International Conference-School

SHILNIKOV WORKSHOP 2020

17-18 December of 2020

Book of Abstracts

Lobachevsky State University of Nizhny Novgorod

Local dynamics of a second order equation with a delay in the derivative

Kashchenko I.S., Maslenikov I.N.

P.G. Demidov Yaroslavl State University

iliyask@uniyar.ac.ru

Consider model of opto-electronic oscillator with nonlinear delayed feedback

$$\varepsilon \frac{dx}{dt} + x + \delta \int_{t_0}^{t} x(s)\, ds = F(x(t-\tau)).$$

Here ε and δ are positive and sufficiently small and satisfy the condition $\delta = k\varepsilon$, $0 < \varepsilon \ll 1$. This equation can be reduced to the form of second-order delay differential equation

$$\varepsilon \frac{d^2y}{dt^2} + \frac{dy}{dt} + \delta y = F(\frac{dy}{dt}(t-\tau)). \tag{1}$$

Let's study it's dynamics in small neighbourhood of zero equilibrium for small enough ε.

It was shown that the critical cases in the equilibrium stability problem have an infinite dimension. As the main results, special nonlinear boundary value problems were constructed whose nonlocal dynamics determine the behavior of the solutions to (1) (for small ε) that belong to a sufficiently small ε-independent neighbourhood of zero.

In particular, if $F'(0) = -(1 + \varepsilon^2\beta)$, then main terms of asymptotic representation of solutions of (1) are determined by solutions of

$$\frac{\partial V}{\partial \tau} = \frac{1}{2}\frac{\partial^2 V}{\partial^2 t} + kV - \frac{k^2}{2}J^2(V) + \beta V + \beta_2 J\left(U_1 \frac{\partial V}{\partial t}\right) + \beta_3 J\left(\left(\frac{\partial V}{\partial t}\right)^3\right),$$

$$V(\tau, t) \equiv -V(\tau, t+1), \qquad \int_0^1 V(\tau, t)dt = 0.$$

Here $J(V)$ are the antiderivative of the function V with zero average value: $(J(V(\tau,t))'_t \equiv V$, and $J^2(V) = J(J(V))$.

This work was funded by RFBR according to the research project No 18-29-10043.