MA1522

Chapter 1

linear equation : standard form

→ $ax_1 + bx_2 + cx_3 + \dots = z$, where $a, b, c, \dots, z$ are constants

linear system

→ set of linear equations in standard form

→ e.g. $\begin{cases} ax_1 + bx_2 = y \\ cx_1 + dx_3 = z \end{cases} \longrightarrow$ Augmented Matrix $\left(\begin{array}{cc|c} a & b & y \\ c & d & z \end{array}\right)$

→ homogeneous → Always consistent

↳ linear system = 0 ⇒ $Ax = 0$

→ non-homogeneous

↳ linear system ≠ 0 ⇒ $Ax = b$

Solutions to linear systems

→ solving for values of $x_1, x_2, \dots$

→ inconsistent linear systems ⇒ NO solutions

↳ NOT able to solve for $x_1, x_2, \dots$

↳ inconsistent iff RHS of augmented matrix is pivot column

→ consistent linear system

↳ able to solve for $x_1, x_2, \dots$

↳ General solutions

→ Infinite number of solutions with x number of free parameters

→ e.g. $\begin{cases} x + 2y = 5 \\ 2x + 4y = 10 \end{cases}$

General solution : $x = 5 - 2s$, $y = s$, $s \in \mathbb{R}$

→ sub non-pivot columns with free parameters.

↳ Unique solutions

REF (Row Echeleon Form)

-> zero rows on the bottom of matrix

-> leading entry (first non-zero entry from left of non-zero row) moves further right as we go down the rows

RREF (Reduced Row Echeleon Form)

-> adds on to REF

-> leading entries are 1

-> pivot columns (columns with leading entries) contain only the leading entry, the rest are all 0's.

ERO (Elementary Row Operations)

-> Exchange 2 rows : $R_1 \leftrightarrow R_2$

-> Add multiple of one row to another : $R_1 + 2R_2$

-> Multiply a row by a constant : aR_1, $a \neq 0$

* -> ERO do not change the solution to the system.

-> ERO do not commute

↳ $R_1 \leftrightarrow R_2, 2R_2 \neq 2R_2, R_1 \leftrightarrow R_2$

-> Reversing ERO

↳ Exchange 2 rows : $R_1 \leftrightarrow R_2 \Rightarrow R_2 \leftrightarrow R_1$ | $R_1 \leftrightarrow R_2$ (swap again)

↳ Add multiple of one row to another : $R_1 + 2R_2 \Rightarrow R_1 - 2R_2$ (subtract)

↳ Multiply a row by a constant : $aR_1 \Rightarrow \frac{1}{a}R_1$ (divide)

Row Equivalent Matrices.

-> $A \xrightarrow{ERO} B$ => A and B are row equivalent matrices.

-> 2 matrices are row equivalent if they have the same solution

Gaussian Elimination (achieve REF)

1. Find leftmost non-empty column
2. Make sure 1st entry in column is not 0. if it is, row swap with next row with non-zero entry
3. Make the entries below top row in first column 0 by adding a suitable multiple of the top row to the subsequent rows.
4. Ignore first row, repeat steps 1 to 3 for rest of matrix.

Gauss-Jordan Elimination (achieve RREF from Gaussian Elimination)

5. Multiply a suitable constant to each row to make the leading entries 1.
6. From bottom up, locate the first non-zero row. Add suitable multiples of each row to rows above to introduce 0's.

Linear systems with unknowns

→ REF to a point where the unknowns become the leading entries.
→ solve by cases: make the leading entries = 0 and see what happens.