EE2703 : Applied Programming Lab
Assignment 8

Rakshit Pandey
EE20B106

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1.AIM OF THE ASSIGNMENT

The aim of this assignment is to examine the DFT (Digital Fourier Transform) of various functions with the help of fft library in numpy.

2.INTRODUCTION TO THE ASSIGNMENT

In this assignment, we work with the fft library in numpyl, with the help of this library and some knowledge of Digital Signal processing (DSP) we will work with ffts of various functions (both bandlimited and non-bandlimited functions). Further, we will analyze the obtained magnitude and phase spectrum plots and compare it to the analytical theoretical results.

3.ASSIGNMENT QUESTIONS

Importing important libraries

In order to this assignment, it is sufficient to import pylab, this is done in python as follows:

Code

```
from pylab import *
```

1. QUESTION 1

1. Spectrum of $\sin(5t)$

As expected the phase for some values near the peaks is non zero. To fix this we sample the input the signal at an appropriate frequency. We also shift the phase plot so that it goes from -π to π.

$$sin(5t) = 0.5(\frac{e^{j5t}}{j} - \frac{e^{-j5t}}{j})$$

Clearly from the above equation we expect to get to peaks in spectrum plot, one at -5 and second one at +5 with an amplitude of 0.5, The phases of the peaks are $\frac{\pi}{2}$ and -$\frac{\pi}{2}$. The magnitude and phase plot is as shown below:

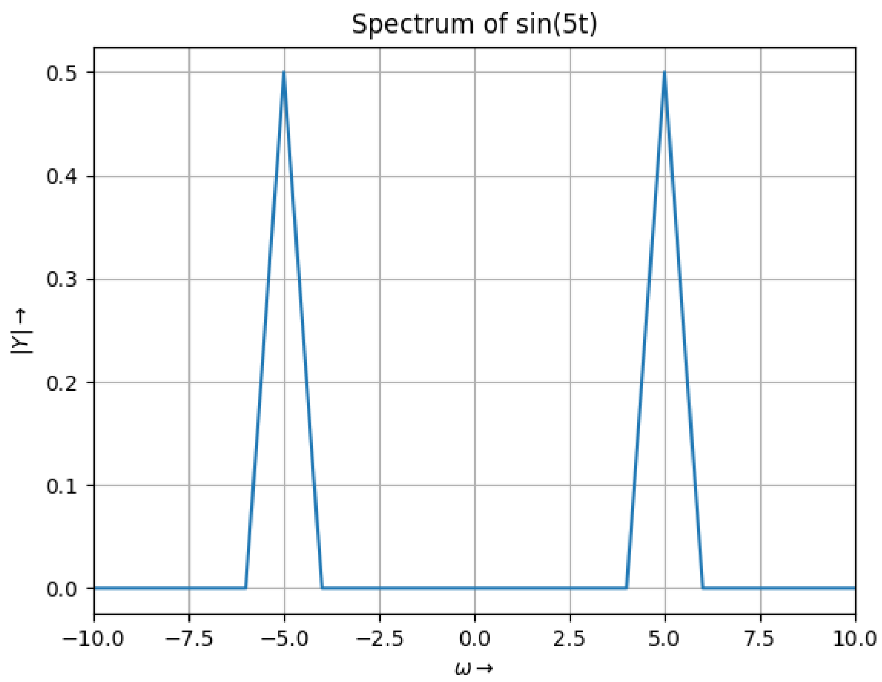

Figure 1: Magnitude spectrum of sin(5t)

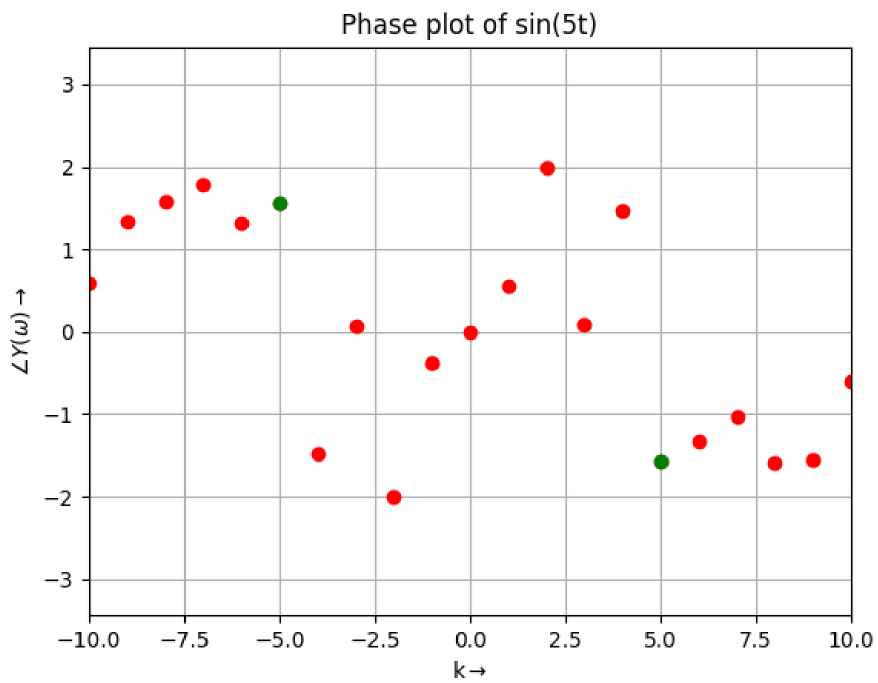

Figure 2: Phase plot of sin(5t)

Code

```
#Generating spectrum of sin(5*t)
N1 = 128
x1 = linspace(0,2.0*pi,N1+1)
x1 = x1[:-1]
y1=sin(5*x1)
Y1=fftshift(fft(y1))/N1
w1=linspace(-64,63,N1)

#Magnitude spectrum plot for sin(5t)
figure(1)
title("Spectrum of sin(5t)")
xlabel(r"$\omega\rightarrow$")
ylabel(r"$\|Y|\rightarrow$")
xlim([-10,10])
plot(w1,abs(Y1))
grid()

#Phase spectrum plot for sin(5t)
figure(2)
title("Phase plot of sin(5t)")
xlabel(r"k$\rightarrow$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlim([-10,10])
plot(w1,angle(Y1),'ro')
ii=where(abs(Y1)>1e-3)
plot(w1[ii],angle(Y1[ii]),'go')
grid()
show()
```

