

# Phase-Space

## Features

- Lower GEM hit coord. along x axis (**x**)
- Lower GEM coordinate along y axis (**y**)
- Difference between GEM hits on upper and lower GEM hits along x axis (**dx**)
- Difference between GEM hits on upper and lower GEM hits along y axis (**dy**)

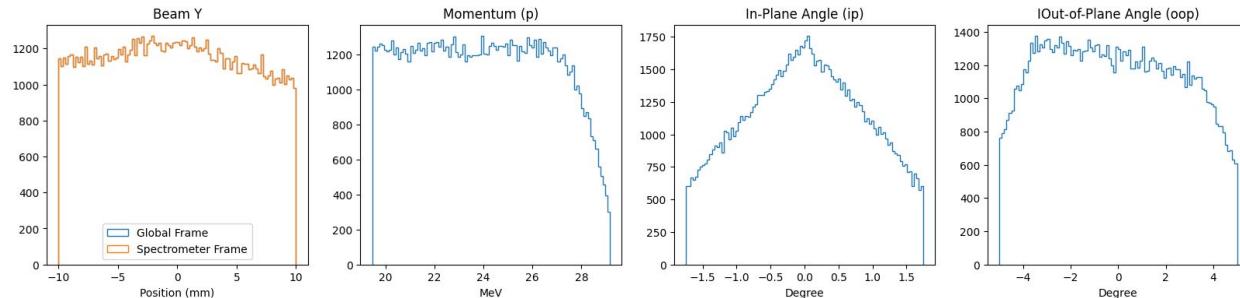
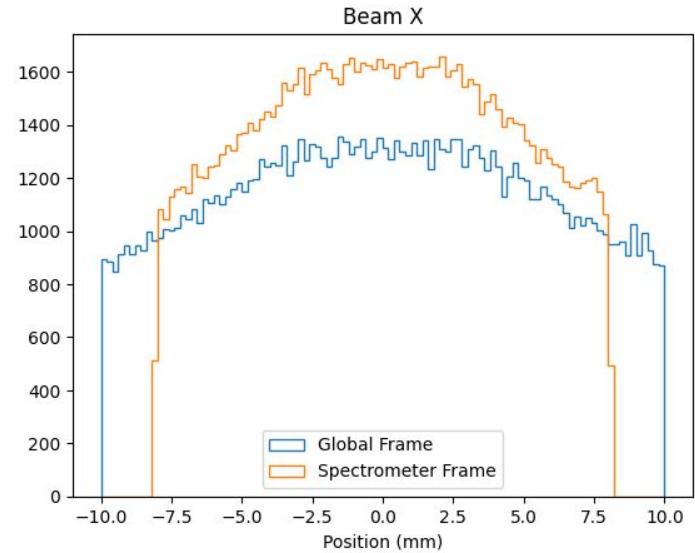


## Targets

- Momentum (**p**)  $\in [-x, x]$  mm
- In-Plane Angle (**ip**)  $\in [-x, x]$  degrees
- Out-of-Plane Angle (**oop**)  $\in [-x, x]$  degrees
- Beam X (**beam\_x**)  $\in [-x, x]$  mm
- Beam Y (**beam\_y**)  $\in [-x, x]$  mm

# 0. Test-Set (eC 25 MeV)

- Constant:
  - Momentum ( $\mathbf{p}$ ) (very small variation!)
- Variable:
  - Beam X (**beam\_x**)
  - In-Plane Angle (**ip**)
  - Of-of-Plane Angle (**oop**)
- Number of events: 114,989
- **beam\_x** range
  - Global Frame: (-10, 10) mm
  - Spectrometer Frame: (-8.1, 8.1) mm

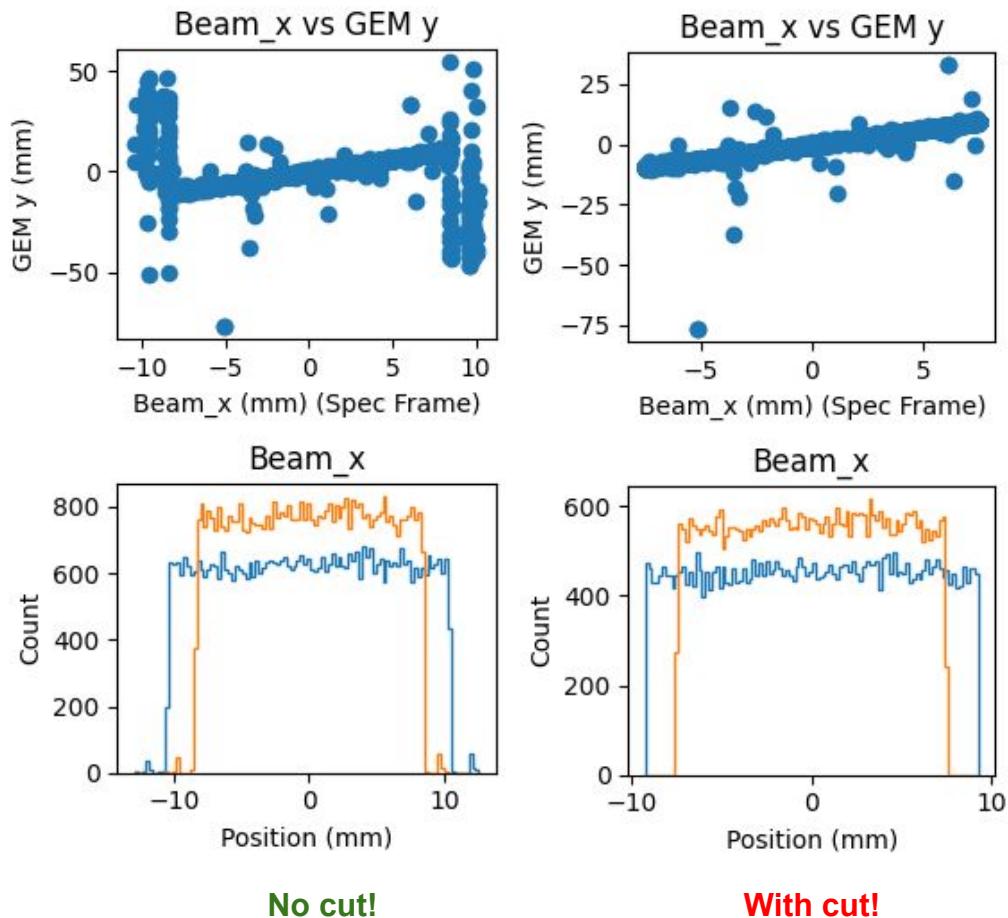


# 1. One DOF: beam\_x

- Constant:
  - Momentum (**p**) = Nominal
  - In-Plane Angle (**ip**) =  $0^\circ$
  - Out-of-Plane Angle (**oop**) =  $0^\circ$
  - Beam Y (**beam\_y**) = 0 mm
- Variable:
  - Beam X (**beam\_x**)  $\sim \mathcal{U}(-20, +20)$  mm
- Number of events: 100k

