Automatic Fingerprint Recognition

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Abstract

Stuff happened.

1 Introduction

Fingerprints have been used to identify people since the 19^{th} century and have been used in criminal investigations since about that time. More recently fingerprints have been used as biometric markers used in boarder control, library stock control and computer and building access control systems. The need for a robust automatic fingerprint recognition system is obvious.

Most fingerprint recognition systems in use are based on the idea of identifying minutiae (points where a ridge ends or joins with another ridge) — in this article a system that uses the greylevel gradient to find minutiae is discussed.

Traditionally, fingerprint analysis consisted of first processing the image in some way, usually either binarization or line thinning. Methods like this have two major drawbacks. Firstly, such alterations to the image can be computationally expensive and therefore take a considerable amount of time for large images, which is a serious problem when used in real time applications. Secondly, non-reversible image manipulation can lead to destruction of information about the fingerprint.

2 Theory and Implementation

The method used in the article does not rely on prior image manipulation. It works by think of the image as a three dimensional object, with the greylevel of time image being the height in the Z-direction. The tangent of this object is then calculated at the position of each pixel in the image. A line can then be traced along the image by taking steps in the direction orthogonal to these tangents. By building up such lines from different starting points in the image, we can classify the entirety of the fingerprint. The two types of minutia we are looking for can be found from this complete classification. Where a line ends without touching another

line or the edge of the image is a termination and points where one line hits the middle of another are bifurcations.

2.1 Point Normal Direction

The point normal function works by taking four mutually adjacent points (*i.e.* a 2×2 array of pixels) and fitting a plane to them. If the greylevel at the pixel (x_k, y_k) is denoted h_k and the level from the fitted plane is denoted p_k then the plane fitting part of the function can be seen as minimising the following expression:

$$\min_{n_1,n_2,c} \sum_k |h_k - p_k|^2 \qquad (1)$$

Which in matrix form can be expresed as:

$$\left| \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{pmatrix} - \begin{pmatrix} -x_1 & -y_1 & 1 \\ -x_2 & -y_2 & 1 \\ -x_3 & -y_3 & 1 \\ -x_4 & -y_4 & 1 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ c \end{pmatrix} \right|^2 \qquad (2)$$

Where n_1, n_2 and c are the x, y and z components of the surface normal respectively. This is a simple least-squares minimisation and after some rearranging we obtain the following expressions for the optimum surface normal:

$$\begin{aligned} n_1 &= \frac{-h_1 + h_2 + h_3 - h_4}{4} \\ n_2 &= \frac{-h_1 - h_2 + h_3 + h_4}{4} \\ c &= \frac{h_1 + h_2 + h_3 + h_4}{4} \end{aligned} \qquad (3)$$

Since we are only concerned with the components in the x, y–plane the function implemented here doesn't bother calculating c (figure 1). Once all 2×2 neighbourhoods have been fitted the function returns n_1 and n_2 then terminates.

As the NPD function needs a four points to fit a plane to the decision has to be made regarding how edges are handled. One possibility is to 'wrap' the edges of the image around, effectively forming a torus — this was not implemented here because it would mean that for two edges the values of n_1 and n_2 would depend on greylevels from the other two edges. The second possibility (implemented here) is to make the output array smaller by 1 pixel in both dimensions, so if the original image is $n \times m$ pixels the array of normals is $(n-1) \times (m-1)$. From figure 1 it can be seen that the PND function implemented here loses the top and right hand pixels.

2.2 Averaged Tangent Direction

The ATD function calculates the x, y–plane tangent that best fits with the surface normals generated by the PND function in a given $n \times n$ neighbourhood. This has the effect of

