Statements of Invertibility

Suppose A is square matrix of order n.

1. A is invertible
2. A^T is invertible.
3. left inverse : $BA = I$
4. right inverse : $AB = I$
5. RREF of A is I $\Rightarrow$ A is row equivalent to I
6. A can be expressed in terms of a product of elementary matrices
   ↳ as elementary matrices are row equivalent to identity matrices
7. $Ax = 0$ has only the trivial solution
8. $Ax = b$ has a unique solution

} as RREF of A is I.

9. $\det(A) \neq 0$
   ↳ $\det(A^{-1}) = \det(A)^{-1} = \frac{1}{\det(A)}$
   ↳ $\det(A) = 0 \Rightarrow A$ is squished to a lower dimension, hence not invertible
10. The columns / rows of A are linearly independent
    columns of A = rows of A^T
    rows of A = columns of A^T
11. The columns / rows of A spans $\mathbb{R}^n$

} the set of vectors in A form a basis for $\mathbb{R}^n$

12. A has full rank $\Rightarrow$ rank$(A) = n$
13. nullity $(A) = 0$

} by Rank-Nullity Theorem, rank (A) + nullity $(A) = n$.

14. 0 is not an eigenvalue of A.
    ↳ $Av = \lambda v = 0$ will non-trivial solution
15. The transformation represented by A is injective.
16. The transformation represented by A is surjective.
17. The transformation represented by A is bijective.