

Phase-Space

Features

- Lower GEM hit coord. along x axis (**x**)
- Lower GEM coordinate along y axis (**y**)
- Difference between GEM hits on upper and lower GEM hits along x axis (**dx**)
- Difference between GEM hits on upper and lower GEM hits along y axis (**dy**)

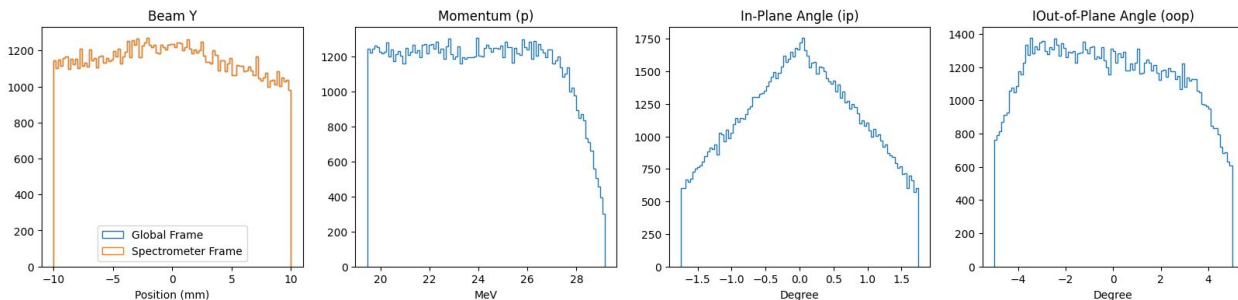
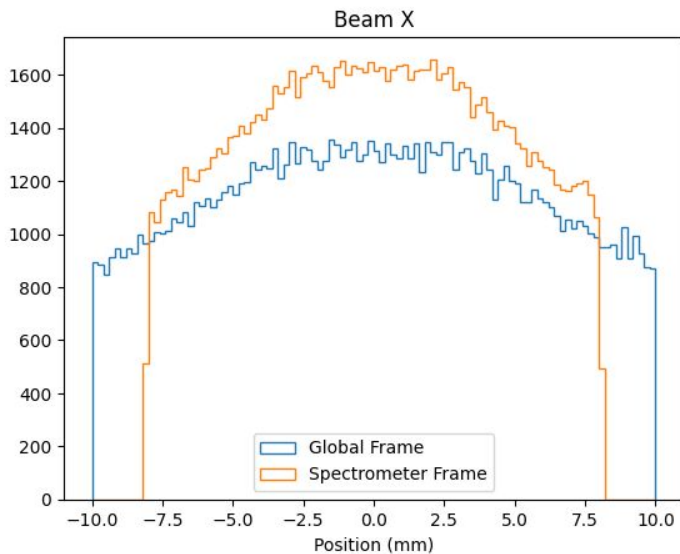


Targets

- Momentum (**p**) $\in [-x, x]$ mm
- In-Plane Angle (**ip**) $\in [-x, x]$ degrees
- Out-of-Plane Angle (**oop**) $\in [-x, x]$ degrees
- Beam X (**beam_x**) $\in [-x, x]$ mm
- Beam Y (**beam_y**) $\in [-x, x]$ mm

0. Test-Set (eC 25 MeV)

- Constant:
 - Momentum (**p**) (very small variation!)
- Variable:
 - Beam X (**beam_x**)
 - In-Plane Angle (**ip**)
 - Of-of-Plane Angle (**oop**)
- Number of events: 114,989
- **beam_x** range
 - Global Frame: (-10, 10) mm
 - Spectrometer Frame: (-8.1, 8.1) mm

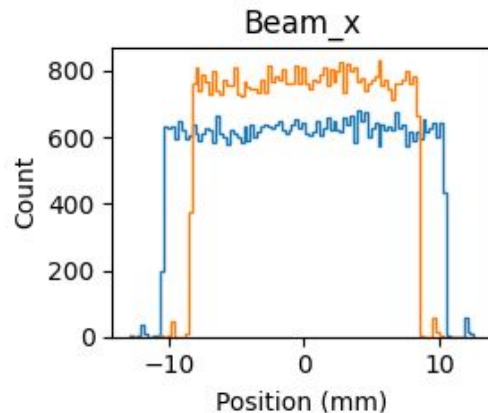
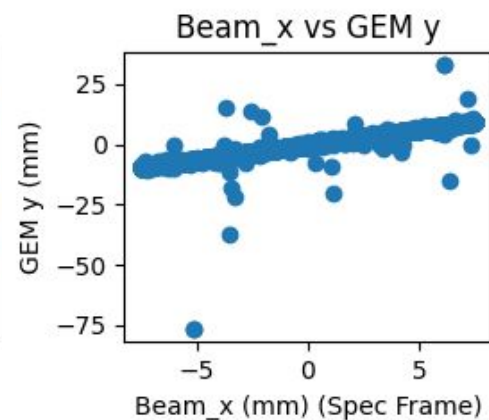
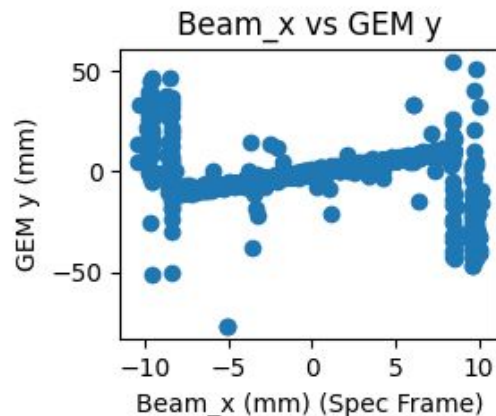


1. One DOF: beam_x

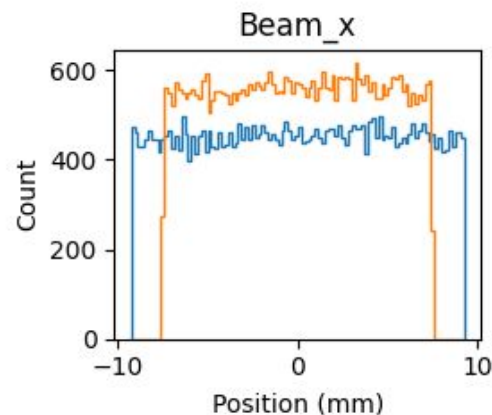
- Constant:
 - Momentum (**p**) = Nominal
 - In-Plane Angle (**ip**) = 0°
 - Out-of-Plane Angle (**oop**) = 0°
 - Beam Y (**beam_y**) = 0 mm
- Variable:
 - Beam X (**beam_x**) $\sim \mathcal{U}(-20, +20)$ mm
- Number of events: 100k

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 - Momentum (**p**) = Nominal
 - In-Plane Angle (**ip**) = 0°
 - Out-of-Plane Angle (**oop**) = 0°
 - Beam Y (**beam_y**) = 0 mm
- Variable:
 - Beam X (**beam_x**) $\sim \mathcal{U}(-20, +20)$ mm
- Number of events: 100k
- **beam_x** range (we have put a hard cut)
 - Global Frame: (-9.3, 9.3) mm
 - Spectrometer Frame: (-7.5, 7.5) mm



No cut!