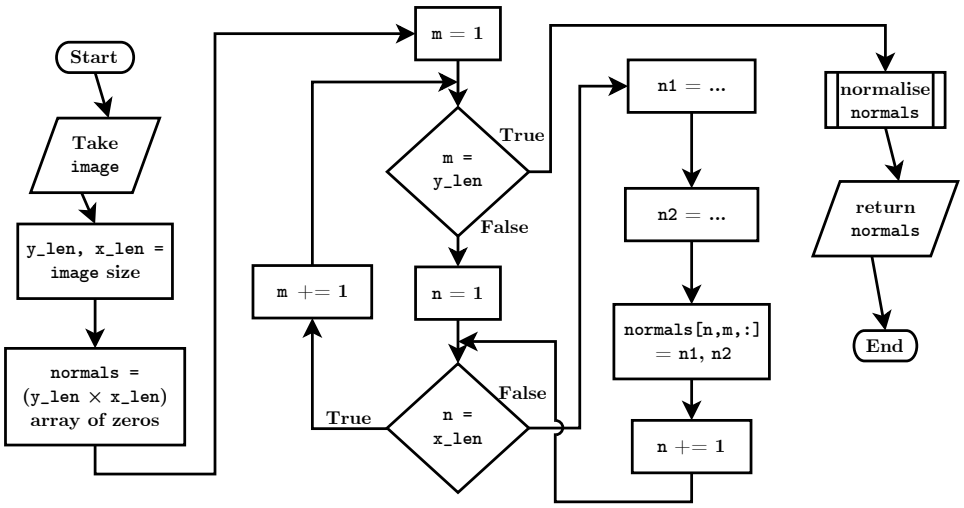


Figure 1: Flow chart of the PND function as implemented.

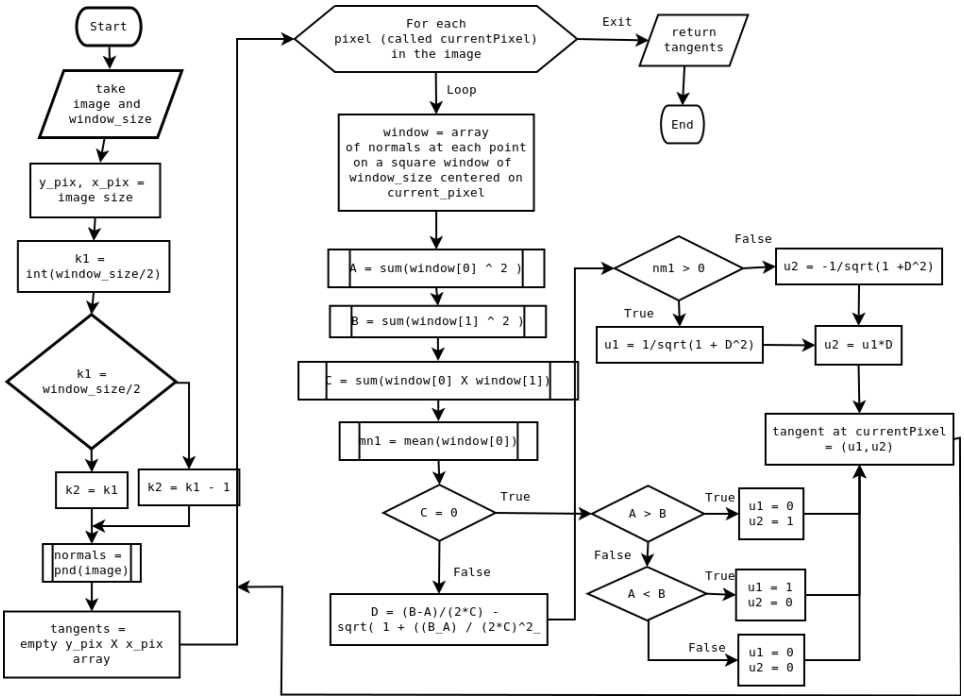


Figure 2: Flow chart of the ATD function as implemented.

smoothing out (plane normal angle) noise but, as with analogous smoothing operations, can obscure features with a size comparable to that of the kernel chosen. The fitting is a least squares minimisation of the row-by-row dot product:

$$\min_{u_1,u_2} \sum_k |(n_{1,k}, n_{2,k}) \cdot (u_1, u_2)|^2 = U(u_1, u_2) \tag{4}$$

Where $n_{1,k}$ is the k^{th} n_1 value as defined above *e.t.c.* and (u_1, u_2) is the normalised fitted vector. If we define the following:

$$\begin{aligned} A &= \sum_k (n_{1,k})^2 \\ B &= \sum_k (n_{2,k})^2 \\ C &= \sum_k n_{1,k} n_{2,k} \end{aligned} \tag{5}$$

then we can express $U(u_1, u_2)$ as

$$U(u_1, u_2) = (u_1, u_2) \begin{pmatrix} A & C \\ C & B \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{6}$$