2. Spectrum of $(1 + 0.1\cos(t))\cos(10t)$

We are given an amplitude modulated signal :

$$f(t) = (1 + 0.1cos(t))cos(10t)$$

the given equation can be analytically written as :

$$0.5(e^{j10t} + e^{-j10t}) + 0.025(e^{j11t} + e^{-j11t} + e^{j9t} + e^{-j9t})$$

From the equation we can clearly see that the peaks would come at -10,+10,-9,+9,-11,+11. Now we plot the curve by finding fft of f(t) function.This is executed in python as follows:

Code

```
N2=512
x2=linspace(-4.0*pi,4.0*pi,N2+1)
x2=x2[:-1]
```

```
y2 = (1+0.1*cos(x2))*cos(10*x2)
Y2=fftshift(fft(y2))/N2
w2=linspace(-64,64,N2+1)
w2=w2[:-1]

#Magnitude spectrum plot for (1+0.1cos(t))cos(10t)
figure(3)
title("Spectrum of (1+0.1cos(t))cos(10t)")
xlabel(r"$\omega\rightarrow$")
ylabel(r"$\|Y|\rightarrow$")
xlim([-15,15])
plot(w2,abs(Y2))
grid()

#Phase spectrum plot for (1+0.1cos(t))cos(10t)
figure(4)
title("Phase plot of (1+0.1cos(t))cos(10t)")
xlabel(r"k$\rightarrow$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlim([-15,15])
plot(w2,angle(Y2),'ro')
grid()
show()
```

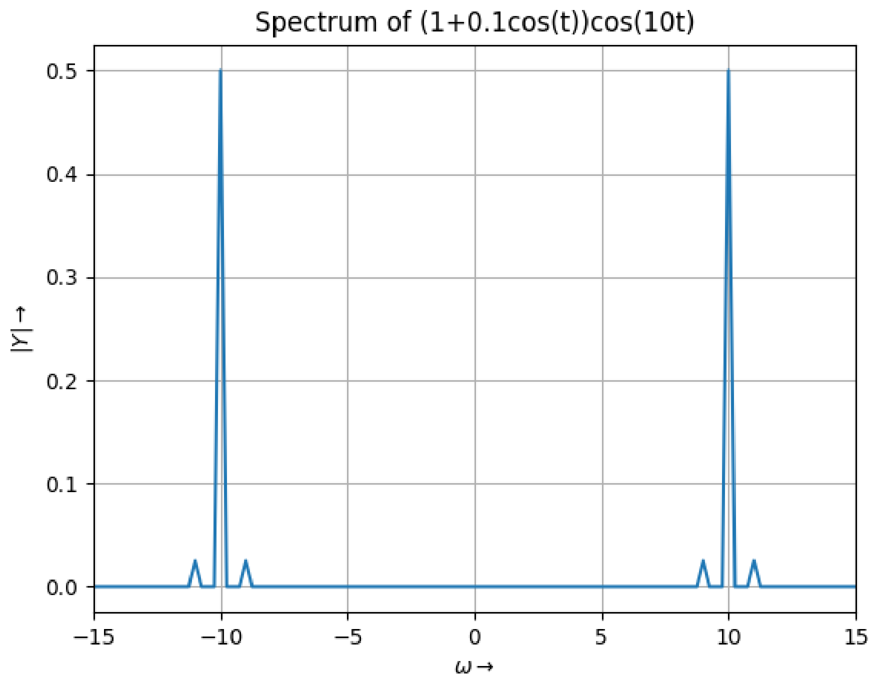

Figure 3: Magnitude spectrum of (1+0.1cos(t))cos(10t)

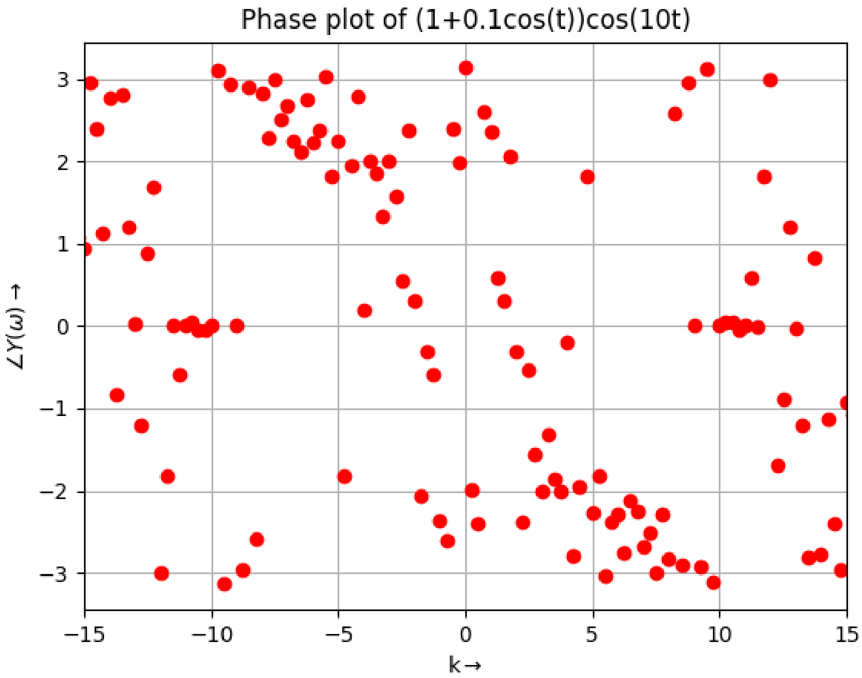

Figure 4: Phase plot of (1++0.1cos(t))cos(10t)

From the curve we can clearly see that we get peaks at -11,-10,-9,+9,+10,+11 as expected.