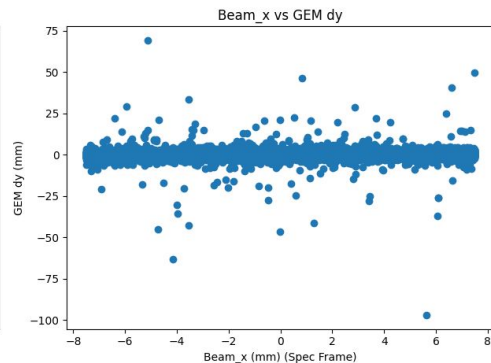
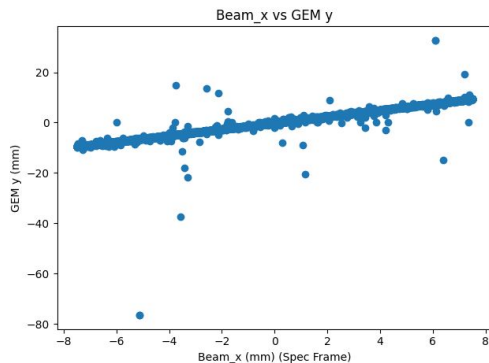


With cut!

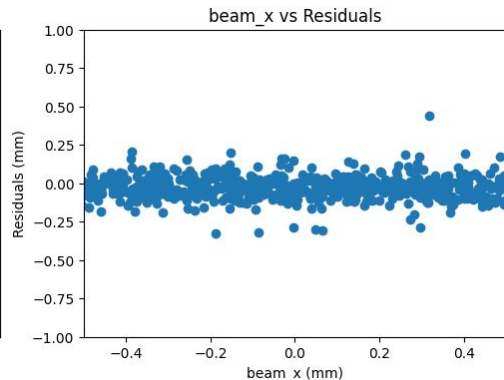
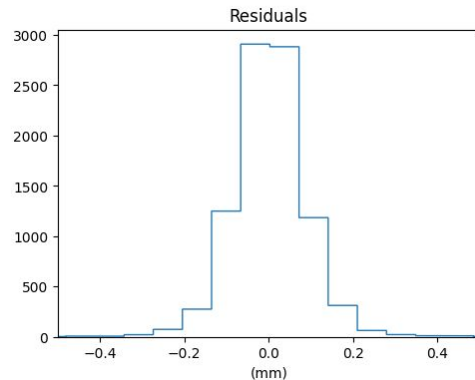
1. One DOF: beam_x

- Constant:
 - Momentum (**p**) = Nominal
 - In-Plane Angle (**ip**) = 0°
 - Out-of-Plane Angle (**oop**) = 0°
 - Beam Y (**beam_y**) = 0 mm
- Variable:
 - Beam X (**beam_x**) $\sim \mathcal{U}(-20, +20)$ mm
- Number of events: 100k
- **beam_x** range (we have put a hard cut!)
 - Global Frame: (-9.3, 9.3) mm
 - Spectrometer Frame: (-7.5, 7.5) mm

beam_x Correlations	
GEM y	1.0
GEM dy	0.17

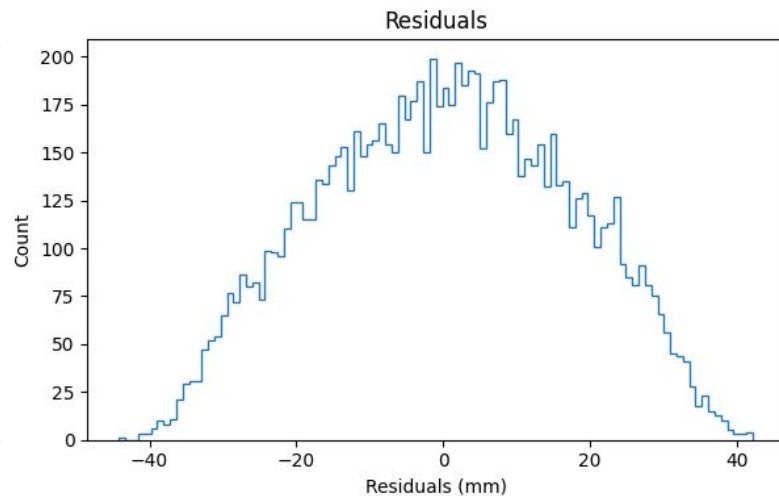
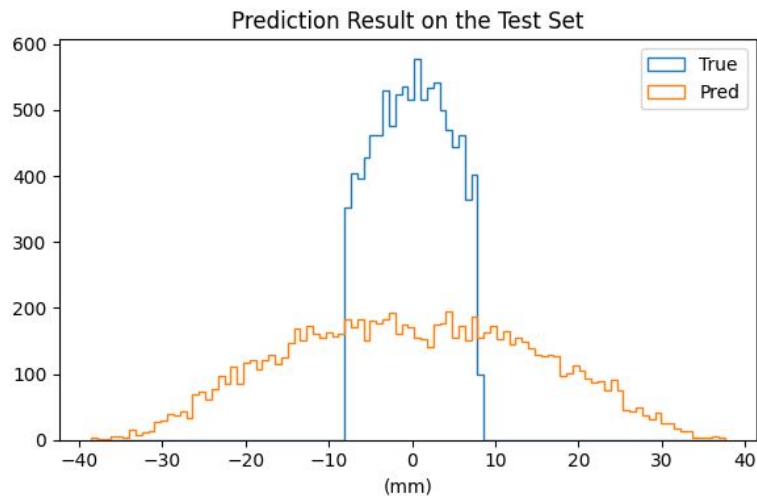


- **OLS Model**: $\text{beam_x} = 0.09 + 0.79 \square y + 0.05 \square dy$
- Out-of-Sample:
 - RMSE: 0.82
 - Standard Deviation: 4.33
 - R^2 : 0.96



