Phase-Space

Features

- Lower GEM hit coord. along x axis (**x**)
- Lower GEM coordinate along y axis (y)
- Difference between GEM hits on upper and lower GEM hits along x axis (**dx**)
- Difference between GEM hits on upper and lower GEM hits along y axis (**dy**)

Targets

- Momentum (**p**) $\in$ [-x, x] mm
- In-Plane Angle (**ip**) $\in$ [-x, x] degrees
- Out-of-Plane Angle (**oop**) $\in$ [-x, x] degrees
- Beam X (**beam_x**) $\in$ [-x, x] mm
- Beam Y (**beam_y**) $\in$ [-x, x] mm

0. Test-Set (eC 25 MeV)

- Constant:
    - Momentum (**p**) (very small variation!)
- Variable:
    - Beam X (**beam_x**)
    - In-Plane Angle (**ip**)
    - Of-of-Plane Angle (**oop**)
- Number of events: 114,989
- **beam_x** range
    - Global Frame: (-10, 10) mm
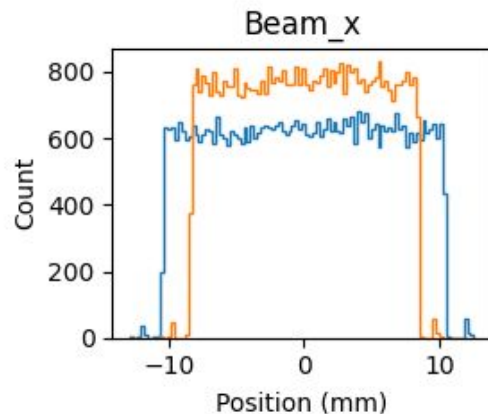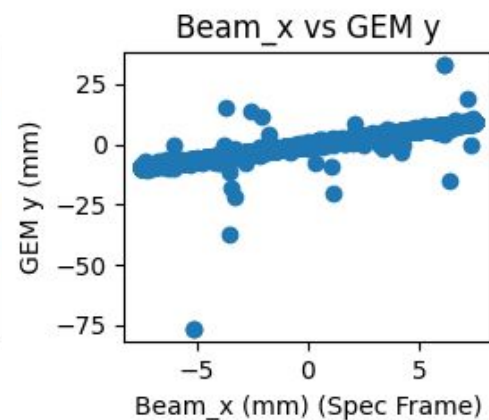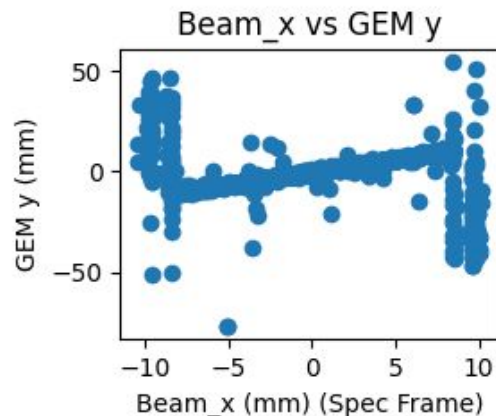   - Spectrometer Frame: (-8.1, 8.1) mm
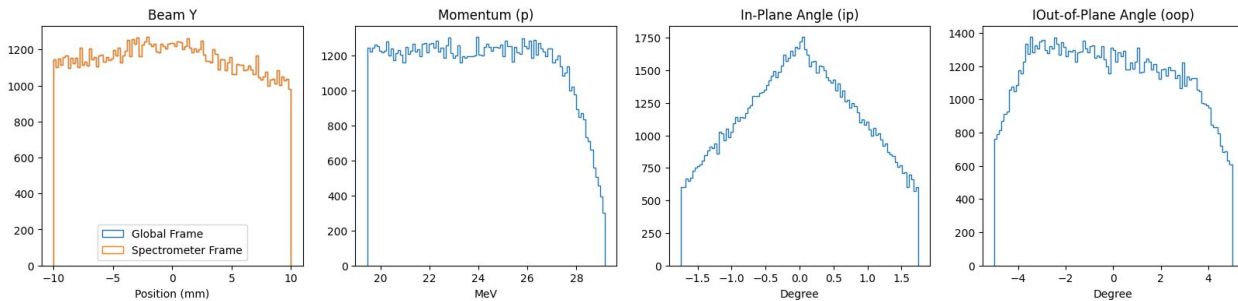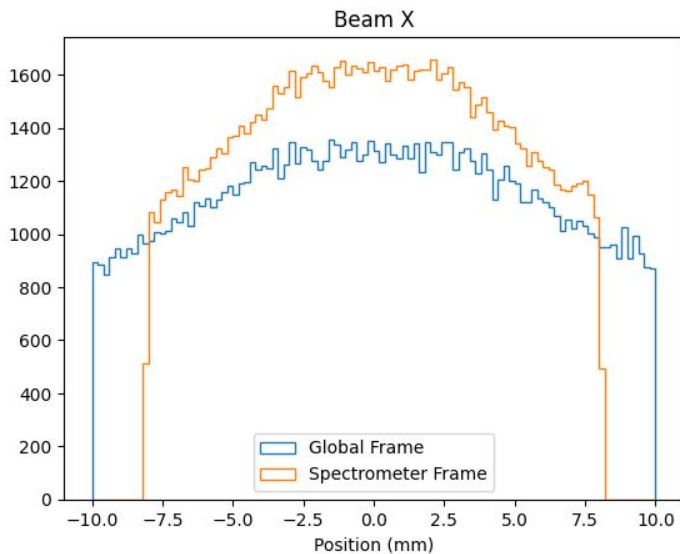
Beam X

