 = R2$. Then the current in both would be the same, and $I1$ and $I2$ would each be $I/2$.

In general, $I1 = V/R1$, and $I2 = V/R2$.
Also, $I = I1 + I2$. (At node A, the current flowing into the node must equal the current flowing out of the node.)

If there is a single resistance (say, $R_{parallel}$) equivalent to the this network, then

$$V/R_{parallel} = V/R1 + V/R2$$

.

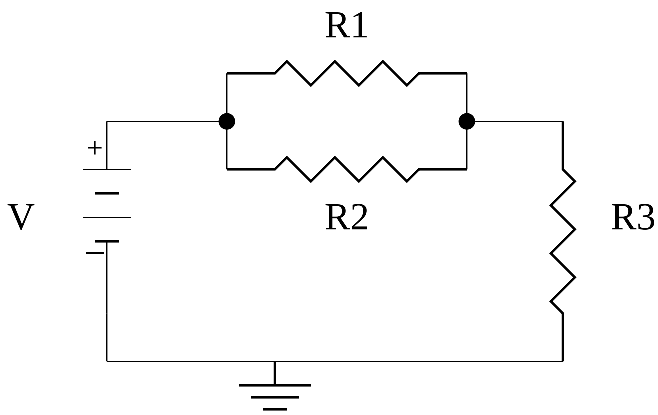
We therefore have the relationship

$$1/R_{parallel} = 1/R1 + 1/R2$$

Clearly, this generalizes for any parallel combination of resistors.

Series and Parallel Equivalent circuits

Clearly, in analyzing a circuit, we can replace parallel and series resistance combinations with equivalent resistances.
We can use this idea to simplify more complex circuits:



Here, the parallel components, R1 and R2, can be replaced by their equivalent resistance, $(R1 \times R2/(R1 + R2))$
This can then be added to the series resistance, R3.
The net result is a single equivalent resistance.

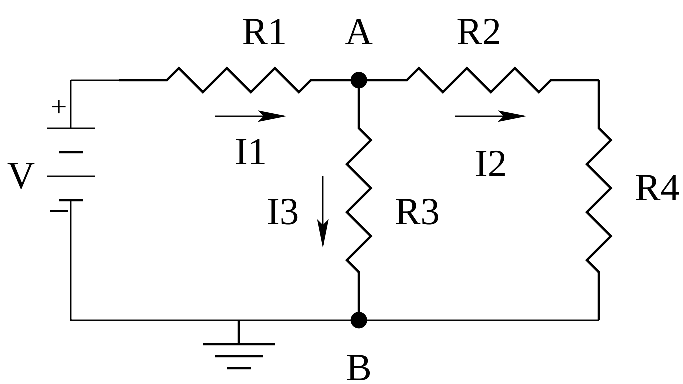
Can this be done for all resistance circuits?
No, so how can more complex circuits be analyzed?

Analyzing complex circuits — Kirchoff's laws

There are two useful laws for analyzing an electrical circuit, both based on conservation laws in physics.

The first states that at any node in a circuit (a node is where two or more components are connected) the current flowing into the node is exactly equal to the current flowing out of the node. This follows from the *conservation of charge.*

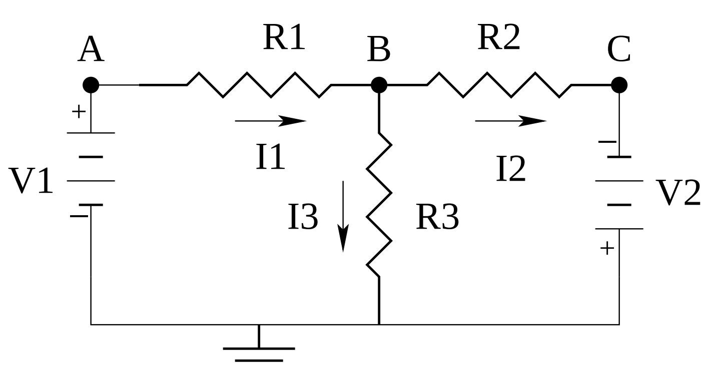
In the following, at node A, $I1 = I2 + I3$



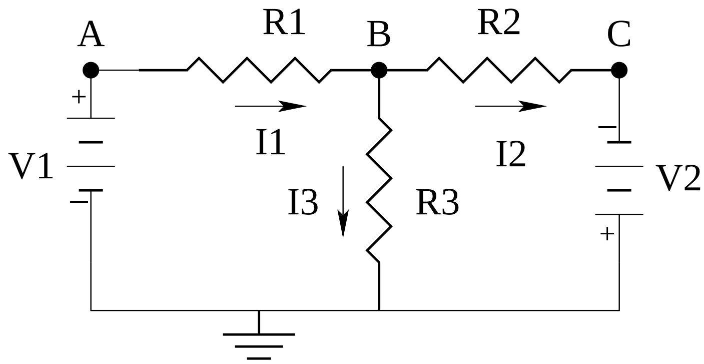
where I1 is the current through R1, I2 is the current through R2 and R4, and I3 is the current through R3.

