

## CHROMATIC MODEL FITTING BIAS

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### ABSTRACT

This is the abstract.

#### 1. INTRODUCTION

Cosmic shear experiments aim to constrain cosmological parameters by measuring the small departure from statistical isotropy of distant galaxy shapes induced by the gravitational lensing from foreground large scale structure. The shapes of galaxy images collected from telescopes, however, are not only affected by cosmic shear (typically a  $\lesssim 1\%$  effect), but also by the combined point spread function (PSF) of the atmosphere (for ground-based experiments), telescope optics, and the image sensor (together often a few % effect). The shape of this additional convolution kernel is typically constrained from the shapes of stars, which are effectively point sources before being smeared by the PSF. Galaxy images can then be deconvolved with the estimated convolution kernel. Implicit in this approach is the assumption that the galactic kernel is the same as the stellar kernel. Effects that make the PSF dependent on wavelength will violate this assumption, as stars and galaxies at different redshifts have different spectral energy distributions, and hence different PSFs.

Examples of wavelength-dependent PSF contributions include:

- atmospheric differential chromatic refraction (DCR)
- atmospheric seeing
- telescope optics DCR
- sensor photoconversion depth and subsequent charge diffusion

This note will focus on PSF mis-estimations due to the first of the above effects, though the conclusions are applicable to any situation in which the bias due to using the wrong PSF is estimated using a “ring test” with a simple model parameterization.

#### 2. ANALYTIC EXPECTATIONS

Plazas & Bernstein (2012, hereafter PB12) derived analytic expressions for the bias expected in weak lensing measurements due to DCR. We summarize the main points of their argument here.

Let  $R(\lambda; z_a)$  be the refraction towards the zenith of a photon with wavelength  $\lambda$  and true zenith angle (before refraction)  $z_a$ . The refraction can be factored into

$$R(\lambda; z_a) = h(\lambda)\tan(z_a), \quad (1)$$

where  $h(\lambda)$  implicitly depends on air pressure, temperature, and the partial pressure of water vapor, and can be obtained

from Edlén (1953) and Coleman et al. (1960). For monochromatic sources, the only effect is to move the apparent position of the object. For sources that are not monochromatic (i.e., all real sources), having a wavelength density of surviving photons (i.e., the product of the source photon density and the total system throughput function) given by  $p_\lambda(\lambda)$ , the mismatched displacements of different wavelengths introduces a convolution kernel in the zenith direction that can be written in terms of the inverse of Equation 1:

$$k(R) = \frac{p_\lambda(\lambda(R; z_a)) \left| \frac{d\lambda}{dR} \right|}{\int p_\lambda(\lambda) d\lambda} \quad (2)$$

This kernel can largely be characterized in terms of its first and second central moments given by:

$$\bar{R} = \int k(R) R dR = \frac{\int R(\lambda; z_a) p_\lambda(\lambda) d\lambda}{\int p_\lambda(\lambda) d\lambda} \quad (3)$$

$$V = \int k(R) (R - \bar{R})^2 dR = \frac{\int (R(\lambda; z_a) - \bar{R})^2 p_\lambda(\lambda) d\lambda}{\int p_\lambda(\lambda) d\lambda} \quad (4)$$

Galaxy shapes can be characterized by the quadrupole moments of their light distribution given by:

$$I_{\mu\nu} = \frac{1}{f} \int dx dy I(x, y) (\mu - \bar{\mu})(\nu - \bar{\nu}) \quad (5)$$

$$\bar{\mu} = \frac{1}{f} \int dx dy I(x, y) \mu \quad (6)$$

$$f = \int dx dy I(x, y) \quad (7)$$

In particular, galaxy size and 2-component ellipticity are frequently defined as:

$$r^2 = I_{xx} + I_{yy} \quad (8)$$

$$e_1 = \frac{I_{xx} - I_{yy}}{r^2} \quad (9)$$

$$e_2 = \frac{2I_{xy}}{r^2} \quad (10)$$

For the case of DCR, we can set, without loss of generality, the  $x$  direction to be towards zenith. The effect of DCR is then to take  $I_{xx} \rightarrow I_{xx} + V$ , which also takes  $r^2 \rightarrow r^2 + V$ , but leaves  $I_{yy}$  and  $I_{xy}$  unchanged. The ellipticity and size parameterizations are defined in terms of the quadrupole moments before the galaxy light distribution is smeared by the PSF, but we only have access to the light distribution after convolution. The second moments before ( $I^g$ ) and after ( $I^o$ ) convolution are related via the second moments of the PSF

