The laws of logic

Let p be any proposition, T_0 be tautology & F_0 be contradiction.

1). **Law of double negation:**

$$\sim(\sim p) \Leftrightarrow p$$

2). **Idempotent laws:**

$$p \vee p \Leftrightarrow p \quad , \quad p \wedge p \Leftrightarrow p$$

3). **Identity laws:**

$$p \vee F_0 \Leftrightarrow p \quad , \quad p \wedge T_0 \Leftrightarrow p.$$

4). **Inverse laws:**

$$p \vee \sim p \Leftrightarrow T_0 \quad , \quad p \wedge \sim p \Leftrightarrow F_0$$

5). **Domination laws:**

$$p \vee T_0 \Leftrightarrow T_0 \quad , \quad p \wedge F_0 \Leftrightarrow F_0$$

6). **Commutative laws:**

$$p \vee q \Leftrightarrow q \vee p \quad , \quad p \wedge q \Leftrightarrow q \wedge p$$

7). **Absorption laws:**

$$[p \vee (p \wedge q)] \Leftrightarrow p \quad , \quad [p \wedge (p \vee q)] \Leftrightarrow p$$

8). **Demorgan's laws:**

$$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q \quad , \quad \sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$$

9). **Associative laws:**

$$p \vee (q \vee r) \Leftrightarrow (p \vee q) \wedge (p \vee r), \quad (p \wedge (q \wedge r)) \Leftrightarrow (p \wedge q) \wedge r$$

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10). <u>Distributive laws :</u>

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$, $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

11). <u>Law for the negation of conditional :</u>

$\sim (p \rightarrow q) \Leftrightarrow p \wedge (\sim q)$

$p \rightarrow q \Leftrightarrow \sim p \vee q$.

- Simplify the following compound propositions using laws of logic.

OR.

Simplify the following logical statements without using truth table.

1). $(p \vee q) \wedge \{\underbrace{\sim(\sim p \wedge q)}_{\text{De morgan's law}}\}$

$\Leftrightarrow (p \vee q) \wedge \{\underbrace{\sim(\sim p)}_{\text{Double negation}} \vee \sim q\}$

$\Leftrightarrow \underbrace{(p \vee q) \wedge \{p \vee \sim q\}}_{\text{Distributive law}}$

$\Leftrightarrow p \vee \underbrace{(q \wedge \sim q)}_{\text{Inverse law}}$

$\Leftrightarrow \underbrace{p \vee F_0}_{\text{Identity law}}$.

$\Leftrightarrow p$.

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2). Prove that $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$.

Sol∴ Consider $p \vee [p \wedge (p \vee q)]$

$\Leftrightarrow p \vee p$. [By absorption law]

$\Leftrightarrow p$ [Idempotent law]

3). prove that $[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$

Sol∴ Consider $[(p \vee q) \wedge (p \vee \sim q)] \vee q$

$\Leftrightarrow [p \vee (q \wedge \sim q)] \vee q$ [By distributive law]

$\Leftrightarrow [p \vee F_0] \vee q$ [By inverse law]

$\Leftrightarrow p \vee q$ [By identity law]

4). P.T $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \longrightarrow r$

Sol∴ Consider $p \rightarrow (q \rightarrow r)$

w.k.t $\boxed{p \rightarrow q \Leftrightarrow \sim p \vee q}$

$\therefore p \rightarrow (q \rightarrow r) \Leftrightarrow \sim p \vee (q \rightarrow r)$ [negation of condⁿ]

$\Leftrightarrow \sim p \vee (\sim q \vee r)$ "

$\Leftrightarrow (\sim p \vee \sim q) \vee r$ [Asso law]

$\Leftrightarrow \sim (p \wedge q) \vee r$ [Demorg]

$\Leftrightarrow (p \wedge q) \longrightarrow r$.

5). Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology by using laws of logic & establish the result through truth table also.

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Sol Consider $(p \wedge q) \to (p \vee q)$

$\Leftrightarrow \sim(p \wedge q) \vee (p \vee q)$ [neg of cond^n]

$\Leftrightarrow (\sim p \vee \sim q) \vee (p \vee q)$ [Demor]

$\Leftrightarrow (\sim p \vee p) \vee (\sim q \vee q)$ [By regrouping]

$\Leftrightarrow T_0 \vee T_0$

$\Leftrightarrow T_0$.

TT :

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
0	0	0	0	1
0	1	0	1	1
1	0	0	1	1
1	1	1	1	1

$\therefore$ From the truth table $(p \wedge q) \to (p \vee q)$ is a T_0.

