Qualitative Disease Models, including lock downs and health care system distress

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1 Introduction

Only those that have truly been living under rocks haven't heard the terms "flatten the curve", "Lock down" and "health care system distress". We all know what these words mean but we may not have any sort of instinctual of what this means. Here in South Africa, our government has lifted restrictions from level 5 (or hard lock down) to level 4 (or a softer, friendlier hard lock down). When it comes to modeling pandemics (you can get a master class on this online these days), what does this mean for infection rates and more importantly, fatality rates. Here, I'll be examining and comparing a number of qualitative disease models to understand what this means. There is a caveat though. 'Qualitative' in this context means that the models behave in more or less the correct fashion but the actual numbers do not reflect reality. While qualitative models are not useful in any predictive sense, they do help us understand and get a feeling for the world we live in. For those that are interested in more details, the OpenModelica models, Octave scripts, batch scripts and the tex source for this document can be found by following this link: `https://github.com/DarkHorse84/CovidModels/tree/master/Qualitative`.

2 The Models

We'll start with a very basic model. For the basics, we'll take a population and divide it into 4 parts, the yet to be infected (u), and three groups that represent those that have or have had the disease (the total of these three groups will be called the infected and be represented by i), the sick (s), the recovered (r) and the dead (d). Now it should be clear that if we take the uninfected and add the sick, the recovered and the dead, we'll get the total number of people in our population (T) which is all the people that will ever get infected, or as an equation $T = u + s + d + r = u + i$. The number of people that are currently sick can be found by taking the total infected and subtracted the recovered and the dead, or $s = i - r - d$. This model deals with how people move from one group into the other, so lets start with people getting infected. Consider that for every interaction between an actively infected (or sick) and an uninfected person, there is a probability that the uninfected person will become infected. This can be modeled using the logistic model:

$$\dot{i} = a \times i \times s$$

The rate of change of infections ($\dot{i}$) is zero when there are no infections ($i = 0$) or when there are no sick people ($s = 0$) and reaches a maximum when the number of infected people is half the total number of people or when $i = \frac{T}{2}$. The multiple a is a constant of proportionality. The dot above the i means the rate of change, so $\dot{i}$ can be read as 'the rate of change of i'. There will also be people recovering from the illness and people dying. in both cases the rate of change of these values is proportional to the number of people that are currently sick:

$$\dot{r} = b \times s$$
$$\dot{d} = c \times s$$

Like a, b and c are constants of proportionality. When trying to model a real system we would choose a,b and c so that our models match reality. If we know the values of those constants and the initial size of each group, we can solve these equations and get a mathematical picture of what is happening. If we assume that there is 1 infected person in a population of 15 million, no recovered or dead people and with

$$a = 0.02$$
$$b = 0.7$$
$$c = 0.015$$

the graphs look like the graph in figure 1.

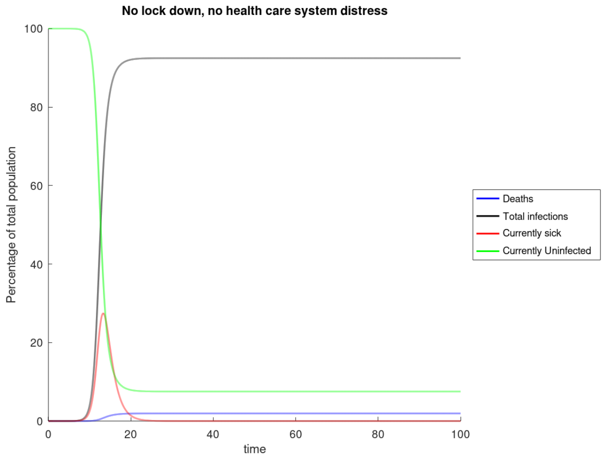

Figure 1: The Basic Model

We'll use this as a baseline to compare other models later.

What happens if the health care system were to become distressed? This can be modeled by increasing the death rate and decreasing the recovery rate if the number of sick people are above a certain number. Here if the number of sick people are over 10% of the population, the death rate doubles (people die at a faster rate) and the recovery rate becomes three quarters what it is if there is no distress (people take longer to recover):

$$b = \begin{cases} \frac{3}{4} \times 0.7 & \text{if } s > 10 \\ 0.7 & \text{otherwise} \end{cases}$$

$$c = \begin{cases} 2 \times 0.015 & \text{if } s > 1 \\ 0.015 & \text{otherwise} \end{cases}$$

In South Africa, at the time of writing, the government has enforced a period of hard lock down (some have called this shelter in place and the unflattering, house arrest) and has then entered a period of lower restrictions. This phased release of lock down can be modeled by changing the infection rate in a similar way as the recovery and death rate. Here the recovery rate is initially high, drops during hard lock down and goes back up but not all the way to the original value when restrictions are lifted.

$$a = \begin{cases} 0.02 & \text{if } t < 3 \text{ (the time before the initial lock down)} \\ 0.0005 & \text{if } 3 \leq t < 11 \text{ (hard lock down)} \\ 0.01 & \text{otherwise (lightly lifted restrictions)} \end{cases}$$

These can be assembled into four different models that can be compared:

- The basic model, with no lock down and no health care system distress.
- The basic model, with lock down.
- The basic model, with health care system distress.
- The basic model, with lock down and health care system distress.