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( $I^*$ ) like  $I^g = I^o - I^*$ . If the SEDs of galaxies and stars were the same, then the effect of DCR would be to add  $V$  to both  $I^o$  and  $I^*$ , which would then cancel when computing  $I^g$  and subsequently galaxy shape parameters. The differences between stellar and galactic SEDs, however, will introduce a small error  $\Delta V \ll r^2$  into  $I_{xx}$  and  $r^2$ , which leads to biases in the derived ellipticity parameters:

$$e_1 \rightarrow \frac{I_{xx} + I_{yy} + \Delta V}{r^2 + \Delta V} \approx e_1 \left(1 - \frac{\Delta V}{r^2}\right) + \frac{\Delta V}{r^2} \quad (11)$$

$$e_2 \rightarrow \frac{2I_{xy}}{r^2 + \Delta V} \approx e_2 \left(1 - \frac{\Delta V}{r^2}\right) \quad (12)$$

Under the expectation that  $\langle e_i \rangle \approx 2\gamma_i$  and defining the shear calibration parameters  $m$  and  $c$  such that  $\hat{\gamma} = \gamma(1 + m) + c$ , where  $\hat{\gamma}$  indicates the estimator for the true shear  $\gamma$ , we reach the expectation that  $m_1 = m_2 = m = -\frac{\Delta V}{r^2}$ ,  $c_1 = -m/2$ , and  $c_2 = 0$ . Note that nowhere in the above analysis is there any assumption on the profile of the galaxy in question.

### 3. RING TEST

An alternative way to estimate the bias in  $\hat{\gamma}$  induced by DCR is to simulate galaxy images using the “true” (galactic) PSF and then attempt to recover the simulated galactic ellipticities while using the “wrong” (stellar) PSF. A ring test (Nakajima & Bernstein 2007) is a specific prescription for such a suite of simulations designed to rapidly converge to the correct (though biased) average value of  $\hat{\gamma}$ . The test gets its name from the arrangement of galaxy shapes used in the simulated images, which form a ring in ellipticity space centered at the origin (i.e.,  $|e|$  is constant), before any shear is applied. By choosing intrinsic ellipticities that exactly average to zero, the results of the test converge faster than for randomly (but isotropically) chosen ellipticities that only average to zero statistically.

The general procedure can be implemented as follows:

1. Choose an input “true” reduced shear  $g$
2. Choose a pre-sheared ellipticity  $e^s = (e_1^s, e_2^s)$  on the ring
3. Compute the sheared ellipticity from  $e^o = \frac{e^s + g}{1 + g^* e^s}$
4. Generate a “truth” image by convolving the galaxy model having ellipticity  $e^o$  with the “true” PSF (galactic in this case)
5. Using a different “reconstruction” PSF for convolution (stellar in this case), search the model parameter space to find the best fitting model image to the “truth” image, and record the measured ellipticity from the model parameters
6. Repeat steps 3-5 using the opposite pre-sheared ellipticity  $e^s \rightarrow -e^s$
7. Repeat steps 2-6 for as many values around the ellipticity ring as desired. Typically these are uniformly spaced around the ring.
8. Average all recorded output ellipticity values. This is the shear estimate  $\hat{g}$

9. Repeat steps 1-8 to map out the relation  $g \rightarrow \hat{g}$

10.  $1 + m$  and  $c$  are then the slope and intercept of the best-fit linear relation between  $g$  and  $\hat{g}$  (note we assume that  $g \approx \gamma$ )

As described above, the ring test requires some prescription for creating galaxy images with a given ellipticity. Here we investigate using a single Sersic profile as the galaxy model. The Sersic profile has 7 parameters: the  $x$  and  $y$  coordinates of the center, the total flux, the effective radius  $r_e$  (also called the half-light radius), the two component ellipticity  $e$ , and the Sersic index  $n$ . Using  $r$  as an elliptical radial coordinate, the profile shape is:

$$I(r) \propto e^{[-k(r/r_e)^2]^{\frac{1}{2n}}} \quad (13)$$

The constant  $k \approx 1.9992n - 0.3271$  is chosen such that  $r_e$  is the half-light radius for a circularized profile. Specific cases of the Sersic profile include the Gaussian profile which has  $n = 0.5$ , an exponential profile which has  $n = 1.0$ , and a de Vaucouleurs profile which has  $n = 4.0$ . The profile ranges from smoothly peaked with small tails at low  $n$  (such as an  $n = 0.5$  Gaussian), to very sharply peaked with heavy tails (such as an  $n = 4.0$  de Vaucouleurs profile).