I1 flows into node A, and I2 and I3 flow out of node A.

This law applies even if there are several voltage sources:



The second law states that, in any closed loop in a circuit, the net voltage difference is always zero. (Since voltage is potential energy, this is simply conservation of energy.)
It is often stated as "the sum of the voltages in a closed loop is zero."



Here there are three possible loops; considering only the outer loop, and traversing the loop in a clockwise direction,

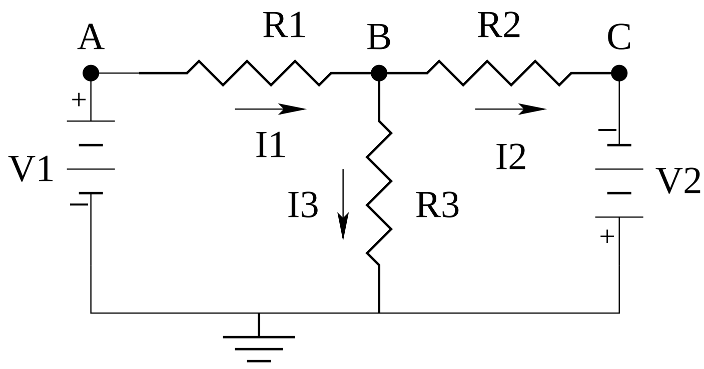
$$-V1 + I1 \times R1 + I2 \times R_2 - V2 = 0$$

Note that the sign here is the sign from the sources in the order in which they are encountered in the traversal of the loop.
If we use either of the inner loops, we can use the fact that $I3 = I1 - I2$.

Either one of these laws can be used to derive a general method for solving for voltages and/or currents in a circuit.

Node Analysis

Consider the following circuit:



Let R1 be 2K ohms, R2 be 3K ohms, and R3 be 1K ohms, V1 be 3 Volts, and V2 be 10 volts.

Node A is at 3V, node C is at -10V, and only node B is unknown.

We wish to solve for the currents, $I1$, $I2$, and $I3$.

Of course, we need only find any two of those, since $I1 = I2 + I3$.

From Ohm's law:

$I1 = (V1 - VB)/R1 = (3 - VB)/2000$

$I2 = (VB - V2)/R2 = (VB - (10))/3000$

$I3 = VB/R3 = VB/1000$

Since, at node B, $I1 = I2 + I3$, we can write a single equation for the unknown voltage at VB as

$(3 - VB)/2000 = (VB - (10))/3000 + VB/1000$

or VB = -1.0V

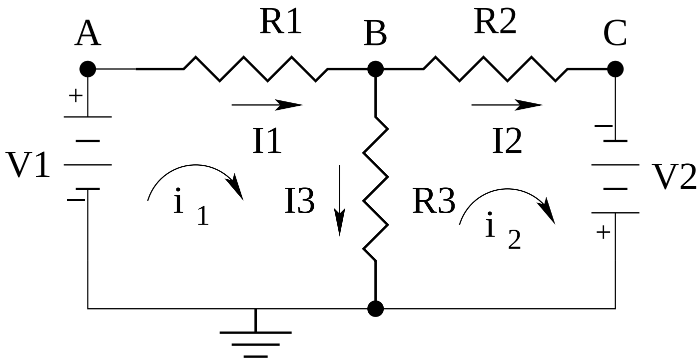
The currents can be found by substituting for VB, giving

$I3 = VB/1000 = -0.001$ amp $= 1$ ma

similarly, $I2 = 3$ ma and $I1 = 2$ ma

Mesh Analysis

Consider the same circuit:



Again, R1 is 2K ohms, R2 is 3K ohms, and R3 is 1K ohms, V1 is 3 Volts, and V2 is 10 volts.

