Phase-Space

Features

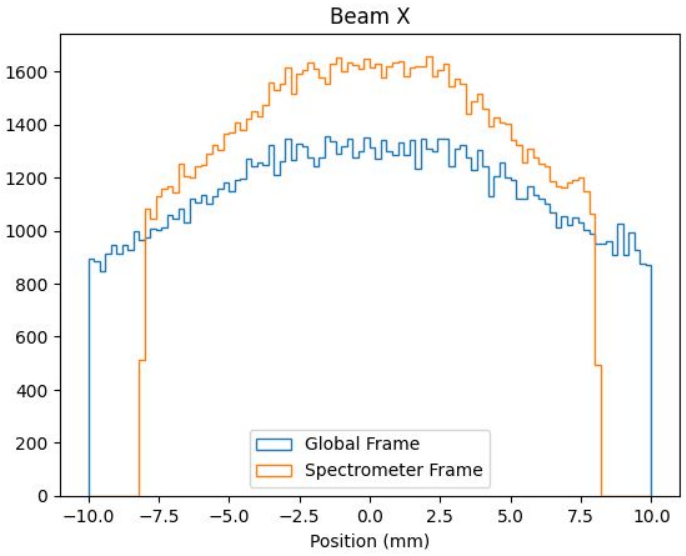
- Lower GEM hit coord. along x axis (**x**)
- Lower GEM coordinate along y axis (y)
- Difference between GEM hits on upper and lower GEM hits along x axis (**dx**)
- Difference between GEM hits on upper and lower GEM hits along y axis (**dy**)

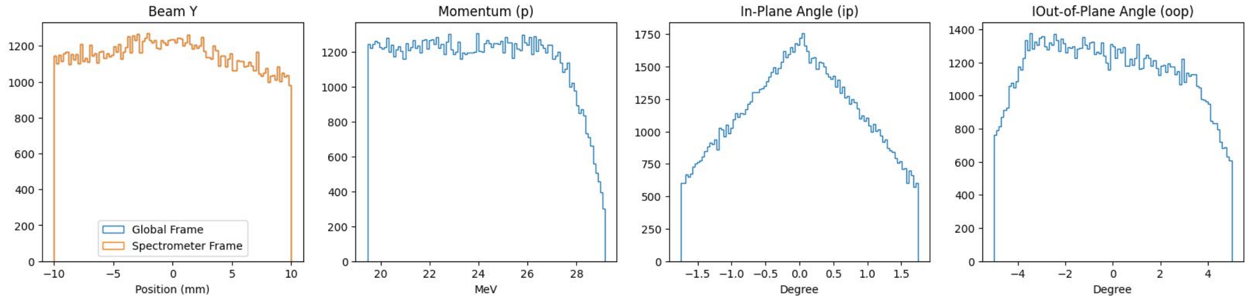
Targets

- Momentum (**p**) ∈ [-x, x] mm
- In-Plane Angle (**ip**) ∈ [-x, x] degrees
- Out-of-Plane Angle (**oop**) ∈ [-x, x] degrees
- Beam X (**beam_x**) ∈ [-x, x] mm
- Beam Y (**beam_y**) ∈ [-x, x] mm

0. Test-Set (eC 25 MeV)

- Constant:
  - Momentum (**p**) (very small variation!)
- Variable:
  - Beam X (**beam_x**)
  - In-Plane Angle (**ip**)
  - Of-of-Plane Angle (**oop**)
- Number of events: 114,989
- **beam_x** range
  - Global Frame: (-10, 10) mm
  - Spectrometer Frame: (-8.1, 8.1) mm



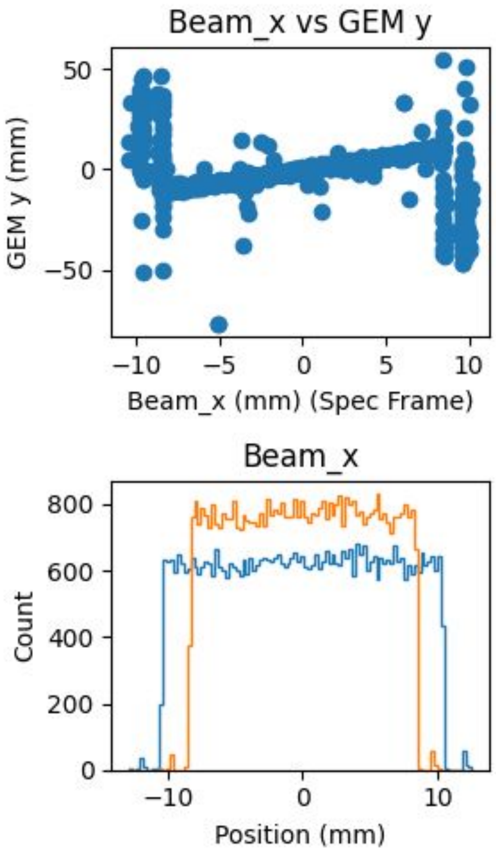


1. One DOF: beam_x

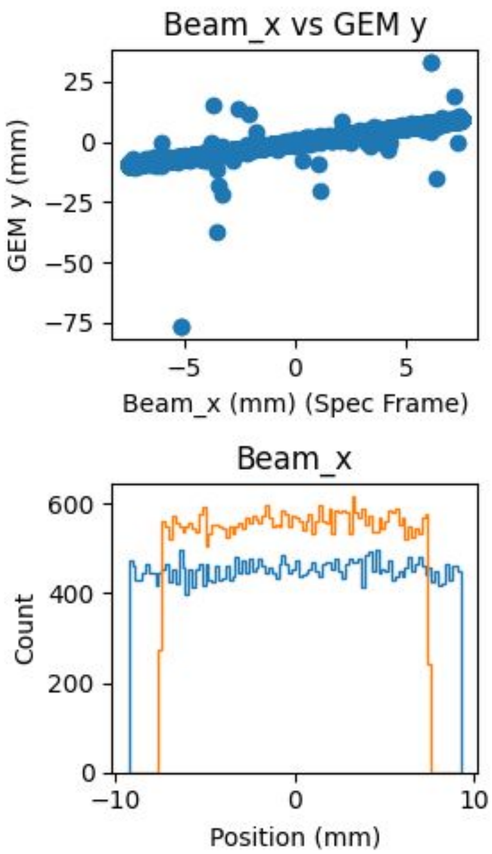
- Constant:
  - Momentum (**p**) = Nominal
  - In-Plane Angle (**ip**) = 0°
  - Out-of-Plane Angle (**oop**) = 0°
  - Beam Y (**beam_y**) = 0 mm
- Variable:
  - Beam X (**beam_x**) $\sim \mathcal{U}(-20, +20)$ mm
- Number of events: 100k

1. One DOF: beam_x

- Constant:
  - Momentum (**p**) = Nominal
  - In-Plane Angle (**ip**) = 0°
  - Out-of-Plane Angle (**oop**) = 0°
  - Beam Y (**beam_y**) = 0 mm
- Variable:
  - Beam X (**beam_x**) ~ $\mathcal{U}$(-20, +20) mm
- Number of events: 100k
- **beam_x** range (we have put a hard cut)
  - Global Frame: (-9.3, 9.3) mm
  - Spectrometer Frame: (-7.5, 7.5) mm



No cut!

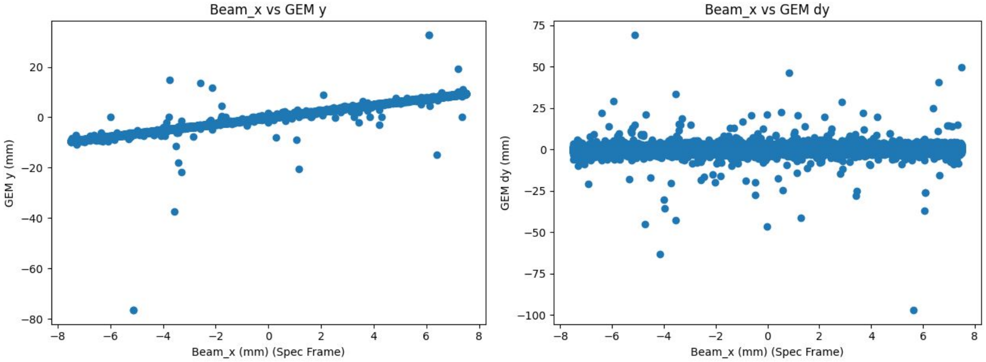


With cut!