In addition to a galaxy model, the ring test also requires a model for the PSF. We have described above the PSF kernel that describes the contribution of DCR, but other contributions to the PSF also exist, due to atmospheric turbulence, telescope optics, and the detector for example. Ground based telescope PSFs are usually dominated by atmospheric turbulence, and are frequently modeled by a Moffat profile:

$$I_p(r) \propto \left(1 + \left(\frac{r}{\alpha}\right)^2\right)^{-\beta} \quad (14)$$

$$\alpha = \frac{\text{FWHM}}{2\sqrt{2^{1/\beta} - 1}} \quad (15)$$

The PSF used in the ring test in this note is based on a Moffat profile with a FWHM of  $0''.7$  and softening parameter  $\beta = 2.6$ , values which were chosen based on fits to simulated LSST stars via the photon-by-photon simulation package phoSim. This base PSF is then convolved (in the zenith direction only) with the DCR kernel given in Equation 2. The “true” PSF uses a DCR kernel derived from a galaxy SED, while the “reconstruction” PSF uses a DCR kernel derived from a stellar SED.

### 4. SERSIC INDEX DEPENDENCE

The results of the ring test for a set of Sersic galaxies simulated at a zenith angle of 30 degrees are shown in Figure 1. The galaxy size quantified by  $r^2$  is the same for each Sersic index. The ring test results match the analytic formulae well when Gaussian galaxies ( $n = 0.5$ ) are used, but diverge by factors of 2-3 when de Vaucouleurs profiles ( $n = 4.0$ ) are used. The PB12 analytic formalism has no dependence on the profile of the galaxy, but only on the assumption that the difference between stellar and galactic dispersion kernels is much smaller than the size of the unsmeared galaxy ( $\Delta V \ll r^2$ ). This condition is met identically for each profile by construction, suggesting either that something is flawed with the PB12 formalism or with the ring test.