1. One DOF: beam_x Test Set Result

RMSE	16.9645
STD	4.404
R^2	-13.83

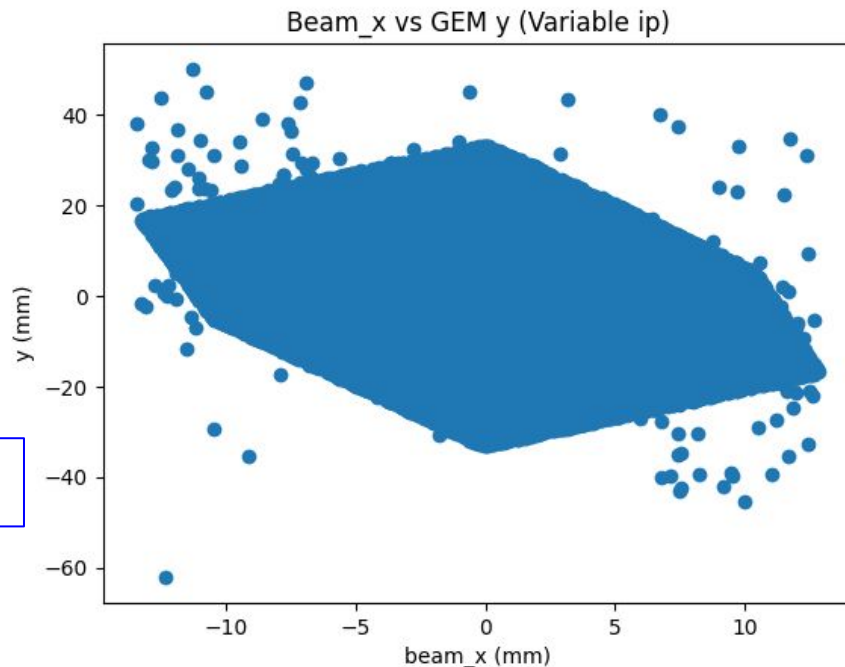


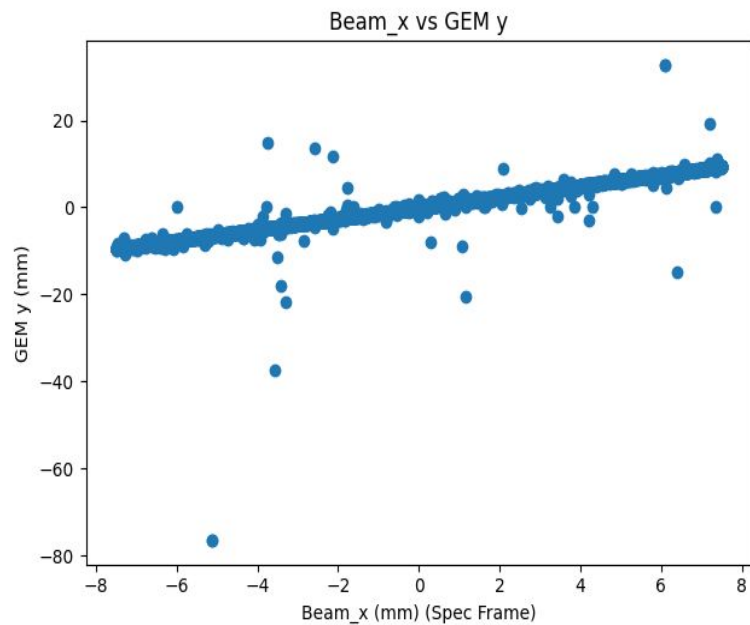
2. Two DOF: ip, beam_x

- Constant:
 - Momentum (**p**) = Nominal
 - Out-of-Plane Angle (**oop**) = 0°
 - Beam Y (**beam_y**) = 0 mm
- Variable:
 - Beam X (**beam_x**) $\sim \mathcal{U}(-20, +20)$ mm
 - In-Plane Angle (**ip**) $\sim \mathcal{U}(-1.35^\circ, +1.35^\circ)$
- Number of events: 100k
- Model 1:

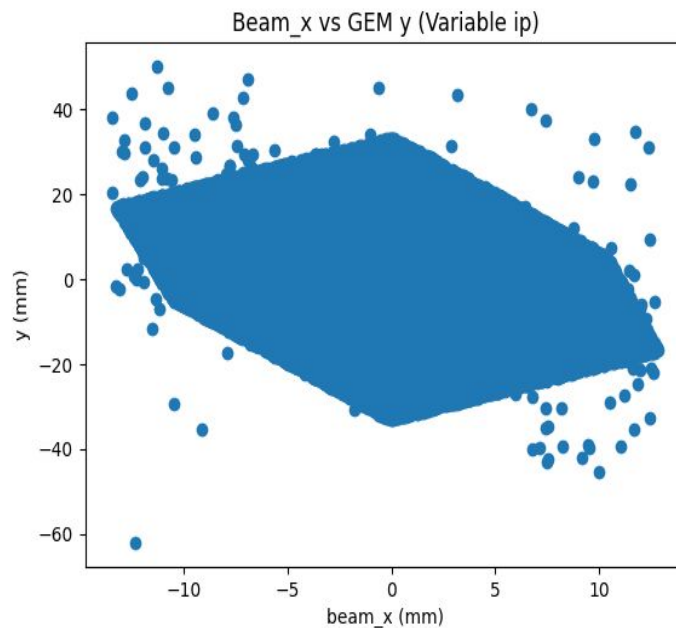
$$\text{beam_x} = \beta_0 + \beta_1 y + \beta_2 dy$$

RMSE	5.68
STD	5.99
R ²	0.101

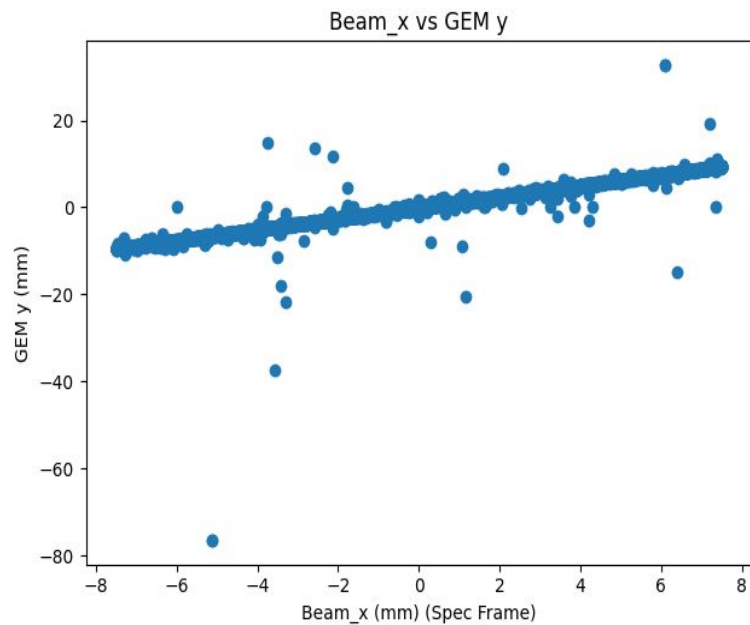




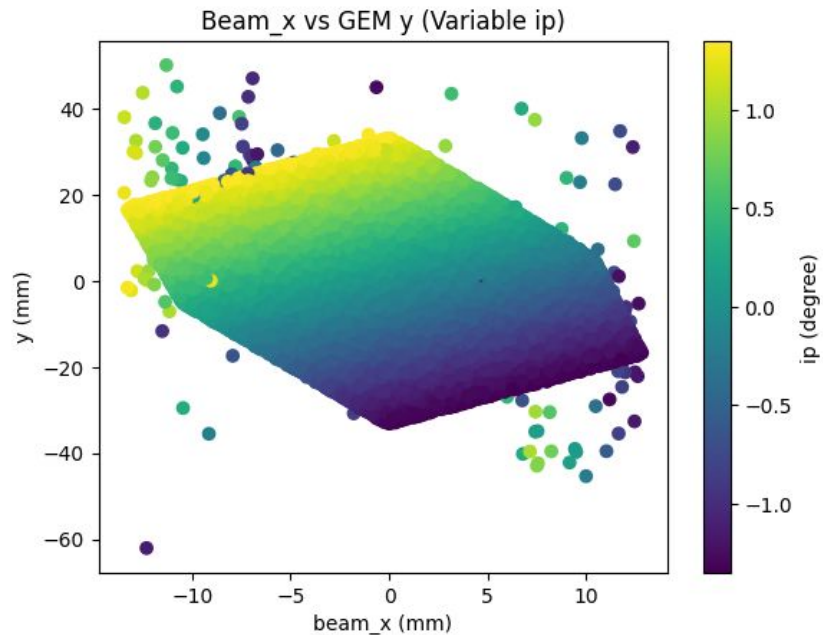
ip fixed!