Beam Y | Momentum (p) | In-Plane Angle (ip) | IOut-of-Plane Angle (oop)

1. One DOF: beam_x
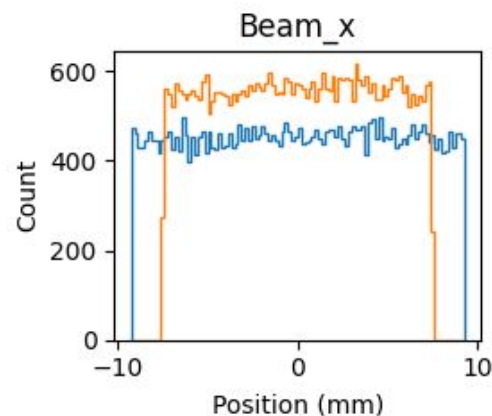
- <u>Constant</u>:
    - Momentum (**p**)   = Nominal
    - In-Plane Angle (**ip**)   = 0°
    - Out-of-Plane Angle (**oop**) = 0°
    - Beam Y (**beam_y**)   = 0 mm
- <u>Variable</u>:
    - Beam X (**beam_x**) ∼ $\mathcal{U}$(-20, +20) mm
- <u>Number of events</u>: 100k

1. One DOF: beam_x

- <u>Constant</u>:
  - Momentum (**p**)   = Nominal
  - In-Plane Angle (**ip**)   = 0°
  - Out-of-Plane Angle (**oop**) = 0°
  - Beam Y (**beam_y**)   = 0 mm
- <u>Variable</u>:
  - Beam X (**beam_x**) $\sim \mathcal{U}$(-20, +20) mm
- <u>Number of events</u>: 100k
- **beam_x** range (<u>we have put a hard cut</u>)
  - Global Frame: (-9.3, 9.3) mm
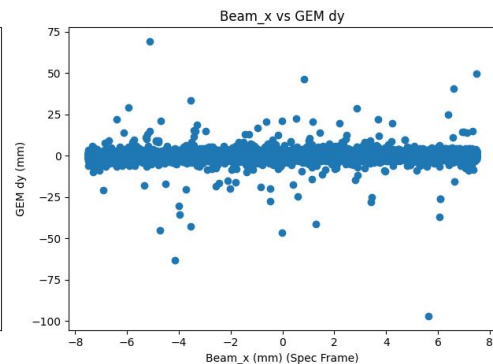 - Spectrometer Frame: (-7.5, 7.5) mm

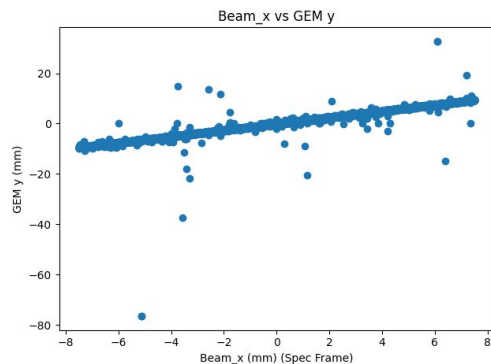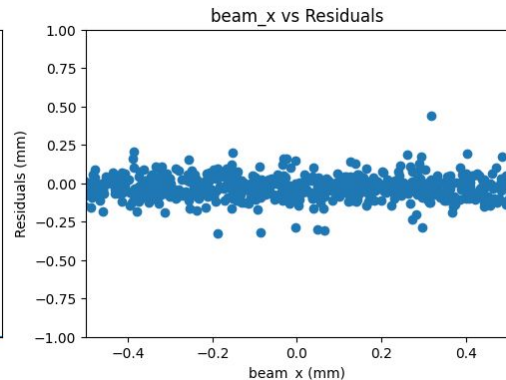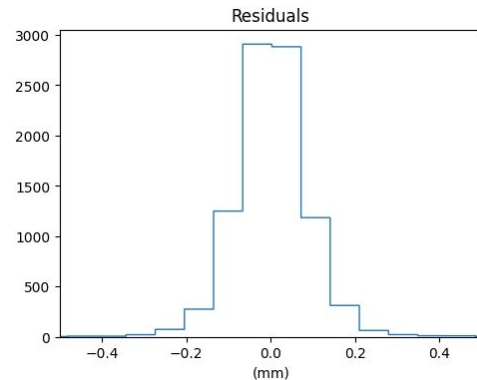**No cut!**   **With cut!**

