UNIT-1 : Logic I & II

- **Proposition** - A proposition is a declarative statment which is either true or false, but not both.

=> Propositions are usually denoted by small letters such as p, q, r, s ....

=> If the proposition is true then it is denoted by 1 or T

=> If the proposition is false then it is den-oted by 0 or F

eg - $\sqrt{5}$ is an integer - False [0]

- **Logical connectives & truth tables**

=> The words/ Phrases like not, and, or, iff, if then etc.... are called logical connectives

=> Any statement containing these logical conn-ectives is called compound proposition

=> The proposition which is free from logical connectives is called simple proposition

- **Negation [NOT]**

If P is a proposition then negation of P is "NOT P" & it is denoted by "$\sim p$" or "$\neg p$".

If the truth value of P is 1 then the truth value of "not P" is 0 & vice versa.

Truth table :

P	~P
0	1
1	0

- Conjunction [AND]

Let P & q be simple propositions, then conjunction of P & q is "P AND q" & is denoted by "$P \wedge q$".

The truth value of "$P \wedge q$" is 1 if both p & q are 1 & otherwise it is '0'.

T.T :

P	q	$P \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

- Disjunction [OR]

Let p & q be two simple propositions then disjunction of p & q is "P OR q" & is denoted by "$p \vee q$"

The truth value of "$p \vee q$" is 1 if any one of p & q is 1 otherwise if both are 0 then $p \vee q$ is 0.

TT :

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

- <u>Exclusive disjunction</u>

Let p & q be two simple propositions then ~~its~~ exclusive disjunction of p & q is "either p or q but not both" & is denoted by "$p \veebar q$". The truth values of $p \veebar q$ is 1 if any one of p or q is 1 but 0 if both p & q are '0' or '1'.

TT :

p	q	$p \veebar q$
0	0	0
0	1	1
1	0	1
1	1	0

- <u>Conditional (if then)</u>

Let p & q be simple propositions then conditional of p & q is "if p then q" & is denoted by "$p \to q$". The truth value of $p \to q$ is 0 if p is 1 & q is 0 & otherwise it is 1

TT :

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

- Bi-Conditional [iff]

Let p & q be simple propositions then bi-conditional of p & q is "p iff q" or "p if & only if q" or "If p then q & if q then p" & is denoted by "$p \leftrightarrow q$" or "$(p \to q) \wedge (q \to p)$".

The truth value of $p \leftrightarrow q$ is true only when both p & q are 0 or 1.

TT:

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

- Tautology (T_0)

A compound proposition which is true for all possible truth values of its components is called tautology.

- Contradiction (F_0)

A compound proposition which is false for all possible truth values of its components is called contradiction.

- Contingency

A compound proposition that can be true or false i.e neither tautology nor contradiction, depending upon the truth values of its components is called contingency.

- **Logical Equivalence**

Let u & v be two compound propositions which are said to be logically equivalent whenever u & v have the same truth value. It is denoted by " $u \Leftrightarrow v$ ".

- **Duality**

Let u be a compound proposition involving the connectives " $\wedge$ & $\vee$ ", a new proposition is obtained by replacing $\wedge$ & $\vee$ by $\vee$ & $\wedge$ is called dual of given proposition.

- **NAND & NOR**

Let p & q be two propositions then "NOT (p & q)" is called NAND connective & is denoted by $p \uparrow q$.

$$p \uparrow q \Leftrightarrow \sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q.$$

Let p & q be two propositions then " NOT (P or q) " is called NOR connective & is denoted by $p \downarrow q$.

$$p \downarrow q \Leftrightarrow \sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q.$$

- **Converse, Inverse & Contrapositive of condinal**

Let $p \to q$ be a conditional then.

1). $q \to p$ is called converse.

2). $\sim p \to \sim q$ is called Inverse.

3). $\sim q \to \sim p$ is called contrapositive.

- Logical Implication

Consider the conditional $p \to q$, here q is true whenever p is true then we say that p logically implies q which is denoted by $p \Rightarrow q$.

- Problems

1). Construct the truth table for the following compound propositions.

1). $p \vee \sim q$

p	q	$\sim q$	$p \vee \sim q$
0	0	1	1
0	1	0	0
1	0	1	1
1	1	0	1

2). $p \to \sim q$

p	q	$\sim q$	$p \to \sim q$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	0	0

3). $\sim p \veebar \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \veebar \sim q$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

4). $[(p \wedge q) \vee \sim r] \leftrightarrow p$

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee (\sim r)$ = A	$A \leftrightarrow p$
0	0	0	0	1	1	0
0	0	1	0	0	0	1
0	1	0	0	1	1	0
0	1	1	0	0	0	1
1	0	0	0	1	1	1
1	0	1	0	0	0	0
1	1	0	1	1	1	1
1	1	1	1	0	1	1

5). Show that for any proposition p & q the compound proposition $p \rightarrow (p \vee q)$ is a tautology & the compound proposition $p \wedge (\sim p \wedge q)$ is a contradiction.

