MathLib Domentation

Arjun Sarao

March 16, 2021

Contents

Abstract

I made this library to understand and test the ways computers calculate various mathematical functions and constants. Most of the answers will be approximations due to the natural inaccuracy of the methods of estimation.

1 Euler's Number

There are a few constants that are included in this library.

- Euler's Number (e)
- Pi (π)
- Golden Ratio (ϕ)

Euler's Number appears everywhere.It is an irrational number and can be derived as follow:

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = 2.7182812\ldots \tag{1}$$

While there are other ways to derive e such as the sum of 1 over a incrementing factorial, this is easier to calculate.

2 Pi

Pi is one of the most common irrational number constants. It describes the ratio of a circles circumference to its diameter. I chose to use the *Ramanujan-Sato Series* for deriving Pi as i felt it would present an interesting challenge and that it uses elements such as factorials, square roots, and powers; which are functions in my library. The equation is:

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{99^2} \sum_{k=0}^{\infty} \frac{(4k)!}{k!^4} \frac{26390k + 1103}{369^{4k}} = 0.318309886\ldots \tag{2}$$

3 Golden Ratio

This derivation is fairly tame as compared to the previous 2. The *Golden Ratio* is when 2 numbers added to each other and divided by the larger are equal to one number over the other.
Let a and b represent any two numbers where $a > b > 0$

$$\frac{a+b}{a} = \frac{a}{b} = \phi \tag{3}$$

We can also calculate the numerical value:

$$\phi = \frac{1 + \sqrt{5}}{2} = 1.618033988\ldots \tag{4}$$

4 Powers and Square roots

The process for calculating square roots is as follows:

$$\sqrt[a]{b} = e^{\frac{1}{a}\ln(b)} \tag{5}$$

This is how most pocket calculators solve for any roots. The same can be said for any type of powers using:

$$a^b = e^{b\ln(a)} \tag{6}$$

5 Fibonacci Sequence

The *Fibonacci Sequence* is a recursive series where the next number is the sum of the previous 2 numbers.

$$F_n = F_{n-1} + F_{n-2} \tag{7}$$

We can also determine the n^{th} term of the sequence using Phi and the following formula

$$F_n = \frac{\phi^n - \Phi^n}{\phi - \Phi} = \frac{(\frac{1+\sqrt{5}}{2}) - (\frac{1-\sqrt{5}}{2})}{\sqrt{5}} \tag{8}$$

6 Factorials

Factorials are useful operation in many places, in this library they are used in the definition of pi, permutations, and combinations. Factorials are defined as:

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1 \tag{9}$$

While there is no neat formula I found, factorial calculations can be done easily using a recursive function.

7 Logarithms

Natural logs are useful in calculating powers and square roots and in regular logarithms of variable bases. A useful estimation is:

$$\ln(x) = \lim_{a\to\infty} ax^{\frac{1}{a}} - a \tag{10}$$

From an estimation of logarithms we can calculate logarithms of variable bases using the change of base formula:

$$\log_b(x) = \frac{\ln(x)}{\ln(b)} \tag{11}$$

8 Permutations and Combinations

Permutations and *Combinations* are very useful in probability questions. Permutations can be calculated as follows:

$$_nP_r = \frac{n!}{(n-r)!} \tag{12}$$

And combinations:

$$_nC_r = \frac{n!}{r!(n-r)!} \tag{13}$$