Optimization of RFM's Structure Based on PSO Algorithm and Figure Condition Analysis

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***Abstract*— Rational function model (RFM) faces difficulty in extracting accurate geometric information from remotely sensed images, which is mainly due to the problems of overparameterization and ill-posedness. These problems can be addressed via variable selection methods, in which an optimum subset of rational polynomial coefficients is identified via an optimization algorithm, usually metaheuristic methods [e.g., genetic algorithm and particle swarm optimization (PSO)]. In this letter, we propose a PSO-based method that benefits from a novel cost function. The proposed cost function applies a figure condition analysis, where the sum of estimated errors for the entire image's pixels is regarded as the cost value. The main advantages of the proposed method, in comparison to its alternatives of the same type, are as follows: 1) in contrast to other metaheuristic-based methods, it can be applied even with a limited number of control points (CPs); 2) since the proposed cost function is a global and continuous one, it yields an appealing RFM from the generalization capability viewpoint (i.e., it leads to satisfying positional accuracies for the entire image, even for those pixels far from CP); and 3) the method is much more stable, which means that it gives very similar results in successive runs. Our experiments, conducted over four real data sets, demonstrate that the proposed method addresses the aforementioned problems, namely, overparameterization and ill-posedness. In addition, it outperforms the typical PSO-based method by 37% on average and also achieves RFM's structures that are more reliable and more stable than those identified by its alternative.**

***Index Terms*— Figure condition analysis (FCA), particle swarm optimization (PSO) algorithm, rational function model (RFM).**

I. Introduction

RATIONAL function model (RFM) is a wieldy used generic sensor model facilitating the extraction of accurate geospatial products from remote sensing images. RFM, which attempts to define the mathematical relationship between the object and image spaces, has been widely used by the satellite image vendors as a replacement for the rigorous models [1]. Despite the availability of various rigorous models, RFM is easy to implement and independent of sensor parameters and ephemeris data [1]–[3], which makes it a suitable model for various applications [4]–[7].

Apart from all the mentioned advantages, RFM is commonly criticized for its numerous variables, known as rational polynomial coefficients (RPCs), which are unnecessary and highly correlated [1], [8]. This causes RFM to be burdened with severe errors due to problems of overparameterization and ill-posedness.

A vast number of methods have been proposed in the literature to address the mentioned problems including l_1-norm regularization technique [1], and variable selection methods based on a nested regression [9], significance test [10], scattering matrix [1], stepwise-then-orthogonal regression [11], statistical solution [12], and metaheuristic optimization algorithms (MOAs) [3], [8], [13].

Within the framework of tackling the problems of overparameterization and ill-posedness, the MOAs have been applied to identify optimally and globally a suitable subset of RPCs. Zoej *et al.* [3] applied a genetic algorithm to find an optimum structure of RFM designed to minimize a cost function of root-mean-square error (RMSE). They divided control points (CPs) into three categories: 1) ground CPs (GCPs) for RPC estimation; 2) dependent check points (DCPs) in charge of the cost function estimation; and 3) independent check points (ICPs) that are only applied to assess the quality of the optimum RFM found through their method. This research was later reinforced in [13], where a particle swarm optimization (PSO) algorithm was developed to improve the positional accuracy of [3] and also mitigate its run-time problem.

The methods in [3] and [13], which are the same in concept, benefit from the capability of doing a global search. However, they used the RMSE as the cost function that has the following disadvantages.

1) The estimation of the RMSE requires some devoted CPs, termed as DCPs. In fact, one has to provide an extra set of CPs, except those of GCPs, to estimate the RMSE as the cost function. This, as a result, increases the need for more CPs, which is costly and time consuming [14],
2) Due to the fact that the RMSE is a point-based/pixel-based measure, the optimum RFM found through this method is only optimum for those pixels in the vicinity of DCPs. Therefore, the resulting RFM may not necessarily be optimum for the whole image. This situation gets even worse if a limited number of GCPs and DCPs are employed.

The purpose of this letter is to address the disadvantages of [3] and [13] by replacing the RMSE with a novel cost function in the PSO algorithm. To accomplish this, we consider the sum of estimated errors obtained through figure condition analysis (FCA) as the cost function. Hence, from now on, we call our proposed method FCA-PSO.

FCA-PSO can address the previously mentioned limitations of metaheuristic-based methods, such as those mentioned

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in [3] and [13], because the proposed cost function is a global and continuous measure of positional accuracy, and more importantly it needs no additional CPs. In other words, FCA, in contrast to the RMSE, does not require any DCPs to measure the positional accuracy. Being driven by the positional accuracy assessment in the entire image, FCA provides a global cost function, and therefore, the resulting RFM via FCA-PSO is optimum for the whole image.