# 1. One DOF: beam\_x

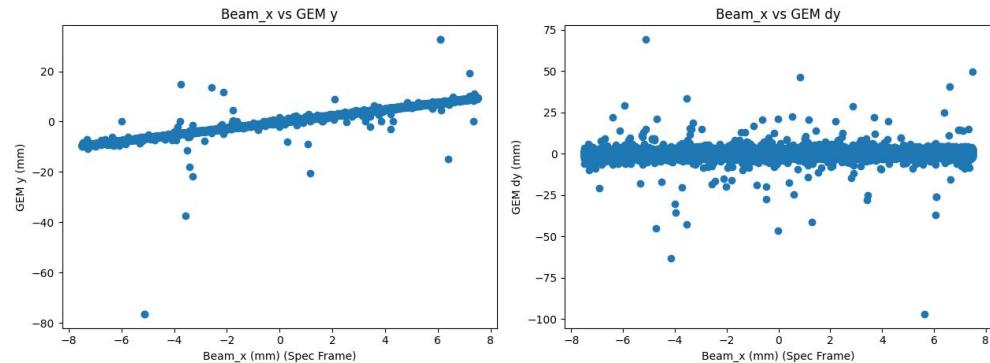
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  - In-Plane Angle (**ip**) =  $0^\circ$
  - Out-of-Plane Angle (**oop**) =  $0^\circ$
  - Beam Y (**beam\_y**) = 0 mm
- Variable:
  - Beam X (**beam\_x**)  $\sim \mathcal{U}(-20, +20)$  mm
- Number of events: 100k
- **beam\_x** range (we have put a hard cut)
  - Global Frame: (-9.3, 9.3) mm
  - Spectrometer Frame: (-7.5, 7.5) mm



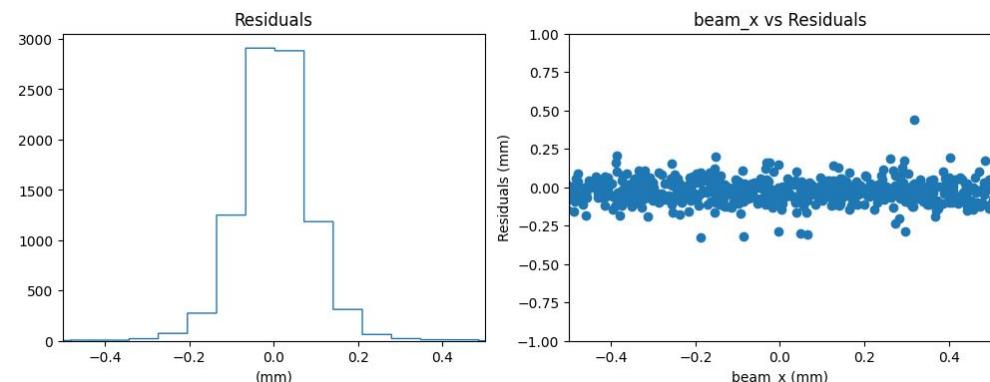
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- Variable:
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- Number of events: 100k
- **beam\_x** range (we have put a hard cut!)
  - Global Frame: (-9.3, 9.3) mm
  - Spectrometer Frame: (-7.5, 7.5) mm

beam_x Correlations	
GEM y	1.0
GEM dy	0.17

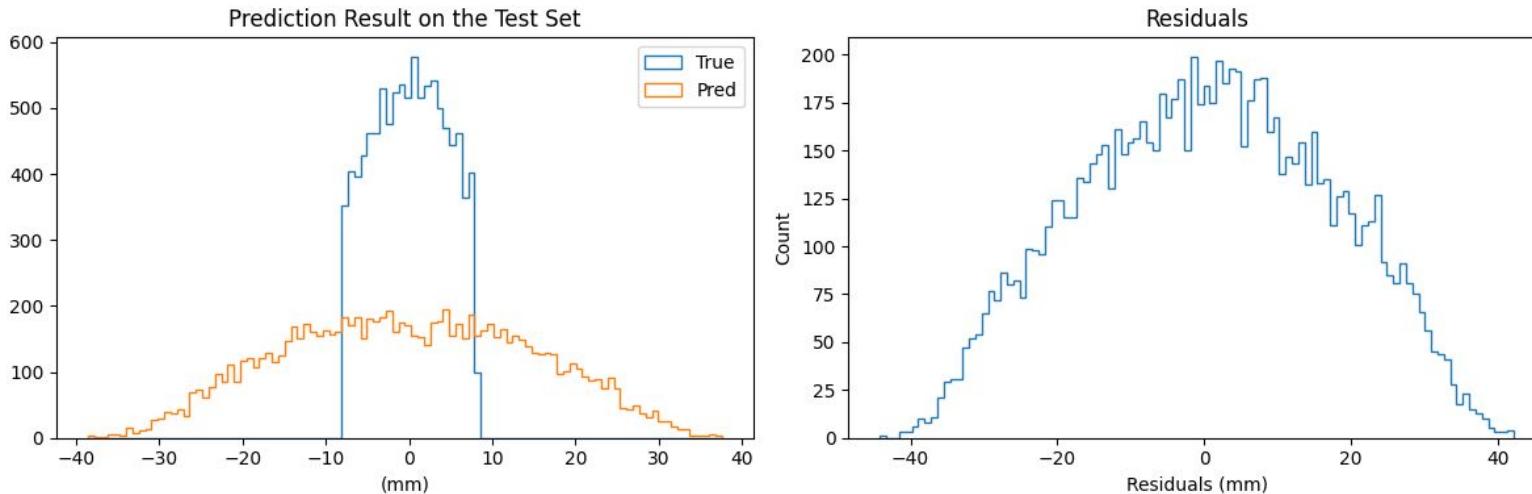


- **OLS Model:**  $\text{beam}_x = 0.09 + 0.79 \square y + 0.05 \square dy$
- Out-of-Sample:
  - RMSE: 0.82
  - Standard Deviation: 4.33
  - $R^2$ : 0.96