1. One DOF: beam_x

- <u>Constant</u>:
  - Momentum (**p**) = Nominal
  - In-Plane Angle (**ip**) = 0°
  - Out-of-Plane Angle (**oop**) = 0°
  - Beam Y (**beam_y**) = 0 mm
- <u>Variable</u>:
  - Beam X (**beam_x**) ~ $\mathcal{U}$(-20, +20) mm
- <u>Number of events</u>: 100k
- **beam_x** range (we have put a hard cut!)
  - Global Frame: (-9.3, 9.3) mm
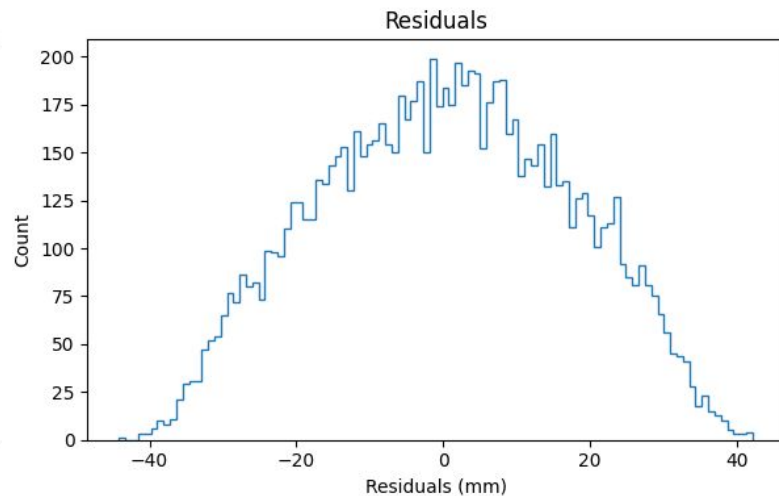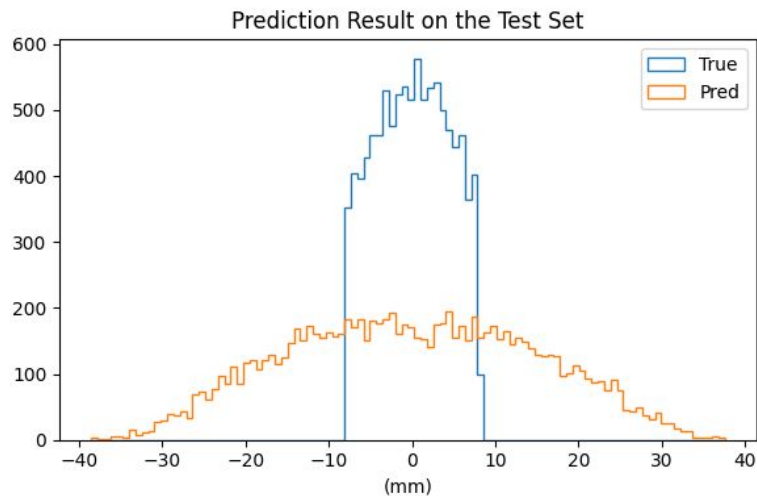 - Spectrometer Frame: (-7.5, 7.5) mm

beam_x Correlations	
GEM y	1.0
GEM dy	0.17

- **OLS Model**: beam_x = 0.09 + 0.79 □y + 0.05□dy
- <u>Out-of-Sample</u>:
  - RMSE: 0.82
  - Standard Deviation: 4.33
  - R²: 0.96