The eigenvalues of the above 2×2 matrix, λ_1 and λ_2, are the maximum and minimum values of U respectively. It is trivial to determine that the two eigenvalues are given by:

$$\begin{aligned} \lambda_1 &= \frac{A + B + \sqrt{(A-B)^2 + 4C^2}}{4} \\ \lambda_2 &= \frac{A + B - \sqrt{(A-B)^2 + 4C^2}}{4} \end{aligned} \tag{7}$$

With some further rearranging of equation (6) we find that:

$$u_1 = \sqrt{\frac{1}{1 + \left(\frac{\lambda_2 - A}{C}\right)^2}} \tag{8}$$

$$u_2 = u_1 \left(\frac{\lambda_2 - A}{C}\right) \tag{9}$$

In implementing these equations several special cases have to be taken into account. The first special case is when $C = 0$, which would require deviding by zero. To cope with this case the function is set to test if $C = 0$ and if so the function will set $(u_1, u_2) = (1, 0)$ or $(u_1, u_2) = (0, 1)$ depending on weather $A < B$ or $B > A$ respectively. The second special case is when $\lambda_1 = \lambda_2$, which the function tests for after determining if $C = 0$. In this case there is no optimum orientations thus $(u_1, u_2) = (0, 0)$.

A final problem with implementing equation (8) is the fact that u_1 can only ever be positive. This ensures that in the x–direction one cannot distinguish between the two sides edges of a ridge. To solve this the function calculates the mean of the n_1 values then checks if this mean is negative and if so it makes u_1 negative.

2.3 The Ridge Follower

The ridge following system takes a slice (of width $2\sigma + 1$ and height 1 pixel centred around pixel (x_c, y_c)) of the image, finds the peak of the ridge and moves to it, calculates the direction of the ridge (ϕ_c) then moves forward by μ pixels before starting the process again (figure 3). In this manor the ridge follower follows the ridge, and since the function logs where it's been it's possible to determine where the minutiae are.

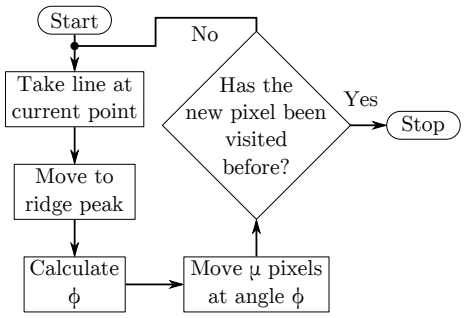


Figure 3: Simple version of the ridge following algorithm.

Taking a line at a given point is simple enough: the current pixel, (x_c, y_c), is the centre of the line and by simple trig the coordinates of the pixels that make up the line are

$$\begin{pmatrix} x_{l,k} \\ y_{l,k} \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + (k - \sigma) \begin{pmatrix} \cos(\phi_c + \pi/2) \\ \sin(\phi_c + \pi/2) \end{pmatrix} \quad (10)$$

where $x_{l,k}$ is x–coordinate of the k^{th} pixel of the line *e.t.c.* and k ranges from zero to $2\sigma+1$. Using these coordinates one can extract the relevant greylevels and plane normals.

Identifying the peak of the ridge is somewhat involved if noise tolerance is needed. One method is to find the point midway between the two angles that are closest to the end of the lines. Another method is to try and smooth out the ridge to ensure that there is just one peak in it - by taking the average of a line in front of and behind the current point then convolving along the line with a Gaussian mask one can effectively find the 'weak' maxima of the ridge. This weak maxima may not be the actual centre of the ridge but it should be near enough.

In the algorithm employed here a more simplistic (and not as noise tolerant) method that simply finds the darkest point in the line. This method is vulnerable to local minima due to noise and has no way of coping with plateau ridges.

Calculating ϕ_c is trivial but demands that some care to be taken. Using the result of the PND or ATD functions one can easily determine the angle of the greylevel tangent and add or subtract $pi/2$ from the angles to get ϕ_c. In the algorithim implemented here the two possible values of ϕ_c are compared to the old value of ϕ_c and the one with the smallest difference is chosen.

Moving forward is trivial. Using basic trigonometry one can see that the following is true:

$$\begin{pmatrix} x_{c+1} \\ y_{c+1} \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + \mu \begin{pmatrix} \cos\phi_c \\ \sin\phi_c \end{pmatrix} \quad (11)$$

3 Results & Analysis

In order to test the various functions we use several images, shown in figure 4. Each of these images present their own challenges: 4a, 4b and 4d all have quite wide ridges that plateau while 4b and 4c both have significant levels of noise that, if not properly dealt with, could easily lead to the ridge follower missing the true path of a ridge.