3 Analysis

3.1 Total Infection and Active Infection Comparison

Let's start comparing the impact that that lock down and health care system distress has on overall infections rates.

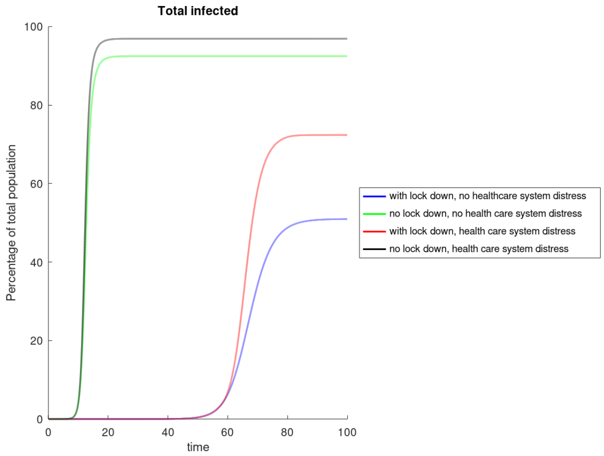

Figure 2: Total Infections For All Models

Figure 2 shows the total number of infections at any time for the four different models. Unsurprisingly, the infection rate is sensitive (depends on) both whether or not a lock down is present and if the health care systems experience distress or not. It can be seen that while it is better preserve the health care system than to let it collapse, lock down is far more effective at reducing overall infection rates.

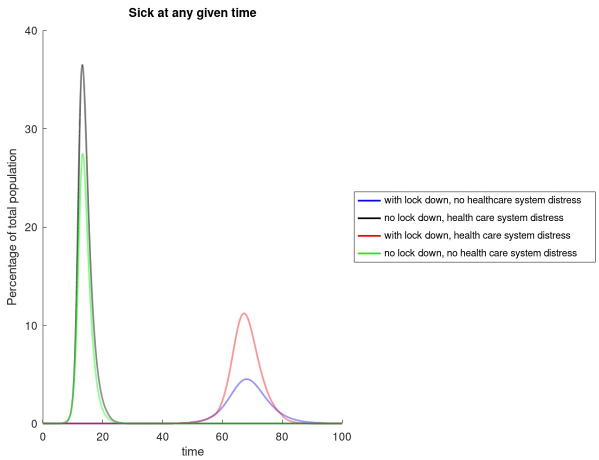

Figure 3: Active Infections (Currently Sick) For All Models

Figure 3 shows how many people are actively infected (or sick) at any given time. Here, it's clear that the number of people that are sick is sensitive to both the lock down and health care system distress, however lock down dominates. The reason for this is that, initially, the rate of people getting ill is higher than the rate of people dying or recovering. Lock down reduces this rate, if the goal is to reduce the peak number of sick people, then a lock down is clearly a successful strategy (this could be replaced with voluntary measures but that comparison will not be made here).

3.2 Total Death Comparison

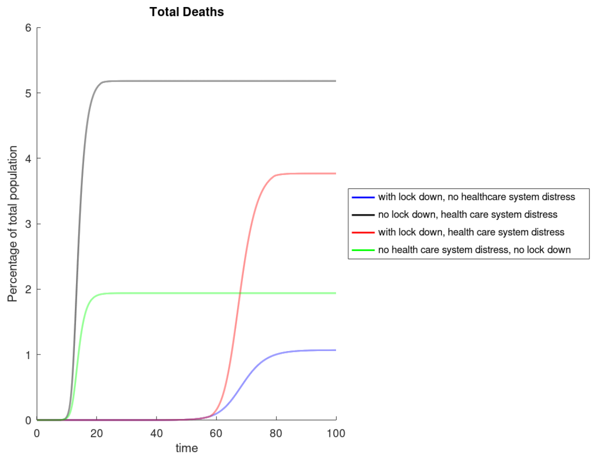

Figure 4: Deaths For All Models

The death rate tells a different story. When health care systems are distressed, the death rate is far higher than if there is no distress, however lock down still reduces the number of deaths.

3.3 The Hidden Feature

There is a hidden feature not implicitly modeled. The rate of change of active infections is

$$\dot{s} = (a \times u - c - b) \times s$$

As the number of sick people is always positive, if $c + b \geq a \times u$ then the number of sick people will decrease. This can reduce the rate of new infections during a lock down and can also lead to the scenario where there are uninfected people in the population but not enough sick people to infect them (the disease burns itself out). This feature helps the success of lock downs. It provides an early boost to the number of people that have recovered, reducing the rate of infection later on. Reducing the infection rate reduces the total count of infections, which in turn reduces the total death count. Also, as lock downs reduce the peak number of active infections, if successful enough, they can keep the number of active infections below the threshold that causes the health care system distress.

4 Parting Shots

When I built these models, I was attempting to understand the impact of health care system distress. What I did not expect to discover was the value of lock downs and their ability to reduce death rates. This can be amplified if lock downs are used to increase the health care systems capacity, something not included in these models. Another simplifying feature in these models is that the health care system is perfectly fine and suddenly is distressed when a certain threshold is breached. This won't be the case in reality. The health care system will become increasingly less effective as the active infection count increases. It also won't suddenly recover once the active infection count reduces below the threshold. The health care system will take time to recover. This will increase the death rate if the health care system is strained. It is also useful to remember that these model all assumed a total population size of 15 million people. 2% is 300,000 hypothetical people, 5% is three quarters of a million.