1. One DOF: beam_x Test Set Result

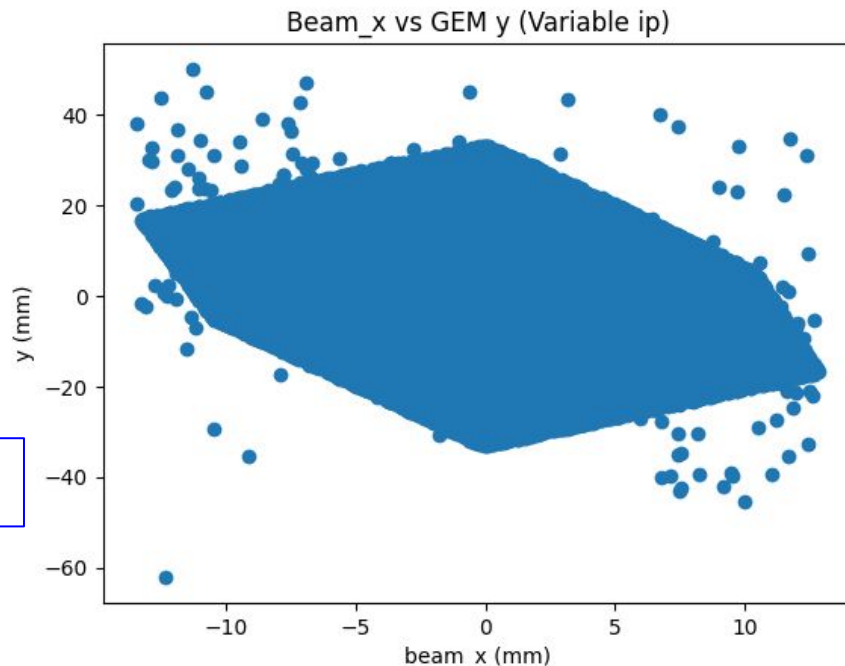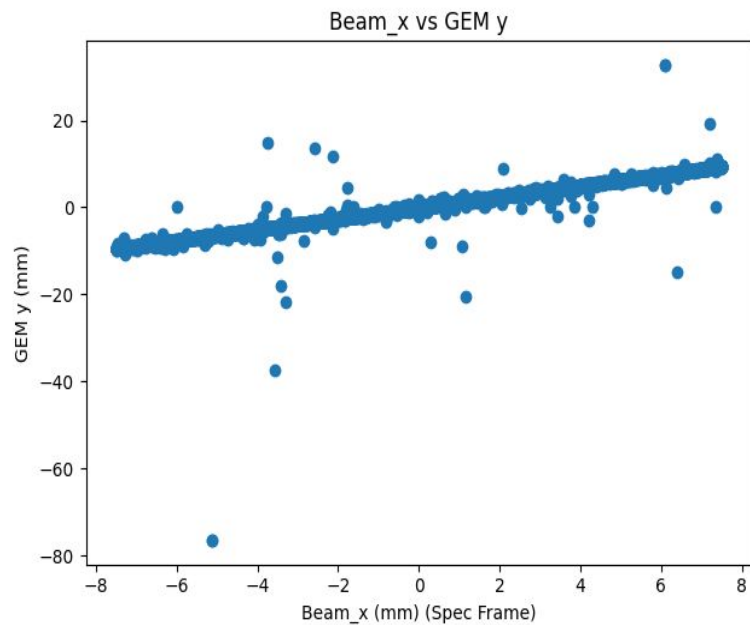| RMSE	16.9645
STD	4.404
R^2	-13.83

2. Two DOF: ip, beam_x

- Constant:
    - Momentum (**p**) = Nominal
    - Out-of-Plane Angle (**oop**) = 0°
    - Beam Y (**beam_y**) = 0 mm
- Variable:
    - Beam X (**beam_x**) $\sim \mathcal{U}$(-20, +20) mm
    - In-Plane Angle (**ip**) $\sim \mathcal{U}$(-1.35°, +1.35°)
- Number of events: 100k

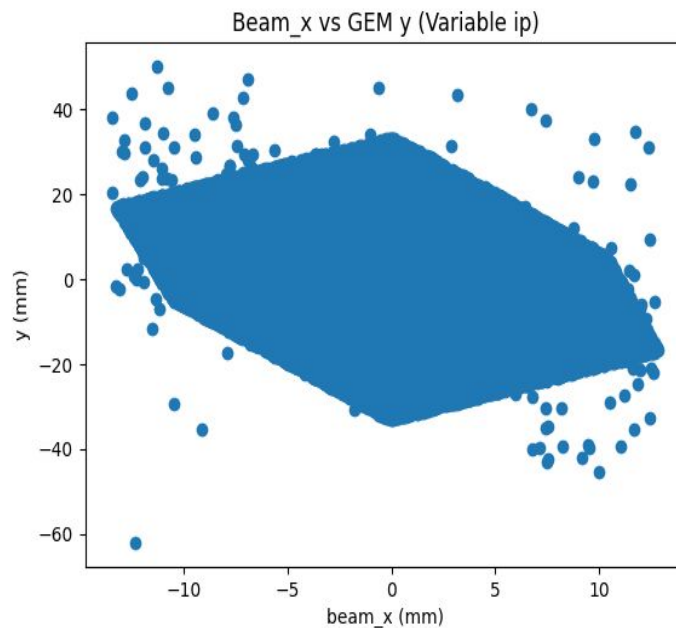- **Model 1:** $$\text{beam\_x} = \beta_0 + \beta_1 y + \beta_2 dy$$

RMSE	5.68
STD	5.99
R^2	0.101

Beam_x vs GEM y (Variable ip)