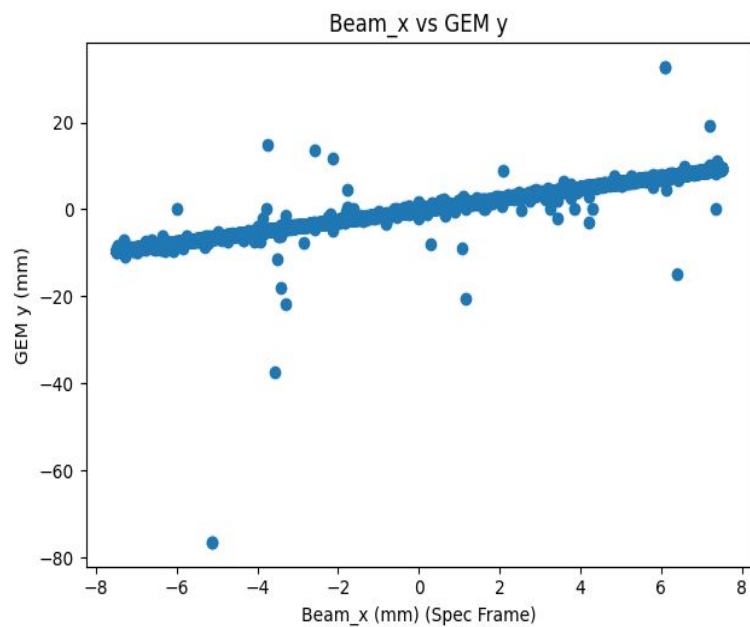
ip free!



ip fixed!

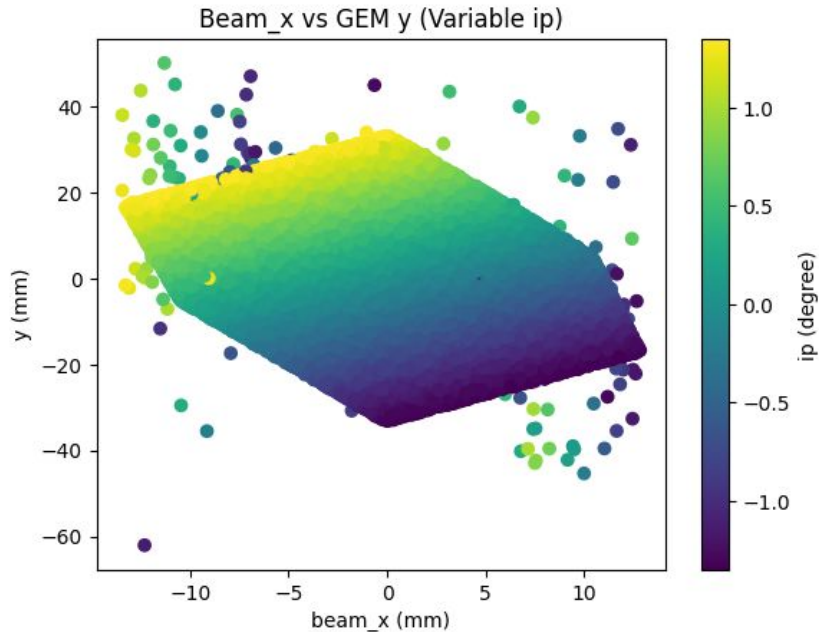


ip free!



ip fixed!

$$\begin{aligned} \hat{ip} &= f(y, dy) \\ \text{beam_x} &= g(y, dy, \hat{ip}) \end{aligned}$$

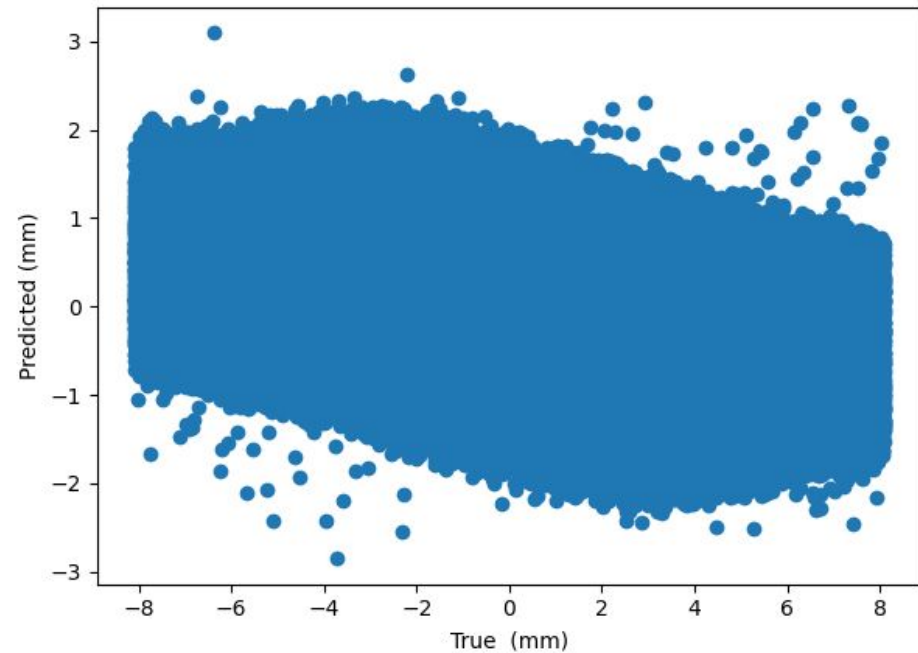


ip free!

