This handout includes space for every question that requires a written response. Please feel free to use it to handwrite your solutions (legibly, please). If you choose to typeset your solutions, the `README.md` for this assignment includes instructions to regenerate this handout with your typeset LaTeX solutions.

4.a

For each singular vector $\mathbf{v}_j$, $\mathbf{A}\mathbf{v}_j = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T \mathbf{v}_j$. Since any vector $\mathbf{v}$ can be expressed as a linear combination of the singular vectors plus a vector perpendicular to the $\mathbf{v}_i$, $\mathbf{A}\mathbf{v} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T \mathbf{v}$, and matrices A and B are identical if and only if for all vectors v, $\mathbf{A}\mathbf{v} = \mathbf{B}\mathbf{v}$, thus, $\mathbf{A} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T$

4.b

$$\mathbf{u}_i = \frac{1}{\sigma_i}\mathbf{A}\mathbf{v}_i \tag{1}$$

4.c

4.d