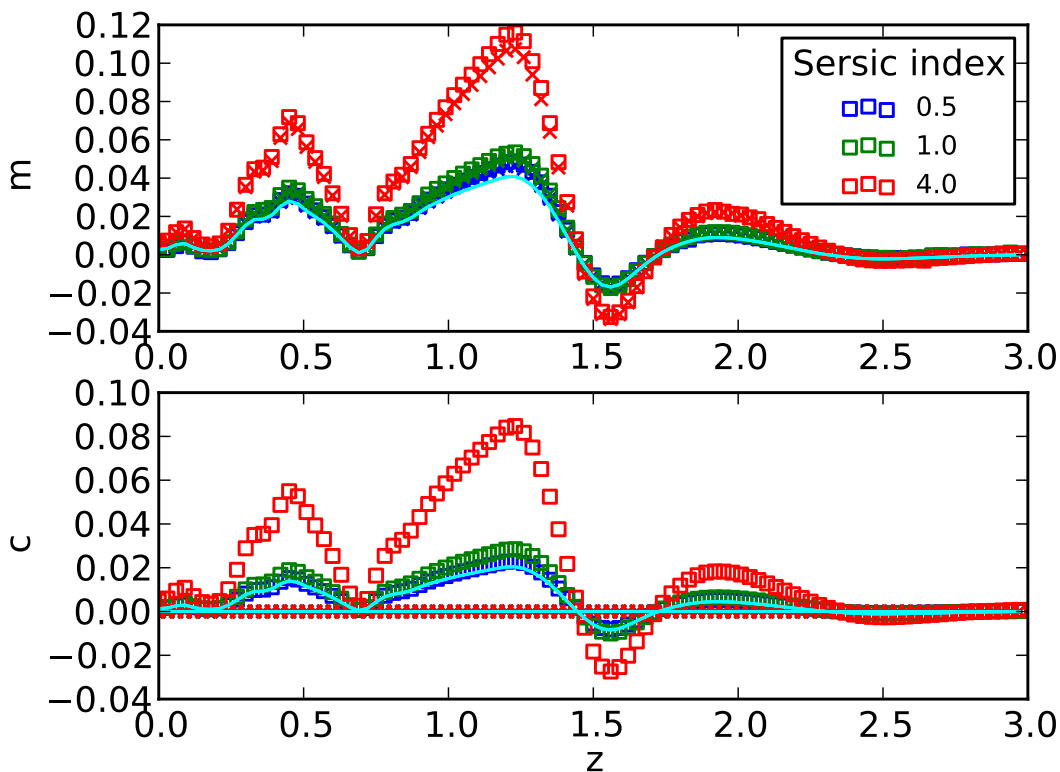


FIG. 1.— The shear calibration parameters  $m$  and  $c$  plotted against redshift. The results are obtained from the ring test assuming a true PSF derived from an elliptical galaxy spectrum and reconstruction PSF derived from a G5v stellar spectrum. The true galaxy images are simulated using the values:  $r_e = 1.1$ , and  $|e| = 0.2$ . Three values of the galaxy Sersic parameter  $n$  are shown:  $n = 0.5$ , corresponding to a Gaussian profile (blue),  $n = 1.0$ , corresponding to an exponential profile (green), and  $n = 4.0$ , corresponding to a de Vaucouleurs profile (red). Squares are used for  $m_1$  and  $c_1$ , and “x’s” for  $m_2$  and  $c_2$ . The cyan line in each panel shows the analytic result from the PB12 formalism. The zenith angle is 30 degrees.

## 5. RING TEST CASE STUDY

To investigate the ring test in more detail, we have conducted a case study focusing on the redshift 1.25 galaxy that shows the largest  $m$  and  $c$  values in both the ring test results and the PB12 results. For this study, we have constructed a series of diagnostic images showing the results of the fits in Step 5 of the ring test for each of the Sersic indices studied. These diagnostic images are shown in Figures 2, 3, and 4. Note that the diagnostic figures were generated for a zenith angle of 60 degrees for better visualization, though the qualitative results are the same at 30 degrees.

In Figures 2, 3, and 4, the “truth” image is shown in the top right panel. The panel immediately below this one is the best fit to the “truth” image possible using the “reconstruction” PSF, which in this case is the PSF generated from the G5v stellar spectrum. The leftmost panel in the second row shows the above-atmosphere galaxy model used to create this “best fit” image. The rightmost panel in the third row shows the residual between the “truth” image and the “best fit” image, and is an indication of the quality of the fit in Step 5 of the ring test. If the fit were perfect, then the residual image would be entirely zeros.

The reason the residual image is not entirely filled with zeros is due to “model bias” (Melchior et al. 2009; Voigt & Bridle 2010; Bernstein 2010). The fitting step can be viewed as an attempt at deconvolution of the “truth” image by the “reconstruction” PSF under the assumption that the functional

form of the deconvolved image is known. The best fit model (second row, first column) is the result of this attempted deconvolution, and the ellipticity parameters can then simply be read off as the parameters that generated the model. In practice, however, the true deconvolved image (i.e. the true deconvolution of the “truth” image by the “reconstruction” PSF, not computed or shown in any of the Figure panels) will not always be an instance of the functional form of the fit. More generally, the true deconvolution may not even have elliptical isophotes, precluding a solution by simply adding degrees of freedom to the radial profile of the functional form. (Degrees of freedom can, of course, be added *azimuthally* to get a perfect deconvolution, but then the ellipticity can no longer be simply read off as a model parameter).

The degree to which the ring test results are biased by model fitting depends on how well the deconvolution is able to function. Larger residuals to the fit are more susceptible to bias. Comparing Figures 2, 3, and 4, we see visually that the residuals grow with the Sersic index, which explains how the  $n = 4.0$  curve of Figure 1 became so different than the analytic prediction. All of the curves are subject to both model bias and bias from DCR, but the  $n = 4.0$  curve is the most sensitive to model bias. The visual comparison is backed up by the data in Table 1. As the Sersic index increases, the best fit  $\chi^2$  also increases indicating a worse fit. From the quadrupole moments of the residuals, one can define a Fig-

ure of Demerit:

$$\text{FoD} = I_{xx}^r + I_{yy}^r \quad (16)$$

where  $I^r$  is the residual image. The Figure of Demerit also increases with Sersic index, indicating an increasing model bias.

