Exercise 532
Electron diffraction on polycrystalline graphite lattice.

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Abstract

This report presents examination of de Broglie's postulate concerning material particles and calculating of inter-planar spacing in graphite material.

1 Introduction

The aim of this exercise was to determine interplanar spacing of graphite using Broglie's postulate about consideration beam of electrons as wave and, as source of data, electron diffraction vacuum tube.

2 Theory and measurement

In 1924, physicist Louis de Broglie proposed extension of Einstein's suggestion that a quantum of light has linear momentum in the way that not only photons, but also electrons obey this phenomena [1]. According to this, electron's wavelength can be calculated from formula

$$\lambda = \frac{h}{p} \tag{1}$$

where h is the Planck's constant and λ is called de Broglie wavelength of the moving particle.

Considering beam of electrons as wave and using graphite as diffraction grading - accelerated by electron gun electrons are diffracted from a polycrystalline layer of graphite and shown on screen in form of luminous rings (Fig. 1), we can calculate, from the diameter of the rings and accelerating voltage interplanar spacing of the structure.

The momentum p of particles can by calculated form velocity v

$$E = \frac{mv^2}{2} = \frac{p^2}{2m} \tag{2}$$

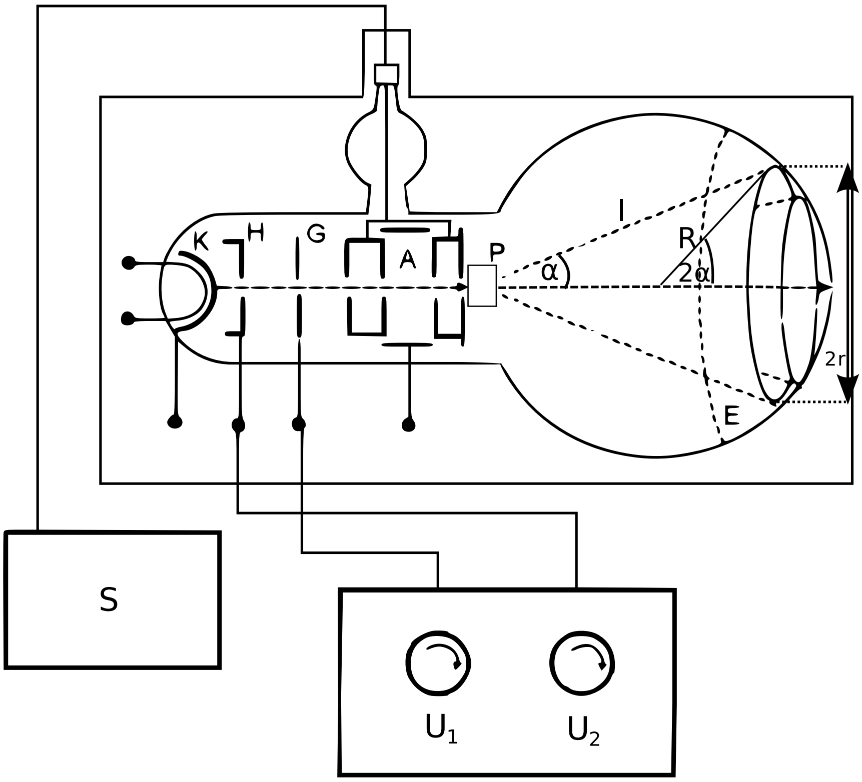

Figure 1: Electron diffraction vacuum tube [2]. It consists of: graphite lattice (P); electron gun: cathode (K), Wehnelt cylinder (H), electronic lens system (G), focusing voltage regulator (U_1), stopping voltage regulator (U_2), anode (A); screen (E) and high voltage power supply (S).

where m is rest mass of electron. Energy can by calculated also from acceleration voltage U_A

$$E = eU_A \tag{3}$$

where e is the electron charge. By combining equations 1, 2 and 3 we can obtain following dependency.

$$\lambda = \frac{h}{\sqrt{2mU_A}} \tag{4}$$

Bragg equation (5) describes the reflection of the electron beam on the structural lattice of crystal

$$2d \sin\theta = n\lambda \tag{5}$$

where d is the spacing between layers of carbon's atoms and θ is the angle between incident beam and lattice planes (Bragg angle). According to figure 1

$$\sin 2\alpha = \frac{r}{R} \tag{6}$$

where r is the radius of the interference ring and R is the radius of the glass bulb. Assuming that for small angles of θ: $\sin\alpha = \sin 2\theta \approx 2\sin\theta$ we can obtain

$$r = \frac{2R}{d} n\lambda \tag{7}$$

3 Results

From equation 4 we can calculate wavelength, assuming that $h = 6.625 \cdot 10^{-34}$ Js and $m = 9.109 \cdot 10^{-31}$ kg. And from equation 6 radii of the ring, assuming that the radius of the glass bulb, $R = 65 \cdot 10^{-3}$ m.

Table 1: Measured angles and calculated radii and wavelengths for setted voltages.