# 1. One DOF: beam\_x Test Set Result

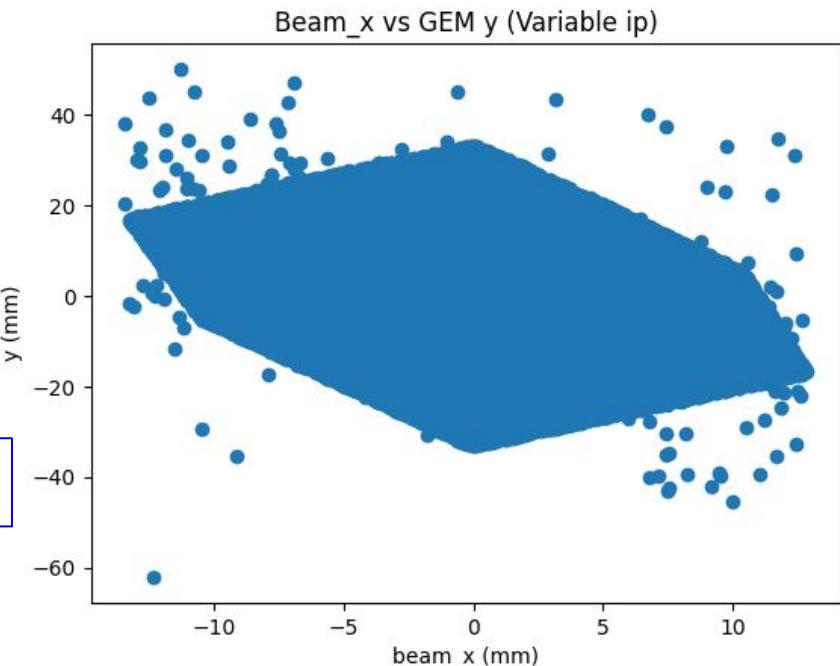
RMSE	16.9645
STD	4.404
$R^2$	-13.83

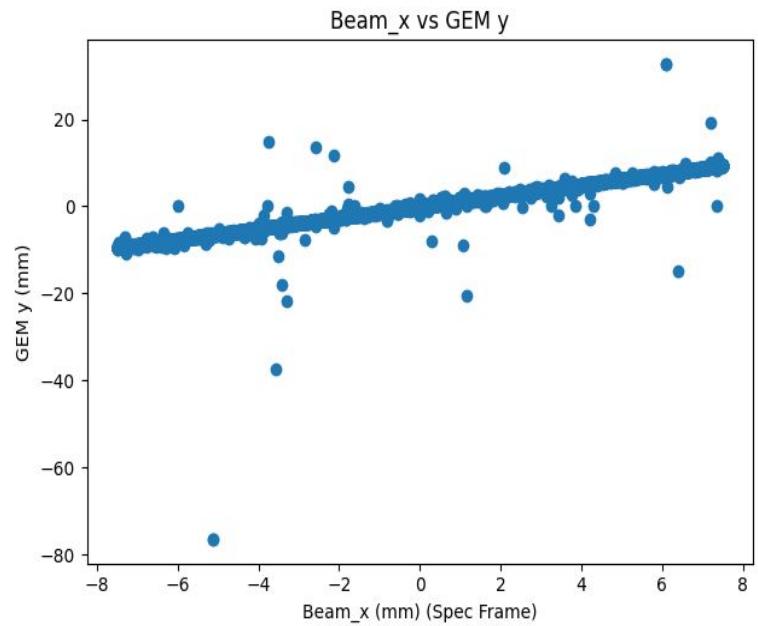


## 2. Two DOF: ip, beam\_x

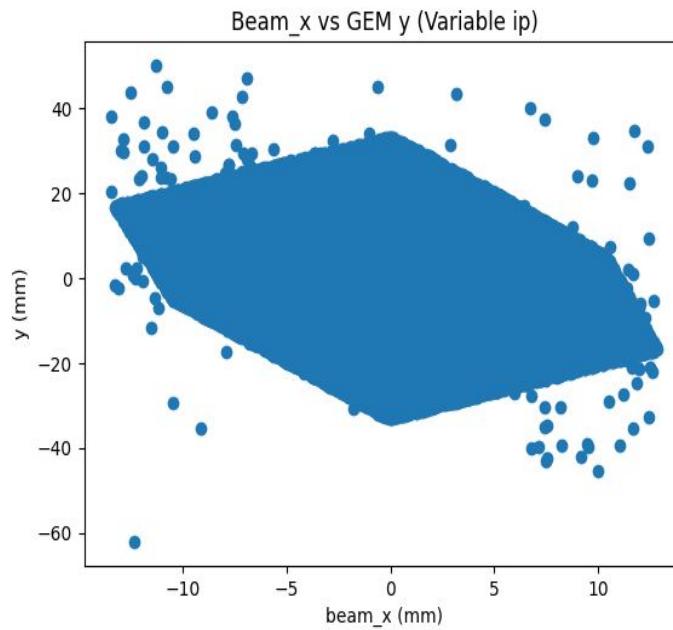
- Constant:
  - Momentum (**p**) = Nominal
  - Out-of-Plane Angle (**oop**) =  $0^\circ$
  - Beam Y (**beam\_y**) = 0 mm
- Variable:
  - Beam X (**beam\_x**)  $\sim \mathcal{U}(-20, +20)$  mm
  - In-Plane Angle (**ip**)  $\sim \mathcal{U}(-1.35^\circ, +1.35^\circ)$
- Number of events: 100k
- Model 1: 
$$\boxed{\text{beam}_x = \beta_0 + \beta_1 y + \beta_2 dy}$$

RMSE	5.68
STD	5.99
$R^2$	0.101

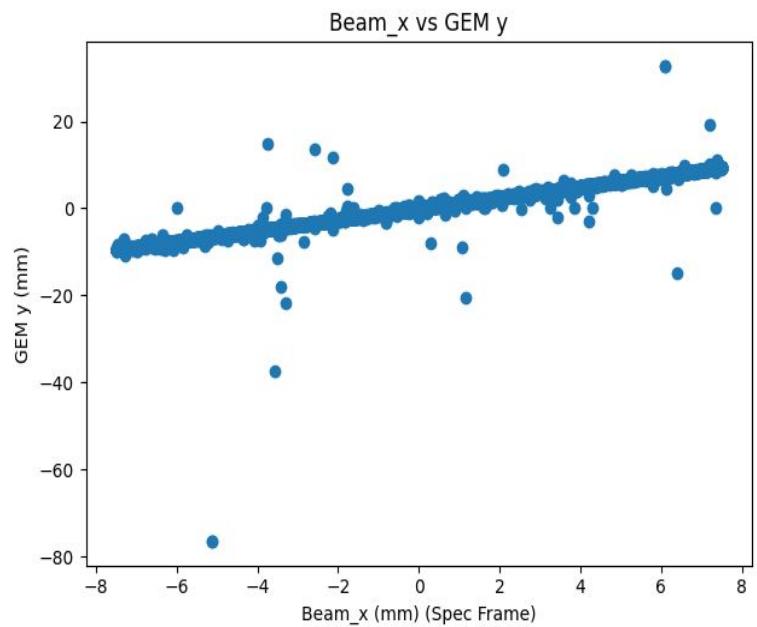




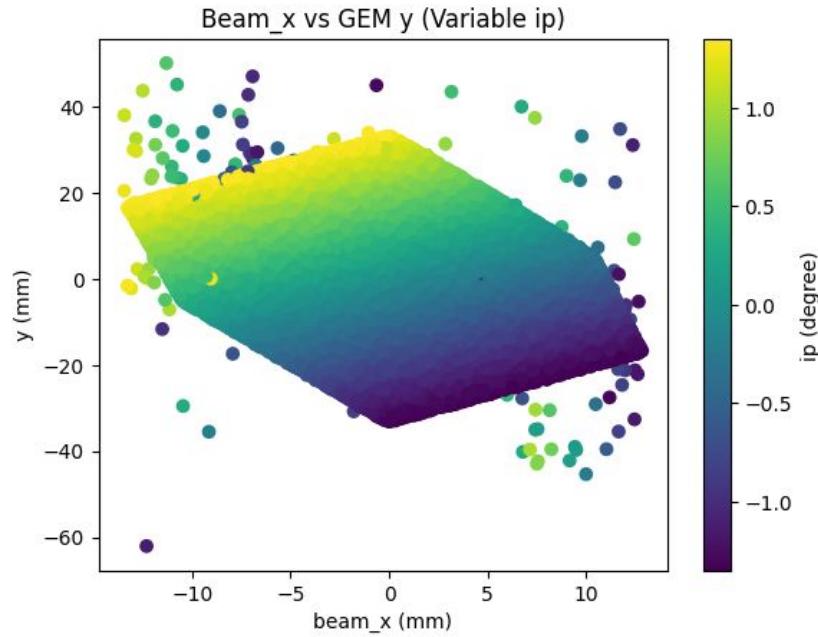
ip fixed!



