Fiche d'entraînement : calculs de primitives

Dans chacun des cas suivants, déterminer une primitive de la fonction proposée :

1) $f_1(x) = x + x^2$

2) $f_2(x) = 7 - 8x^3$

3) $f_3(x) = \dfrac{1}{x} + 2$

4) $f_4(x) = x - \dfrac{7}{x}$

5) $f_5(x) = \dfrac{x^2+1}{x^2}$

6) $f_6(x) = x^{-3}$

7) $f_7(x) = \mathrm{e}^{2x}$

8) $f_8(x) = \cos x + \sin x$

9) $f_9(x) = \cos(3x)$

10) $f_{10}(x) = 2x\,\mathrm{e}^{x^2}$

11) $f_{11}(x) = (2x-1)\,\mathrm{e}^{x^2-x}$

12) $f_{12}(x) = 2x\left(x^2+1\right)^3$

13) $f_{13}(x) = \dfrac{2x}{x^2+1}$

14) $f_{14}(x) = 11x + 3x^{-2}$

15) $f_{15}(x) = -7x^2 + \dfrac{4}{x^3}$

16) $f_{16}(x) = 12x\,\mathrm{e}^{x^2-3}$

17) $f_{17}(x) = \dfrac{x}{x^2+1}$

18) $f_{18}(x) = \sin^2 x \cos x$

19) $f_{19}(x) = \dfrac{6x+3}{x^2+x+1}$

20) $f_{20}(x) = \dfrac{x}{\left(x^2+1\right)^2}$

21) $f_{21}(x) = x\left(x^2+4\right)^{-3}$

22) $f_{22}(x) = x\,\mathrm{e}^{-x^2}$

23) $f_{23}(x) = \dfrac{\mathrm{e}^x}{\mathrm{e}^x+1}$

24) $f_{24}(x) = \dfrac{8x}{\sqrt{2x^2+1}}$

25) $f_{25}(x) = \dfrac{\ln x}{x}$

26) $f_{26}(x) = \dfrac{5x}{\left(x^2+1\right)^4}$

27) $f_{27}(x) = \dfrac{\sin x}{\cos x}$

28) $f_{28}(x) = \dfrac{\mathrm{e}^{-x}}{\mathrm{e}^{-x}+3}$

29) $f_{29}(x) = \dfrac{7x}{3x^2+3}$

Solutions

1) $F_1(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3$

2) $F_2(x) = 7x - 2x^4$

3) $F_3(x) = \ln(x) + 2x$

4) $F_4(x) = \frac{1}{2}x^2 - 7\ln(x)$

5) $f_5(x) = \frac{x^2+1}{x^2} = \frac{x^2}{x^2} + \frac{1}{x^2} = 1 + \frac{1}{x^2}$ donc $F_5(x) = x - \frac{1}{x}$

6) $F_6(x) = \frac{x^{-3+1}}{-3+1} = \frac{x^{-2}}{-2} = \frac{1}{-2x^2} = -\frac{1}{2x^2}$ car $x^{-2} = \frac{1}{x^2}$

7) $f_7(x) = \mathrm{e}^{2x} = \frac{1}{2} \times \underbrace{2}_{u'} \underbrace{\mathrm{e}^{2x}}_{\mathrm{e}^u}$ donc $F_7(x) = \frac{1}{2} \underbrace{\mathrm{e}^{2x}}_{\mathrm{e}^u}$

8) $F_8(x) = \sin x - \cos x$

9) $F_9(x) = \frac{1}{3}\sin(3x)$

10) $F_{10}(x) = \mathrm{e}^{x^2}$

11) $f_{11}(x) = \underbrace{(2x-1)}_{u'} \underbrace{\mathrm{e}^{x^2-x}}_{\mathrm{e}^u}$ donc $F_{11}(x) = \underbrace{\mathrm{e}^{x^2-x}}_{\mathrm{e}^u}$

12) $f_{12}(x) = \underbrace{2x}_{u'} \underbrace{\left(x^2+1\right)^3}_{u^3}$ donc $F_{12}(x) = \frac{\overbrace{\left(x^2+1\right)^4}^{u^{3+1}}}{\underbrace{4}_{3+1}}$

13) $f_{13}(x) = \frac{\overbrace{2x}^{u'}}{\underbrace{x^2+1}_{u}}$ donc $F_{13}(x) = \underbrace{\ln\left(x^2+1\right)}_{\ln(u)}$

14) $F_{14}(x) = \frac{11}{2}x^2 + 3 \times \frac{\overbrace{x^{-1}}^{x^{-2+1}}}{\underbrace{-1}_{-2+1}} = \frac{11}{2}x^2 - \frac{3}{x}$ car $x^{-1} = \frac{1}{x}$

15) $f_{15}(x) = -7x^2 + \frac{4}{x^3} = -7x^2 + 4x^{-3}$ donc $F_{15}(x) = \frac{-7}{3}x^3 + 4 \times \frac{\overbrace{x^{-2}}^{x^{-3+1}}}{\underbrace{-2}_{-3+1}} = \frac{-7x^3}{3} - \frac{2}{x^2}$ car $x^{-2} = \frac{1}{x^2}$

16) $f_{16}(x) = 12x\mathrm{e}^{x^2-3} = 6 \times \underbrace{2x}_{u'} \underbrace{\mathrm{e}^{x^2-3}}_{e^u}$ donc $F_{16}(x) = 6 \underbrace{\mathrm{e}^{x^2-3}}_{e^u}$

