Chapter - 02

- **Open statement:**

A statement which becomes a proposition when the values of the variable involved is known as open statement.

It is a sentence which involves one or more variables which turnsout as a proposition when the variables take permissible values.

Eg :- $P(x): x+3=5$, open statement

$$\left.\begin{array}{l} P(2): 2+3=5\ (T) \\ P(1): 1+3=4\ (F) \end{array}\right\} \text{proposition}.$$

- **Quantified statement:**

A proposition involving the idea of a quality is known as quantified statement.

eg: Statement consists of words like for all, for every, for some, there exists etc.

There are two types of quantifiers :-

1). Universal Quantifier – The phrases like for all, for every, for each, are involved in the sentence is called universal quantifier. It is denoted by "$\forall$"

2). Existential Quantifier – The phrases like for some, there exists, at least are involved in the sentence is called existential quantifier. It is denoted by "$\exists$"

- NOTE :-

1). $[\forall x, p(x)]$ is true only when $p(x)$ is true for every $x \in U$.

2) $[\exists x, p(x)]$ is false only when $p(x)$ is false for every $x \in U$

3). Negation of a quantified statement.

$$\sim[\forall x, p(x)] \equiv \exists x[\sim p(x)]$$
$$\sim[\exists x, p(x)] \equiv \forall x[\sim p(x)]$$

- Problems

1). Let the universal set be the set of all integers. Consider the following open statement.

$P(x): x \leq 3$, $q(x): x+1$ is odd , $r(x): x \leq 0$

Find the truth value of the following.

i). $P(2)$

Sol: Consider $P(x): x \leq 3$

put $x = 2$, $P(2): 2 \leq 3$

$P(2)$: True (1)

ii). $P(3) \vee \sim r(3)$

Sol: Consider $P(x): x \leq 3$

$P(3): 3 \leq 3$ = True(1)

Consider $r(x): x \leq 0$

$r(3): 3 \leq 0$

$r(3)$: False (0).

$\sim r(3)$: True (1)

Again consider $P(3) \vee \sim r(3) \Leftrightarrow 1 \vee 1 \Leftrightarrow 1$ [True].

iii). $\sim p(4) \wedge \sim q(5)$.

Sol Consider $p(x) : x \leq 3$

$P(4) : 4 \leq 3$: False

$\sim p(4)$: True

Consider $q(x) : x+1$ is odd.

$q(5) : 5+1$ is odd : False

$\sim q(5)$: True.

$\therefore \sim p(4) \wedge \sim q(5) \equiv 1 \wedge 1 \equiv 1$

iv). $p(-1) \wedge q(1)$.

Sol Consider $P(-1) \wedge q(1)$

$\equiv (-1 \leq 3) \wedge (1+1 \text{ is odd})$

$\equiv T \wedge F$

$\equiv 1 \wedge 0$

$\equiv 0$

v). $P(0) \rightarrow q(0)$.

Consider $P(0) \rightarrow q(0)$

$\equiv (0 \leq 3) \rightarrow (0+1 \text{ is odd})$

$\equiv T \rightarrow T$

$\equiv 1 \rightarrow 1 \equiv 1$

2). For the set of all integers

Let $P(x) : x > 0$

$q(x) : x$ is ~~a perfect square~~ even

$r(x) : x$ is a perfect square

$s(x) : x$ is divisible by 3

$t(x) : x$ is divisible by 7

- Using the above propositions write down the following quantified statements in symbolic form.

i). Atleast one integer is even.

Sol: $\exists x, q(x)$.

ii). There exists a positive integer & that is even.

Sol: $\exists x, [p(x) \wedge q(x)]$

iii). Some even integers are divisible by 3.

Sol. $\exists x, [q(x) \wedge s(x)]$

iv). Every integer is either even or odd but not both

Sol. $\forall x, [q(x) \veebar \sim q(x)]$

v). If x is even & a perfect square then x is not divisible by 3.

Sol. $\forall x, [(q(x) \wedge r(x)) \rightarrow \sim s(x)]$

vi). If x is odd or x is not divisible by 7 then x is divisible by 3.

Sol. $\forall x, [(\sim q(x) \vee \sim t(x)) \rightarrow s(x)]$

- Express the following symbolic statements in words & indicate the truth value.

1) $\forall x, [r(x) \longrightarrow p(x)]$

Sol: For all integer x, if x is a perfect square then $x > 0$.

For $x = 0$, $r(0) = 0$ is a perfect square $=$ True

$p(0) = 0 > 0 =$ False.

$\therefore r(0) \to p(0) = 1 \longrightarrow 0 = 0$.

$\therefore$ Truth value $= 0$.

2) $\exists x, [s(x) \wedge \sim q(x)]$

Sol: for some integer x, x is divisible by 3 & x is not even.

for $x = 9$, $s(9) = 9$ is divisible by 3 $=$ True

$\sim q(9) = 9$ is not even $=$ True

$\therefore s(9) \wedge \sim q(9) = 1 \wedge 1 = 1$

$\therefore$ Truth value $= 1$

3) $\forall x, [\sim r(x)]$

Sol: For any integer x, x is not a perfect square.

For $x = 4$, $\sim r(5) = 4$ is not a perfect square $=$ false

$\therefore$ Truth value $= 0$.

4). $\forall x, [r(x) \vee t(x)]$

Sol: For any integer x, x is a perfect square & x is divisible by 7.

for $x = 3$, $r(3) \vee t(3) =$ (3 is a perfect square) & (3 is divisible by 7).

$= 0 \vee 0 = 0$.

$\therefore$ Truth value $= 0$.