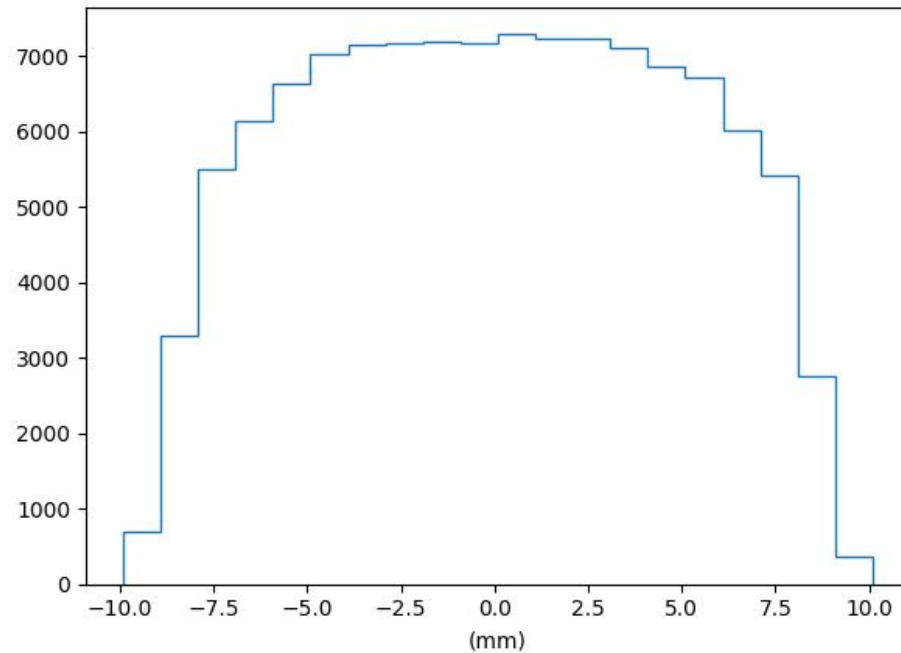
$$\Rightarrow \text{beam_x} = g(y, dy, f(y, dy))$$

2 DOF: beam_x, ip

Beam X Reconstruction



Residuals



RMSE	4.79
STD	4.39
R^2	-0.18

3. Three DOF: ip, oop, beam_x

- Constant:
 - Momentum (**p**) = Nominal
 - Beam Y (**beam_y**) = 0 mm
- Variable:
 - Beam X (**beam_x**) $\sim \mathcal{U}(-20, +20)$ mm
 - In-Plane Angle (**ip**) $\sim \mathcal{U}(-1.8^\circ, +1.8^\circ)$
 - Out-of-Plane Angle (**oop**) $\sim \mathcal{U}(-5^\circ, +5^\circ)$
- Number of events: 500k

$$\hat{ip} = \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy$$

$$\hat{oop} = \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx$$

$$\hat{\text{beam_x}} = \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy$$

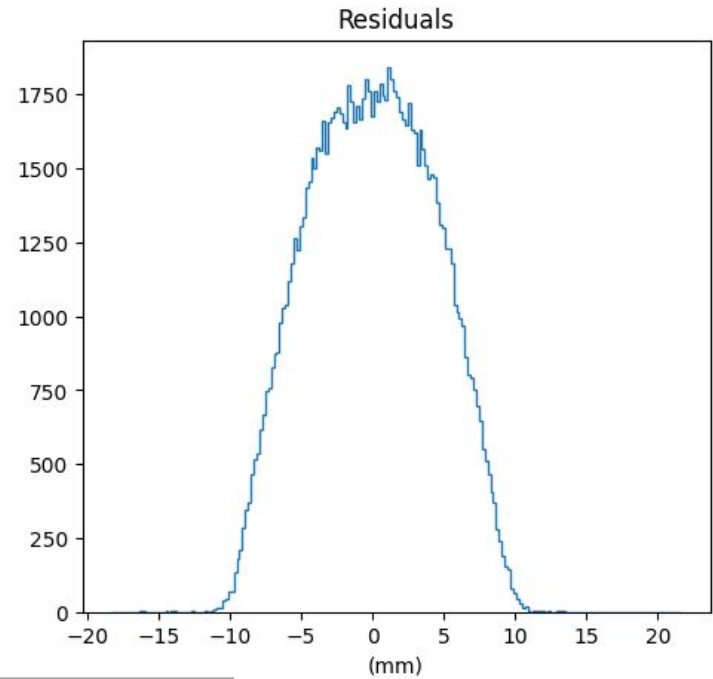
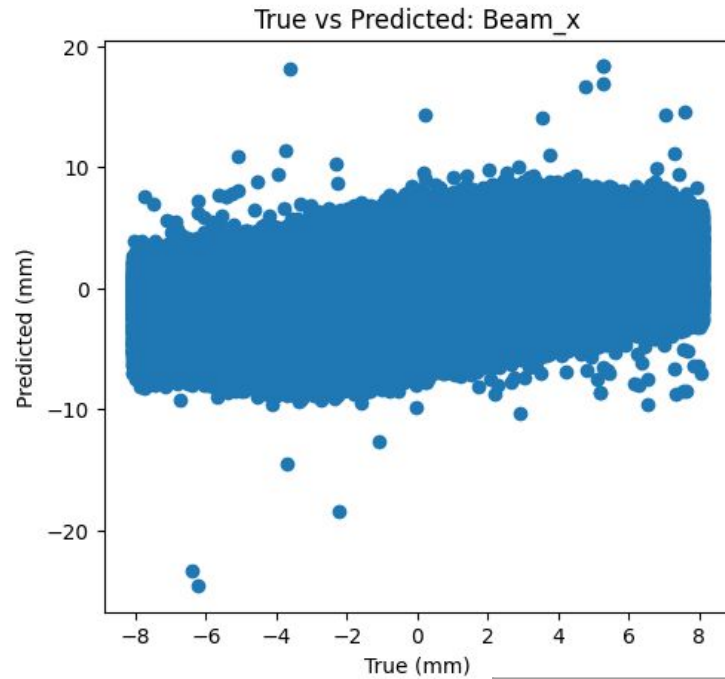
3. Three DOF: ip, oop, beam_x

- Constant:
 - Momentum (**p**) = Nominal
 - Beam Y (**beam_y**) = 0 mm
- Variable:
 - Beam X (**beam_x**) $\sim \mathcal{U}(-20, +20)$ mm
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 - Out-of-Plane Angle (**oop**) $\sim \mathcal{U}(-5^\circ, +5^\circ)$
- Number of events: 500k

$$\begin{aligned}\hat{ip} &= \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy \\ \hat{oop} &= \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx \\ \hat{beam_x} &= \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy\end{aligned}$$