3.1 The PND function

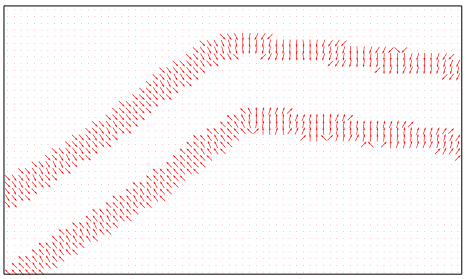


Figure 5: Vectors correctly extracted from a section of figure 4a.

As can be seen in figure 5 the PND function does correctly calculate surface normals for a noise free image. It should be noted that in the system developed here black represents a low pixel value while white is high (as is standard for greyscale images). If PND is applied to 4b the result is not as satisfactory

3.2 The ATD Function

The results of the ATD function are not as encouraging. The primary problem lies with the system that determines how the various cases should be dealt with.

3.3 The Ridge Follower

The results here are more encouraging - for noise free images the ridge follower is able to

4 Conclusion

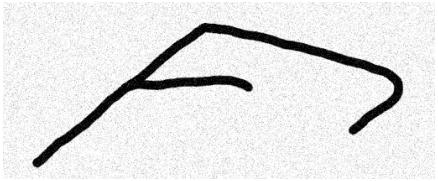
A Code

```
#! /usr/bin/python3

## Basic fragments of code that will probably be useful

## Uses axes, imshow, colorbar, title, savefig, close, figure
## from matplotlib.pyplot
from matplotlib.pyplot import axes, imshow, colorbar, title, \
     savefig, close, figure
def show_pic(image,plot_name='Test Image',colourmap=None):
```

(a) Simple wide curve.



(b) Simple wide curve with noise added.

(c) Scan of a real fingerprint.

(d) Computer generated fingerprint.