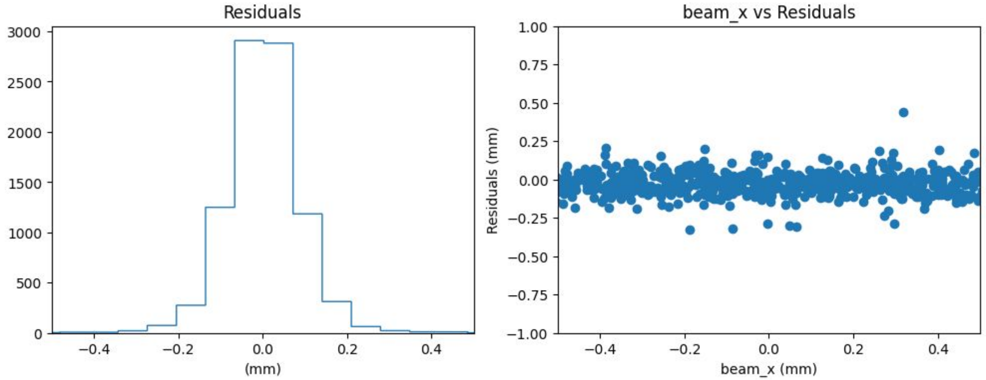
1. One DOF: beam_x

- <u>Constant</u>:
    - Momentum (**p**) = Nominal
    - In-Plane Angle (**ip**) = 0°
    - Out-of-Plane Angle (**oop**) = 0°
    - Beam Y (**beam_y**) = 0 mm
- <u>Variable</u>:
    - Beam X (**beam_x**) ~ $\mathcal{U}$(-20, +20) mm
- <u>Number of events</u>: 100k
- **beam_x** range (we have put a hard cut!)
    - Global Frame: (-9.3, 9.3) mm
    - Spectrometer Frame: (-7.5, 7.5) mm

beam_x Correlations	
GEM y	1.0
GEM dy	0.17

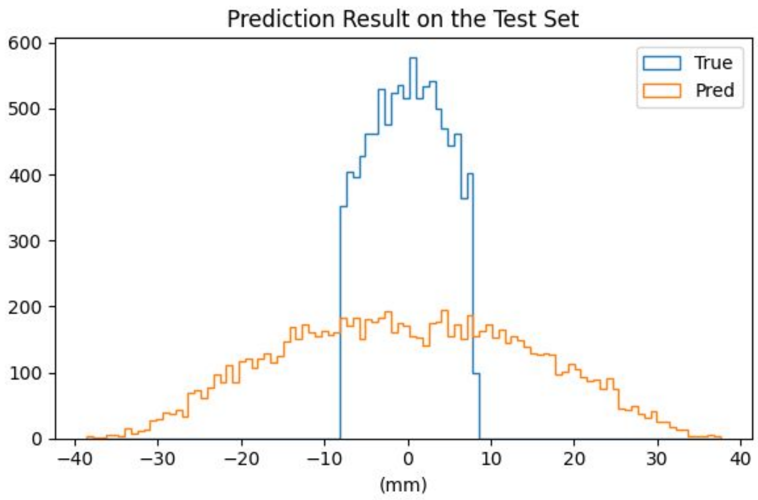


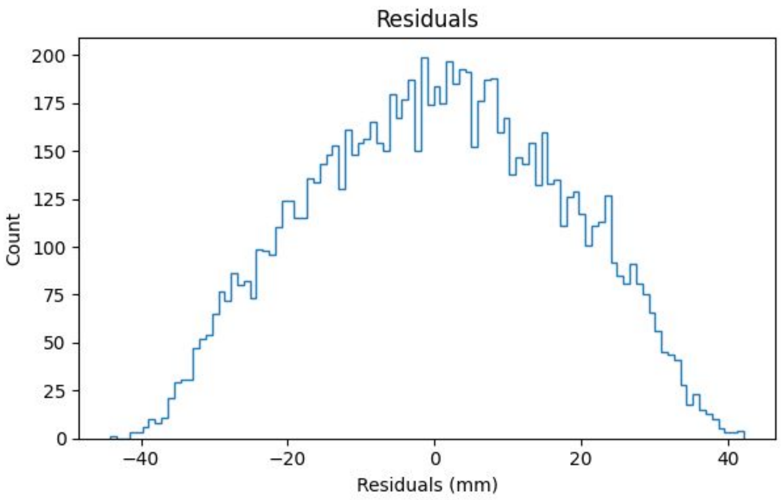
- **OLS Model**: beam_x = 0.09 $\boxed{+ 0.79}$ □y + 0.05□dy
- <u>Out-of-Sample</u>:
    - RMSE: 0.82
    - Standard Deviation: 4.33
    - R^2: 0.96



1. One DOF: beam_x Test Set Result

RMSE	16.9645
STD	4.404
R^2	-13.83



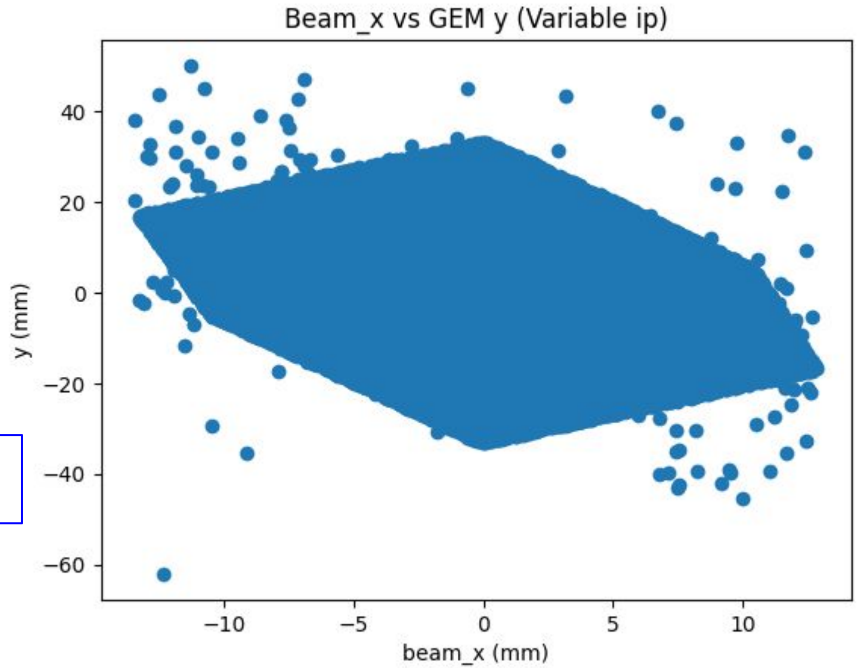


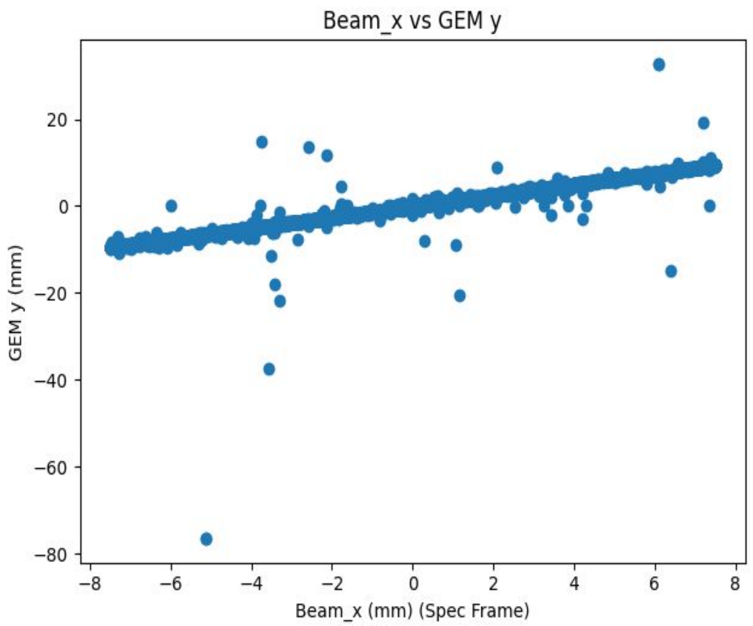
2. Two DOF: ip, beam_x

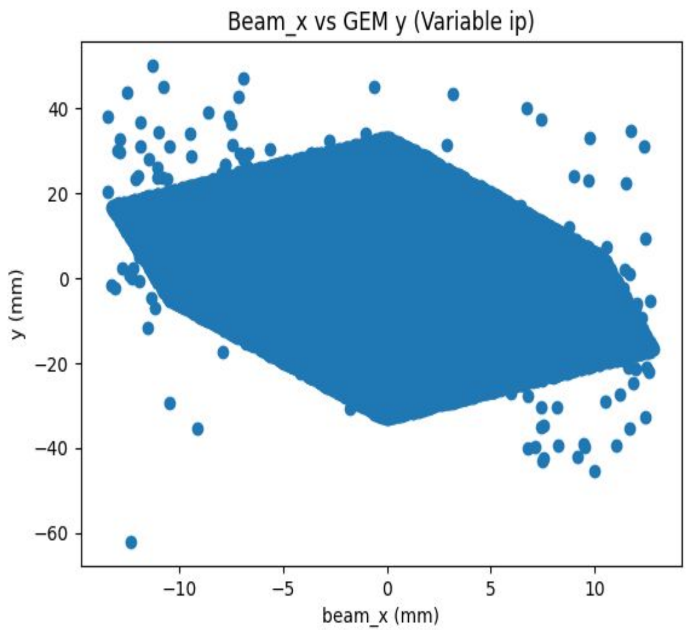
- Constant:
  - Momentum (p) = Nominal
  - Out-of-Plane Angle (oop) = 0°
  - Beam Y (beam_y) = 0 mm
- Variable:
  - Beam X (beam_x) ~ $\mathcal{U}$(-20, +20) mm
  - In-Plane Angle (ip) ~ $\mathcal{U}$(-1.35°, +1.35°)
- Number of events: 100k