17) $f_{17}(x) = \frac{x}{x^2+1} = \frac{\frac{1}{2} \times \overbrace{2x}^{u'}}{\underbrace{x^2+1}_{u}} = \frac{1}{2} \times \frac{\overbrace{2x}^{u'}}{\underbrace{x^2+1}_{u}}$ donc $F_{17}(x) = \frac{1}{2} \underbrace{\ln\left(x^2+1\right)}_{\ln(u)}$

18) $f_{18}(x) = \underbrace{\sin^2 x}_{u^2} \underbrace{\cos x}_{u'}$ donc $F_{18}(x) = \frac{\overbrace{\sin^3 x}^{u^{2+1}}}{\underbrace{3}_{2+1}}$

19) $f_{19}(x)=\dfrac{6x+3}{x^2+x+1}=\dfrac{3\times\overbrace{(2x+1)}^{u'}}{\underbrace{x^2+x+1}_{u}}=3\times\dfrac{\overbrace{(2x+1)}^{u'}}{\underbrace{x^2+x+1}_{u}}$ donc $F_{19}(x)=3\underbrace{\ln\left(x^2+x+1\right)}_{\ln(u)}$

20) $f_{20}(x)=\dfrac{x}{\left(x^2+1\right)^2}=x\times\left(x^2+1\right)^{-2}=\dfrac{1}{2}\times\underbrace{2x}_{u'}\times\underbrace{\left(x^2+1\right)^{-2}}_{u^{-2}}$ donc $F_{20}(x)=\dfrac{1}{2}\times\dfrac{\overbrace{\left(x^2+1\right)^{-1}}^{u^{-2+1}}}{\underbrace{-1}_{-2+1}}=\dfrac{-1}{2\left(x^2+1\right)}=\dfrac{-1}{2x^2+2}$

car $\left(x^2+1\right)^{-1}=\dfrac{1}{x^2+1}$

21) $f_{21}(x)=x\left(x^2+4\right)^{-3}=\dfrac{1}{2}\times\underbrace{2x}_{u'}\times\underbrace{\left(x^2+4\right)^{-3}}_{u^{-3}}$ donc $F_{21}(x)=\dfrac{1}{2}\times\dfrac{\overbrace{\left(x^2+4\right)^{-2}}^{u^{-3+1}}}{\underbrace{-2}_{-3+1}}=\dfrac{-1}{4\left(x^2+4\right)^2}$ car $\left(x^2+4\right)^{-2}=\dfrac{1}{\left(x^2+4\right)^2}$

22) $f_{22}(x)=x\,\mathrm{e}^{-x^2}=\dfrac{-1}{2}\times\underbrace{-2x}_{u'}\,\underbrace{\mathrm{e}^{-x^2}}_{\mathrm{e}^{u}}$ donc $F_{22}(x)=\dfrac{-1}{2}\,\underbrace{\mathrm{e}^{-x^2}}_{\mathrm{e}^{u}}$

23) $f_{23}(x)=\dfrac{\overbrace{\mathrm{e}^{x}}^{u'}}{\underbrace{\mathrm{e}^{x}+1}_{u}}$ donc $F_{23}(x)=\underbrace{\ln\left(\mathrm{e}^{x}+1\right)}_{\ln(u)}$

24) $f_{24}(x)=\dfrac{8x}{\sqrt{2x^2+1}}=\dfrac{2\times\overbrace{4x}^{u'}}{\underbrace{\sqrt{2x^2+1}}_{\sqrt{u}}}=2\times\dfrac{\overbrace{4x}^{u'}}{\underbrace{\sqrt{2x^2+1}}_{\sqrt{u}}}$ donc $F_{24}(x)=2\times\underbrace{2\sqrt{2x^2+1}}_{2\sqrt{u}}=4\sqrt{2x^2+1}$

25) $f_{25}(x)=\dfrac{\ln x}{x}=\underbrace{\dfrac{1}{x}}_{u'}\times\underbrace{\ln x}_{u}$ donc $F_{25}(x)=\dfrac{\overbrace{(\ln x)^2}^{u^{1+1}}}{\underbrace{2}_{1+1}}$

26) $f_{26}(x)=\dfrac{5x}{\left(x^2+1\right)^4}=\dfrac{5}{2}\times\underbrace{2x}_{u'}\times\underbrace{\left(x^2+1\right)^{-4}}_{u^{-4}}$ donc $F_{26}(x)=\dfrac{5}{2}\times\dfrac{\overbrace{\left(x^2+1\right)^{-3}}^{u^{-4+1}}}{\underbrace{-3}_{-4+1}}=\dfrac{-5}{6\left(x^2+1\right)^3}$ car $\left(x^2+1\right)^{-3}=\dfrac{1}{\left(x^2+1\right)^3}$

27) $f_{27}(x)=\dfrac{\sin x}{\cos x}=-\dfrac{\overbrace{-\sin x}^{u'}}{\underbrace{\cos x}_{u}}$ donc $F_{27}(x)=-\underbrace{\ln\left(|\cos x|\right)}_{\ln|u|}$

28) $f_{28}(x)=\dfrac{\mathrm{e}^{-x}}{\mathrm{e}^{-x}+3}=-\dfrac{\overbrace{-\mathrm{e}^{-x}}^{u'}}{\underbrace{\mathrm{e}^{-x}+3}_{u}}$ donc $F_{28}(x)=-\underbrace{\ln\left(\mathrm{e}^{-x}+3\right)}_{\ln(u)}$

29) $f_{29}(x)=\dfrac{7x}{3x^2+3}=\dfrac{7}{6}\times\dfrac{\overbrace{6x}^{u'}}{\underbrace{3x^2+3}_{u}}$ donc $F_{29}(x)=\dfrac{7}{6}\,\underbrace{\ln\left(3x^2+3\right)}_{\ln(u)}$