Beam_x vs GEM y

Beam_x vs GEM y (Variable ip)

ip fixed!　　　　　　　　**ip free!**

Beam_x vs GEM y — **ip fixed!**

Beam_x vs GEM y (Variable ip) — **ip free!**
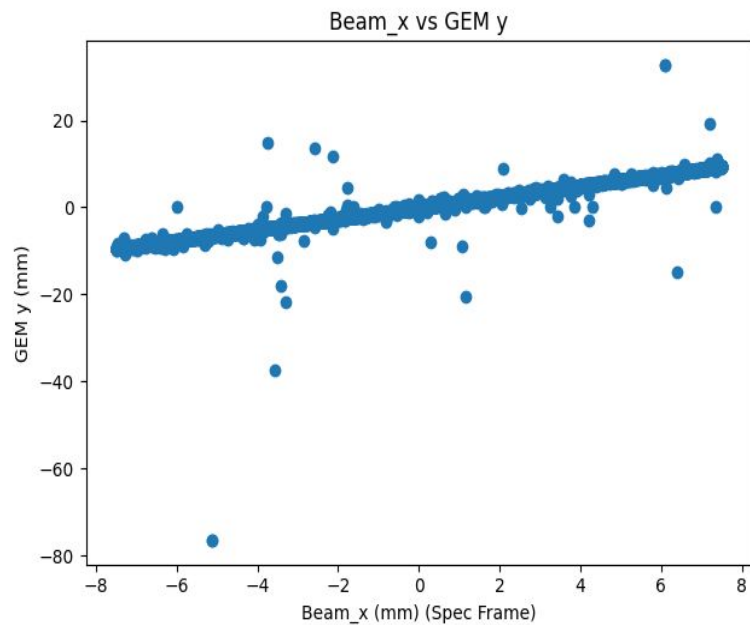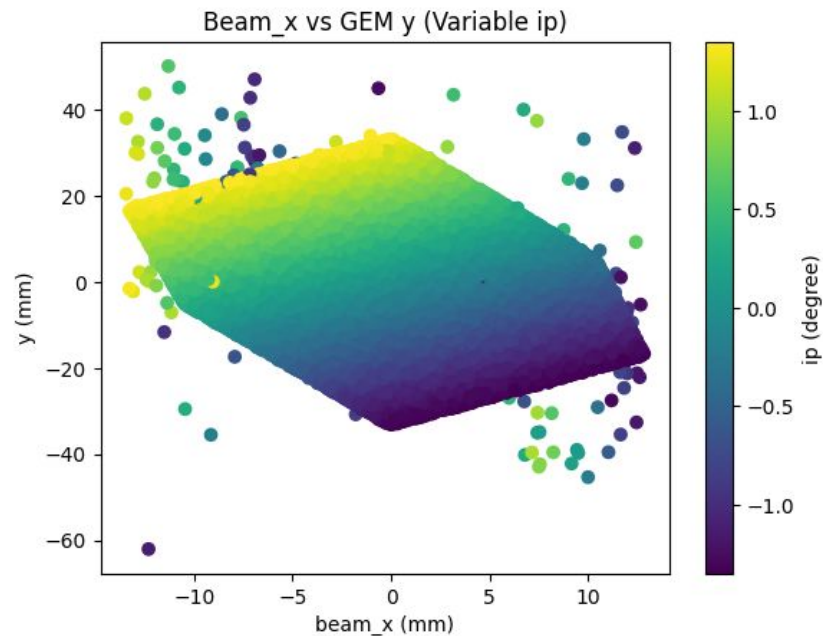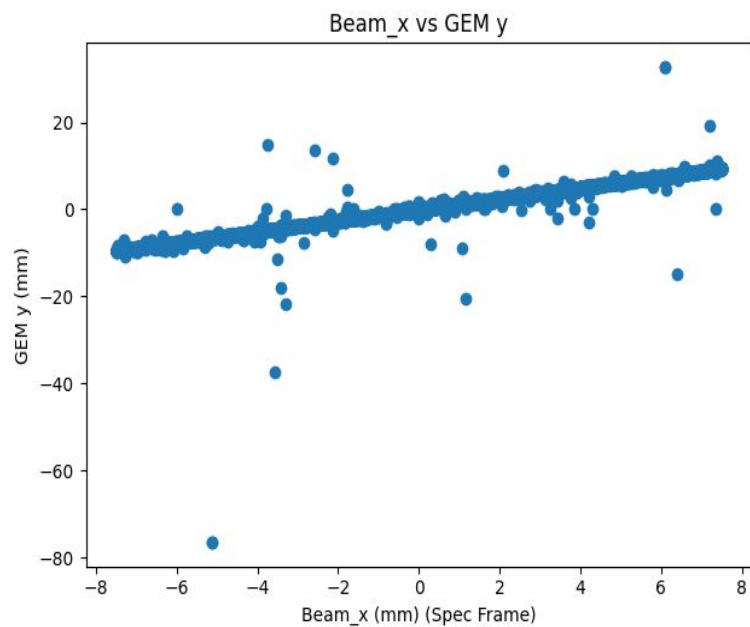
ip fixed! **ip free!**