- **Model 1:** $$\boxed{\text{beam\_x} = \beta_0 + \beta_1 y + \beta_2 dy}$$

RMSE	5.68
STD	5.99
R^2	0.101

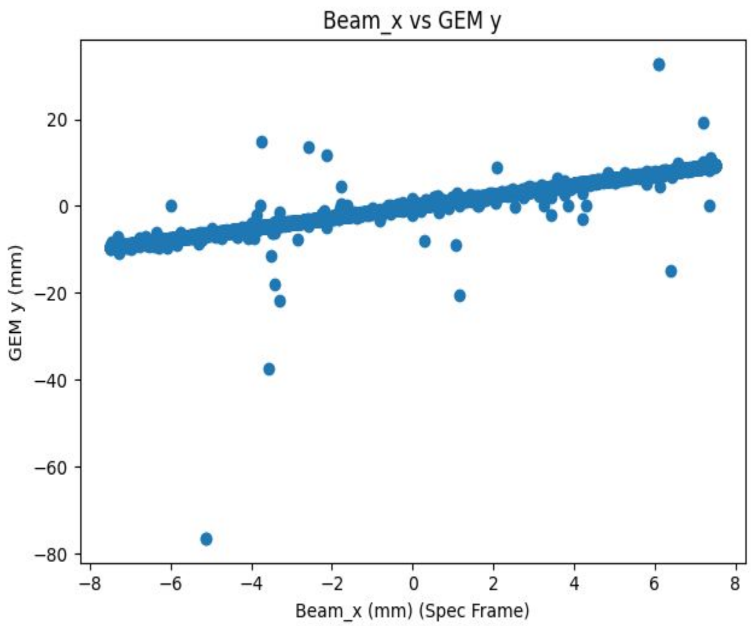




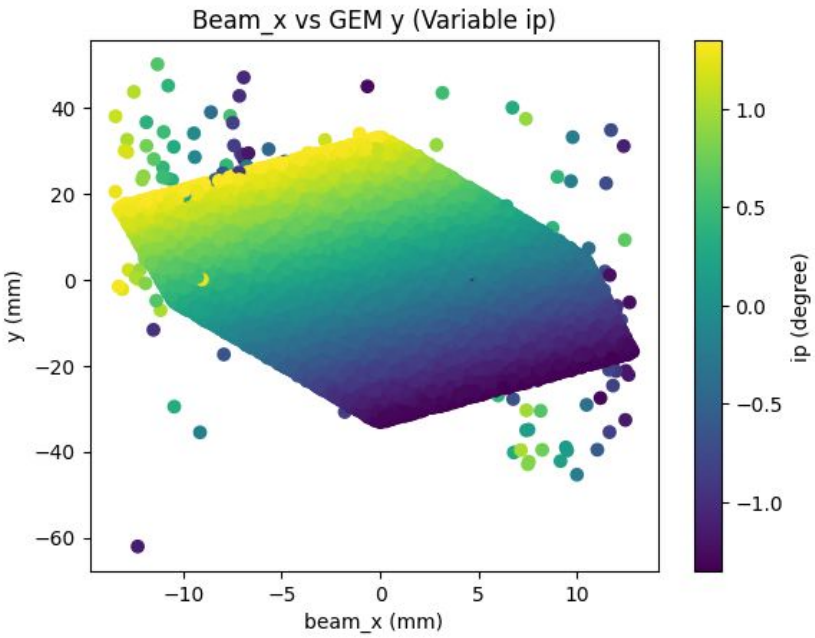


ip fixed!

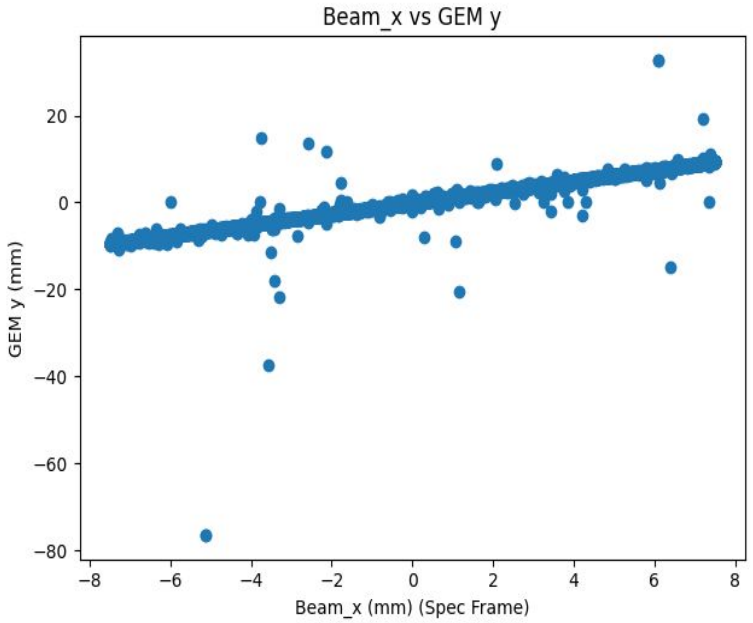
ip free!

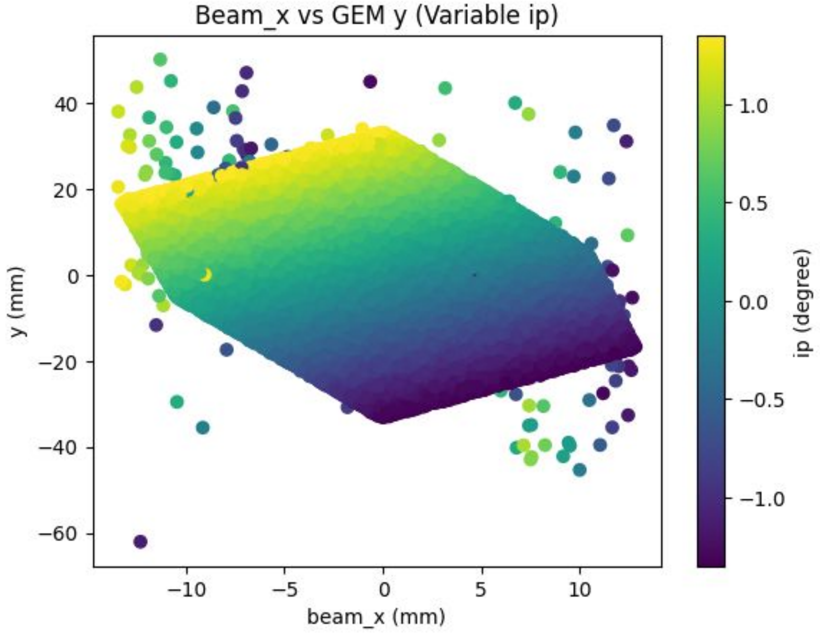


ip fixed!



ip free!



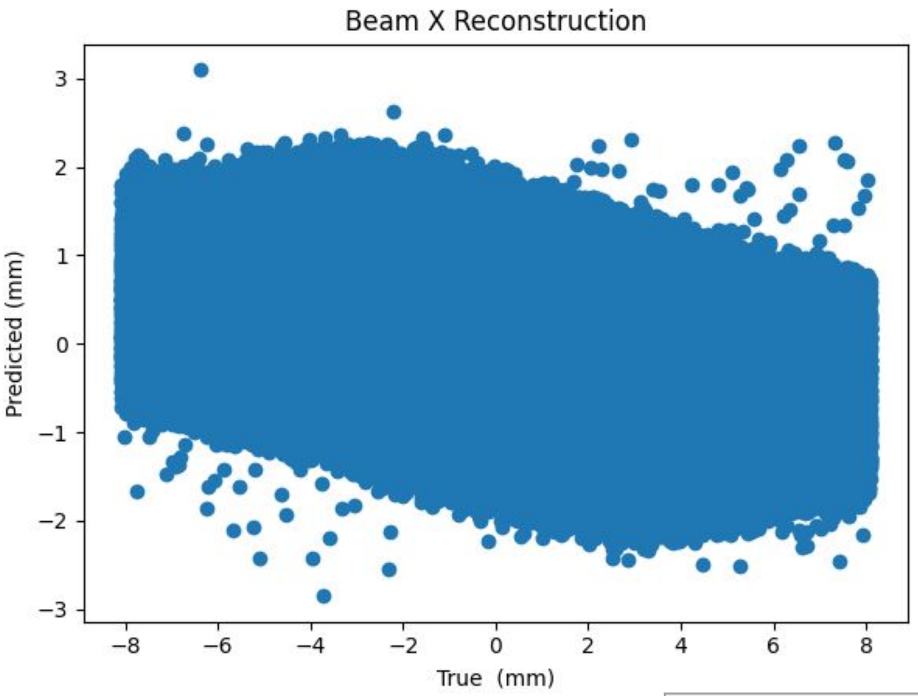


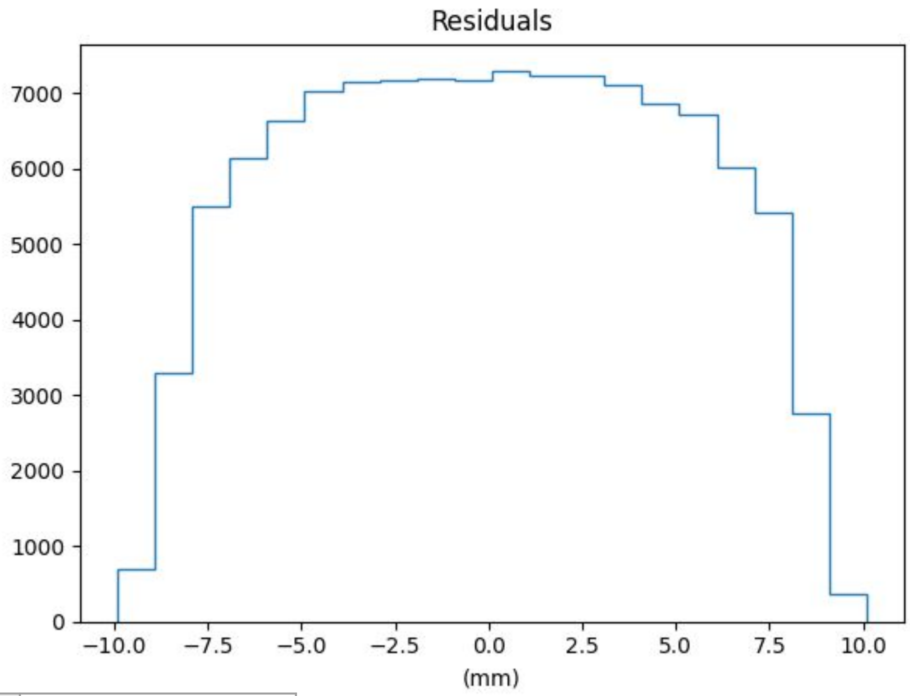
ip fixed!