6). Write the dual of the following :-

i). $[(p \wedge q) \vee [(\sim p \vee q) \wedge (\sim r \vee s)] \vee (r \wedge s)]^d$

Sol $(p \vee q) \wedge [(\sim p \wedge q) \vee (\sim r \wedge s)] \wedge (r \vee s)$.

ii). $[(p \vee T_0) \wedge (q \vee F_0)] \vee [(r \wedge s) \wedge T_0]^d$.

Sol $[(p \wedge F_0) \vee (q \wedge T_0)] \wedge [(r \vee s) \vee F_0]$

iii). $[(p \to q) \to r]^d$

Sol $[(p \to q) \to r]^d \Leftrightarrow [\sim(p \to q) \vee q]^d$

$\Leftrightarrow [\sim(\sim p \vee q) \vee q]^d$

$\Leftrightarrow [(\sim(\sim p) \wedge \sim q) \vee q]^d$

$\Leftrightarrow [(p \wedge \sim q) \vee q]^d$

$\Leftrightarrow (p \vee \sim q) \wedge q$.

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7). P.T $\sim(p \downarrow q) \Leftrightarrow \sim p \uparrow \sim q$

Sol Consider $\sim(p \downarrow q) \Leftrightarrow \sim[\sim(p \vee q)]$ (NOR)

$\Leftrightarrow \sim[\sim p \wedge \sim q]$ [De mor]

$\Leftrightarrow \sim p \uparrow \sim q$ [Nand]

HW 8). P.T $\sim(p \uparrow q) \Leftrightarrow \sim p \downarrow \sim q$

9). PT $(p \downarrow q) \downarrow r \Leftrightarrow (p \vee q) \wedge \sim r$

Sol Consider $(p \downarrow q) \downarrow r$

$\Leftrightarrow \sim[(p \downarrow q) \vee r]$ [NOR]

$\Leftrightarrow \sim(p \downarrow q) \wedge \sim r$ [De mor]

$\Leftrightarrow [\sim\{\sim(p \vee q)\} \wedge \sim r]$ [NOR]

$\Leftrightarrow (p \vee q) \wedge \sim r$ [Double neg]

- Excercise.

1). PT $\sim(p \uparrow q) \Leftrightarrow \sim p \downarrow \sim q$.

2). PT $\sim[\sim\{(p \vee q) \wedge r\} \vee \sim q] \Leftrightarrow q \wedge r$.

3). Find the dual of $p \rightarrow q$

4). Simplify $(p \vee q) \wedge [\sim(\sim p \wedge q)]$

5). Prove the following both by using laws of logic & truth tables.

i). $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

ii). $[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$.

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Rules of Inference.

Consider a set of proposition $P_1, P_2 \dots P_n$ & q then the compound proposition $(P_1 \wedge P_2 \wedge \dots \wedge P_n) \rightarrow q$ $[\Rightarrow q]$ is called an argument.
Here $P_1, P_2 \dots P_n$ are called premises & q is called conclusion.

Note – An argument is generally written as follows

$$\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \\ \hline \therefore q \end{array} \quad , \quad (P_1 \wedge P_2 \dots \wedge P_n) \Rightarrow q.$$

We use the following rules known as the rules of inference to check the validity of an argument.

1). Rule of conjunctive simplification.

$$\begin{array}{c} p \wedge q \\ \hline \therefore p \end{array} \quad , \quad p \wedge q \Rightarrow p \quad \text{(OR)} \quad \begin{array}{c} p \wedge q \\ \hline \therefore q \end{array} \quad , \quad p \wedge q \Rightarrow q$$

2). Rule of ~~disjunctive addition / Amplification.~~

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array} \quad , \quad p \Rightarrow p \vee q$$

3). Rule of Syllogism.

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array} \quad , \quad [(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow p \rightarrow r.$$

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4). Modus ponens : [Rule of detachment]

$$\begin{array}{l} p \to q \\ \underline{\quad p \quad} \\ \therefore q \end{array} \qquad [(p \to q) \land p \Rightarrow q]$$

5). Modus Tollens [Method of denying]

$$\begin{array}{l} p \to q \\ \underline{\quad \sim q \quad} \\ \therefore \sim p \end{array} \quad , \quad [(p \to q) \land \sim q] \Rightarrow \sim p$$

6). Rule of disjunctive syllogism.

$$\begin{array}{l} p \lor q \\ \underline{\quad \sim p \quad} \\ \therefore q \end{array} \quad , \quad [(p \lor q) \land \sim p] \Rightarrow q .$$

7). Rule of contradiction

$$\begin{array}{l} \underline{\sim p \to F_0} \\ \therefore \; p \end{array} \quad , \quad \sim p \to F_0 \Rightarrow p .$$

8). Rule of Resolution

$$\begin{array}{l} p \lor q \\ \underline{\sim p \lor r} \\ \therefore q \lor r \end{array} \quad , \quad [(p \lor q) \land (\sim p \lor r)] \Rightarrow (q \lor r) .$$

Thus the validity is established with aid of above argument. But in some cases we also use laws of logic, logical equivalence & tautologies. If its not possible to prove using the above rules then we use truth tables.