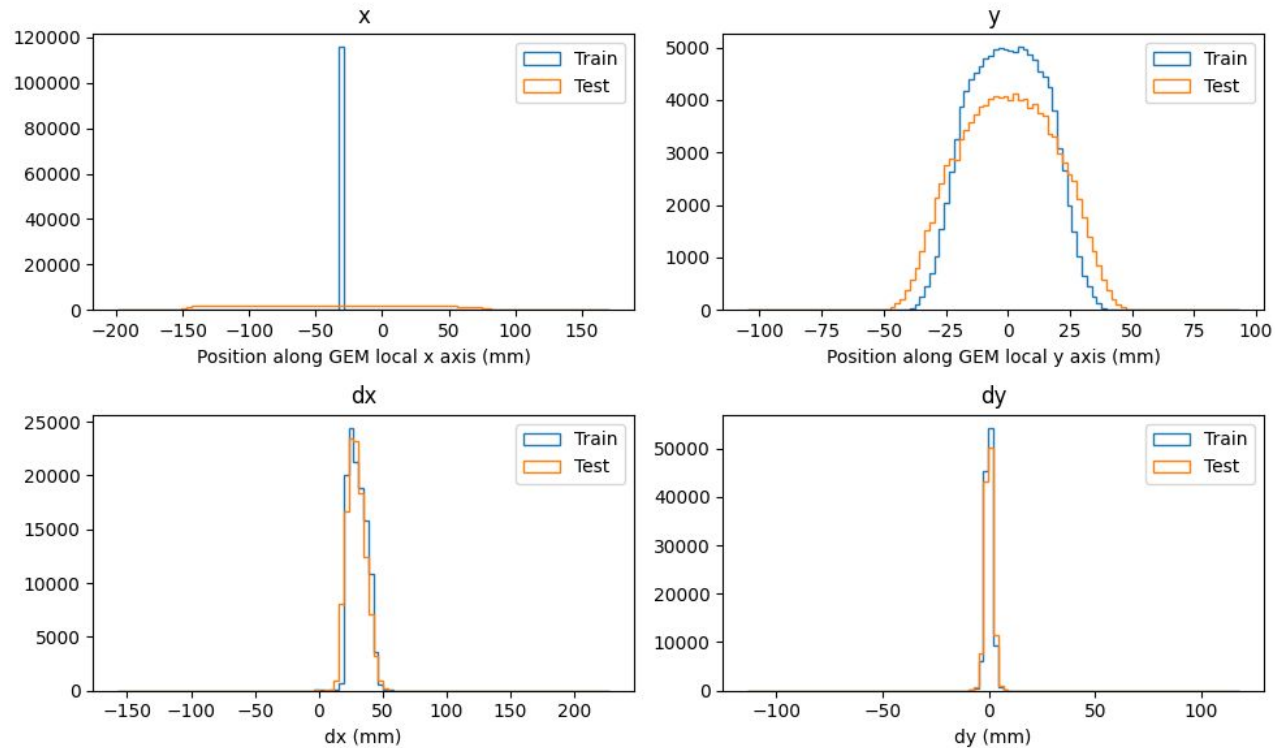
Model 1: Predicting ip	
RMSE	0.29
STD	0.88
R ²	0.88

Model 2: Predicting oop	
RMSE	2.24
STD	2.7
R ²	0.32

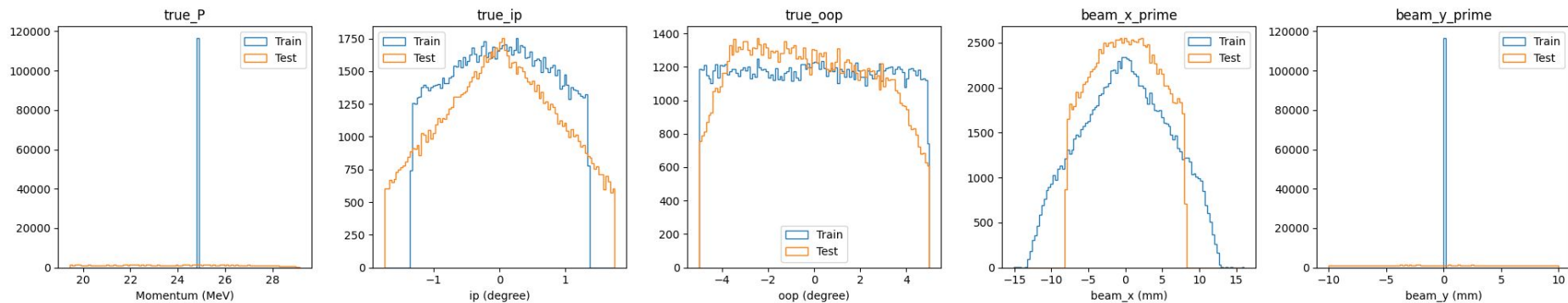


Model 3: Predicting beam_x	
RMSE	4.38
STD	4.39
R^2	0.005

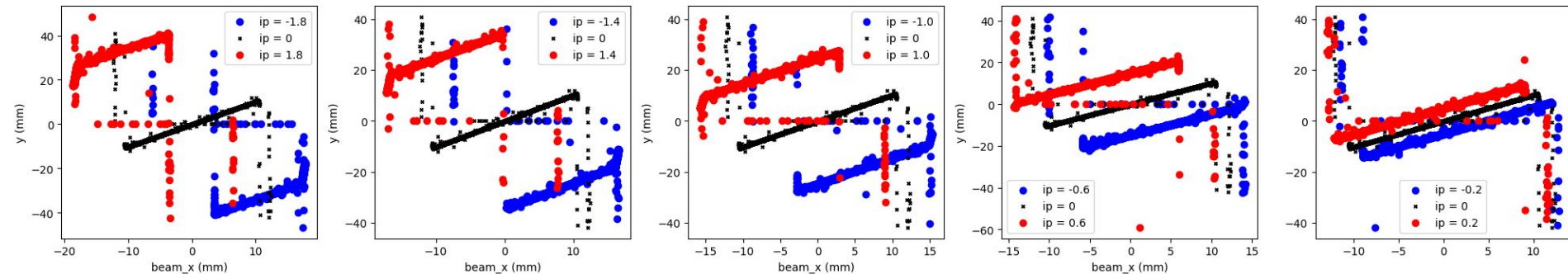
Data Distribution in 3 DOF Case (Features)



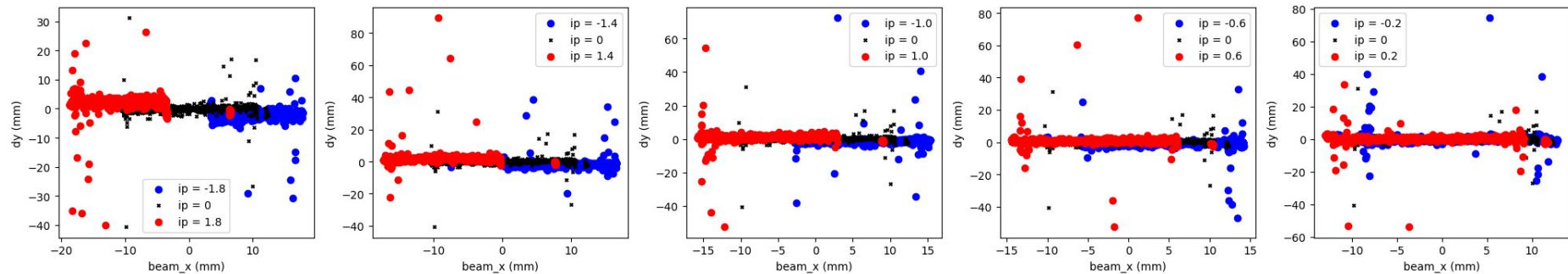
Data Distribution in 3 DOF Case (Targets)



beam_x vs y (constant ip)



beam_x vs dy (constant ip)