Sol:

p	q	$\sim p$	$p \vee q$	$p \rightarrow p \vee q$	$p \wedge (\sim p \vee q)$
0	0	1	0	1	0
0	1	1	1	1	0
1	0	0	0	1	0
1	1	0	0	1	0

$\therefore$ $\forall$ possible truth values of compound proposition $p \rightarrow (p \vee q)$ is a tautology & $p \wedge (\sim p \wedge q)$ is a contradiction.

3). Prove that $[(\sim q) \wedge (p \to q)] \to \sim p$ is a tautology

Sol :

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$ (A)	$A \to \sim p$
0	0	1	1	1	1	1
0	1	1	0	1	0	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

Since all the values of $(\sim q \wedge (p \to q)] \to \sim p$ are 1, the given proposition is a tautology.

4). P.T $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a T_0.

Sol :

p	q	r	$p \to q$ (a)	$q \to r$ (b)	$a \wedge b$	$p \to r$ (c)	$(a \wedge b) \to c$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

5). P.T $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is T_0.

p	q	r	$p \to q$ (a)	$q \to r$ (b)	$p \to r$ (c)	$p \to b$ (d)	$a \to c$ (e)	$d \to e$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

6). Using truth table establish the following logical equivalence.

<1>. $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$.

p	q	$p \vee q$	$\sim p$	$\sim q$	(A) $\sim(p \vee q)$	(B) $\sim p \wedge \sim q$
0	0	0	1	1	1	1
0	1	1	1	0	0	0
1	0	1	0	1	0	0
1	1	1	0	0	0	0

$\therefore$ From the truth table $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$.

<2>. $p \veebar q \Leftrightarrow (p \vee q) \wedge (\sim p \vee \sim q)$.

p	q	$\sim p$	$\sim q$	(a) $p \vee q$	(b) $\sim p \vee \sim q$	(A) $a \wedge b$	(B) $p \veebar q$
0	0	1	1	0	1	0	0
0	1	1	0	1	1	1	1
1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	0

$\therefore A \Leftrightarrow B$.

7). Let p & q be primitive statements for which the conditional $p \rightarrow q$ is false. Determine the truth values of the following compound proposition.

i). $p \wedge q$ ii). $\sim p \vee q$ iii). $q \rightarrow p$ iv). $\sim q \rightarrow \sim p$.

Sol. Given — $p \to q = 0$.

$\therefore \Rightarrow p = 1$ & $q = 0$.

$\therefore$ 1). $p \wedge q \Leftrightarrow 1 \wedge 0 \Leftrightarrow 0$

2). $\sim p \vee q \Leftrightarrow 0 \vee 0 \Leftrightarrow 0$

3). $q \to p \Leftrightarrow 0 \to 1 \Leftrightarrow 0$

4). $\sim q \to \sim p \Leftrightarrow 1 \to 0 \Leftrightarrow 0$.

8). Given p is true & q is false. Find the truth value of.

(i) $\sim(p \wedge q) \vee \sim(q \leftrightarrow p)$ (ii) $(p \to q) \vee \sim(p \leftrightarrow \sim q)$

Sol. Given $p = 1$ & $q = 0$.

Consider (i) $\sim(p \wedge q) \vee \sim(q \leftrightarrow p)$

$\Leftrightarrow \sim(1 \wedge 0) \vee \sim(0 \leftrightarrow 1)$

$\Leftrightarrow \sim(0) \vee \sim(0)$

$\Leftrightarrow 1 \vee 1$

$\Leftrightarrow 1$ [True]

(ii) $(p \to q) \vee \sim(p \leftrightarrow \sim q)$

$\Leftrightarrow (1 \to 0) \vee \sim(1 \leftrightarrow \sim 0)$

$\Leftrightarrow 0 \vee \sim(1 \leftrightarrow 1)$

$\Leftrightarrow 0 \vee \sim(1)$

$\Leftrightarrow 0 \vee 0$

$\Leftrightarrow 0$ [False]

9). Indicate how many rows are needed in the truth table for the compound proposition $(p \vee \sim q) \leftrightarrow [(\sim r \wedge s) \to t]$. Find the truth value of proposition if p & r are true & q, s, t are false.

Sol The given compound statement contains $n = 5$ propositions.

$\therefore$ The no of rows required in the truth table is $2^n = 2^5 = 32$.