Figure 4: Example images used to test the functions written.

```
    """Method to simply shwo the image. Useful for tests"""
    figure(facecolor='white', figsize=(5,4))

    # Remove axes from the plot
    ax = axes(frameon=False)
    ax.get_yaxis().set_visible(False)
    ax.get_xaxis().set_visible(False)

    # Give the image a title
    title(plot_name)

    if colourmap != None:
        # Make image with a colourmap
        imshow(image, interpolation='none',cmap=colourmap)
        colorbar()
    else:
        # Make an image without a colourmap
        imshow(image, interpolation='none')


def save_pic(image, plot_name, colourmap=None):
```

```
    """A function that nicley abstracts producing an image object"""
    figure(facecolor='white', figsize=(5,4))

    # Remove axes from the plot
    ax = axes(frameon=False)
    ax.get_yaxis().set_visible(False)
    ax.get_xaxis().set_visible(False)

    # Give the image a title
    title(plot_name)

    if colourmap != None:
        # Make image with a colourmap
        imshow(image, interpolation='none',cmap=colourmap)
        colorbar()
    else:
        # Make an image without a colourmap
        imshow(image, interpolation='none')
    savefig("{:s}.pdf".format(plot_name).replace(' ', '_'))
    close()

## Uses sqrt, sum
def normalise(array):
    """Normalises a 3D array"""
    from numpy import sqrt
    yl = len(array)
    xl = len(array[0])
    for n in range(yl):
        for m in range(xl):
            y = array[n,m,0]
            x = array[n,m,1]

            if x == 0 and y == 0:
                array[n,m,:] = 0 , 0
            else:
                array[n,m,:] = array[n,m,:]*(x**2 + y**2)**(-0.5)

    return array

## Uses zeros and array from numpy
def pnd(image):
    """Returns the point normal determination"""
    from numpy import zeros,array
    # Get image dimensions
```

```
    y_pix = len(image)
    x_pix = len(image[0])

    # Make array to hold normals
    normals = zeros((y_pix-1, x_pix-1,2))

    # Loop through and calculate the normal vector
    for n in range(1, y_pix):
        for m in range(1, x_pix):
            n1 = (-image[n, m] + image[n-1, m] + image[n-1, m-1] \
                - image[n, m-1])/4
            n2 = (-image[n, m] - image[n-1, m] + image[n-1, m-1] \
                + image[n, m-1])/4

            normals[n-1, m-1, :] = n1, n2

    normals = normalise(normals)

    return normals

## Uses zeros, sum, sqrt from numpy
def atd(image, window=9):
    """Returns the averaged tangent diraction of a normal array"""
    from numpy import zeros, sum, sqrt, array, mean
    # Get image dimensions
    y_pix = len(image)
    x_pix = len(image[0])

    # Calculate window half-size
    k1 = int(window/2)
    if k1 == window/2:
        k2 = int(window/2) - 1
    else:
        k2 = k1

    # Make array to hold tangents
    tangents = zeros((y_pix-1, x_pix-1,2))

    # Calcualte normals
    normals = pnd(image)

    # Test to see if a pixel is in the image
    inImageTest = lambda ky,kx : not ((ky < 0) and (kx < 0) and \
                                      (ky > y_pix - k1 - 1) and \
```

```
                                        (kx > x_pix - k1 - 1))

    # Loop through and calculate ATD
    for n in range(1,y_pix-(k1+1)):
        for m in range(1,x_pix-(k1+1)):
            # Extract window from image
            window = [normals[ky,kx] for ky in range(n-k1,n+k2) \
                      for kx in range(m-k1,m+k2) if inImageTest(ky,kx)]

            # Make window a numpy array for easier indexing
            window = array(window)

            # Calculate normalised ATD
            A = sum(window[:,0]**2)
            B = sum(window[:,1]**2)
            C = sum(window[:,0]*window[:,1])
            mn1 = mean(window[:,0])

            # Diagonal matrix case
            if C == 0 and A < B:
                u1 = 1
                u2 = 0

            elif C == 0 and A > B:
                u1 = 0
                u2 = 1

            # Plateau
            elif ((A+B) + sqrt((A-B)**2 + 4*C**2)) == \
                 ((A+B) - sqrt((A-B)**2 + 4*C**2)):
                u1 = 0
                u2 = 0

            # Non-diagonal case
            else:
                D = (B - A)/(2*C) - sqrt(1 + ((B - A)/(2*C))**2)

                # Ensure that u1 points in the correct direction
                if mn1 > 0:
                    u1 = 1/sqrt(1 + D**2)
                else:
                    u1 = -1/sqrt(1 + D**2)

                u2 = u1*D
```

```
                tangents[n,m,:] = u1, u2

    return tangents

def OC1(A, B, C, D, E, xl, yl):
    """Calculates the curvature point via method 1 - needs ADT results"""
    # Straight line case
    if A*B == C**2:
        x = xl/2
        y = yl/2

    # Curved line case
    else:
        x = (B*D - C*E)/(A*B - C**2)
        y = (A*E  - C*D)/(A*B - C**2)

    pc = x, y

    return pc

def OC2(A, B, C, D, E, M, xl, yl):
    """Calculates  the curvature point via method 2 - needs ADT results"""
    # Two points case
    if not M == 0:
        a = D**2/E + (A - B)*D - E
        b = (B - A)*M - D**2 - E**2 + 2*M*D/E
        c = (C*M/E + D)*M

        x = (-b - (b**2 - 4*a*c))/(2*a)
        y = (M - D*x)/E

    # Infinite curvature case
    else:
        x = xl/2
        y = yl/2

    pc = x, y

    return pc

## Uses sum from numpy
def OC_switch(window, threshold=0.1):
    """Decided which OC function to use"""
```

```
from numpy import sum
# Get the window size
yl = len(window)
xl = len(window)

# Calculate vector weighting parameter
r = [n*window[n,m,0] + m*window[n,m,1] for n in range(yl) \
     for m in range(xl)]

# Calculate curvature paramaters
A = sum(window[:,:,0]**2)
B = sum(window[:,:,1]**2)
C = sum(window[:,:,0]*window[:,:,1])
D = sum(window[:,:,0]*r)
E = sum(window[:,:,1]*r)
M = sum(r**2)

# Calculate decision eigenvalue
l = (A + B - ((A - B)**2 + 4*C**2)**0.5)/(2*yl*xl)

# Decide which OC to use
if l > threshold:
    pc = OC1(A, B, C, D, E, xl, yl)
else:
    pc = OC2(A, B, C, D, E, M, xl, yl)

return pc
```