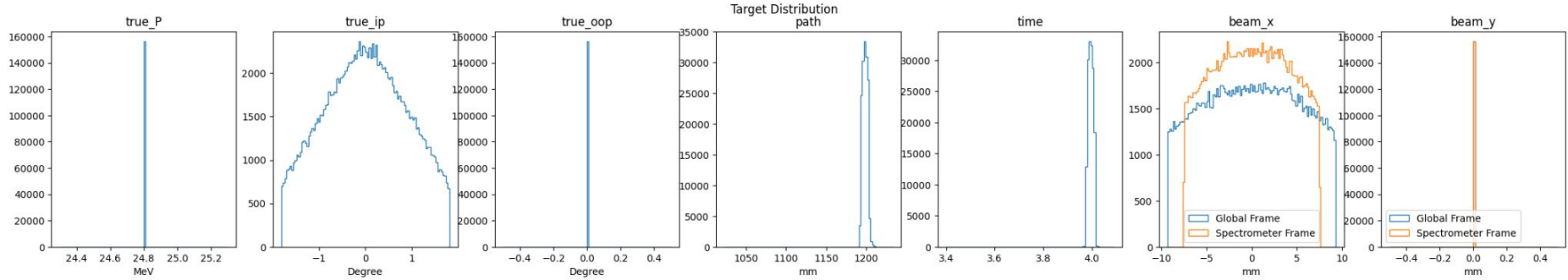


4. Two DOF: `ip`, `beam_x` (Scattering off)

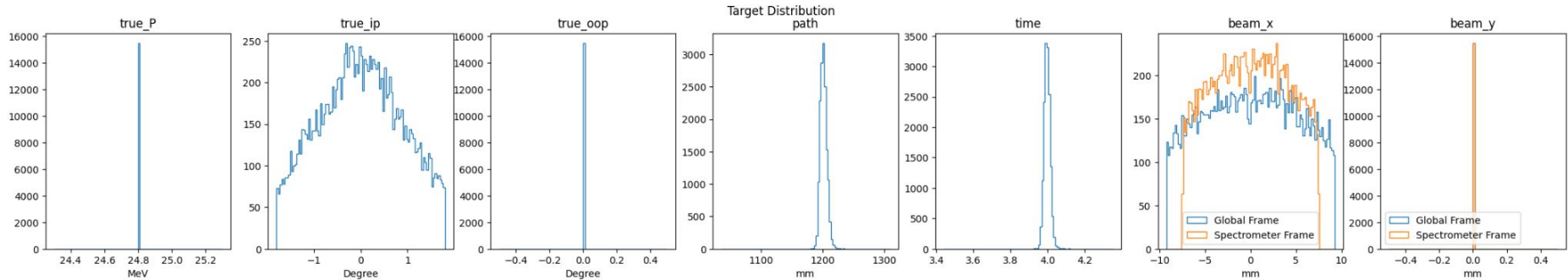
- Constant:
 - Momentum (`p`) = Nominal
 - Out-of-Plane Angle (`oop`) = 0°
 - Beam Y (`beam_y`) = 0 mm
- Variable:
 - Beam X (`beam_x`) $\sim \mathcal{U}(-20, +20)$ mm
 - In-Plane Angle (`ip`) $\sim \mathcal{U}(-1.80^\circ, +1.80^\circ)$
- Number of events: 500k