ip free!

$$\hat{ip} = f(y, dy)$$
$$\text{beam\_x} = g(y, dy, \hat{ip}) \quad \Rightarrow \quad \text{beam\_x} = g\big(y, dy, f(y, dy)\big)$$

2 DOF: beam_x, ip





RMSE	4.79
STD	4.39
R^2	-0.18

3. Three DOF: ip, oop, beam_x

- Constant:
  - Momentum (**p**) = Nominal
  - Beam Y (**beam_y**) = 0 mm
- Variable:
  - Beam X (**beam_x**) $\sim \mathcal{U}$(-20, +20) mm
  - In-Plane Angle (**ip**) $\sim \mathcal{U}$(-1.8°, +1.8°)
  - Out-of-Plane Angle (**oop**) $\sim \mathcal{U}$(-5°, +5°)
- Number of events: 500k

$$\hat{ip} = \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy$$

$$\hat{oop} = \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx$$

$$\hat{\text{beam\_x}} = \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy$$

3. Three DOF: ip, oop, beam_x

- Constant:
  - Momentum (**p**) = Nominal
  - Beam Y (**beam_y**) = 0 mm
- Variable:
  - Beam X (**beam_x**) $\sim \mathcal{U}$(-20, +20) mm
  - In-Plane Angle (**ip**) $\sim \mathcal{U}$(-1.8°, +1.8°)
  - Out-of-Plane Angle (**oop**) $\sim \mathcal{U}$(-5°, +5°)
- Number of events: 500k

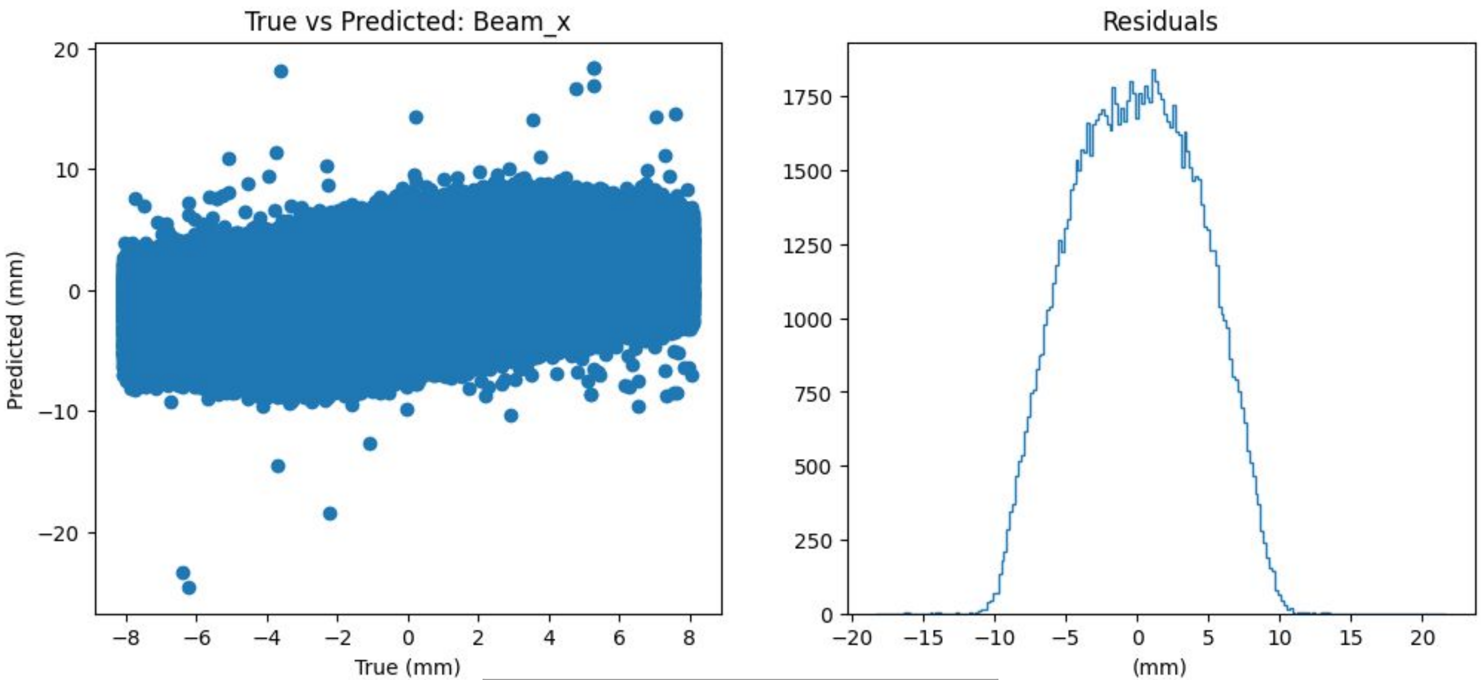
$$\hat{ip} = \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy$$

$$\hat{oop} = \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx$$

$$\hat{\text{beam\_x}} = \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy$$

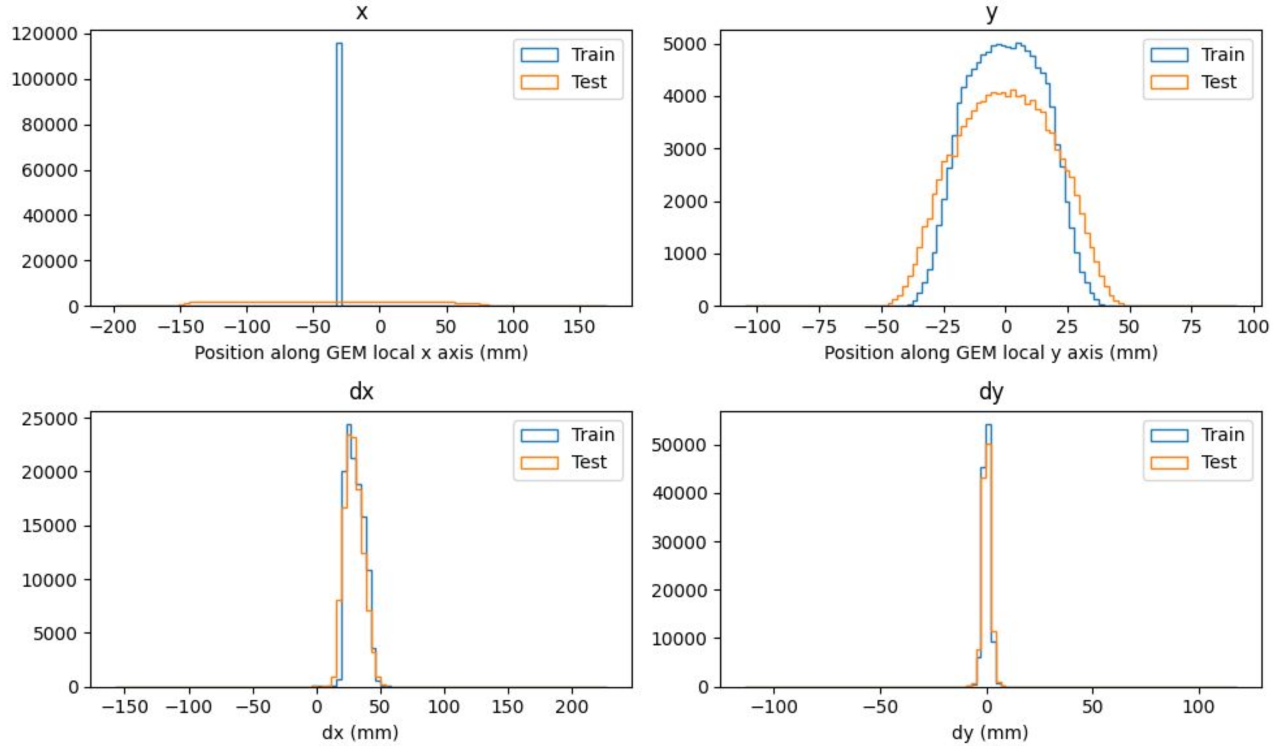
Model 1: Predicting ip	
RMSE	0.29
STD	0.88
R^2	0.88