Here we will use the second law, and consider closed loops (meshes). We will define two mesh currents, i_1 and i_2, and write equations for each mesh:

$-V1 + i_1 \times R1 + (i_1 - i_2) \times R_3 = 0 = -3 + 2000i_1 + 1000(i_1 - i_2)$

$-(i_1 - i_2) \times R_3 + i_2 \times R2 = 0 = -1000(i_1 - i_2) + 3000i_2$

This gives us two equations in two unknowns, but they are simple linear equations and can be solved easily.

Both these techniques can be applied to circuits of any complexity, and result in simple sets of linear equations.

Even more importantly, both creating the equations to be solved and the actual solving of the equations can be done numerically with relatively simple code.

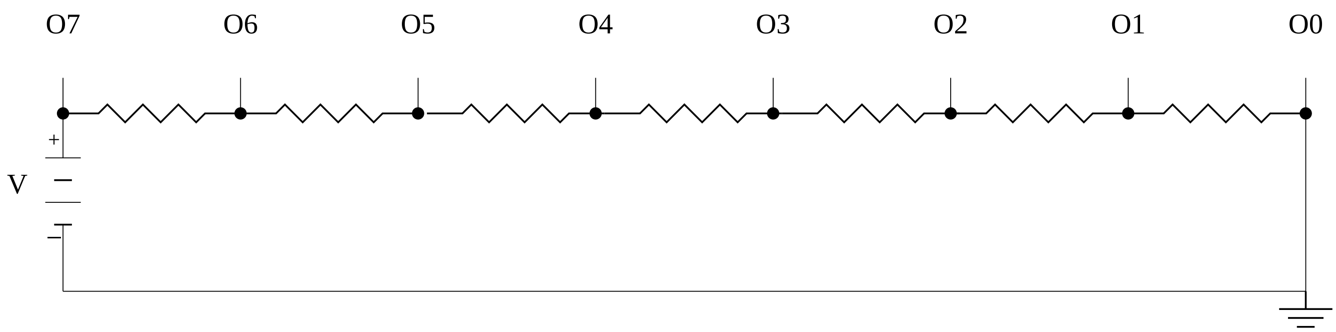
(Insert commercial for CS3731 here)

Using resistors in circuits

Resistors are used in several ways in simple circuits.
We will look at two applications.

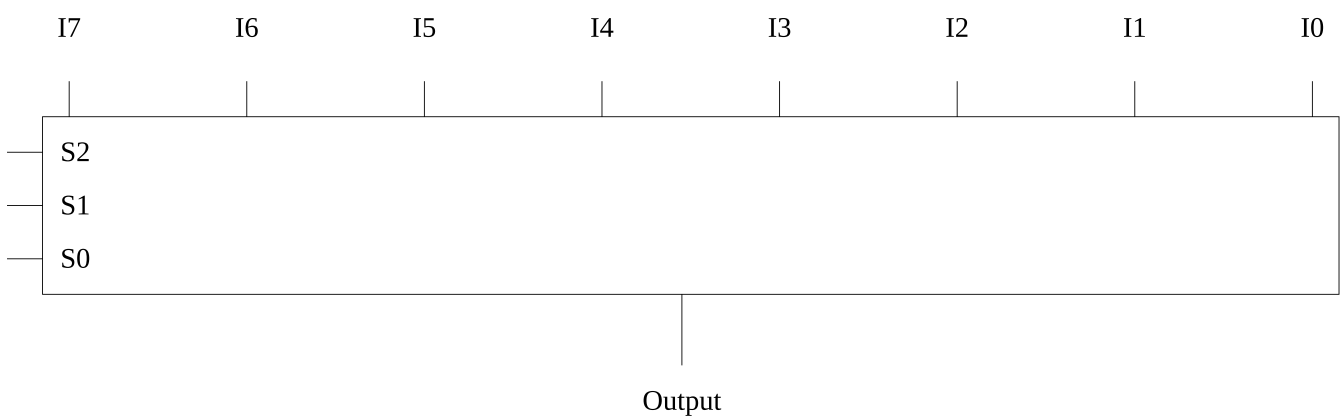
1. A voltage divider:

Suppose we want to divide a voltage into n steps, for example, 8 steps.
Consider the following possibility:

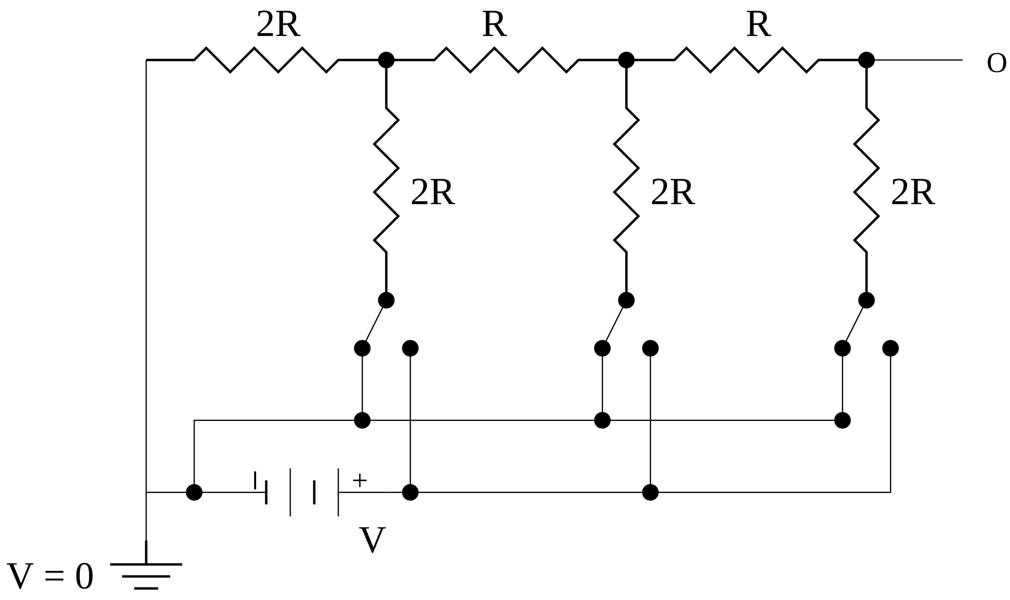


What is the voltage at each of the outputs O0 to O7?

Suppose we had an analog MUX (multiplexor) connected to the outputs O0 to O7. What would the output be then?



Consider the following:



What is the output when the switches are as shown?

When all are switched the other way?

When any one is set "on" (to the right)?

Consider what happens when the center leg is connected to V, and the other legs are connected to ground.

What happens if the resistances are not exactly equal? Suppose there is a 10% variation in the resistors.

2. The second common application is a current limiter:

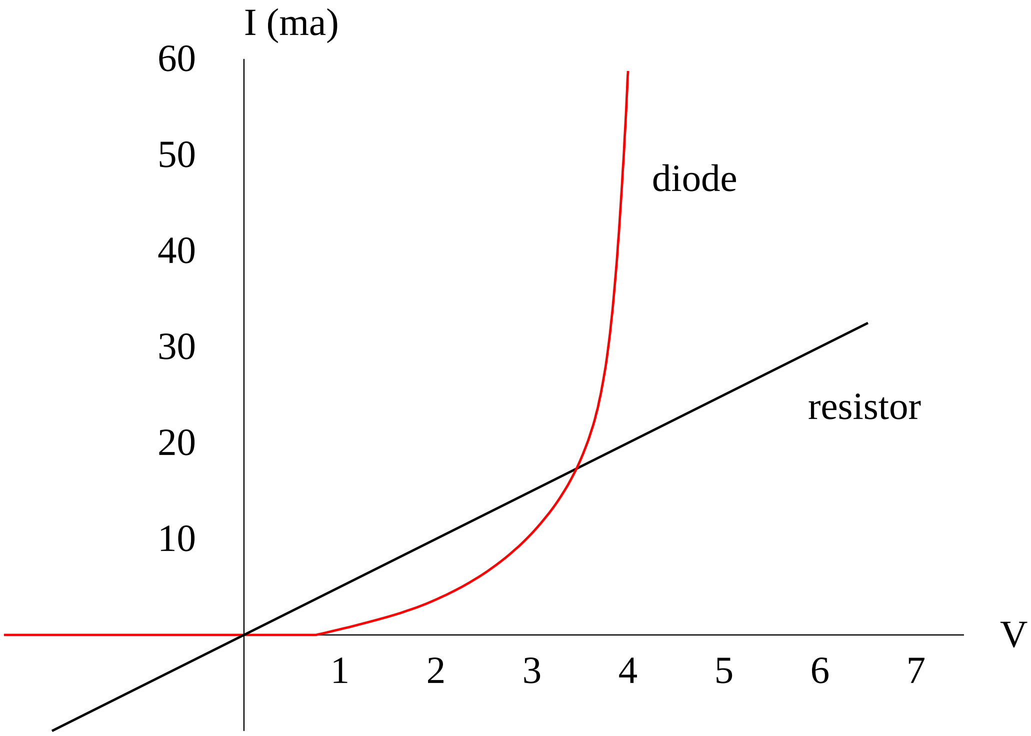
Many devices (e.g., a light emitting diode, or LED) have a very non-linear IV characteristic, and can burn out if too much current is applied. A resistor is commonly used to limit the current in such situations.