Presumably, better ellipticity measurement algorithms would show less dependence on the Sersic index. For in-

stance, the algorithms described in [Bernstein \(2010\)](#) and [Melchior et al. \(2011\)](#) both aim to mitigate model bias. If Step 5 of the ring test were replaced with one of these algorithms for measuring ellipticity, then maybe the effect of DCR could better be isolated from the effects of model bias.

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TABLE 1  
FIT QUALITY

n	$\chi^2$	FoD	m	c	PB12 m	PB12 c
0.5	2.08e-4	5.82e-10	(0.008672, 0.008513)	(0.004213, 0.0)	-0.008111	0.004055
1.0	3.28e-3	8.97e-9	(0.010016, 0.009828)	(0.005755, 0.0)	-0.008111	0.004055
4.0	1.15e-2	9.01e-8	(0.023039, 0.022608)	(0.020024, 0.0)	-0.008111	0.004055

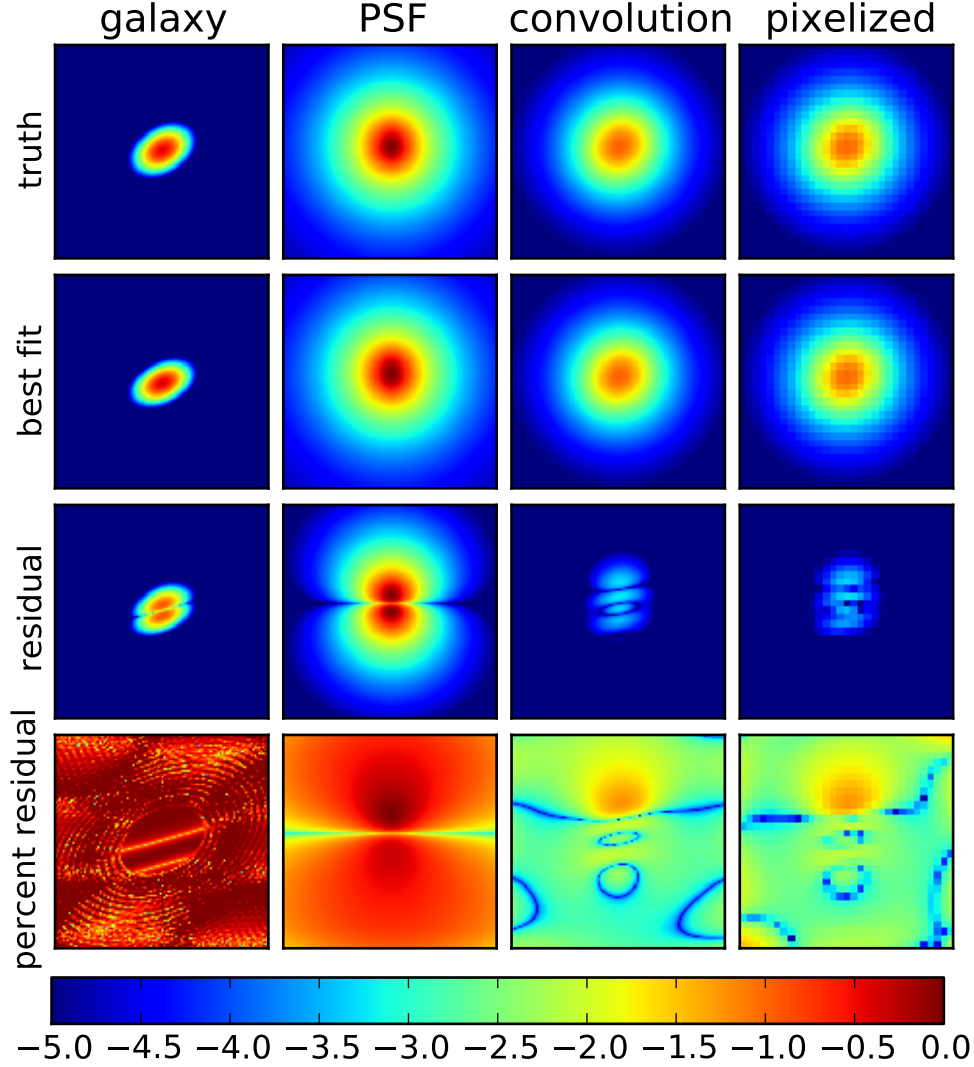


FIG. 2.— Illustration of the simulation and fitting portion of the ring test procedure (steps 4 and 5). The columns from left to right represent (i) the high-resolution model galaxy before PSF convolution, (ii) the PSF, (iii) the galaxy convolved with the PSF, and (iv) the pixelization of (iii). The rows from top to bottom represent (i) the “true” galaxy model, PSF, and convolutions, (ii) the best fit galaxy model, PSF, and convolutions, (iii) the base-10 logarithm of the absolute value of the residual (bestfit – truth), and (iv) the base-10 logarithm of the absolute fractional residual ((bestfit – truth)/truth). The fit is designed to minimize the pixelized residuals in the third row, fourth column. The difference between the truth and best fit galaxy models illustrates the size of the model bias induced by using the “reconstruction” PSF in the fit. The galaxy profile is Gaussian in this figure ( $n = 0.5$ , both in truth image and fixed during the fit), the galaxy PSF is derived from an elliptical spectrum at redshift 1.3, and the stellar PSF is derived from the spectrum of a G5v star. The zenith angle is 60 degrees.