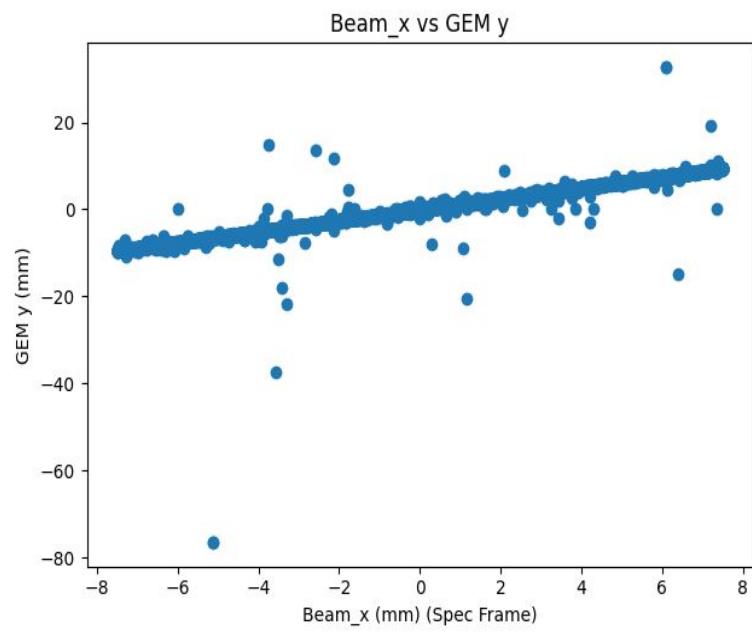
ip free!



ip fixed!



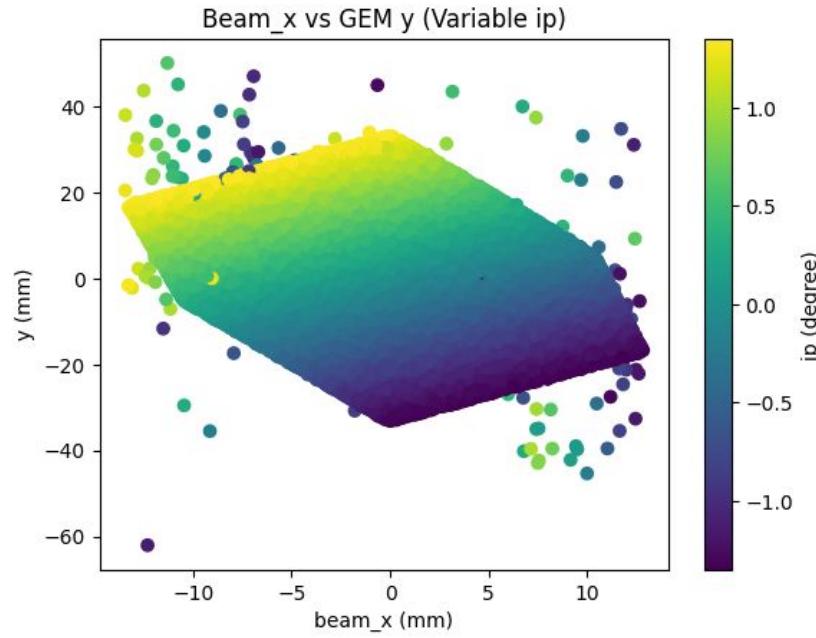
ip free!



ip fixed!

$$\hat{ip} = f(y, dy)$$

$$\text{beam\_x} = g(y, dy, \hat{ip})$$

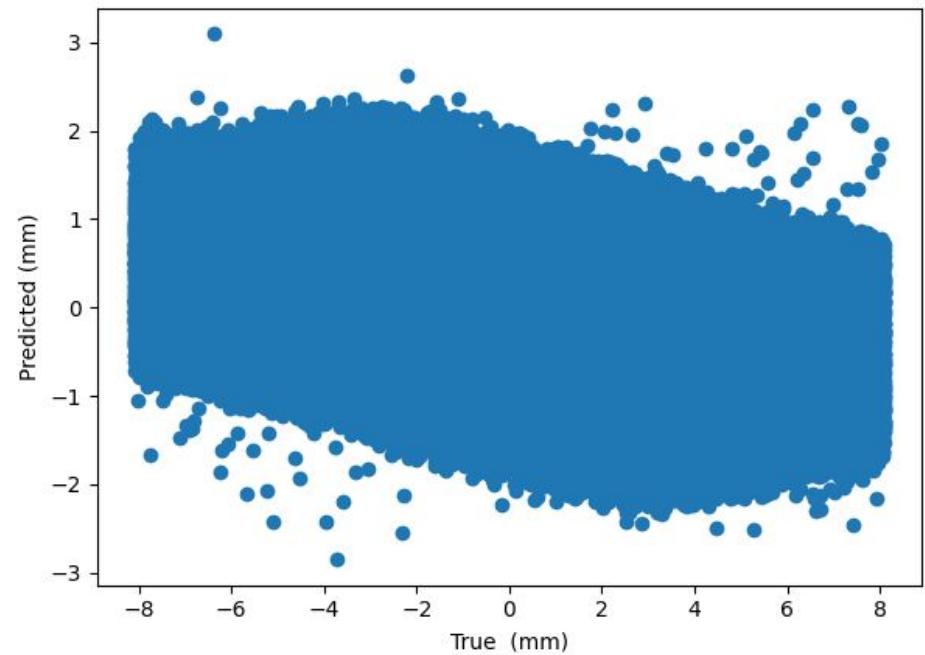


ip free!

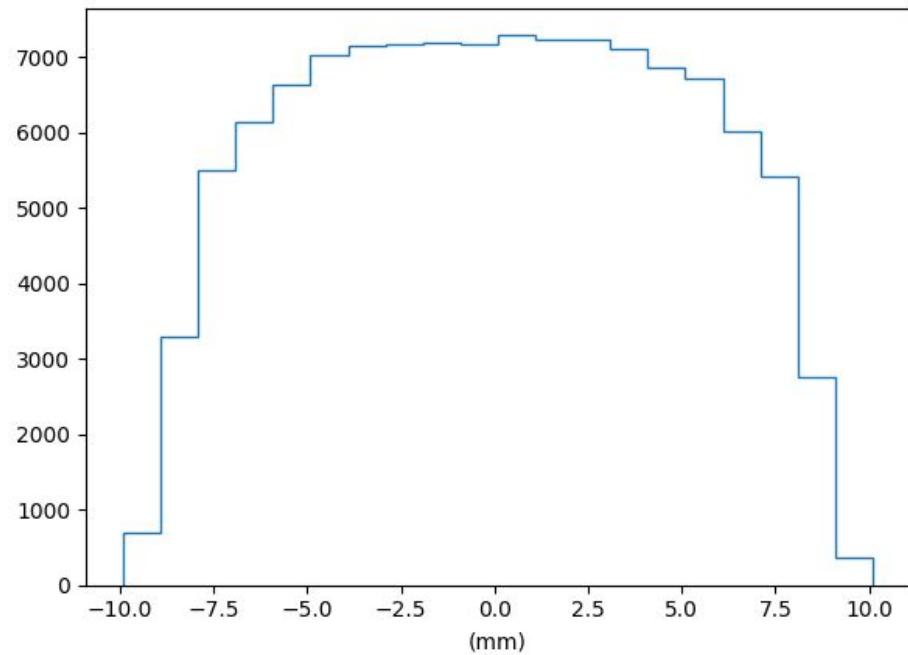
$$\Rightarrow \text{beam\_x} = g(y, dy, f(y, dy))$$