$$\hat{ip} = f(y, dy)$$
$$\text{beam\_x} = g(y, dy, \hat{ip})$$
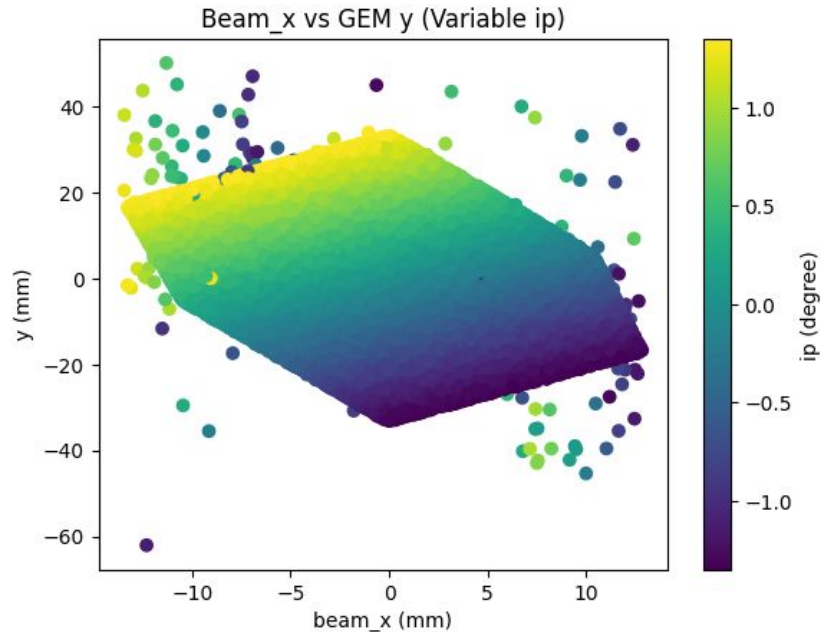
$$\Rightarrow \quad \text{beam\_x} = g\big(y, dy, f(y, dy)\big)$$

2 DOF: beam_x, ip
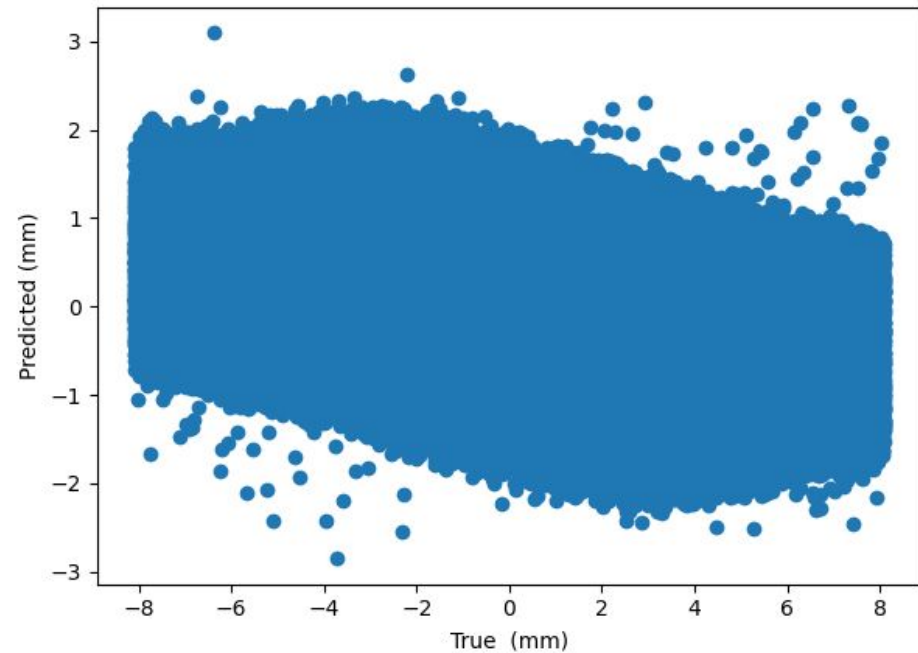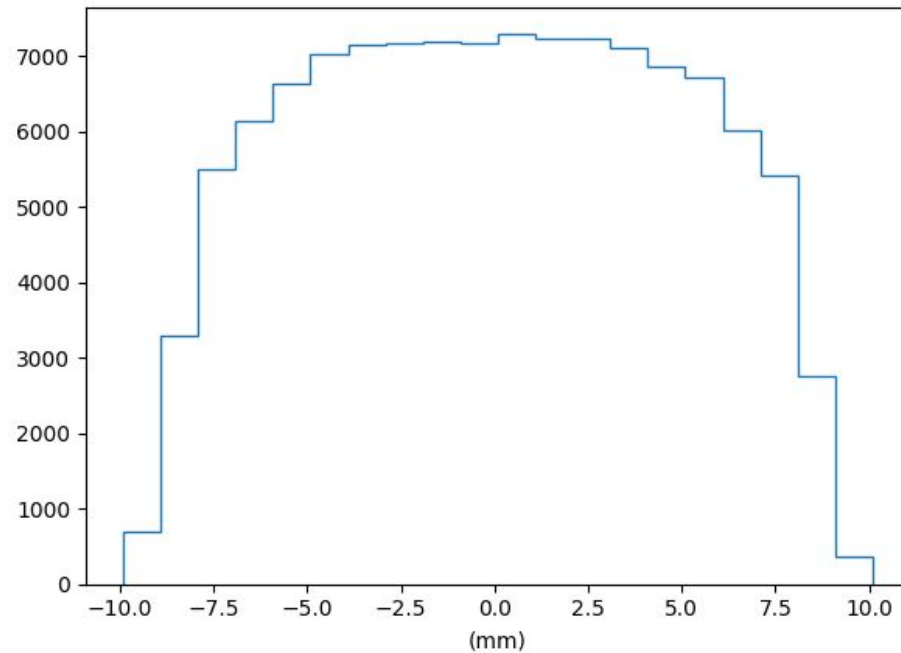
RMSE	4.79
STD	4.39
R^2	-0.18