~~3). Consider the following~~

3). Negate & simplify each of the following.

i). $\exists x, [p(x) \vee q(x)]$

Sol: $\sim [\exists x, [p(x) \vee q(x)]]$

$\Leftrightarrow \forall x, \sim (p(x) \vee q(x))$

$\Leftrightarrow \forall x, \sim p(x) \wedge \sim q(x)$ [By demorgans law].

ii). $\forall x, [p(x) \rightarrow q(x)]$

Sol: $\sim [\forall x, (p(x) \rightarrow q(x))]$

$\Leftrightarrow \exists x, \sim (p(x) \rightarrow q(x))$ [$\because$ neg^n of cond^n $\sim(p \rightarrow q) \Rightarrow p \wedge \sim q$]

$\Leftrightarrow \exists x, p(x) \wedge \sim q(x)$

iii). $\exists x, [(p(x) \vee q(x)) \rightarrow r(x)]$

Sol: $\sim [\exists x, (p(x) \vee q(x)) \rightarrow r(x)]$

$\Leftrightarrow \forall x, \sim [(p(x) \vee q(x)) \rightarrow r(x)]$

$\Leftrightarrow \forall x, (p(x) \vee q(x)) \wedge \sim r(x)$.

4). $\exists x, [(p(x) \vee q(x)) \rightarrow r(x)]$

Sol $\sim [\exists x, ((p(x) \vee q(x)) \rightarrow r(x))]$

$\Leftrightarrow \forall x, \sim[(p(x) \vee q(x)) \rightarrow r(x)]$

$\Leftrightarrow \forall x, [p(x) \vee q(x)] \wedge \sim r(x)$ [$\because$ Negation of cond^n $(p \rightarrow q) \Leftrightarrow p \wedge \sim q$]

4). Let the universal set be the set of all integers given $p(x)$: x is odd.

$q(x)$: $x^2 - 1$ is even.

Express the conditional in both statement & symbolic form & negate it.

Sol Conditional

statement – For all x, if x is odd then $x^2 - 1$ is even.

symbol – $\forall x, [p(x) \rightarrow q(x)]$

Negation – $\sim[\forall x, p(x) \rightarrow q(x)]$

$\Leftrightarrow \exists x, \sim[p(x) \rightarrow q(x)]$ [$\because$ Negation of conditional $\sim(p \rightarrow q) \Leftrightarrow p \wedge \sim q$]

$\Leftrightarrow \exists x, p(x) \wedge \sim q(x)$.

5). Let Z be set of all integers consider the statements $p(x): 2x+1 = 5$, $q(x): x^2 = 9$. Obtain the negation of the quantified statement $\exists x \in Z, \{p(x) \wedge q(x)\}$ & express in words.

Sol: Consider

$$\sim[\exists x \in Z, (p(x) \wedge q(x))]$$

$$\Leftrightarrow \forall x \in Z, \sim[p(x) \wedge q(x)]$$

$$\Leftrightarrow \forall x \in Z, \sim p(x) \vee \sim q(x).$$

$\forall x$, $2x+1 \neq 5$ or $x^2 \neq 9$.

i.e For all integer x, $2x+1 \neq 5$ or $x^2 \neq 9$.

6). Write down the following proposition in symbolic form & find its negation.

i). " All integers are rational numbers & some rational numbers are not integers.

Sol: Let Z be the set of all integers.

Q be the set of all rationals.

$p(x)$: x is a rational no.

$q(x)$: x is an integer.

Given statement in symbolic form.

$\forall x \in Z, p(x) \wedge \exists x \in Q, \sim q(x)$.

Negation of given statement.

$$\Leftrightarrow \sim[\forall x \in Z, p(x) \wedge \exists x \in Q, \sim q(x)]$$

$$\Leftrightarrow \sim[\forall x \in Z, p(x)] \vee \sim[\exists x \in Q, \sim q(x)]$$

(Demorgans law)

$$\Leftrightarrow \exists x \in Z, \sim p(x) \vee \forall x \in Q, q(x).$$

$\therefore$ Some integers are not rational nos or every rational nos is an integer.

Scanned by CamScanner

ii). If k, m, n are any integers, if $(k-m)$ & $(m-n)$ are odd, then $(k-n)$ is even.

Sol: Let Z be the set of all integers

$p(x)$: $k-m$ is odd

$q(x)$: $m-n$ is odd

$r(x) =$ $k-n$ is even.

$\forall k, m, n \in Z, [(p(x) \wedge q(x)) \rightarrow r(x)]$

$\Rightarrow \sim[\forall k, m, n \in Z, [(p(x) \wedge q(x)) \rightarrow r(x)]]$

$\Leftrightarrow \exists k, m, n \in Z, \sim[(p(x) \wedge q(x)) \rightarrow r(x)]$

$\Leftrightarrow \exists k, m, n \in Z, [(p(x) \wedge q(x)) \wedge \sim r(x)]$

$\therefore$ For some integers k, m, n, $(k-m)$ & $(m-n)$ are odd & $(k-n)$ is not even.

- Exercise

1). For $P(x): x \leq 3$, $q(x): (x+1)$ is odd, $r(x): x \leq 0$

Find i). $P(4) \vee \{q(1) \wedge q(3)\}$

ii). $\sim p(5) \vee r(0)$

iii). $p(2) \wedge \{q(0) \vee \sim r(2)\}$

2). Consider the following open statement with the set of all real numbers ~~R~~

$P(x): x \geq 0$, $q(x): x^2 \geq 0$.

$r(x): x^2 - 3x - 4 = 0$, $s(x): x^2 - 3 \geq 0$.

Determine the truth value of the following for $x = 1$

3). Negate − $\forall x, [p(x) \wedge \sim q(x)]$

4). Write down the following proposition in symbolic form & find its negation.

i). "For all integers n, if n is not divisible by 2 then n is odd"

ii). " If all triangles has three sides then some squares are rectangles."