2 DOF: beam\_x, ip

Beam X Reconstruction



Residuals



RMSE	4.79
STD	4.39
$R^2$	-0.18

### 3. Three DOF: ip, oop, beam\_x

- Constant:
  - Momentum (**p**) = Nominal
  - Beam Y (**beam\_y**) = 0 mm
- Variable:
  - Beam X (**beam\_x**)  $\sim \mathcal{U}(-20, +20)$  mm
  - In-Plane Angle (**ip**)  $\sim \mathcal{U}(-1.8^\circ, +1.8^\circ)$
  - Out-of-Plane Angle (**oop**)  $\sim \mathcal{U}(-5^\circ, +5^\circ)$
- Number of events: 500k

$$\hat{ip} = \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy$$

$$\hat{oop} = \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx$$

$$\hat{\text{beam\_x}} = \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy$$

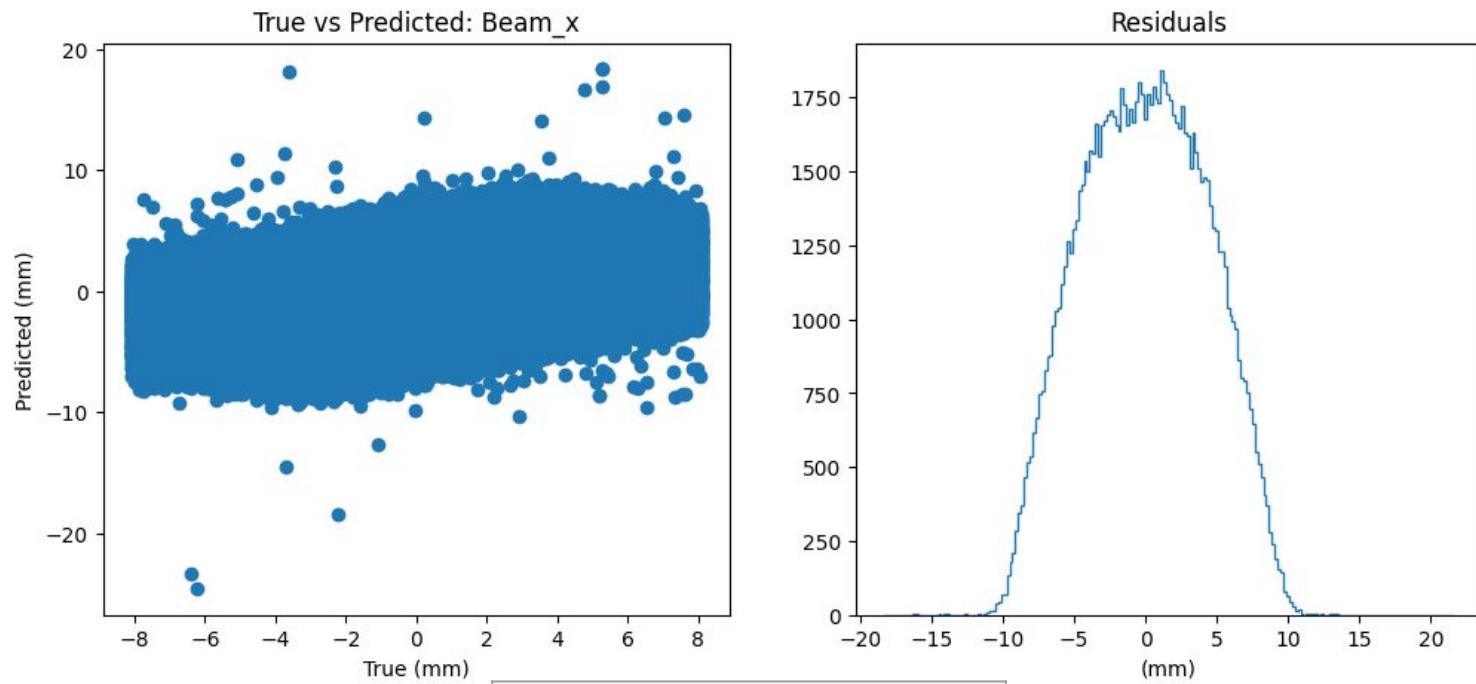
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- Constant:
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  - Beam Y (**beam\_y**) = 0 mm
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  - Beam X (**beam\_x**)  $\sim \mathcal{U}(-20, +20)$  mm
  - In-Plane Angle (**ip**)  $\sim \mathcal{U}(-1.8^\circ, +1.8^\circ)$
  - Out-of-Plane Angle (**oop**)  $\sim \mathcal{U}(-5^\circ, +5^\circ)$
- Number of events: 500k

$$\hat{ip} = \alpha_1 + \alpha_2 \cdot y + \alpha_3 \cdot dy$$
$$\hat{oop} = \beta_1 + \beta_2 \cdot y + \beta_3 \cdot dy + \beta_4 \cdot x + \beta_5 \cdot dx$$
$$\hat{beam\_x} = \gamma_1 + \gamma_2 x + \gamma_3 dx + \gamma_4 y + \gamma_5 dy$$