3. Three DOF: ip, oop, beam_x

- Constant:
    - Momentum (**p**)   = Nominal
    - Beam Y (**beam_y**)   = 0 mm
- Variable:
    - Beam X (**beam_x**) $\sim \mathcal{U}$(-20, +20) mm
    - In-Plane Angle (**ip**) $\sim \mathcal{U}$(-1.8°, +1.8°)
    - Out-of-Plane Angle (**oop**) $\sim \mathcal{U}$(-5°, +5°)
- Number of events: 500k

$$\hat{ip} = \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy$$
$$\hat{oop} = \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx$$
$$\hat{beam\_x} = \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy$$
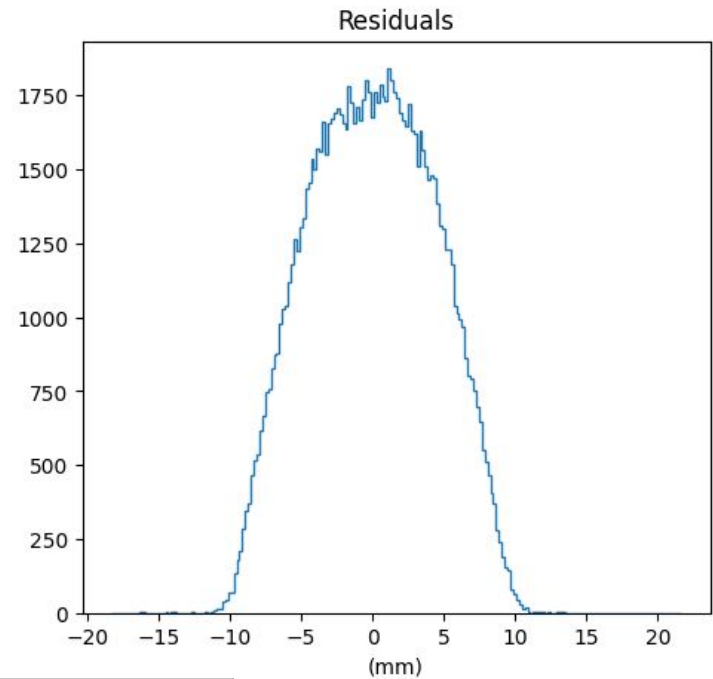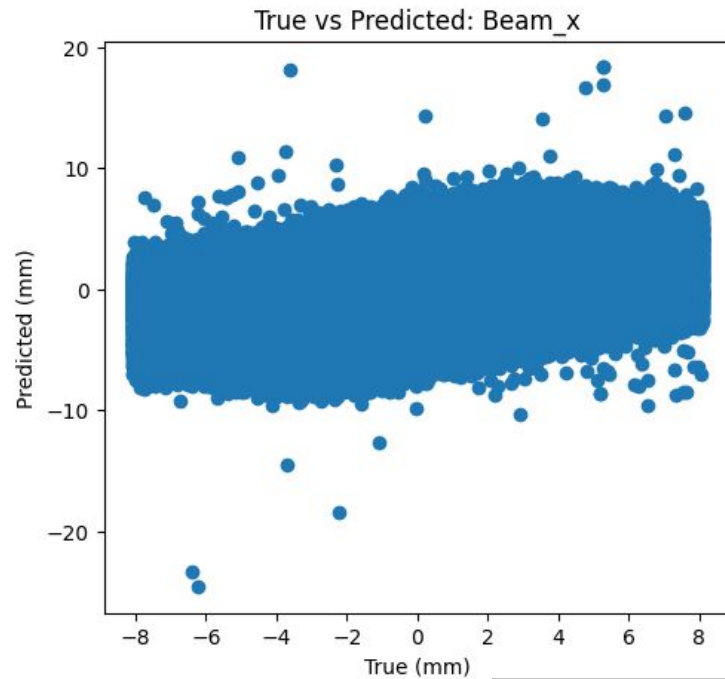
3. Three DOF: ip, oop, beam_x