II. Theoretical Background

A. RFM

The RFM models the existing relationship between an image space (l, s) and an object space (X, Y, Z) using the ratios of two polynomial functions as follows [15]:

$$l = \frac{P_1(X,Y,Z)}{P_2(X,Y,Z)}, s = \frac{P_3(X,Y,Z)}{P_4(X,Y,Z)} \tag{1}$$

$$\begin{aligned} P_i(X,Y,Z) = {} & a_{i,0} + a_{i,1}X + a_{i,2}Y + a_{i,3}Z + a_{i,4}XY \\ & + a_{i,5}XZ + a_{i,6}ZY + a_{i,7}X^2 + a_{i,8}Y^2 \\ & + a_{i,9}Z^2 + a_{i,10}XYZ + \ldots. \end{aligned} \tag{2}$$

The nonlinear form of (1) can be reformulated to a linear one as follows [1], [9], [15]:

$$\begin{aligned} P_1(X,Y,Z) - lP_2(X,Y,Z) = 0 \\ P_3(X,Y,Z) - sP_4(X,Y,Z) = 0. \end{aligned} \tag{3}$$

In order to solve the unknown RPCs (i.e., $a_{i,n}$) in (3), standard linear Gauss–Markov model and least-squares (LS) method can be applied (note that $a_{2,0} = a_{4,0} = 1$)

$$\boldsymbol{y} = A\boldsymbol{x} + \boldsymbol{e} \tag{4}$$

$$\hat{\boldsymbol{x}} = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}\boldsymbol{y} \tag{5}$$

where A is a design matrix, and $\boldsymbol{e}$, $\boldsymbol{y}$, and $\hat{\boldsymbol{x}}$ represent the vector of random errors, observations, and estimated RPCs, respectively. The forgoing method of RPC estimation is known as "direct solution" in [15].

B. Figure Condition Analysis (FCA)

Given (4) as a linear model between two coordinate spaces of $\boldsymbol{x}$ and $\boldsymbol{y}$, in our case, the ground and image spaces, respectively, and (5) as a solution for its unknown parameters, accuracy assessment measures are used to evaluate how fit the obtained model is.

The most common accuracy assessment measure is perhaps the RMSE, which is typically computed over a discrete set of points known as ICPs. Recently, FCA was proposed [14] as a new accuracy assessment measure that, unlike the conventional RMSE, provides users with a continuous and global analysis of accuracy. In fact, FCA takes into account every pixel of an image, which is in contrast to RMSE that is only grounded on a limited number of ICPs. Meanwhile, the prominent advantage of FCA is the fact that it requires no ICPs, which makes it very appealing, since ICPs are of the most critical cost concerns in the remote sensing and photogrammetry disciplines. In the following, the theoretical background of FCA is described.

Equation (4) can be reduced for each point in the image as follows:

$$[l_i, s_i]^{\mathrm{T}} = \boldsymbol{y}_i = A_i\hat{\boldsymbol{x}} + \boldsymbol{e}_i. \tag{6}$$

In (6), A_i is filled with values in $\boldsymbol{g}_i = (X_i, Y_i, Z_i)$ that are the ground coordinates of the ith point, which are typically taken from a digital elevation model (DEM).

The existing error in $\boldsymbol{y}_i$ is caused by the propagated errors existing in $\hat{\boldsymbol{x}}$ and $\boldsymbol{g}_i$ that are represented by $Q_{\hat{\boldsymbol{x}}}$ and $Q_{\boldsymbol{g}_i}$, respectively, as follows:

$$Q_{\hat{\boldsymbol{x}}} = \hat{\sigma}_0^2(A^{\mathrm{T}}A)^{-1} \tag{7}$$

$$Q_{\boldsymbol{g}_i} = \begin{bmatrix} \sigma_{X_i}^2 & 0 & 0 \\ 0 & \sigma_{Y_i}^2 & 0 \\ 0 & 0 & \sigma_{Z_i}^2 \end{bmatrix}^{-1} \tag{8}$$

where $\hat{\sigma}_0^2$ is a posterior estimation of the variance factor [16] and $(\sigma_{X_i}^2, \sigma_{Y_i}^2, \sigma_{Z_i}^2)$ represent an estimate for the existing error in the coordinate values of the ith ground point.