Model 1: Predicting ip	
RMSE	0.29
STD	0.88
R <sup>2</sup>	0.88

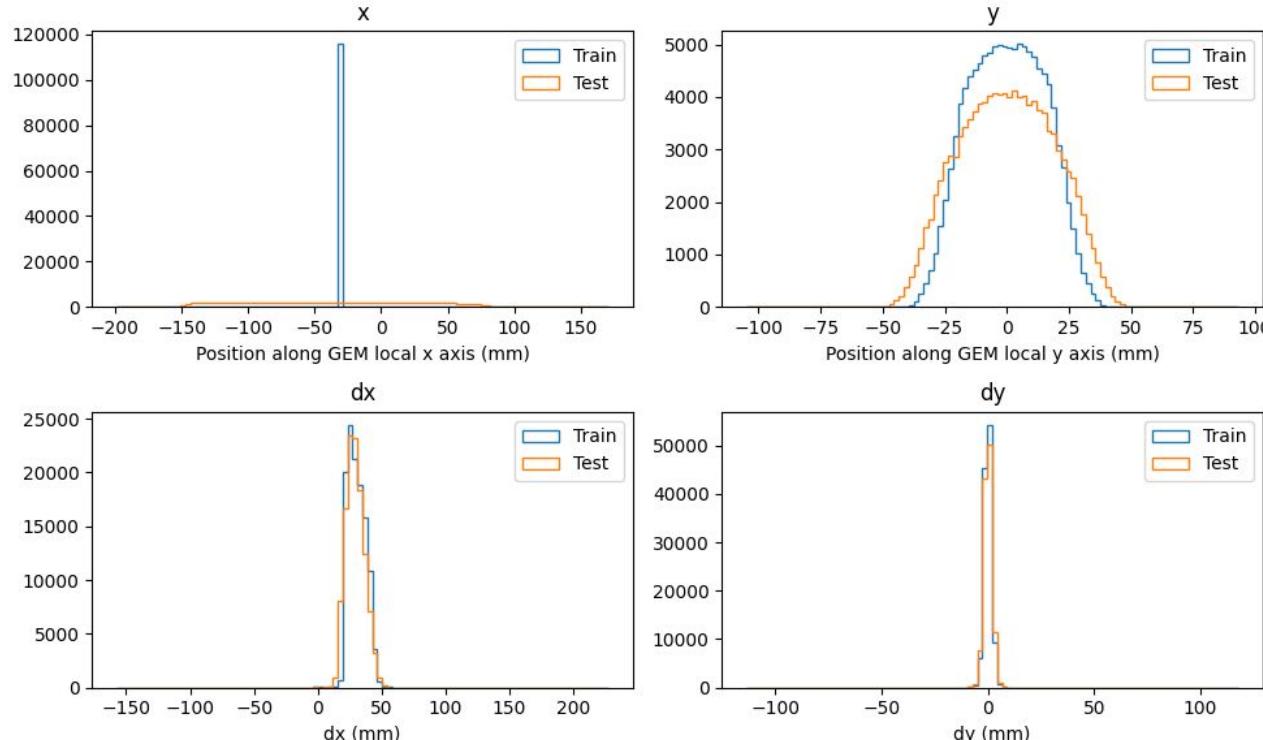
Model 2: Predicting oop	
RMSE	2.24
STD	2.7
R <sup>2</sup>	0.32



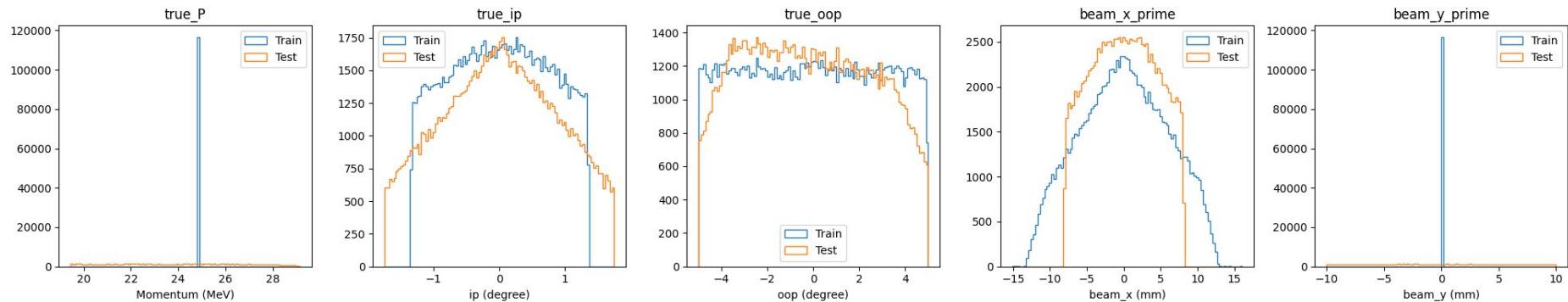
Model 3: Predicting beam\_x

RMSE	4.38
STD	4.39
R <sup>2</sup>	0.005

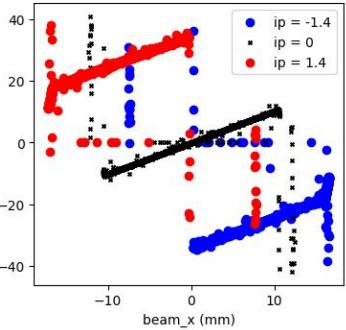
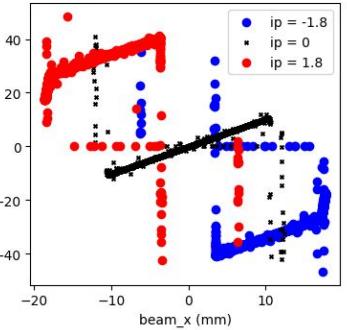
# Data Distribution in 3 DOF Case (Features)



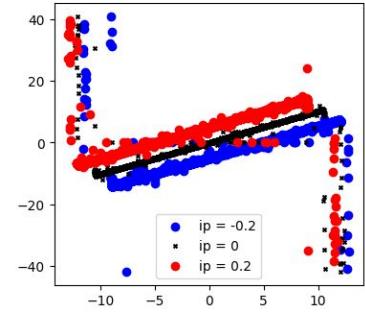
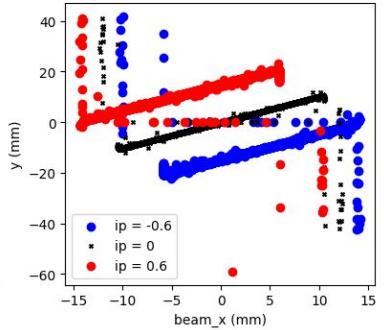
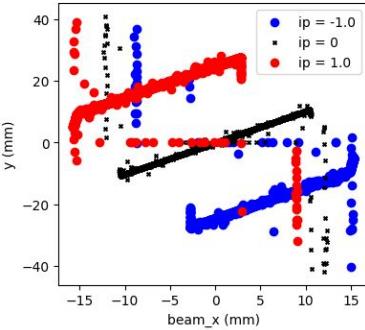
# Data Distribution in 3 DOF Case (Targets)



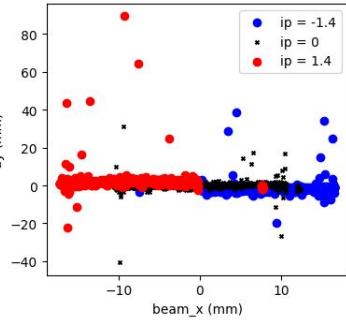
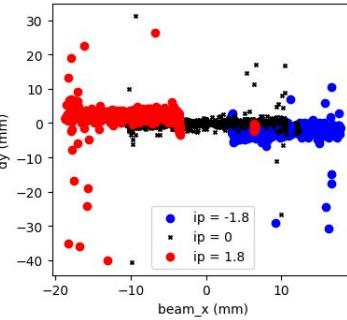
y (mm)