Target Distribution

Scattering: Off



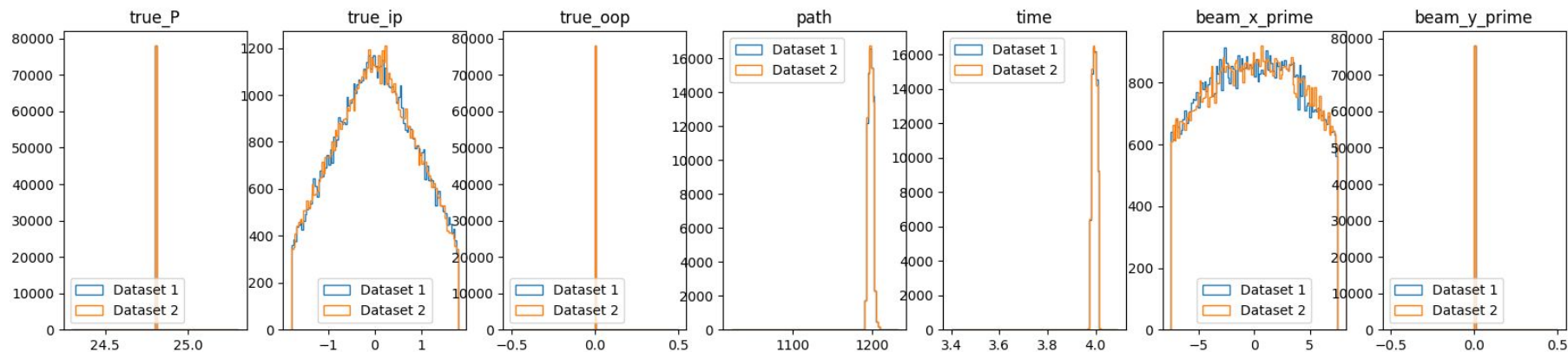
Scattering: On



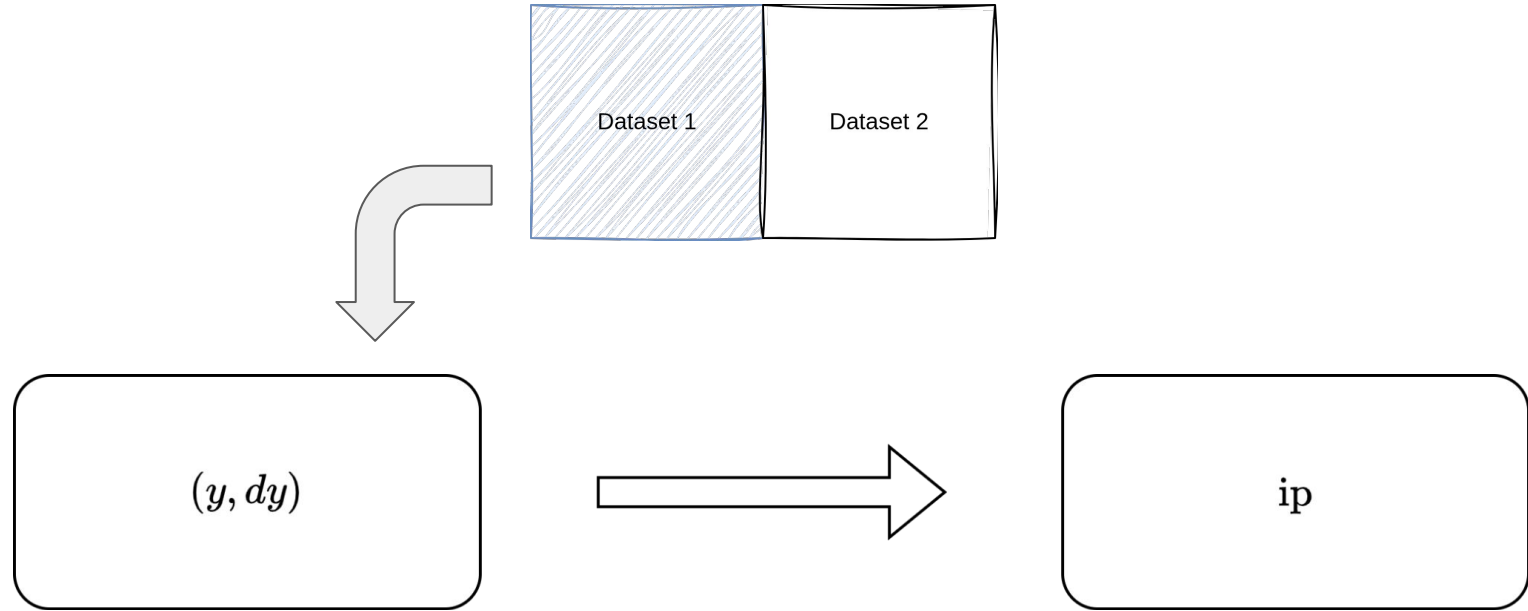
Step 1: Split into two Datasets



Step 1: Split into two Datasets

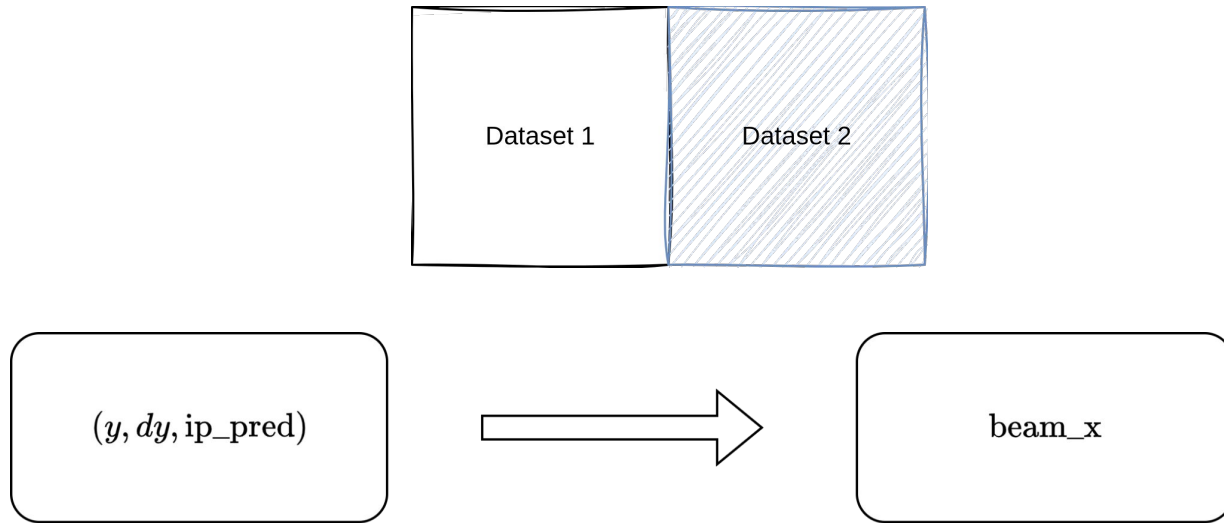


Step 2: Train Model 1 on Dataset 1



Step 3: Predict In-Plane Angle using Model 1 on Dataset 2

Step 4: Train Model 2 on Dataset 2 and predict beam_x on Dataset 2 validation set.



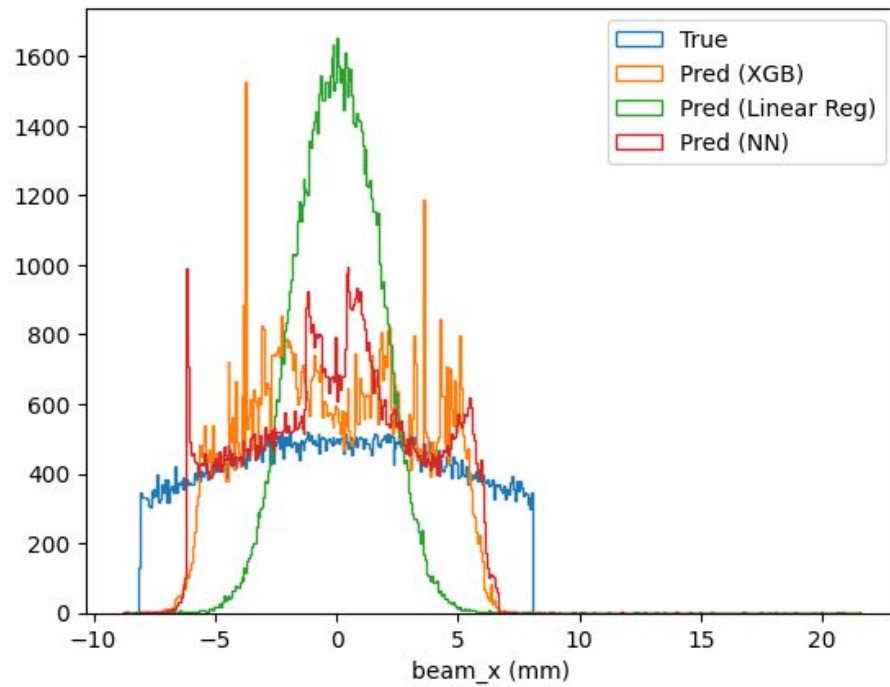
Model 1: Predicts ip

	Linear Regression	XGB	Neural Networks
R^2	0.95	0.96	0.99
RMSE	0.20	0.17	0.10
STD	0.89	0.89	0.89
Norm-RMSE	0.23	0.19	0.11

Model 2: Predicts beam_x

	Linear Regression	XGB	Neural Networks
R^2	0.16	0.85	0.86
RMSE	3.78	1.59	1.51
STD	4.12	4.12	4.12
Norm-RMSE	0.91	0.38	0.37

DOF: 2



5. Alternate Approach - Classification

- Classify range of beam_x into N intervals, call them bins.
- We learn the following mapping:
 - (x, y, dx, dy) \rightarrow (beam_x bins)
- N = 20
- Range of Beam X = (-7.5 mm, 7.5 mm)
- Bin resolution = 0.75 mm

$$T_i^{(K)} = \text{TopK}_{j \in 1, \dots, C} p_{i,j}$$

$$\text{Top-K Accuracy} = \frac{1}{N} * \sum_{i=1}^N 1_{[y_i \in T_i^{(K)}]}$$

K	Accuracy (%)
1 (Exact bin match!)	54.18
3	83.12
4	90.85
10	97.76