- Constant:
    - Momentum (**p**)   = Nominal
    - Beam Y (**beam_y**)   = 0 mm
- Variable:
    - Beam X (**beam_x**) $\sim \mathcal{U}$(-20, +20) mm
    - In-Plane Angle (**ip**) $\sim \mathcal{U}$(-1.8°, +1.8°)
    - Out-of-Plane Angle (**oop**) $\sim \mathcal{U}$(-5°, +5°)
- Number of events: 500k

$$\hat{ip} = \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy$$
$$\hat{oop} = \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx$$
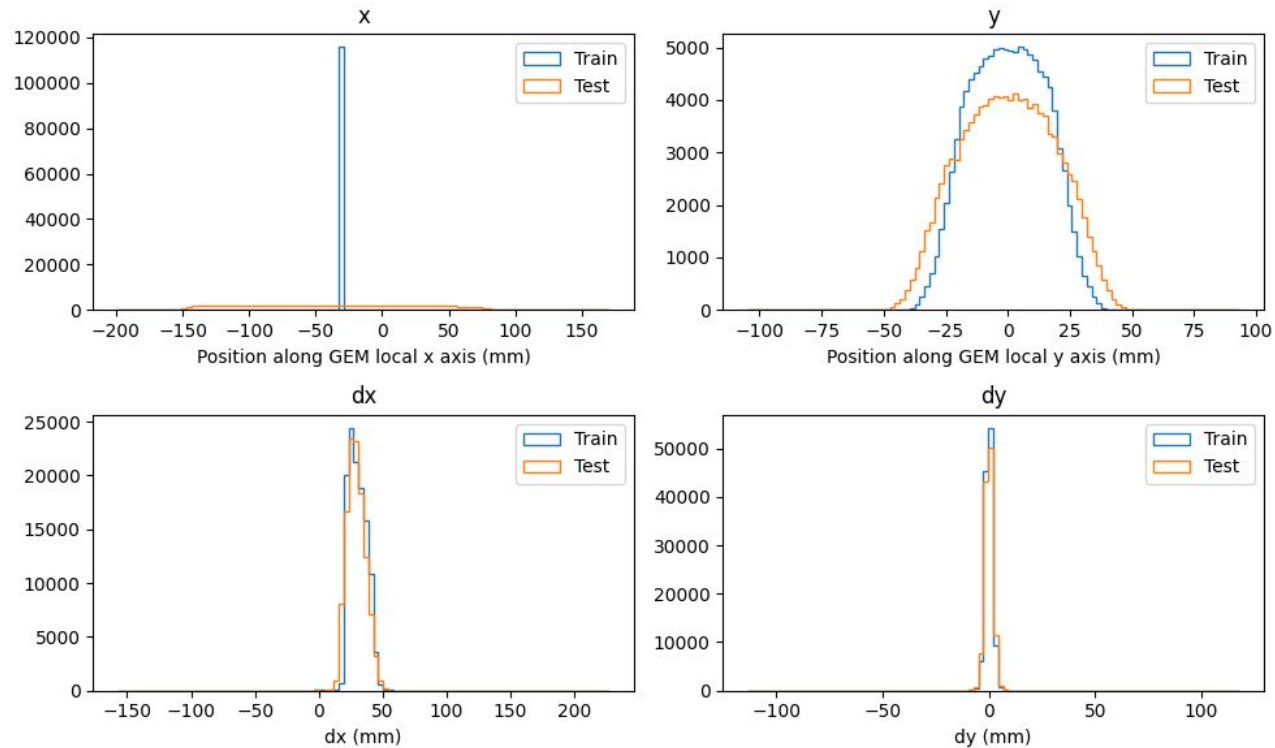$$\hat{beam\_x} = \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy$$

Model 1: Predicting ip	
RMSE	0.29
STD	0.88
R^2	0.88

Model 2: Predicting oop	
RMSE	2.24
STD	2.7
R^2	0.32

Model 3: Predicting beam_x	
RMSE	4.38
STD	4.39
R^2	0.005

Data Distribution in 3 DOF Case
(Features)

Data Distribution in 3 DOF Case
(Targets)