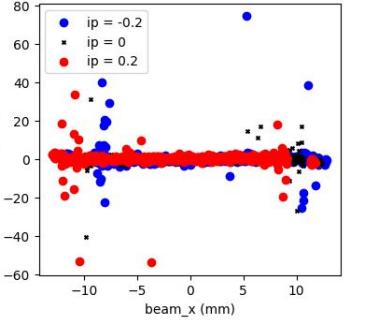
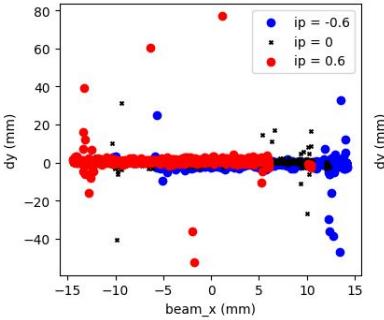
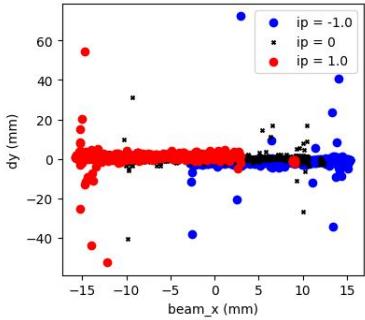
beam\_x vs y (constant ip)



dy (mm)



beam\_x vs dy (constant ip)

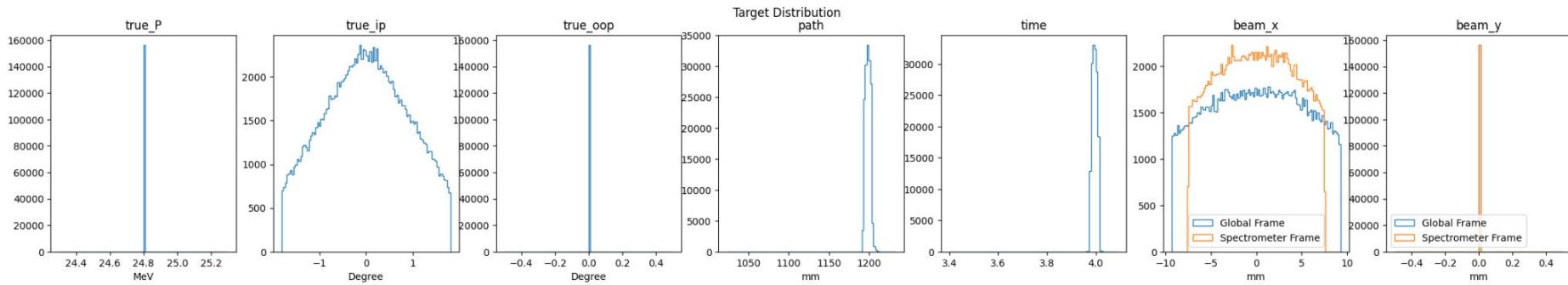


## 4. Two DOF: ip, beam\_x (Scattering off)

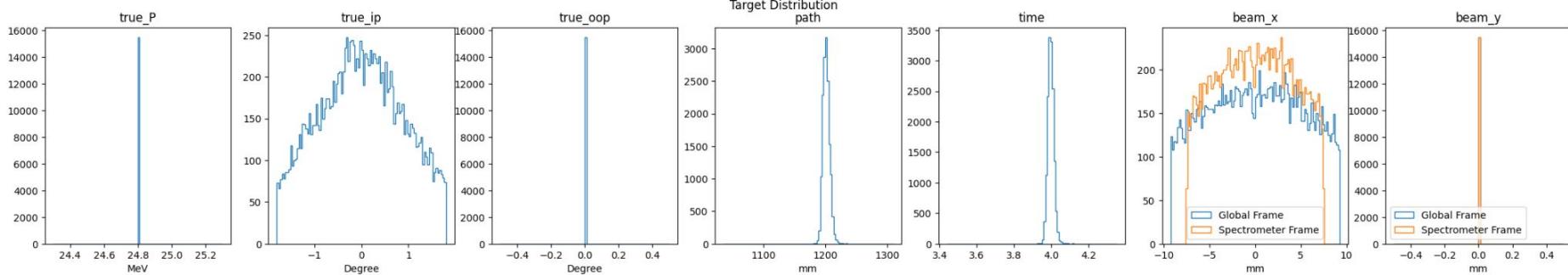
- Constant:
  - Momentum (**p**) = Nominal
  - Out-of-Plane Angle (**oop**) =  $0^\circ$
  - Beam Y (**beam\_y**) = 0 mm
- Variable:
  - Beam X (**beam\_x**)  $\sim \mathcal{U}(-20, +20)$  mm
  - In-Plane Angle (**ip**)  $\sim \mathcal{U}(-1.80^\circ, +1.80^\circ)$
- Number of events: 500k

# Target Distribution

Scattering: Off



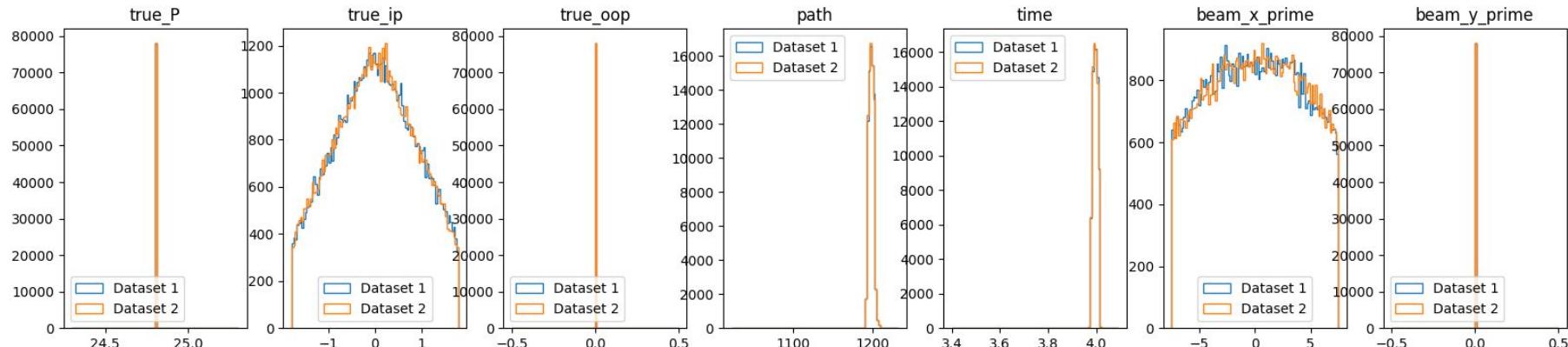
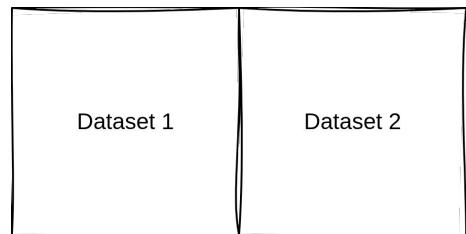
Scattering: On