Model 2: Predicting oop	
RMSE	2.24
STD	2.7
R^2	0.32

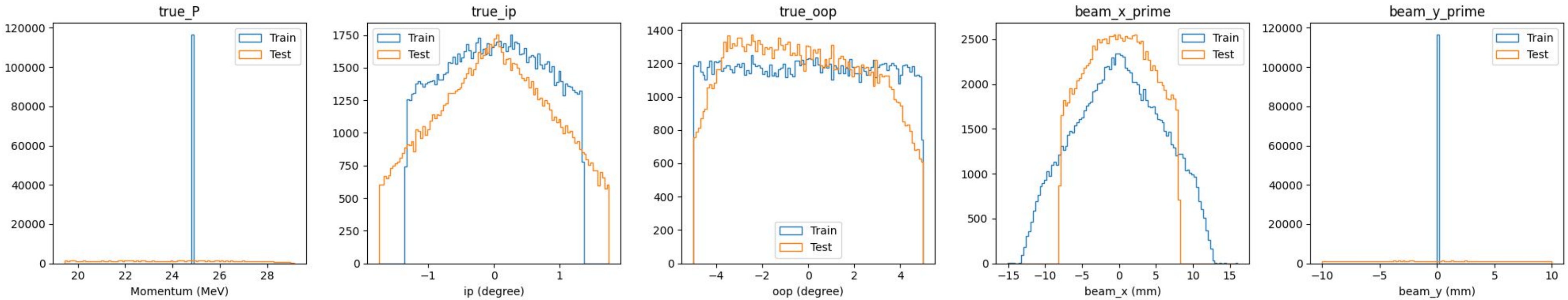


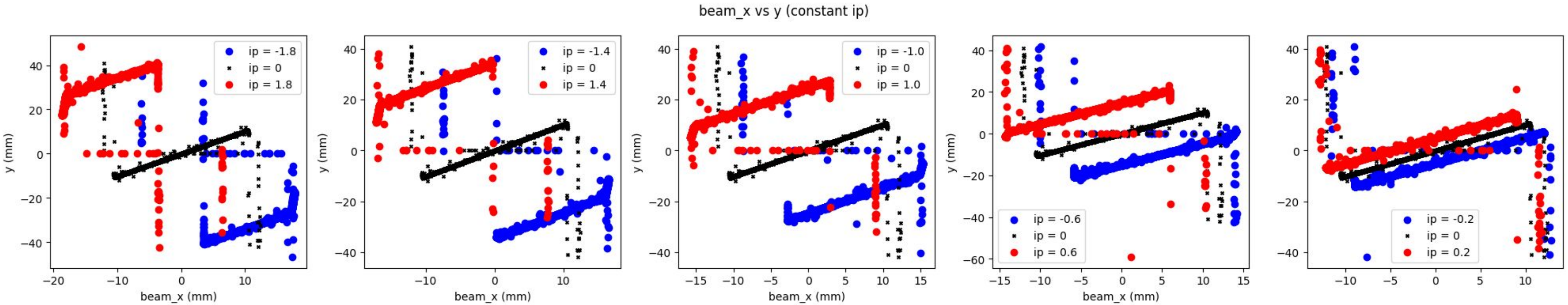
Model 3: Predicting beam_x	
RMSE	4.38
STD	4.39
R^2	0.005

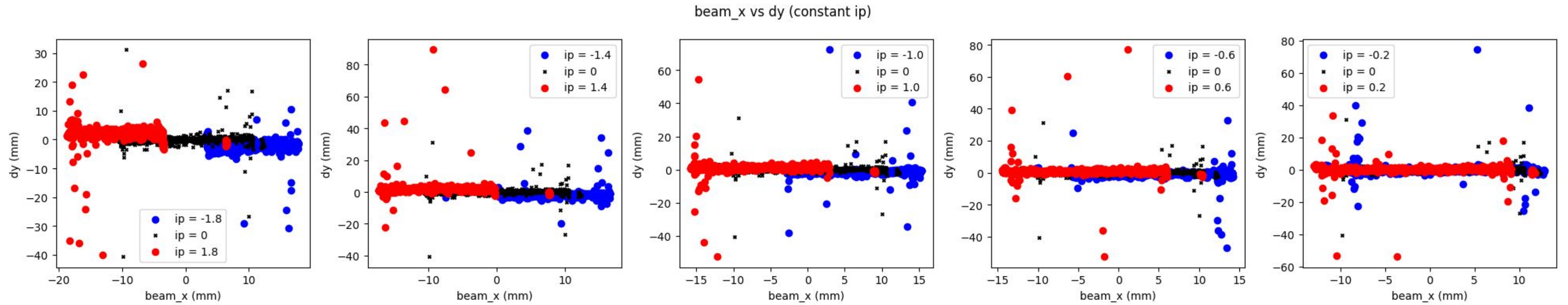
Data Distribution in 3 DOF Case
(Features)



Data Distribution in 3 DOF Case (Targets)




beam_x vs y (constant ip)
ip = -1.8
ip = 0
ip = 1.8
ip = -1.4
ip = 1.4
ip = -1.0
ip = 1.0
ip = -0.6
ip = 0.6
ip = -0.2
ip = 0.2
y (mm)
beam_x (mm)

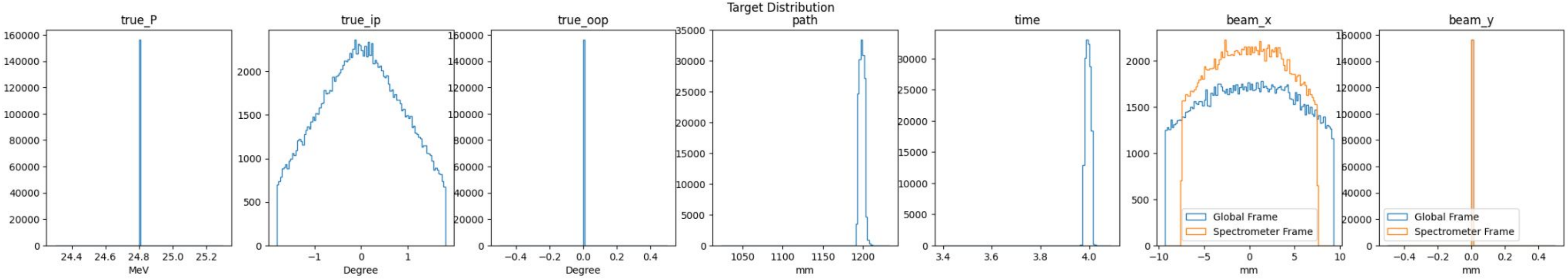

beam_x vs dy (constant ip)
ip = -1.8
ip = 0
ip = 1.8
ip = -1.4
ip = 1.4
ip = -1.0
ip = 1.0
ip = -0.6
ip = 0.6
ip = -0.2
ip = 0.2
dy (mm)
beam_x (mm)

4. Two DOF: ip, beam_x (Scattering off)

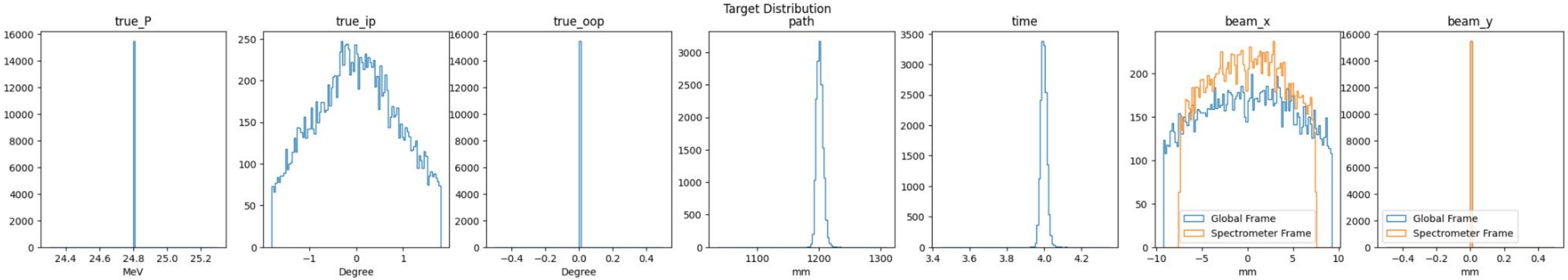
- Constant:
  - Momentum (**p**) = Nominal
  - Out-of-Plane Angle (**oop**) = 0°
  - Beam Y (**beam_y**) = 0 mm
- Variable:
  - Beam X (**beam_x**) $\sim \mathcal{U}$(-20, +20) mm
  - In-Plane Angle (**ip**) $\sim \mathcal{U}$(-1.80°, +1.80°)
- Number of events: 500k