2. QUESTION 2

Spectrum of $sin^3(t)$ and $cos^3(t)$

In this Question we were supposed to analyze the spectrum and phase plots for $\sin^3(t)$ and $\cos^3(t)$.

$$sin^3(t) = \frac{3sin(t)}{4} - \frac{sin(3t)}{4}$$

$$cos^3(t) = \frac{3cos(t)}{4} + \frac{cos(3t)}{4}$$

From the above two equations it becomes clear that we can expect peaks at -3,-1,+1 and +3 in both the cases. The magnitude spectrum and phase plot is generated in python as follows:

Code

```
N3=512
x3=linspace(-4.0*pi,4.0*pi,N3+1)
x3=x3[:-1]
y3=(sin(x3))**3
```

```
Y3=fftshift(fft(y3))/N3
w3=linspace(-64,64,N3+1)
w3=w3[:-1]

#Magnitude spectrum plot for (sin(t))^3
figure(5)
title("Spectrum of $sin^{3}(t)$")
xlabel(r"$\omega\rightarrow$")
ylabel(r"$\|Y|\rightarrow$")
xlim([-10,10])
plot(w3,abs(Y3))
grid()

#Phase spectrum plot for (sin(t))^3
figure(6)
title("Phase plot of $sin^{3}(t)$")
xlabel(r"k$\rightarrow$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlim([-10,10])
plot(w3,angle(Y3),'ro')
grid()

x3=linspace(-4.0*pi,4.0*pi,N3+1)
x3=x3[:-1]
y4=(cos(x3))**3
Y4=fftshift(fft(y4))/N3
w3=linspace(-64,64,N3+1)
w3=w3[:-1]

#Magnitude spectrum plot for (cos(t))^3
figure(7)
title("Spectrum of $cos^{3}(t)$")
xlabel(r"$\omega\rightarrow$")
ylabel(r"$\|Y|\rightarrow$")
xlim([-10,10])
plot(w3,abs(Y4))
grid()

#Phase spectrum plot for (cos(t))^3
figure(8)
title("Phase plot of $cos^{3}(t)$")
xlabel(r"k$\rightarrow$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlim([-10,10])
plot(w3,angle(Y4),'ro')
grid()
show()
```