E.g., consider a LED which we want to supply with 20 ma. current to get an appropriate amount of brightness, and a 4.5 V battery circuit is available.

What resistance is required to limit the current to 20 ma?

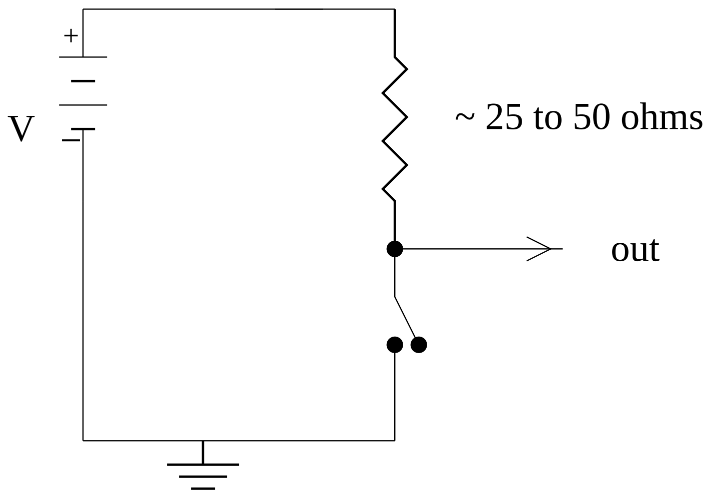
Using Ohm's law, $R = V/I = 4.5\text{V}/(20 \times 10^{-3}\text{A}) = 225$ ohms

Following is a plot of a typical diode and resistor IV characteristic (the resistance is the reciprocal of the slope of the graph):



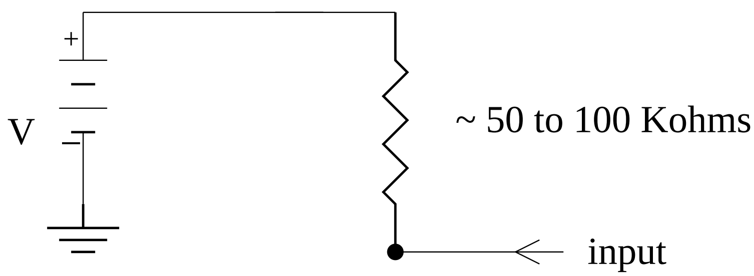
Modeling input pins and output ports

The output ports in the AVR processors have “pull up resistors” which limit the current a port can supply.
Following is (an inexact) model of an output port on an AVR processor:



Actually, it models the output of logic 1, but incorrectly models the output of logic 0.

The model of an input pin is simpler:



Real resistors and voltage sources

In reality, there are no "ideal" voltage sources; all have an internal resistance (although it may be quite small). For our purposes, a conventional battery will be "ideal enough."

Resistors are available as fixed resistors, sold with a particular accuracy, or tolerance, (usually 5% or 10%).

Variable resistors are also available, usually as 3 terminal devices, called potentiometers, or "pots."

These are usually adjusted by turning a dial or screw, and can be either single turn or multi-turn.

Common axial resistors are color coded; see

`http://en.wikipedia.org/wiki/`

`Resistor#Four-band_axial_resistors`

(all on one line) for a description of the color coding.

How are voltage and current measured?

Factoid: An electrical current induces a magnetic field about a coiled wire.

This property can be used to construct a “galvanometer” — a device which measures small electrical current.

Basically, a current passes through a coil of wire, creating a magnetic field, which displaces a small magnet (like a compass needle).

In a more sophisticated variation, the coil is placed in a magnetic field (e.g., between two permanent magnets) and the magnetic force on the coil is measured. Typically, the coil is held by a spring, and the force is measured by the compression of the spring.

To measure voltage, a high resistance is placed in series with the galvanometer, in order to ensure that there is a low current in the coil.

To measure current, a low resistance shunt is placed across the terminals of the coil, so that only a small part of the current flows through the galvanometer.

We will look at how modern digital meters operate later in the course, but the lab illustrates one of the fundamental ideas behind digital meters.

How much power does a circuit consume?

In general, the power consumed by a circuit is

$$Power = V \times I$$

In a DC circuit, this is equivalent to

$$Power = V^2/R = I^2R$$

Note that for a fixed voltage, increasing resistance reduces power consumption.