Target Distribution

Scattering: **Off**



Scattering: **On**



Step 1: Split into two Datasets

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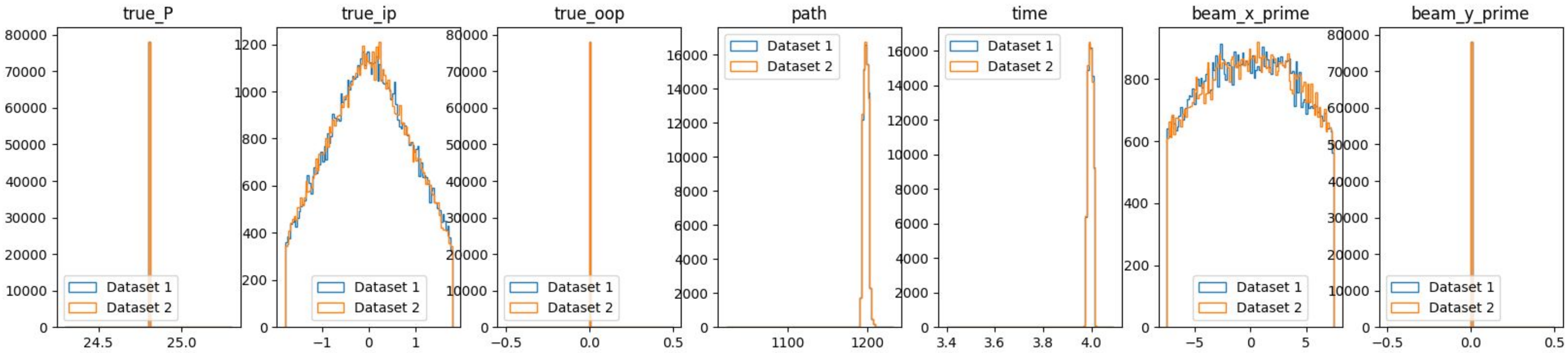
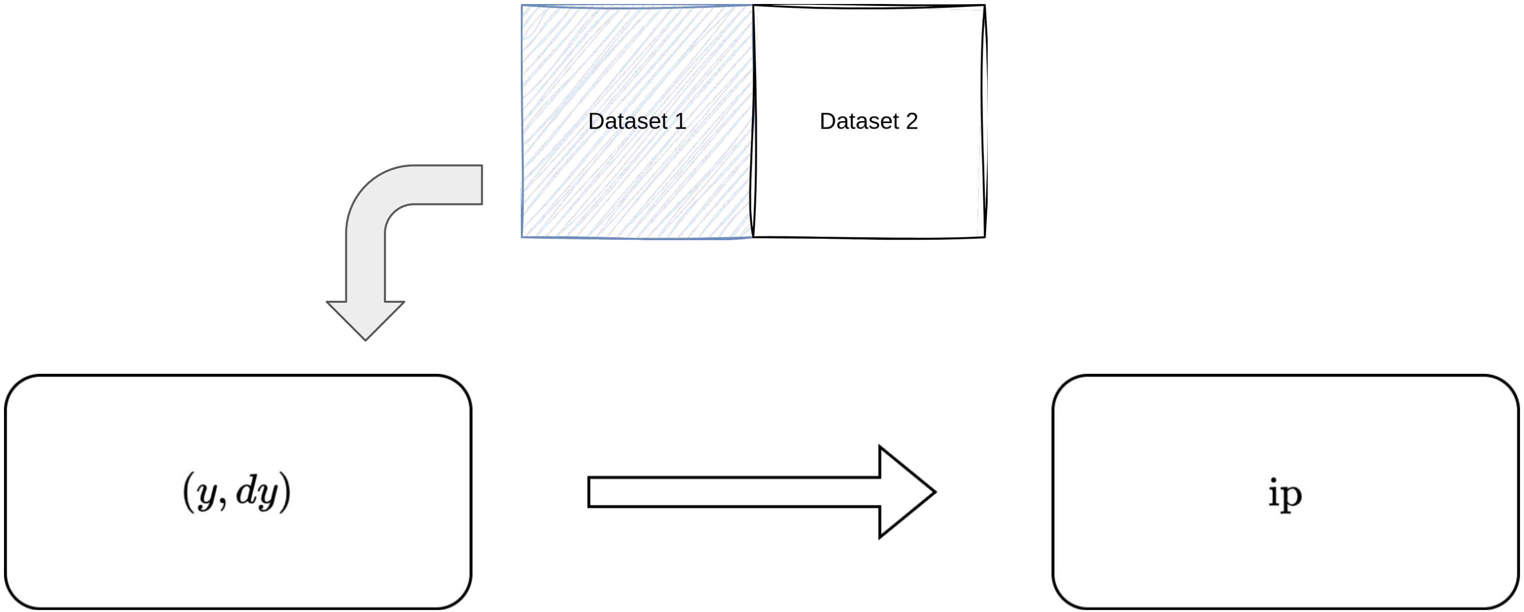
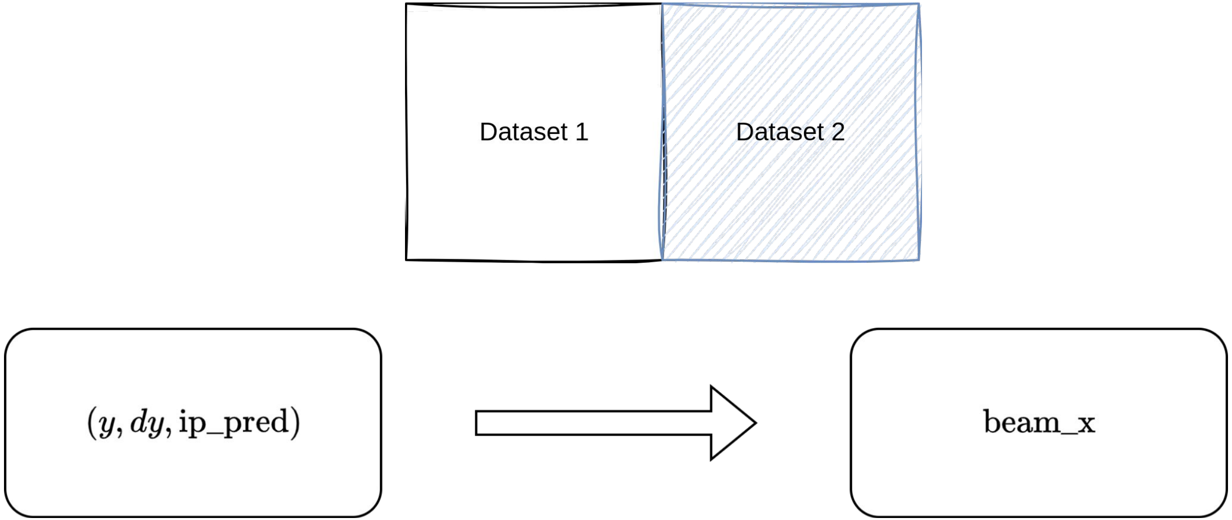


Step 2: Train Model 1 on Dataset 1



Step 3: Predict In-Plane Angle using Model 1 on Dataset 2

Step 4: Train Model 2 on Dataset 2 and predict beam_x on Dataset 2 validation set.

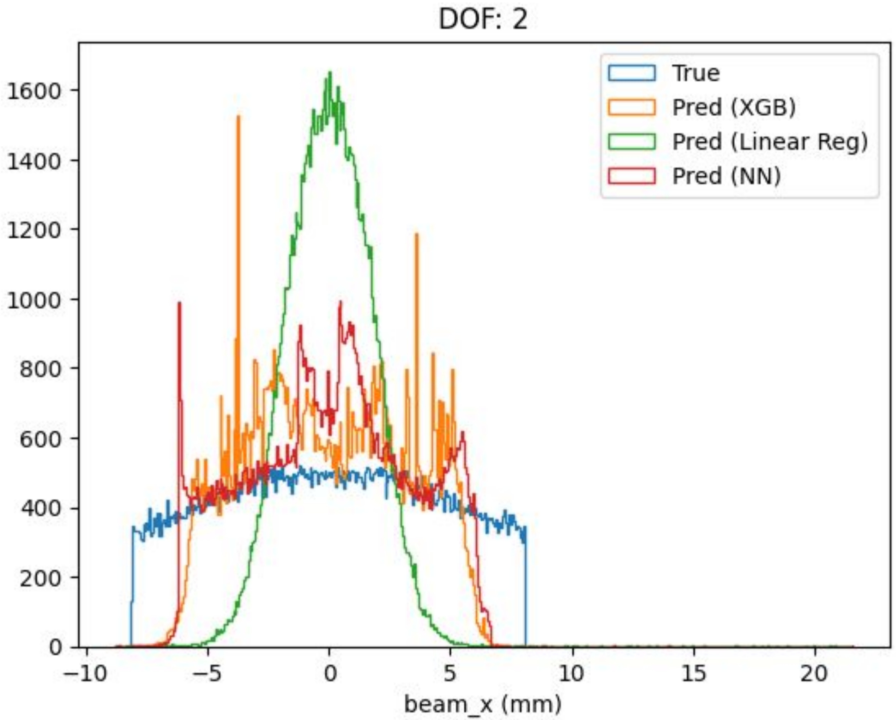


Model 1: **Predicts ip**

	Linear Regression	**XGB**	**Neural Networks**
R^2	0.95	0.96	0.99
RMSE	0.20	0.17	0.10
STD	0.89	0.89	0.89
Norm-RMSE	0.23	0.19	0.11

Model 2: **Predicts beam_x**

	Linear Regression	**XGB**	**Neural Networks**
R^2	0.16	0.85	0.86
RMSE	3.78	1.59	1.51
STD	4.12	4.12	4.12
Norm-RMSE	0.91	0.38	0.37



DOF: 2
True
Pred (XGB)
Pred (Linear Reg)
Pred (NN)
0
200
400
600
800
1000
1200
1400
1600
−10
−5
0
5
10
15
20
beam_x (mm)