According to the error propagation law, one can write the following equations giving in hand an estimate of the existing error in $\boldsymbol{y}_i$:

$$\sigma_i = \sqrt{\mathrm{trace}(Q_{\boldsymbol{y}_i})} \tag{9}$$

$$B_i = \begin{bmatrix} \dfrac{\partial l_i}{\partial X_i} & \dfrac{\partial l_i}{\partial Y_i} & \dfrac{\partial l_i}{\partial Z_i} \\ \dfrac{\partial s_i}{\partial X_i} & \dfrac{\partial s_i}{\partial Y_i} & \dfrac{\partial s_i}{\partial Z_i} \end{bmatrix} \tag{10}$$

$$Q_{\boldsymbol{y}_i} = A_i Q_{\hat{\boldsymbol{x}}} A_i^{\mathrm{T}} + B_i Q_{\boldsymbol{g}_i} B_i^{\mathrm{T}}. \tag{11}$$

The diagonal elements of $Q_{\boldsymbol{y}_i}$ are the estimated errors for l_i and s_i (i.e., image coordinates of the ith point). Therefore, σ_i in (9) gives in hand a measure of the positional accuracy assessment for the ith point. We refer the readers to [12] and [14] for more details on FCA.

III. Methodology

Within the framework of identifying the optimal structure of RFM, we put forward a new method called FCA-PSO. Fig. 1 illustrates the basis of the proposed method.

The algorithm initiates with a set of RFMs, each of which is generated randomly at the first run.

With the purpose of being consistent with [13], which is going to be used in later comparisons, it is assumed that $P_2 = P_4$, and the maximum number of RPCs in P_1, P_3, and P_2 is set to 11, 11, and 10, respectively.

Each RFM is considered as a particle in the PSO algorithm and is represented by a string of 32 binary values (Fig. 2). As shown in Fig. 2, each binary value in the string is associated with a specific RPC while "1" means that the associated RPC participates in the RFM and "0" denotes the opposite.

For every particle, first, the associated RFM is solved (i.e., its RPCs are estimated) using GCPs and the LS method (3)–(5). Afterward and within the framework of FCA, a cost value is assigned to the associated particle. The proposed cost function is given in the following equation:

$$C = \sum_{i=1}^{N} \sigma_i \tag{12}$$

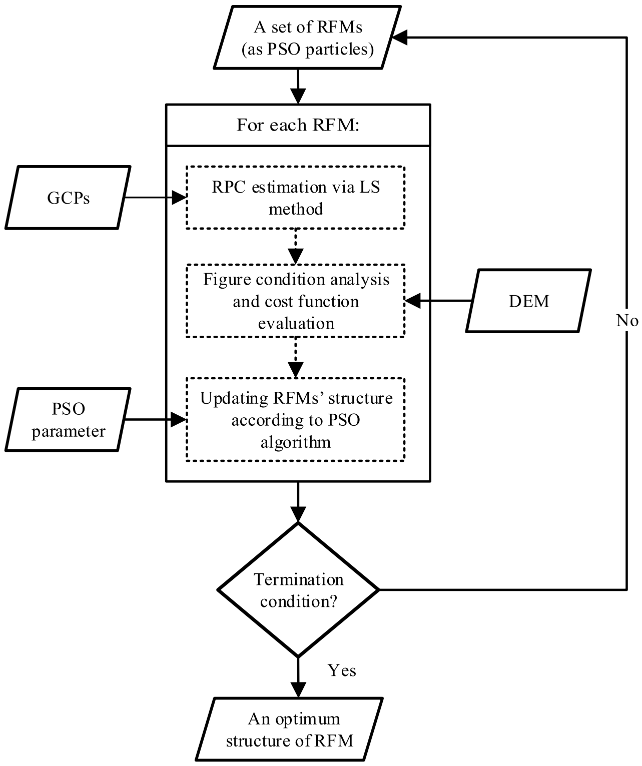

Fig. 1. Flowchart of the FCA-PSO method.

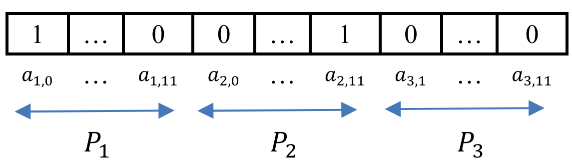

Fig. 2. Particle representation.

where N denotes the total number of discrete points in a DEM, covering the area of interest, and σ_i comes from (9) as a measure of the positional accuracy assessment for the ith point.

Considering the binary representation of RFMs and also having an associated cost for each RFM, one can conduct the binary PSO algorithm to update the current structures of all RFMs. More information about the PSO algorithm is given in [17]. Note that we considered a user-defined maximum number of iterations ($T = 200$) as the termination condition in this letter.

