Checking for linear transformation

1. $T(0) = 0$
2. $T(\alpha u) = \alpha T(u)$
3. $T(u+v) = T(u) + T(v)$

$T_5 : \mathbb{R}^5 \to \mathbb{R} \Rightarrow T_5\left(\begin{pmatrix} x_1 \\ \vdots \\ x_5 \end{pmatrix}\right) = x_3 + 2x_4 - x_5$ for $\begin{pmatrix} x_1 \\ \vdots \\ x_5 \end{pmatrix} \in \mathbb{R}^5$

Recall that linear equations can be written as inner products.

↳ $x_3 + 2x_4 - x_5 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_5 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 1 & 2 & -1 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ \vdots \\ x_5 \end{pmatrix}$

$T : \mathbb{R}^4 \to \mathbb{R}$

Standard matrix of T

→ no. of rows = size of codomain → size of $\mathbb{R}$

→ no. of columns = size of domain → size of $\mathbb{R}^4$

⇒ 1×4 matrix

Another way:

→ Standard matrix

↳ Input : 4×1 matrix

↳ output : 1×1 matrix

↳ Standard matrix must be of size 1×4