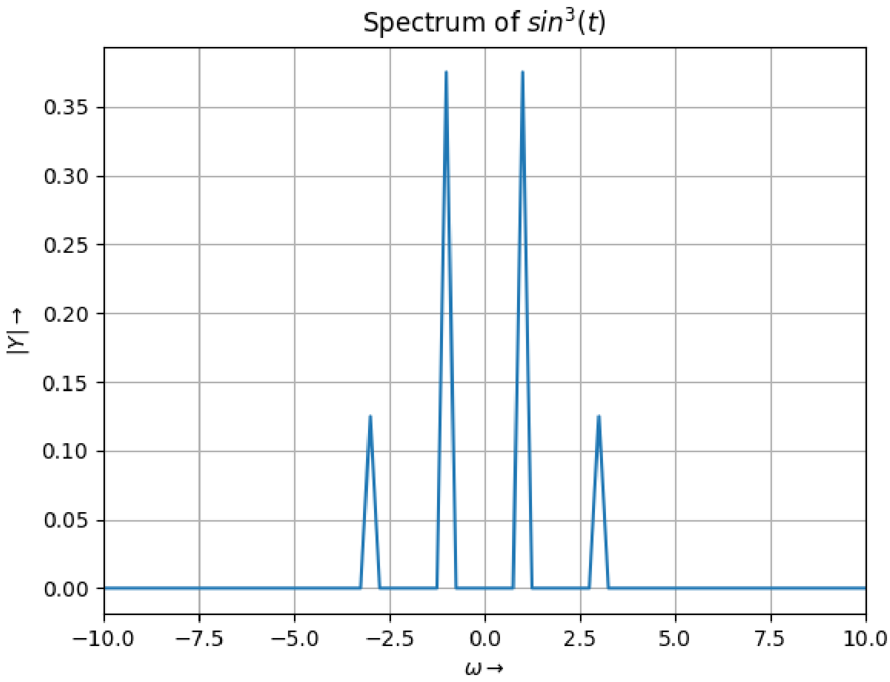

Figure 5: Magnitude spectrum of $\sin^3(t)$

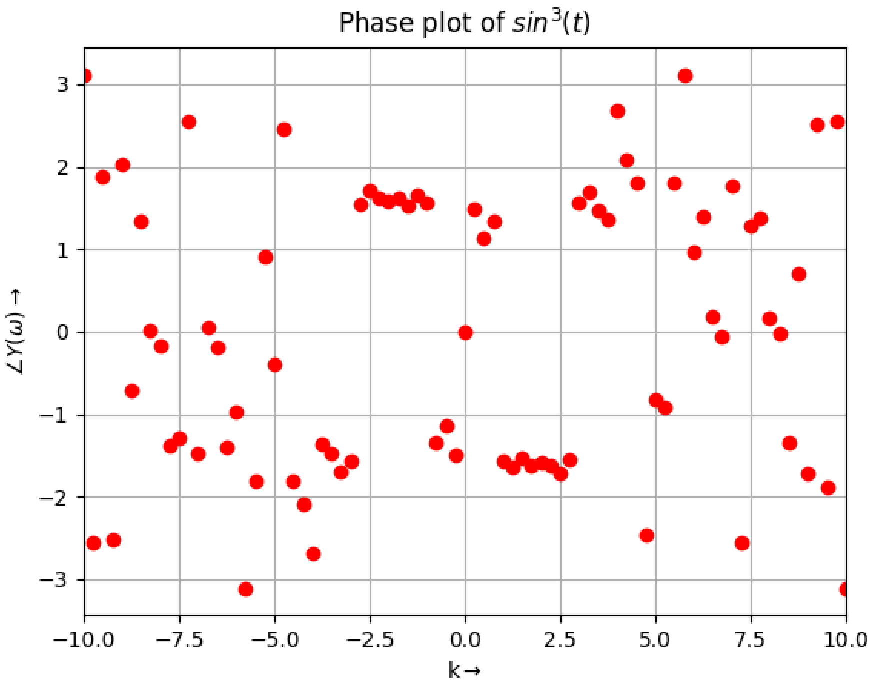

Figure 6: Phase plot of $\sin^3(t)$

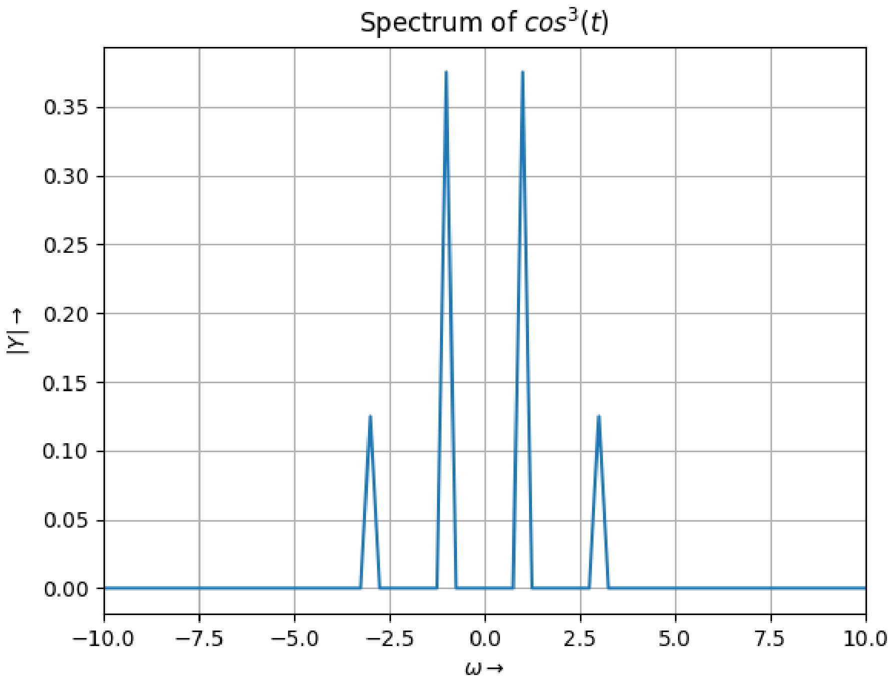

Figure 7: Magnitude spectrum of $\cos^3(t)$

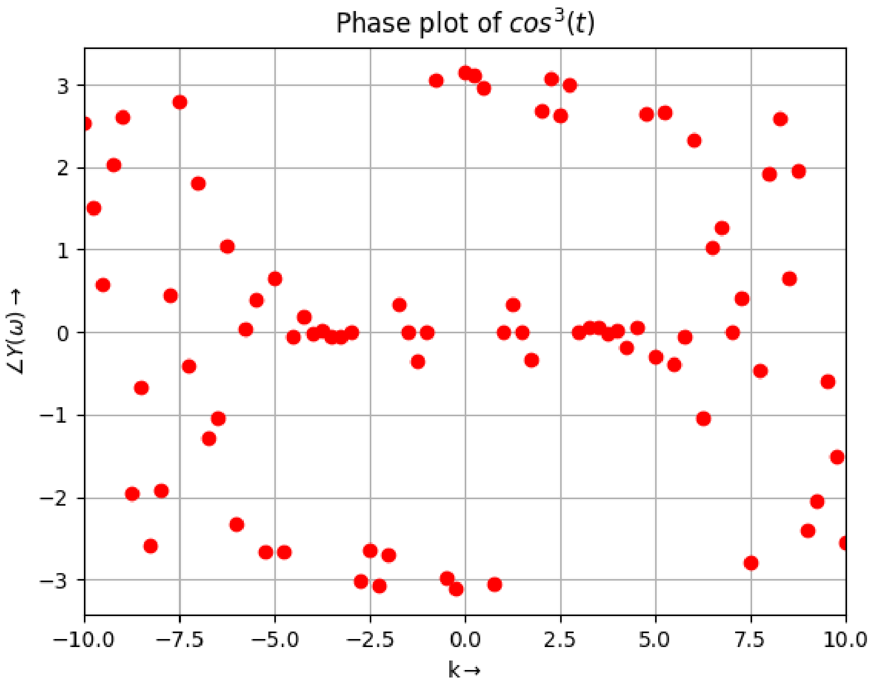

Figure 8: Phase plot of $\cos^3(t)$

From the above graphs we can clearly see that the result that we get analytically matches with the one we get by finding fft of our function.

3. QUESTION 3

In this question we were given a frequency modulated signal g(t)

$$g(t) = cos(20t + 5cos(t))$$

For this frequency modulated signal we were supposed to find the magnitude and phase spectrum plots and analyze them. For phase plot we plot only those points for which $|Y(\omega)| > 10^{-3}$. This is done in python as follows:

Code

```
N5=512
x5=linspace(-4.0*pi,4.0*pi,N5+1)
x5=x5[:-1]
y5=cos(20*x5+5*cos(x5))
Y5=fftshift(fft(y5))/N5
w5=linspace(-64,64,N3+1)
w5=w5[:-1]

#Magnitude spectrum plot for cos(20t+5cos(t))
figure(9)
title("Spectrum of $cos(20t+5cos(t))$")
xlabel(r"$\omega\rightarrow$")
ylabel(r"$\|Y|\rightarrow$")
xlim([-40,40])
plot(w5,abs(Y5))
grid()

#Phase spectrum plot for cos(20t+5cos(t))
figure(10)
title("Phase plot of $cos(20t+5cos(t))$")
xlabel(r"k$\rightarrow$")
ylabel(r"$\angle Y(\omega)\rightarrow$")
xlim([-40,40])
ii=where(abs(Y5)>1e-3)
plot(w5[ii],angle(Y5[ii]),'go')
grid()
show()
```

We obtained the following plots for spectrum magnitude and phase:

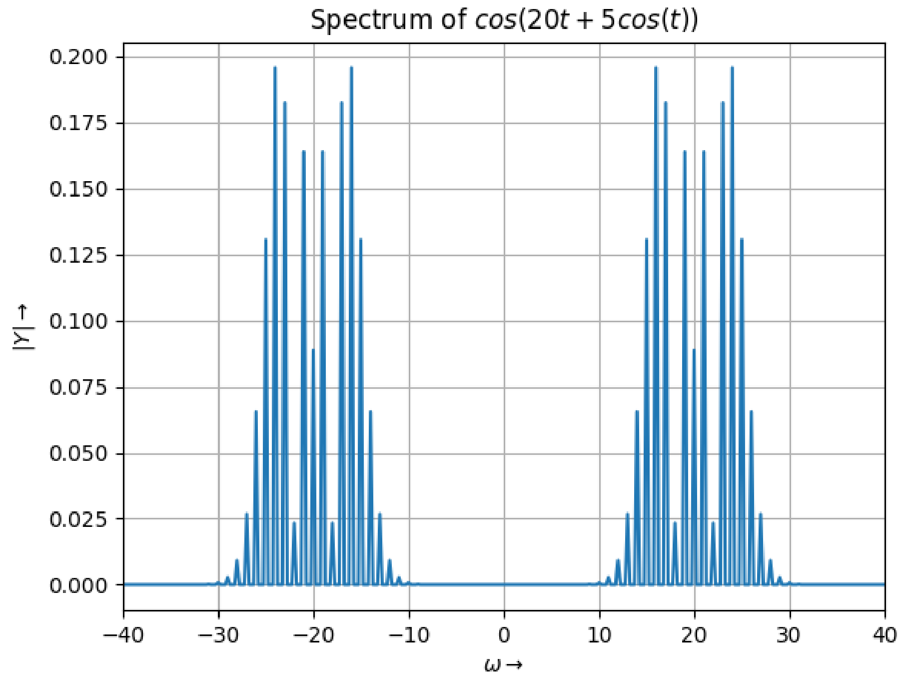

Figure 9: Magnitude spectrum of $\cos(20t + 5\cos(t))$

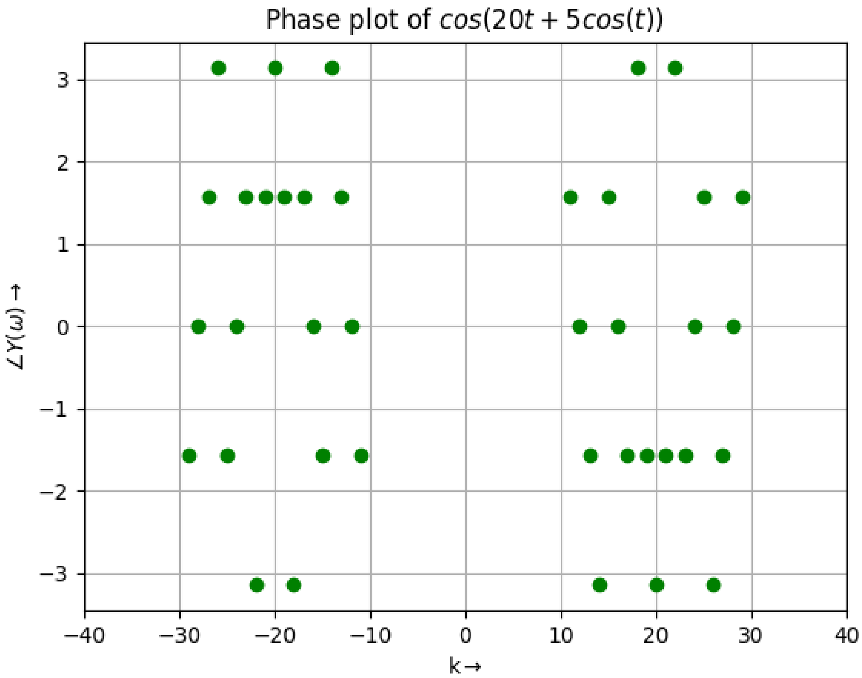

Figure 10: Phase plot of $\cos(20t + 5\cos(t))$

We can clearly see that the number of peaks have increased in case of frequency modulated signal when compared to the amplitude modulated signal, and also the energy of side bands is now comparable to the energy of main band.

The phase spectra is a mix of different phases from $[-\pi, \pi]$ because of phase modulation, i.e. since the phase is changed continuously with respect to time, the carrier signal can represent either a sine or cosine depending on the phase contribution from $\cos(t)$.

Also the strength of the side band frequencies either decays or is very small as they are away from the center frequency or carrier frequency component as we observe from the plot.

4. QUESTION 4

The gaussian $e^{-\frac{t^2}{2}}$ is not bandlimited in frequency. The most interesting thing about a gaussian is that the CTFT (Continous time fourier transform) of a gaussian is a gaussian itself. The Fourier transform of gaussian is :

$$F(j\omega) = \sqrt{2\pi}e^{-\frac{\omega^2}{2}}$$

In order to spectrum accurate to 6th digit our tolerance in the error value is 10^{-6}. We take different values of N(No. of samples) and Sampling time window size and check for the error. While doing this it was found that for N=512 and sampling time window size of 8.0π, the error is of the order of magnitude of 10^{-15}.This is executed in python as follows:

Code

```
def err(N,range_t):
  t = linspace(-range_t,range_t,N+1)
  t=t[:-1]
  y = exp(-0.5*t**2)
  Y = fftshift(abs(fft(y)))/N
  Y=Y*sqrt(2.0*pi)/max(Y)
  w=linspace(-32,32,N+1)
  w=w[:-1]
  Y_ctft = exp(-0.5*w**2)*sqrt(2.0*pi)

  figure()
  title(r"Spectrum of $e^{-t^{2}/2}$")
  xlabel(r"$\omega\rightarrow$")
  ylabel(r"$\|Y|\rightarrow$")
  xlim([-10,10])
  plot(w,abs(Y))
  grid()

  figure()
  title(r"Spectrum of $e^{-t^{2}/2})$")
  xlabel(r"k$\rightarrow$")
  ylabel(r"$\angle Y(\omega)\rightarrow$")
  xlim([-10,10])
```

```
    plot(w,angle(Y),'ro')
    ii=where(abs(Y)>1e-3)
    plot(w[ii],angle(Y[ii]),'go')
    grid()
    show()

    print("For N={} and time window range={}:".format(N,range_t))
    print("The value of maximum error is: {}".format(abs(Y-Y_ctft).max()))
    print("")

err(512,4.0*pi)
err(512,8.0*pi) #The value pair which gives the most precise result
err(512,12.0*pi)
err(256,8.0*pi)
err(1024,8.0*pi)
```

The maximum value of error that we got for different cases is shown below:

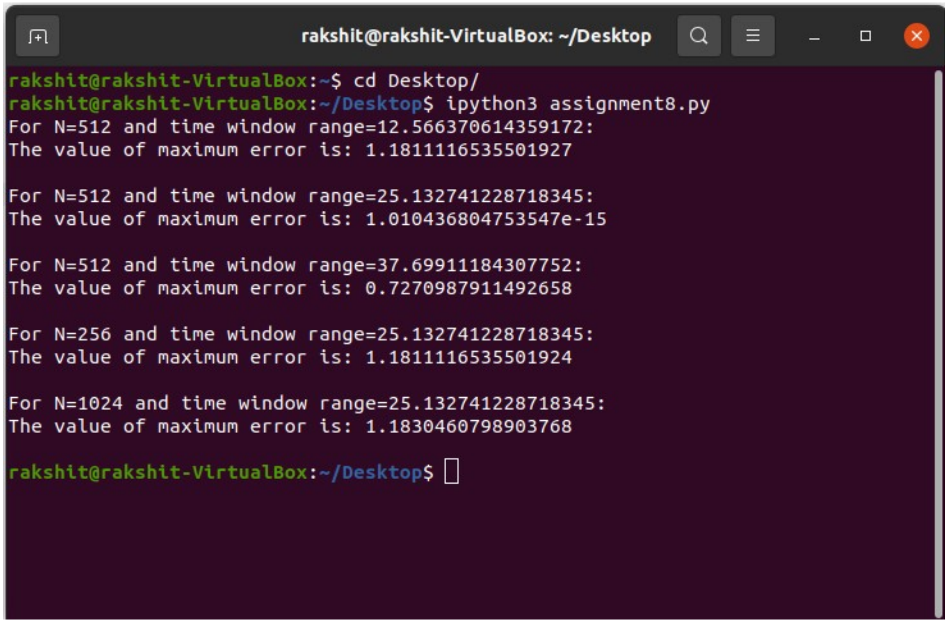

Figure 11: Maximum error values for different N and sampling time window

From the above image it is clear that The most precise results are obtained for N=512 and time window range of 8π, i.e 25.13274122871834 . The following curves were obtained for different cases:

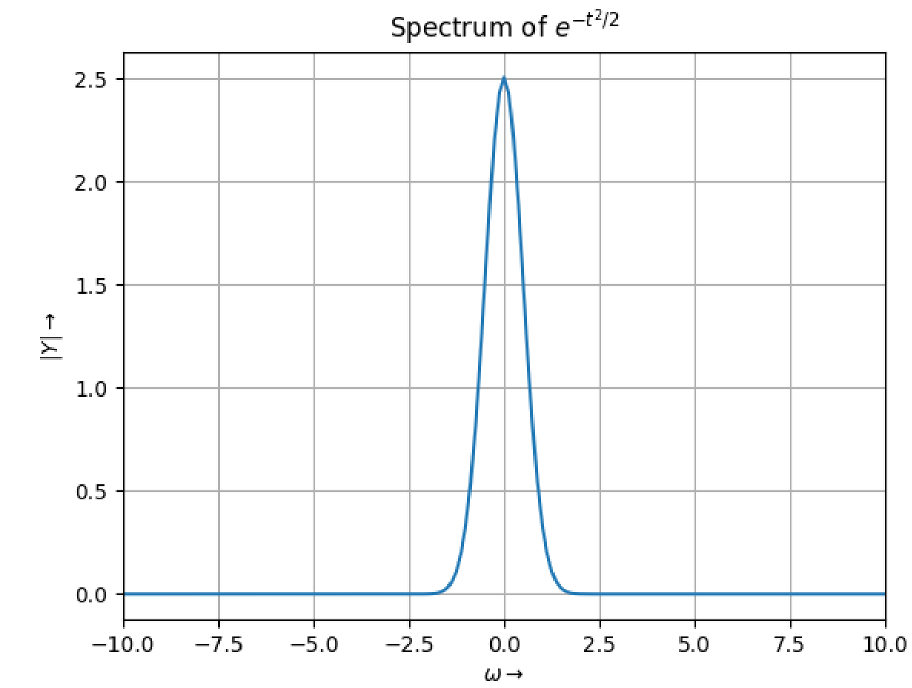

Figure 12: For N=512 and window range = 4π

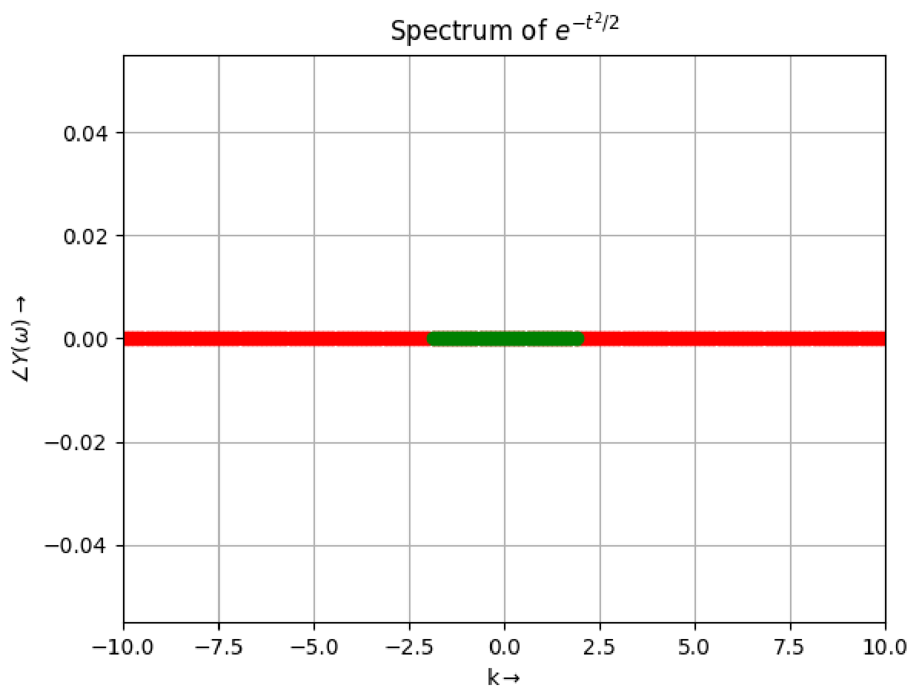

Figure 13: Phase plot For N=512 and window range = 4π

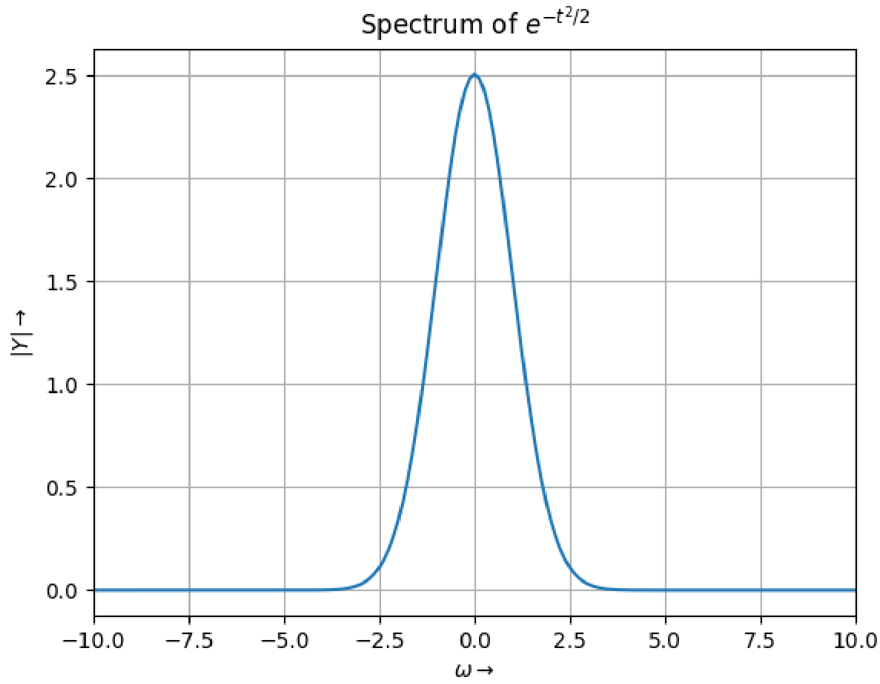

Figure 14: For N=512 and window range = 8π

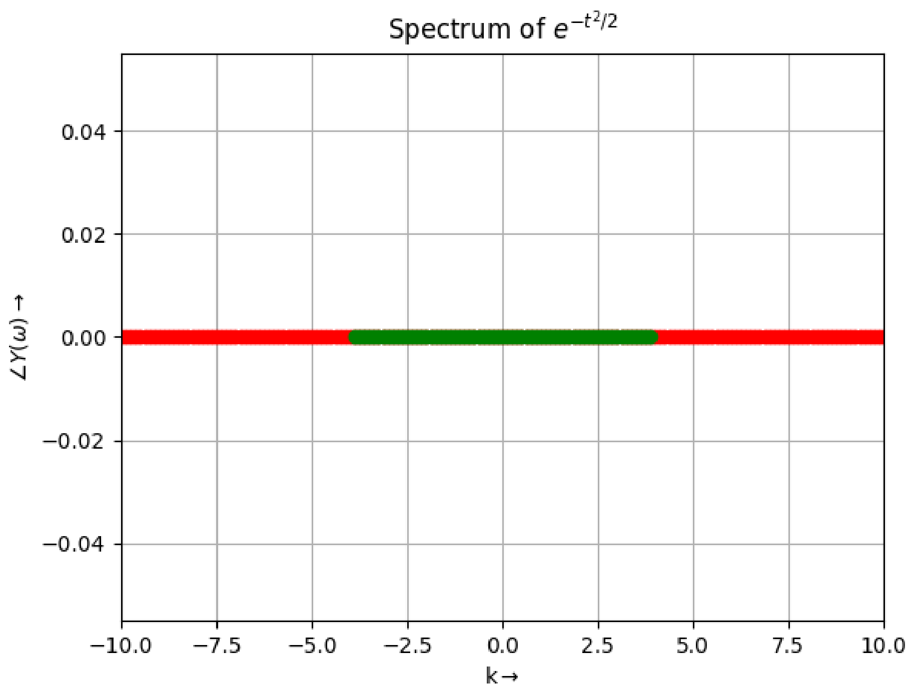

Figure 15: Phase plot for N=512 and window range = 8π

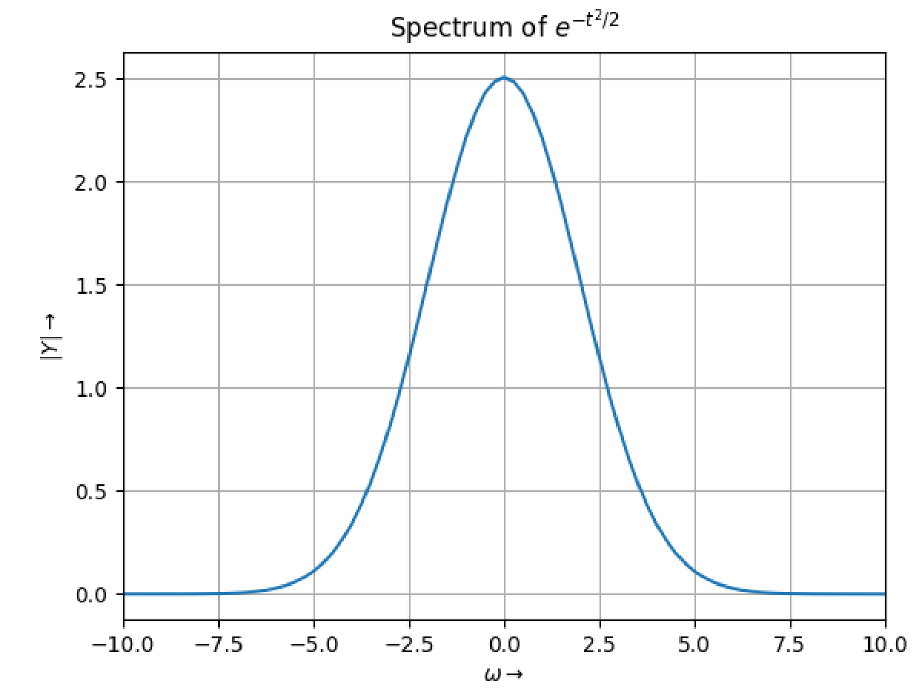

Figure 16: For N=256 and window size = 8π

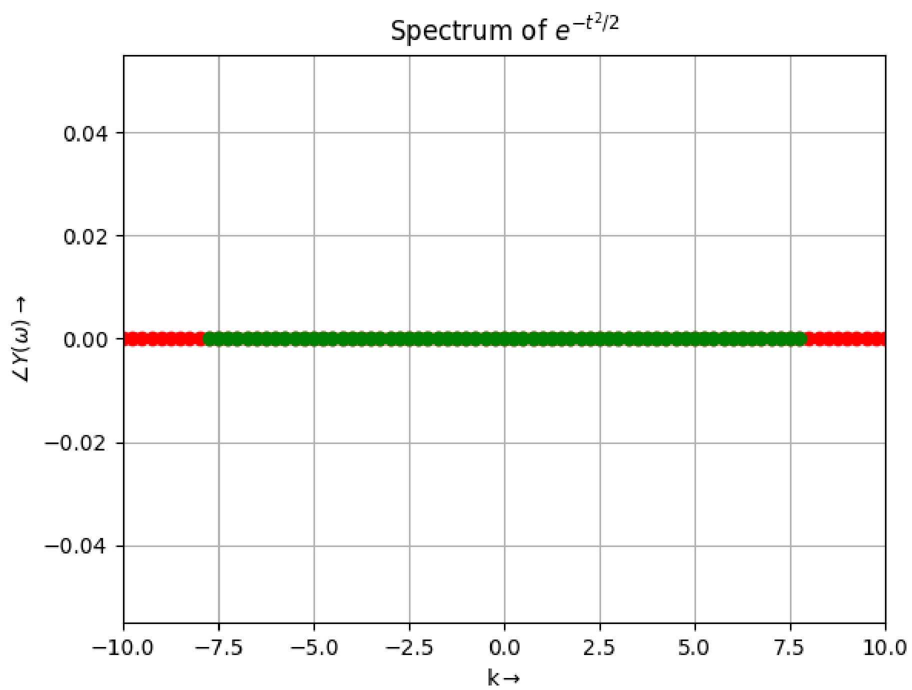

Figure 17: Phase plot For N=256 and window size = 8π

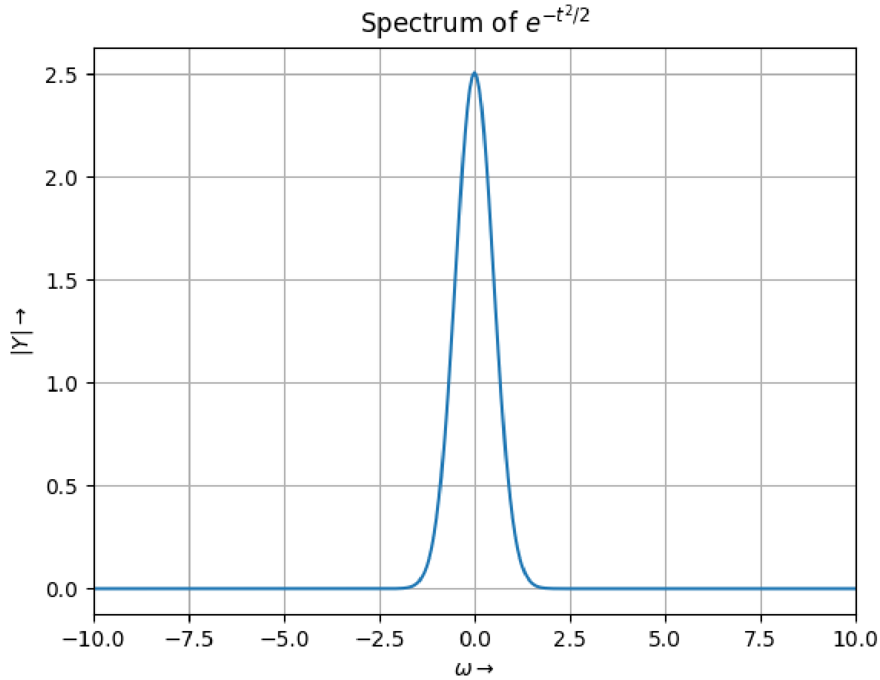

Figure 18: For N=1024 and window size = 8π