IV. Experiments

A. Data Sets

We used four remotely sensed images with different characteristics in our experiments (Table I). CPs for all of the data sets were chosen from distinct features of 1:2000 digital reference maps.

The images used in this letter were acquired over different cities of the country of Iran. C1-KS was collected over a rural area, while the others, i.e., Geo-UR, Geo-ISF, and WV-ISF, were captured over urban areas. These images were all geometrically corrected at a primary level. More rigorously, they were corrected for earth rotation and curvature, mirror look angles, and orbit characteristics, and they were assigned to a geographic coordinate system.

TABLE I

Data Set Specification With Number of Available CPs. ONA and GSD Stand for Off-Nadir Angle and Ground Sampling Distance, Respectively

Data set	Platform	Location	ONA (deg)	GSD (m)	Elevation Range (m)	No. of CPs
C1-KS	Cartosat-1	Kerman Shah	5.0	2.5	570	80
Geo-ISF	GeoEye-1	Isfahan	15.7	0.5	15	50
Geo-UR	GeoEye-1	Urmia	0.8	0.5	185	50
WV-ISF	WorldView-3	Isfahan	1.4	0.5	8	50

As previously mentioned, the implementation of FCA requires a DEM. In our experiments, we applied ASTER global DEMs, which are freely accessible. The horizontal and vertical accuracies of these DEMs were assumed to be 1 arcsecond and 12.62 m [18], respectively.

B. Results and Discussion

In this letter, we applied the same version of PSO that was applied in [13], and set PSO parameters accordingly. In addition, the proposed method in [13], which obtained the highest accuracy among other MOAs of its type, was chosen as the competing method against our proposed one.

Hereinafter, we call this competing method PSORFO, which stands for PSO for rational function optimization [13].

To evaluate the proposed FCA-PSO, we designed some experiments. Different numbers of well-distributed GCPs (i.e., 10, 15, and 20 points) were applied for each data set. These GCPs were also applied in the competing method, namely, PSORFO, while five, seven, and ten well-distributed points were excluded as DCPs from the abovementioned GCPs in that order. It is worth noting that all the existing metaheuristic algorithms designed for RFM optimization, including PSORFO, require a set of DCPs to calculate the RMSE as their cost function.

Due to the fact that all of the metaheuristic algorithm, such as PSO, converges to different solutions in different runs, we repeated the PSO algorithm five times, which in turn resulted in five (sub) optimal solutions in each experiment. The best one with minimum cost function was chosen as the ultimate among the others.

As mentioned earlier, RFMs in our proposed method were solved via the full set of GCPs, while in PSORFO, some of these points were excluded as DCPs to be applied in the estimation of the cost function (i.e., the RMSE). With the purpose of making fair comparisons between the proposed FCA-PSO and the competing PSORFO, the optimal RFM identified via PSORFO was not solved with just its GCPs, and the union of DCPs and GCPs was employed instead.

Table II demonstrates the obtained results through the implementation of the proposed FCA-PSO as well as the PSORFO. Table II provides four measures of assessment for each case including: 1) the RMSE calculated over ICPs for the ultimate solution as a criterion of the positional accuracy;

TABLE II

OBTAINED RESULTS FROM PSORFO AND THE PROPOSED FCA-PSO. SD, DF, AND COND. STAND FOR STANDARD DEVIATION, DEGREES OF FREEDOM, AND THE CONDITION NUMBER OF NORMAL MATRIX, RESPECTIVELY

Data set	GCP/ICP	RMSE (SD) [in pixel]		df		Cond.	
		PSORFO	FCA-PSO	PSORFO	FCA-PSO	PSORFO	FCA-PSO
C1-KS	10/70	3.40 (2E3)	0.89 (0.16)	15	12	2.09	1.98
	15/65	0.65 (18.90)	0.47 (0.09)	22	24	39.77	7.16
	20/60	0.52 (0.05)	0.46 (0.07)	31	34	36.78	4.42
WV3-ISF	10/40	1.0 (100.05)	0.89 (0.00)	16	16	1.86	1.73
	15/35	0.72 (0.05)	0.72 (0.05)	21	25	41.43	3.77
	20/30	0.78 (0.03)	0.75 (0.00)	35	35	40.72	3.77
GEO-UR	10/40	0.83 (3E3)	0.61 (0.00)	10	15	20.45	3.69
	15/35	0.81 (83.36)	0.62 (0.01)	16	23	31.2	14.39
	20/30	0.64 (0.02)	0.64 (0.02)	32	33	161	10.88
GEO-ISF	10/40	2.18 (1.6E3)	0.80 (0.06)	15	15	12.18	13.03
	15/35	1.42 (1.12)	0.98 (0.00)	22	26	164.95	1.61
	20/30	0.85 (0.53)	0.86 (0.11)	32	35	14.16	6.12