$2\alpha_1[°]$ first ring angle	r_1 $[10^{-3}$m] first ring radius	$2\alpha_2[°]$ second ring angle	r_2 $[10^{-3}$m] second ring radius	U_A [kV] anode voltage	λ $[10^{-21}$m] wavelength
28	30.516	46	46.757	2.5	9.817
36	38.206	44	45.153	3.0	8.961
24	26.438	42	43.493	3.5	8.297
24	26.438	38	40.018	4.0	7.761
22	24.349	34	36.348	4.5	7.317
20	22.231	34	36.348	5.0	6.941
18	20.086	32	34.445	5.5	6.618
18	20.086	32	34.445	6.0	6.337
16	17.916	30	32.500	6.5	6.088
16	17.916	28	30.516	7.0	5.867
16	17.916	28	30.516	7.5	5.668

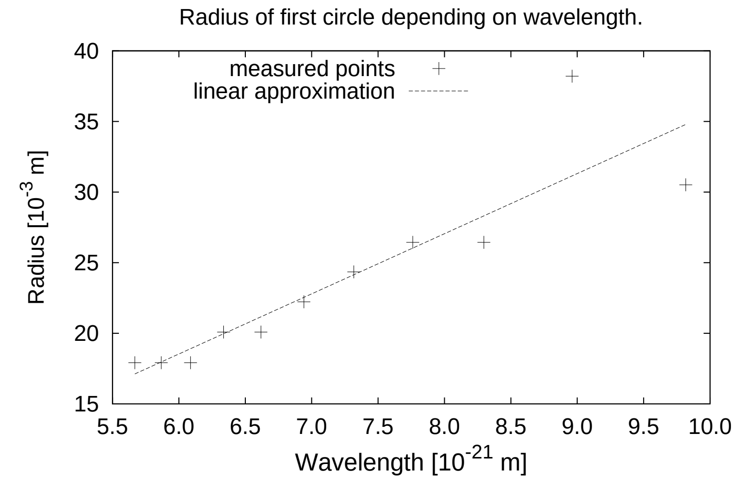

Figure 2: graph of radius of first circle wavelength

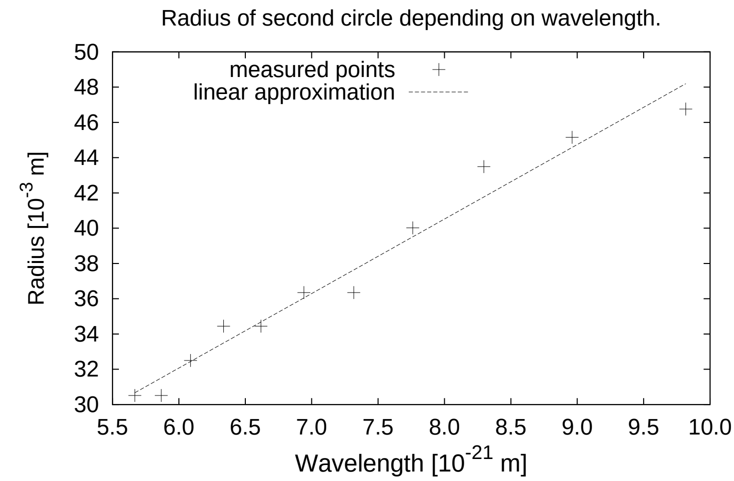

Figure 3: graph of radius of second circle versus wavelength

Slopes of linear approximations calculated by linear regression for radii are respectively $A_1 = 5.952$ and $A_2 = 11.245$ with asymptotic standard error +/- 0.6799 (15.96%) and +/- 0.2401 (5.685%) for first and second graph. To obtain slopes form ratio of appropriate scale we have to multipli both obtained factors (because of different graph scale) by 10^8. From eq. 7 for n = 1 (higher orders interference rings were invisible) we can obtain formula (8) for values of lattice constants d_1 and d_2.

$$A_i = \frac{2R}{d_i} \tag{8}$$

After transformation.

$$d_i = \frac{2R}{A_i} \tag{9}$$

Calculation of propagation of uncertainty

$$\Delta d_i = \left| \frac{d_i(A_i)}{\mathrm{d}A_i} \right| \cdot |\Delta A_i| = \frac{2R}{(A_i)^2} \cdot \Delta A_i \tag{10}$$

Now we can calculate lattice constants d_1 and d_2

$$d_1 = \frac{2 \cdot 65 \cdot 10^{-3}\mathrm{m}}{5.952 \cdot 10^8} \pm \frac{2 \cdot 65 \cdot 10^{-3}\mathrm{m}}{5.952^2 \cdot 10^{16}} \cdot 0.679 \cdot 10^8 = (2.3 \pm 0.3) \cdot 10^{-10}\mathrm{m}$$

$$d_1 = \frac{2 \cdot 65 \cdot 10^{-3}\mathrm{m}}{11.245 \cdot 10^8} \pm \frac{2 \cdot 65 \cdot 10^{-3}\mathrm{m}}{11.245^2 \cdot 10^{16}} \cdot 0.240 \cdot 10^8 = (1.4 \pm 0.3) \cdot 10^{-10}\mathrm{m}$$

4 Conclusions

As it is shown on figures 2 and 3 measured points almost cover linear approximation of graph. Moreover, for obtained uncertainties, results cover data available in literature. It was possible thanks to assumption that for small angles $\sin 2\theta \approx 2 \sin \theta$. When it comes to comparison to literature values ($d_1 = 0.213$nm and $d_2 = 0.123$nm) [2], then obtained values are are similar, even considering fact, that values read from protractor could be not very accurate (rigs of which we measured values were quite wide).

References

[1] *Fundamentals of physics* (2011) [ebook]. David Halliday, Robert Resnick, Jearl Walker. 9th ed. ISBN 978-0-470-46908-8

[2] *Experiment 21. Electron diffraction.* (2005) [online]. Bogdan Żółtowski. Łódź. Available online at: http://phys.p.lodz.pl/materialy/mdems/523.pdf. Accessed 01.04.2013.