5. Alternate Approach - Classification

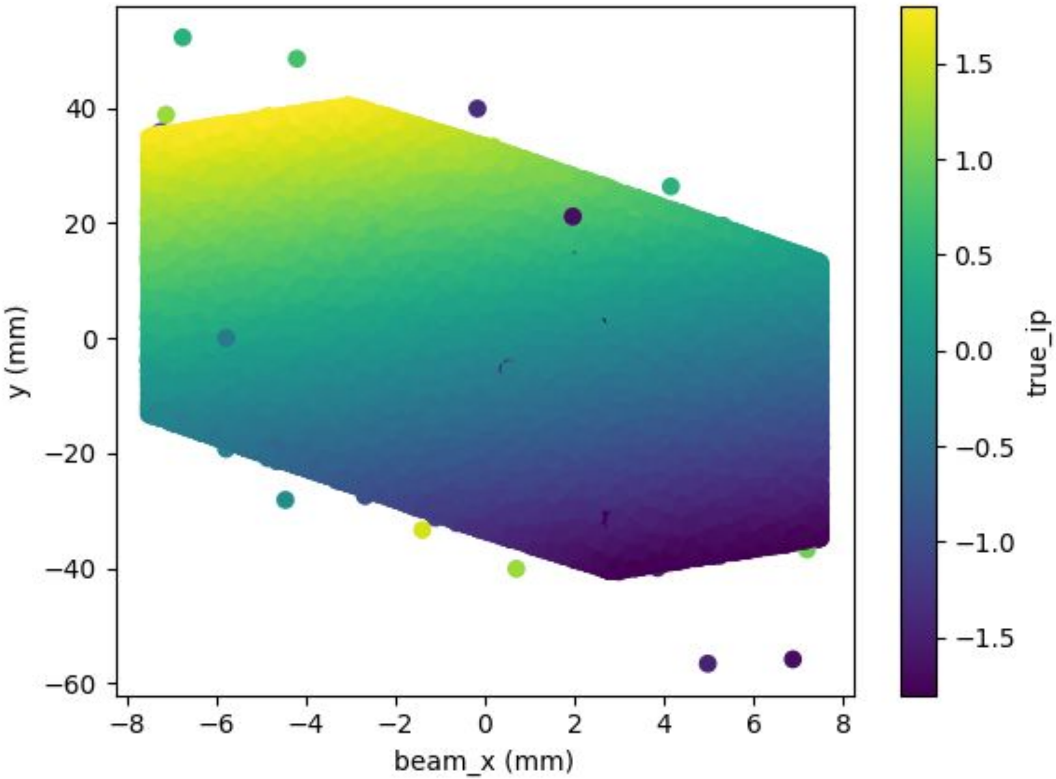
- Classify range of beam_x into N intervals, call them bins.
- We learn the following mapping:
  - (x, y, dx, dy) —-> (beam_x bins)
- N = 20
- Range of Beam X = (-7.5 mm, 7.5 mm)
- Bin resolution = 0.75 mm

$$T_i^{(K)} = \mathrm{TopK}_{j \in 1, \ldots, C} p_{i,j}$$

$$\text{Top-K Accuracy} = \frac{1}{N} * \sum_{i=1}^{N} 1_{[y_i \in T_i^{(K)}]}$$

K	Accuracy (%)
1 (Exact bin match!)	54.18
3	83.12
4	90.85
10	97.76

January 15



y (mm)
40
20
0
−20
−40
−60
−8
−6
−4
−2
0
2
4
6
8
beam_x (mm)
true_ip
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5

Model: Two Stage Conditional Regression Model

Task: Build the mapping:

$$(x, y, dx, dy) \to \text{beam\_x}$$

Model: Two Stage Conditional Regression Model

Task: Build the mapping:

$$(x, y, dx, dy) \to \text{beam\_x}$$

The idea is the following:

Stage 1: Predict ip using (x, y, dx, dy).

Model: Two Stage Conditional Regression Model

Task: Build the mapping:

$$(x, y, dx, dy) \rightarrow \text{beam\_x}$$

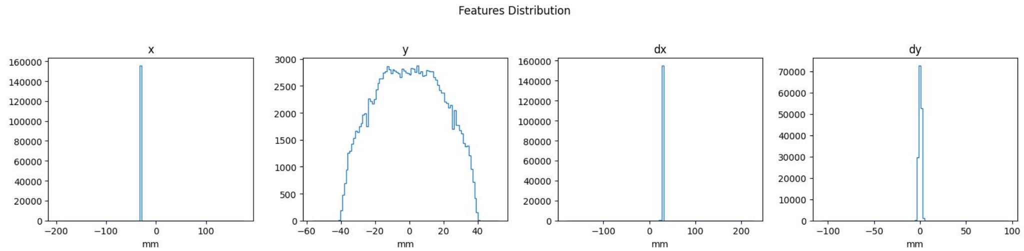
The idea is the following:

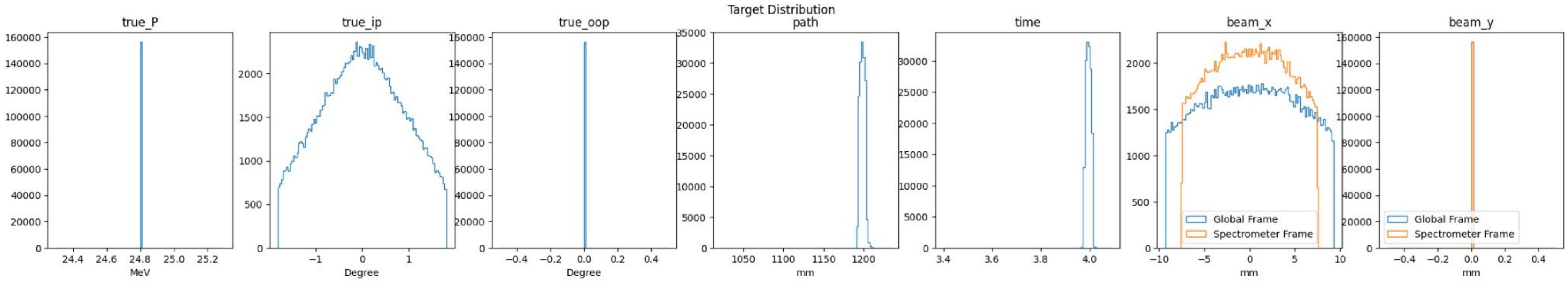
Stage 1: Predict ip using (x, y, dx, dy).

Stage 2: Use (x, y, dx, dy, pred_ip) to predict beam_x.

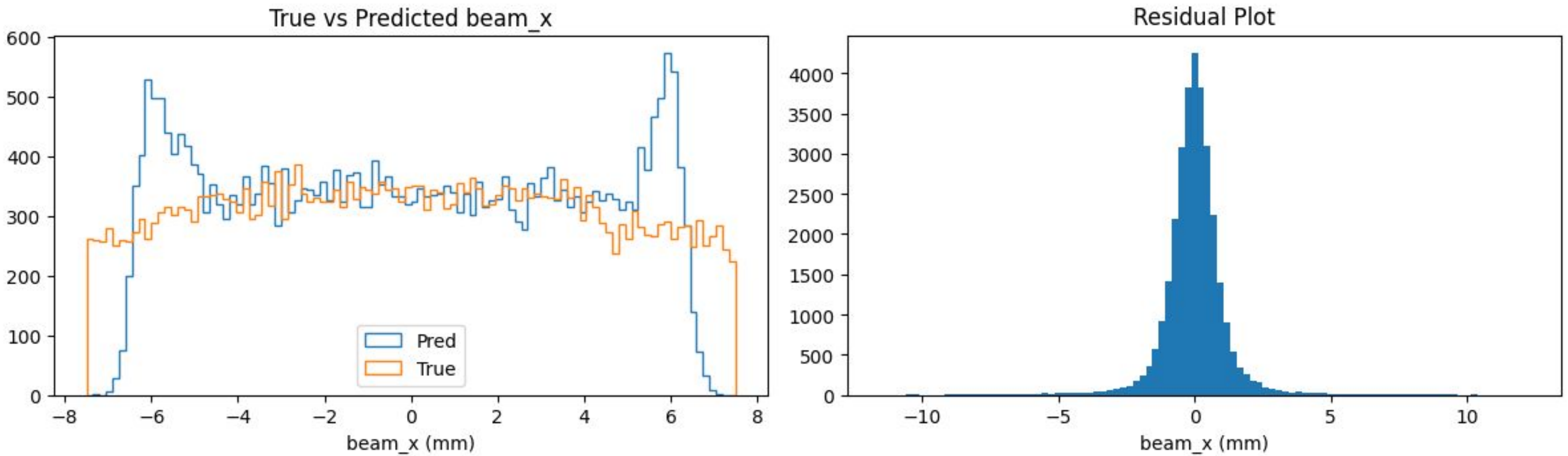
Data:

- Constant:
    - Momentum (**p**) = Nominal
    - Out-of-Plane Angle (**oop**) = 0°
    - Beam Y (**beam_y**) = 0 mm
- Variable:
    - Beam X (**beam_x**) ~ $\mathcal{U}$(-20, +20) mm
    - In-Plane Angle (**ip**) ~ $\mathcal{U}$(-1.80°, +1.80°)
- Number of events: 500k