2) standard deviation (SD) of RMSEs in five runs of PSO, which indicates the stability of the solutions; 3) the degrees of freedom (df) that is a criterion of the models' reliability; and 4) the condition number of normal matrix ($A^{\mathrm{T}}A$) that is a measure of the ill-conditioning problem [19].

In order to compare the positional accuracy of the proposed FCA-PSO and PSORFO, their RMSEs presented, respectively, in the third and fourth columns of Table II were considered. The difference between average RMSEs in each column reveals that FCA-PSO outperforms PSORFO by 37% on average. This improvement was statistically confirmed for being significant at a 99% significance level using a Mann–Whitney U test. As apparent from Table II, FCA-PSO also decreases the need for more GCPs since it can achieve subpixel accuracy with 10 GCPs for all case studies.

The best benefit of FCA-PSO is perhaps revealed from the stability point of view, which is presented in terms of SD in Table II. The small values of SD in the fourth column, associated with the FCA-PSO method, indicate that successive runs of this method lead to almost the same results, which is in contrast to PSORFO, especially in cases that small numbers of GCPs were applied.

Our proposed method yields optimum RFMs with smaller number of RPCs. This can be concluded from the larger values of df when compared with those of PSORFO (Table II). This indicates the improvement of FCA-PSO from not only the model's prediction capability point of view [20], but also from the model's reliability viewpoint. The smaller number of RPCs in an RFM avoids the common problem of overparameterization and thus improves the generalization capability of that RFM.

Another prominent achievement of the proposed method can be inferred from a detailed comparison between the seventh and eighth columns in Table II, which are the condition numbers associated with the PSORFO and FCA-PSO methods, respectively. The lower values of the condition number indicate more effectiveness in addressing the ill-posedness problem [19]. Promising values of the condition number resulting from the proposed method can be thought as the success of this method to tackle the problem of ill-posedness. More rigorously, the proposed method prevents the coexistence of highly correlated RPCs, which is the main reason of the ill-posedness problem [21], in the identified RFM.

The primary reason behind the superiority of FCA-PSO lies in its cost function. We applied FCA as a novel measure to be employed in the cost function. This cost function benefits from two important aspects. First, FCA is a continuous and global measure because it takes into account the positional accuracy of all pixels in the entire image. As a result, the obtained RFM can be generalized to the whole image. This is in sharp contrast to the optimum RFM identified through PSORFO that is only optimum over a limited number of pixels/points (i.e., the DCPs). Second, FCA requires no DCPs, and therefore it can exploit the full potential of all existing GCPs to locate an optimum RFM. Just to remember, PSORFO suffers from the limitation that one has to exclude some DCPs for the estimation of the cost function.

Despite all the mentioned advantages, FCA-PSO was slower than PSORFO: average processing times for one run of FCA-PSO and PSORFO were 39 and 6 s, respectively. This is because the cost function of FCA-PSO is more time demanding than that of PSORFO as it is calculated based on a considerable number of points extracted from a DEM.

V. CONCLUSION

Suffering from the problems of overparameterization and ill-posedness, RFM may cause inaccurate geospatial products such as thematic maps and digital surface models. These problems are caused due to the presence of numerous RPCs, most of which are highly correlated and unnecessary. However, these problems can be addressed if a suitable subset of RPCs is selected. To this end, we presented the FCA-PSO method that exploits the PSO algorithm with a novel cost function, that is, the sum of estimated errors calculated via FCA for the entire image.

The proposed method was compared with another PSO-based method, i.e., PSORFO. The cost function of FCA-PSO was advantageous over that of PSORFO because of the following.

1) It did not require any additional CPs in order to evaluate its cost function.
2) It was a global and continuous cost function, which in turn resulted in an RFM that is optimum and can be generalized for the entire image.

Experimental results demonstrated the superiority of FCA-PSO over PSORFO since it could address the mentioned problems. Rigorously speaking, the FCA-PSO method outperformed PSORFO from the positional accuracy (by 37% on average), reliability, and stability points of view. Moreover, FCA-PSO required less number of GCPs than did PSORFO in order to achieve the subpixel accuracy.

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