Consider $(p \vee \sim q) \leftrightarrow [(\sim r \wedge s) \rightarrow t]$

$\Rightarrow (1 \vee 1) \leftrightarrow [(0 \wedge 0) \rightarrow 0]$

$\Rightarrow 1 \leftrightarrow [0 \rightarrow 0]$

$\Rightarrow 1 \leftrightarrow 1$

$\Rightarrow 1$ [True]

10) If a proposition q has truth value 1, determine all truth value for primitive p, r & s for which the truth value of the following compound proposition is 1

$[q \rightarrow \{(\sim p \vee r) \wedge \sim s\}] \wedge [\sim s \rightarrow (\sim r \wedge q)]$

Sol Given - $[q \rightarrow \{(\sim p \vee r) \wedge \sim s]] \wedge [\sim s \rightarrow (\sim r \wedge q)] = 1$

$\Rightarrow [q \rightarrow \{(\sim p \vee r) \wedge \sim s\}] = 1$ —— 1

& $[\sim s \rightarrow (\sim r \wedge q)] = 1$ —— 2

Consider 1 & since it is true also given q is 1, $(\sim p \vee r) \wedge \sim s$ must be 1

$\therefore \sim p \vee r$ is 1 & $\sim s$ is 1

$\Rightarrow \sim p \vee r = 1$ —— 3 & $\sim s = 1$

$\boxed{s = 0}$

Consider ② & its truth value is 1

also $\sim s$ is 1, given $q = 1$

$\therefore \sim r \wedge q$ must be 1

As $q = 1$, $\sim r$ must be 1

$\therefore \boxed{r = 0}$.

Subs $r = 0$ in ③.

$\therefore \sim p \vee r = 1$

$\sim p \vee 0 = 1$

$\therefore \sim p = 1 \Rightarrow \boxed{p = 0}$

11) Prove that for any proposition p, q, r

$[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$

Sol:

p	q	r	(a) $p \vee q$	(A) $a \rightarrow r$	(b) $p \rightarrow r$	(C) $q \rightarrow r$	(B) $b \wedge C$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

$\therefore$ From (A) & (B) the compound proposition is logically equal.

12) P.T $[(p \vee q) \to r] \leftrightarrow [\sim r \to \sim(p \vee q)]$ is a T_0.

Sol

p	q	r	$p \vee q$ (a)	$a \to r$ (A)	$\sim r$	$\sim(p \vee q)$ (b)	$\sim r \to b$ (B)	$A \leftrightarrow B$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	0	1	1	1
0	1	0	1	0	1	0	0	1
0	1	1	1	1	0	0	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	0	1	0	0	1
1	1	1	1	1	0	0	1	1

∴ From A & B the compound proposition is logical equal.

13). Write converse, inverse & contrapositive of conditional.

⟨i⟩ p: Quadrilateral is a parallelogram.

q: Diagonals of quadrilateral bisects each other.

Sol

$p \to q$: If a quadrilateral is a parallelogram then diagonals of quadrilateral bisects each other.

$q \to p$: If diagonals of quadrilateral bisects each other then quadrilateral is a parallelogram

$\sim p \to \sim q$: If a quadrilateral is not a parallelogram then diagonals of a quadrilateral does not bisect each other.

$\sim q \to \sim p$: If diagonals of a quadrilateral does not bisect each other then quadrilateral is not a parallelogram.

14) If a triangle is not isosceles then it is not equilateral.

Sol: p: Triangle is not isosceles

q: Triangle is not equilateral.

$q \to p$: If triangle is not equilateral then it is not isosceles.

$\sim p \to \sim q$: If Δ^{le} is isosceles then it is equilateral.

$\sim q \to \sim p$: If Δ^{le} is equilateral then it is isosceles.

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- Exercise

1). Construct the truth table for the following

① $(p \vee q) \wedge r$

② $p \vee (q \wedge r)$

③ $(p \wedge q) \rightarrow \sim r$

④ $q \wedge (\sim r \rightarrow p)$.

2). Prove that $[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$ is T_0.

3). Using truth table prove that the following compound propositions are logically equivalent.

①. $p \leftrightarrow q \Leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$

② $[(p \vee q) \rightarrow r] \Leftrightarrow [\sim r \rightarrow \sim (p \vee q)]$

③ $[(p \rightarrow q) \wedge (p \rightarrow \sim q)] \Leftrightarrow \sim p$.

④ $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow q)]$

4). Prove that $[(p \vee q) \rightarrow r] \leftrightarrow [\sim r \rightarrow \sim (p \vee q)]$ is T_0.

5). Write converse, inverse & contrapositive of the conditional.

"If she passes in exam, the professor will resign & the college will close".