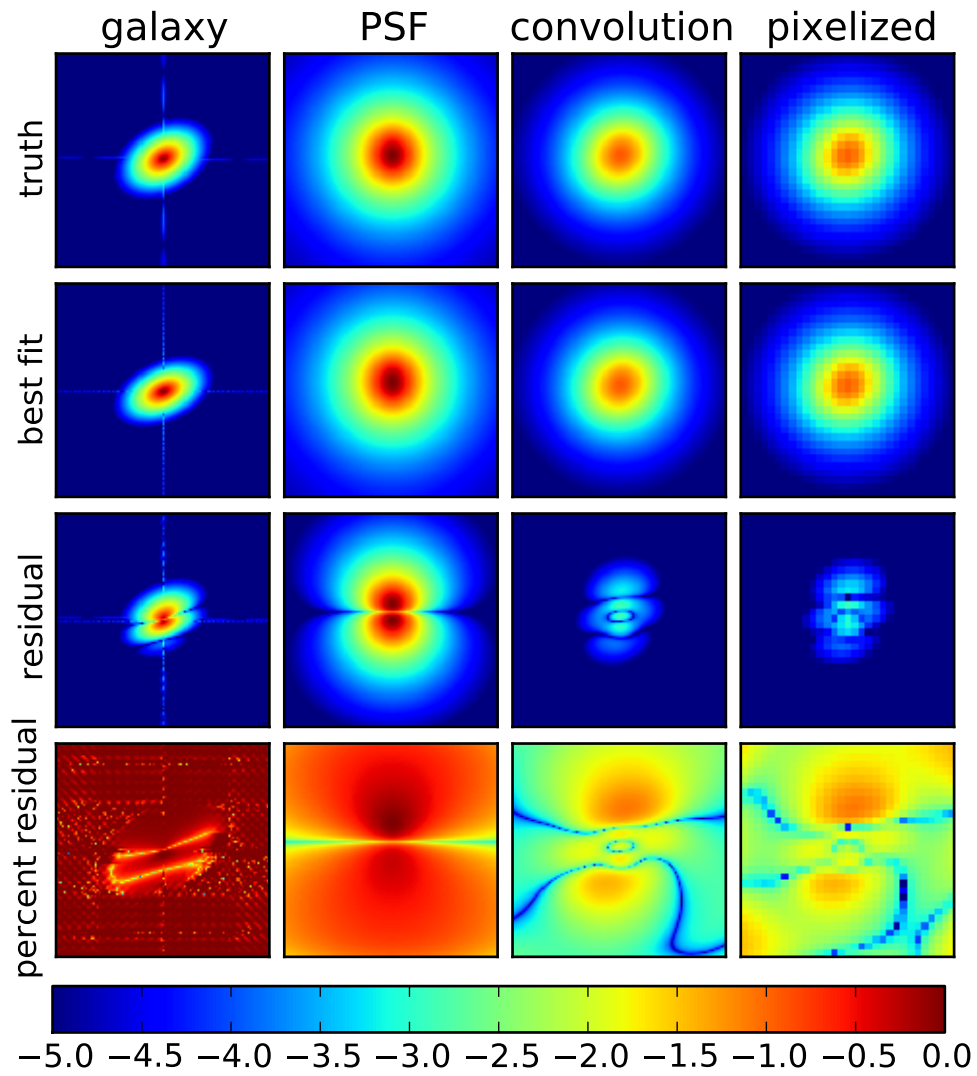


FIG. 3.— The same as Figure 2, but the galaxy profile is changed to be exponential ( $n = 1.0$ , in both the truth image and fixed during the fit).

