Saptarshi's Math Notes

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1 Equations with Complex Solutions

$$\sin(x)^{\sin(x)} = 2$$
$$\implies \sin(x)\ln\sin(x) = \ln(2)$$

Let's assume $\sin(x) = e^a$

$$\therefore ae^a = \ln(2)$$
$$\implies a = W(\ln(2))$$

W is the Lambert W function

$$\therefore \sin(x) = e^{W(\ln(2))}$$
$$\sin(x) = e^{W(0.6931)} = 1.5596 \approx 1.56$$

Clearly this has no real solution. But there might be a complex one:

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} = 1.56$$

Let's assume $u = e^{ix}$

$$\therefore u^2 - u^{-1} = 3.12i$$
$$\implies u^2 - 3.12iu - 1 = 0$$

Taking $a = 1, b = -3.12i, c = -1$

$$\therefore u = \frac{3.12i \pm \sqrt{-9.7344 + 4}}{2}$$
$$\therefore u = \frac{3.12i \pm 2.4i}{2} = 1.56i \pm 1.2i$$
$$\therefore e^{ix} = 1.56i \pm 1.2i$$
$$ix = \ln(1.56i \pm 1.2i) = \ln(i) + \ln(1.56 \pm 1.2)$$

We know, $i = e^{i\frac{\pi}{2}}$

$$\therefore ix = \frac{\pi}{2}i + \ln(1.56 \pm 1.2)$$
$$\implies \boxed{x = \frac{\pi}{2} - \ln(1.56 \pm 1.2)i}$$