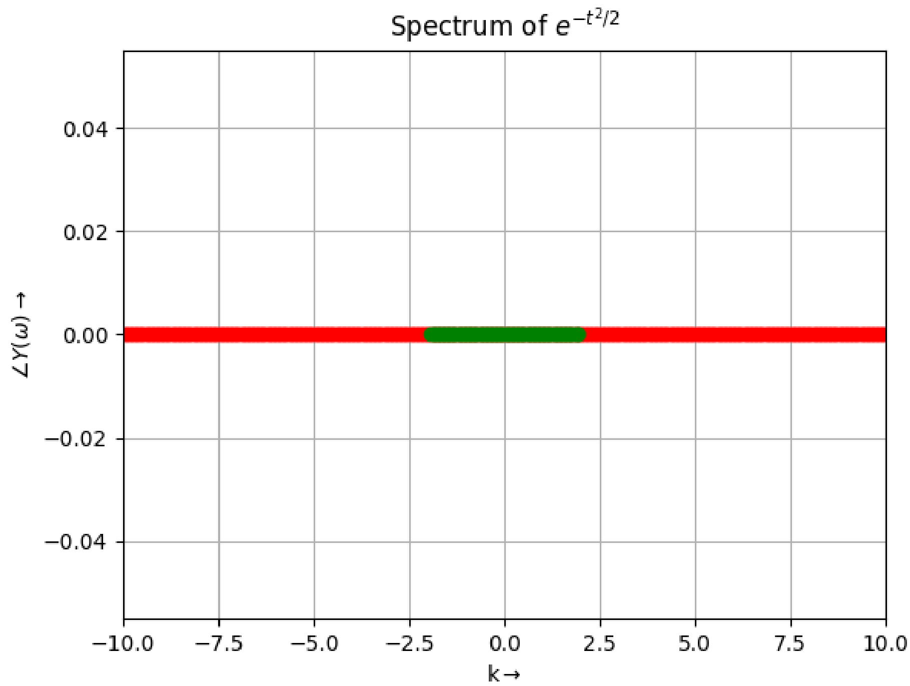

Figure 19: Phase plot for N=1024 and window size = 8π

4.CONCLUSION

- Through this assignment we got to work with the fft library of numpy for finding the DFT(Digital Fourier Tranform) of various sampled Continous time functions.
- We found the DFT of simple sinusoid function, then we did Amplitude modulated siganl followed by a frequency modulated signal, and then we finally did DFT analysis for non-bandlimited gaussian function.
- We used the fast fourier transform method to compute DFT as it improves the computation time.
- We also realized the need to use fftshift and ifftshift to fix the distorted phase responses.