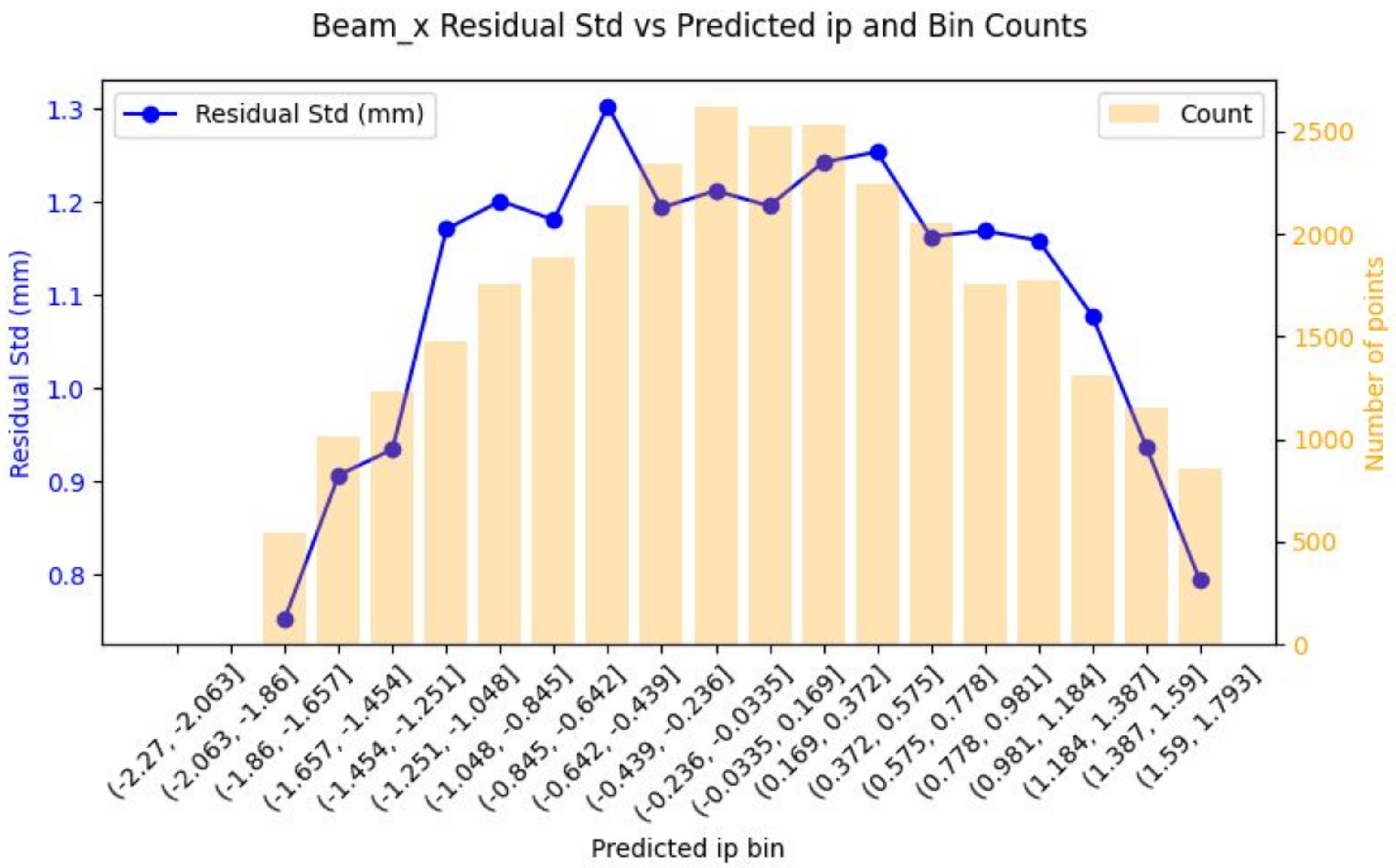




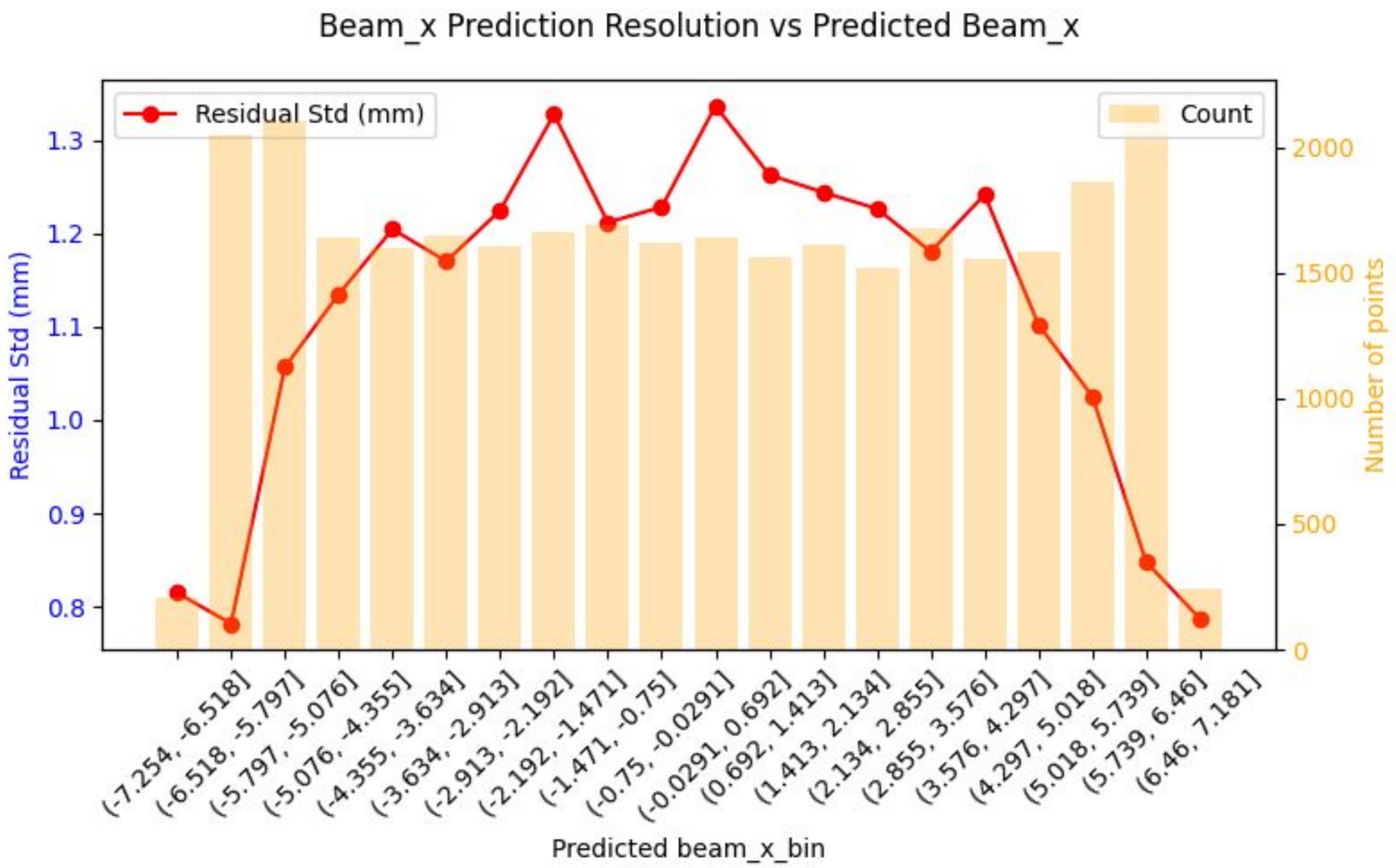
Predicting beam_x




```
 ----------- ip_model Result -----------
Train RMSE: 0.0667, R2: 0.9944
Test  RMSE: 0.0753, R2: 0.9927
 ----------- beam_x_model Result -----------
Train RMSE: 0.0667, R2: 0.9355
Test  RMSE: 0.0753, R2: 0.9218
```



Beam_x Residual Std vs Predicted ip and Bin Counts
Residual Std (mm)
Count
Number of points
Predicted ip bin
1.3
1.2
1.1
1.0
0.9
0.8
2500
2000
1500
1000
500
0
(-2.27, -2.063]
(-2.063, -1.86]
(-1.86, -1.657]
(-1.657, -1.454]
(-1.454, -1.251]
(-1.251, -1.048]
(-1.048, -0.845]
(-0.845, -0.642]
(-0.642, -0.439]
(-0.439, -0.236]
(-0.236, -0.0335]
(-0.0335, 0.169]
(0.169, 0.372]
(0.372, 0.575]
(0.575, 0.778]
(0.778, 0.981]
(0.981, 1.184]
(1.184, 1.387]
(1.387, 1.59]
(1.59, 1.793]



Beam_x Prediction Resolution vs Predicted Beam_x
Residual Std (mm)
Count
Residual Std (mm)
Number of points
Predicted beam_x_bin
0.8
0.9
1.0
1.1
1.2
1.3
0
500
1000
1500
2000
(-7.254, -6.518]
(-6.518, -5.797]
(-5.797, -5.076]
(-5.076, -4.355]
(-4.355, -3.634]
(-3.634, -2.913]
(-2.913, -2.192]
(-2.192, -1.471]
(-1.471, -0.75]
(-0.75, -0.0291]
(-0.0291, 0.692]
(0.692, 1.413]
(1.413, 2.134]
(2.134, 2.855]
(2.855, 3.576]
(3.576, 4.297]
(4.297, 5.018]
(5.018, 5.739]
(5.739, 6.46]
(6.46, 7.181]