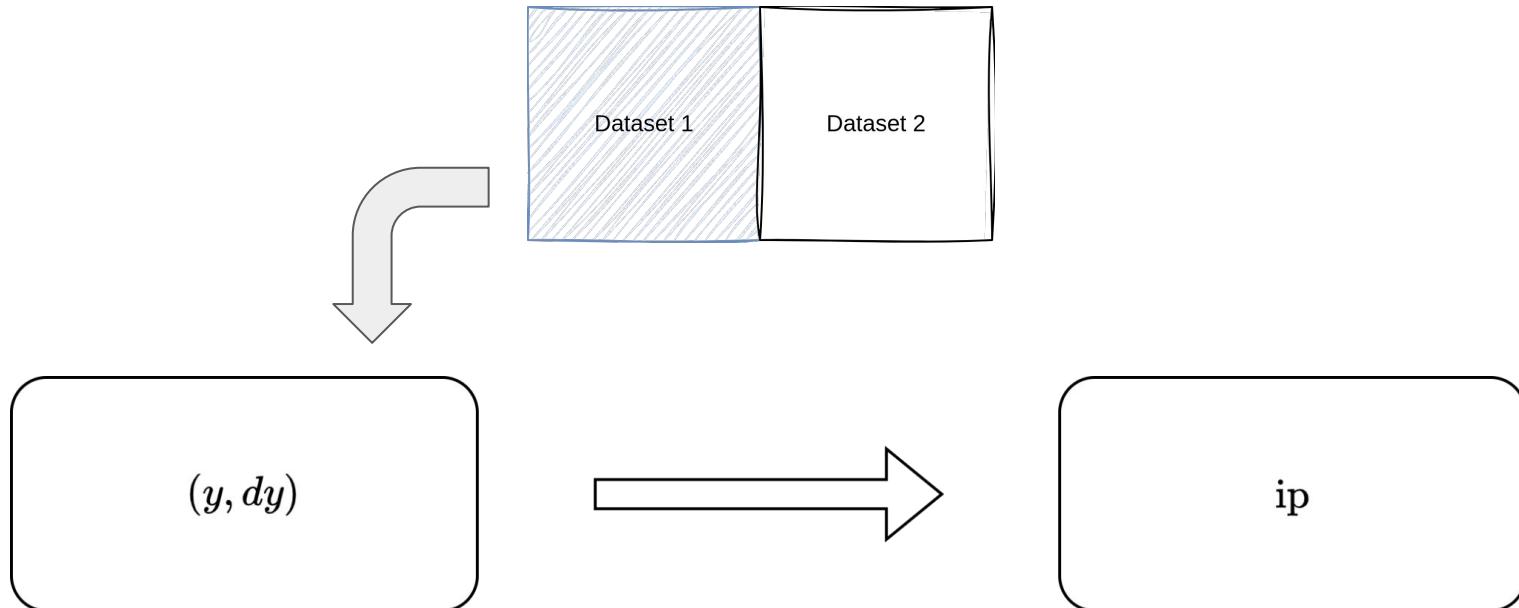
# Step 1: Split into two Datasets



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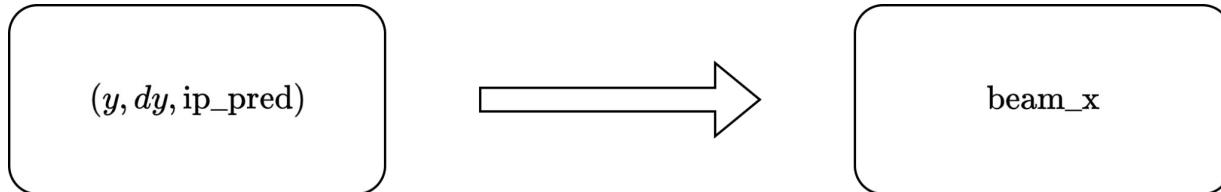
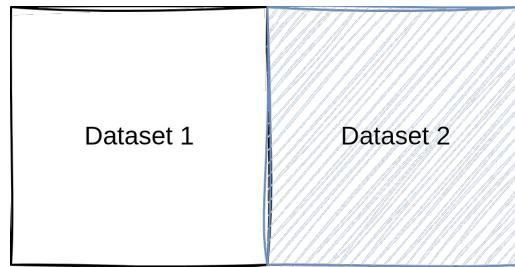


## Step 2: Train Model 1 on Dataset 1



Step 3: Predict In-Plane Angle using Model 1 on Dataset 2

Step 4: Train Model 2 on Dataset 2 and predict beam\_x on Dataset 2 validation set.



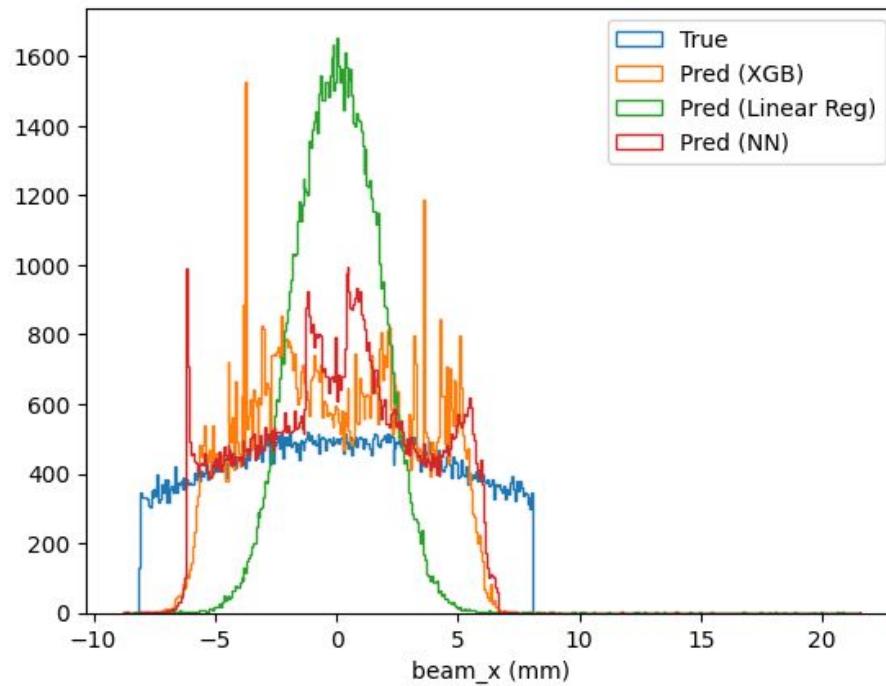
[Model 1: Predicts ip](#)

	<b>Linear Regression</b>	<b>XGB</b>	<b>Neural Networks</b>
R <sup>2</sup>	0.95	0.96	0.99
RMSE	0.20	0.17	0.10
STD	0.89	0.89	0.89
Norm-RMSE	0.23	0.19	0.11

[Model 2: Predicts beam\\_x](#)

	<b>Linear Regression</b>	<b>XGB</b>	<b>Neural Networks</b>
R <sup>2</sup>	0.16	0.85	0.86
RMSE	3.78	1.59	1.51
STD	4.12	4.12	4.12
Norm-RMSE	0.91	0.38	0.37

DOF: 2



## 5. Alternate Approach - Classification

- Classify range of beam\_x into N intervals, call them bins.
- We learn the following mapping:
  - $(x, y, dx, dy) \rightarrow (\text{beam\_x bins})$
- $N = 20$
- Range of Beam X = (-7.5 mm, 7.5 mm)
- Bin resolution = 0.75 mm

$$T_i^{(K)} = \text{TopK}_{j \in 1, \dots, C} p_{i,j}$$

$$\text{Top-K Accuracy} = \frac{1}{N} * \sum_{i=1}^N \mathbf{1}_{[y_i \in T_i^{(K)}]}$$

K	Accuracy (%)
1 (Exact bin match!)	54.18
3	83.12
4	90.85
10	97.76