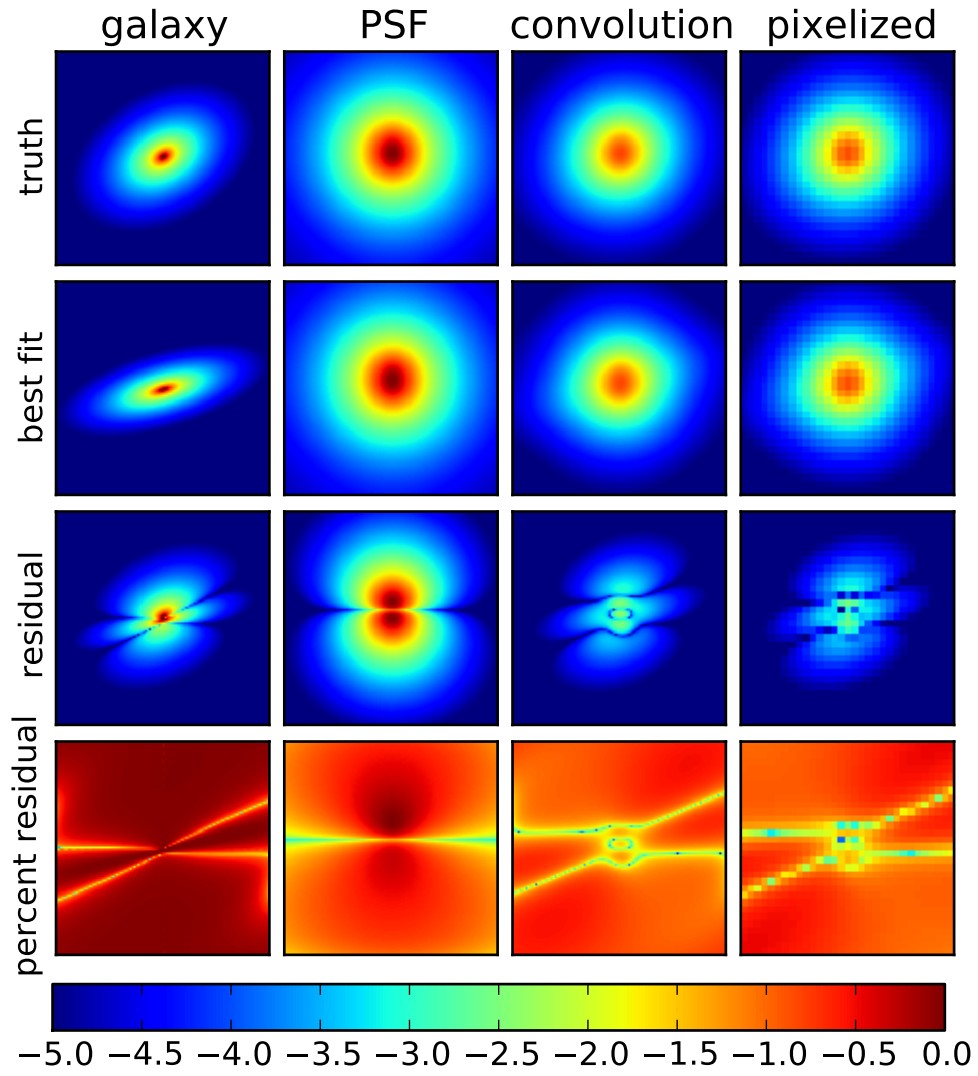


FIG. 4.— The same as Figure 2, but the galaxy profile is changed to be de Vaucouleurs ( $n = 4.0$ , in both the truth image and fixed during the fit).