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- Problems

1). Symbolise the following statement & find its negation :-
"If triangle has three sides then squares are rectangle".

Sol: let p : triangle has three sides
q : squares are rectangle.

$\therefore p \to q$.

Negation $\Rightarrow \sim(p \to q)$

$\Leftrightarrow \sim[\sim p \vee q]$

$\Leftrightarrow \sim(\sim p) \wedge \sim q$

$\Leftrightarrow p \wedge \sim q$.

$\therefore$ Triangle has three sides & squares are not rectangle.

II Test the validity of the following arguments.

1). Andrea can program in C++ & she can program in Java. Therefore Andrea can program in C++.

Sol: p : Andrea can program in C++.
q : Andrea can program in Java.

The argument is as follows :

$$\frac{p \wedge q}{\therefore p} \quad , \quad p \wedge q \Rightarrow p.$$ [By rule of conjunctive simplification]

Thus we conclude that the argument is valid.

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2). If Sachin hits a century, he gets a free car
Sachin does not get a free car

$\therefore$ Sachin has not hit a century.

Sol: p: Sachin hits a century

q: Sachin gets a free car

The argument follows as:

$$\begin{array}{l} p \to q \\ \underline{\sim q} \\ \therefore \sim p \end{array} \quad , \quad (p \to q) \wedge \sim q \Rightarrow \sim p .$$

In the view of modus tollens rule the argument is valid.

3). If ravi hits a century, he get a free car.
Ravi gets a free car

$\therefore$ Ravi has hit a century.

Sol: Let p : Ravi hits a century

q : Ravi gets a free car

$\therefore$ The given argument is as follows as

$$\begin{array}{l} p \to q \\ \underline{q} \\ \therefore p \end{array} \quad , \quad (p \to q) \wedge q \Rightarrow p .$$

we can not verify the validity of given argument using any rules of inference.
$\therefore$ We construct a truth table using compound proposition $[(p \to q) \wedge q] \Longrightarrow p$.

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p	q	$(p \to q)$	$(p \to q) \wedge q$	$\cancel{[(p \to q}$
0	0	1	0	
0	1	1	1	
1	0	0	0	
1	1	1	1	

$\therefore$ The given argument is not valid as the compound proposition is not a tautology.

4). I will become famous or I will not become a musician.
I will become a musician

$\therefore$ I will become famous.

Sol Let p: I will become famous
q: I will become musician

$$\therefore \quad \begin{array}{c} p \vee \sim q \\ q \\ \hline \therefore p \end{array} \overset{\text{commutative}}{\Longleftrightarrow} \begin{array}{c} \sim q \vee p \\ q \\ \hline \therefore p \end{array} \overset{\text{negative of cond}^n.}{\Longleftrightarrow} \begin{array}{c} q \to p \\ q \\ \hline \therefore p \end{array}$$

$$\Leftrightarrow (p \vee \sim q) \wedge q \Rightarrow p$$
$$\Leftrightarrow (\sim q \vee p) \wedge q \Rightarrow p$$
$$\Leftrightarrow (q \to p) \wedge q \Rightarrow p.$$

In view of modus ponens rule the argument is valid.

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5). If I study, I will not fail in exam.
If I don't watch TV in the evening, I will study.

I failed in the examination.

∴ I must have watched TV in the evening.

Sol:
p : I study
q : I fail in the exam
r : I watch TV in the evening.

Then, $p \rightarrow \sim q$ contractive $q \rightarrow \sim p$ Rule of syllogism $q \rightarrow r$

$$\begin{array}{l} \sim r \rightarrow p \\ \hline q \\ \therefore r \end{array} \Leftrightarrow \begin{array}{l} q \rightarrow \sim p \\ \sim p \rightarrow r \\ \hline q \\ \therefore r \end{array} \Leftrightarrow \begin{array}{l} q \rightarrow r \\ \hline q \\ \therefore r \end{array}$$

∴ By modus ponens rule. The argument is valid.

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- Exercise :

1). If a person is poor, he is unhappy
If a person is unhappy, he dies young :

∴ poor person die young.

2). If Ram gets distinction in exam, then his father will get him a bike.
Ram achieves distinction

∴ Ram gets bike.

3). If there is a strike by students, the examination will be postponed.
There was no strike by students

∴ The examination was not postponed.

4). If Ravi goes out with friends, he will not study.
If Ravi does not study, his father will become angry.
His father is not angry.

∴ Ravi has not gone out with friends.

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