Assignment 3

Vibhavasu Pasumarti - EP20BTECH11015

Download all python codes from

```
https://github.com/VIB2020/AI1103/blob/main/
    Assignment%203/code/Assignment%203.py
```

and latex-tikz codes from

```
https://github.com/VIB2020/AI1103/blob/main/
    Assignment%203/Assignment%203.pdf
```

1 GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

(A) $\frac{5}{11}$

(B) $\frac{1}{2}$

(C) $\frac{7}{13}$

(D) $\frac{6}{11}$

2 Solution

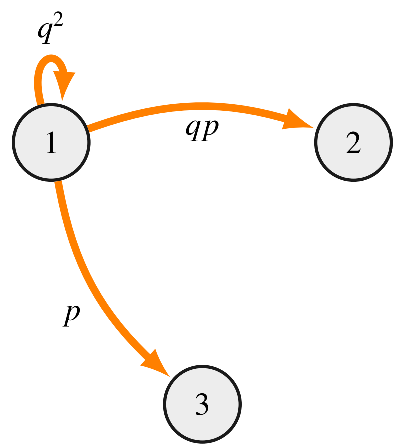
- Given the die is fair.
- So, for any given throw by A or B:
  The probability of getting 6 = $\frac{1}{6}$ = p (say)
  The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

Constraint: A starts the game and A should win.
$\implies$ Until A wins, both A and B cannot win.

State	Corresponding description
1	A and B both lose.
2	A loses and B wins.
3	WINNER

Table 1: State description table

Corresponding Markov chain is:



Definition 2.1. 2 and 3 are ABSORBING states.
Transition/Stochastic matrix is: T

$$\begin{array}{cc} & \text{(To)} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{(From)} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} q^2 & qp & p \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = T \end{array}$$

Theorem 2.1. Every transition matrix can be partitioned as $\left(\begin{array}{c|c} Q & R \\ \hline O & J \end{array}\right)$
where Q arise from transition probabilities between non-absorbing states
R arises from Transition probability from non-absorbing state to absorbing state.
O = Null matrix, J = Identity matrix
T^k approaches $\overline{T}$ as k increases. $\overline{T}$ is the limiting matrix and $\overline{T} = \begin{pmatrix} O & NR \\ O & J \end{pmatrix}$
where N is the fundamental matrix. $N = (I - Q)^{-1}$

Theorem 2.2.
The matrix D = NR gives the probability of ending up in the absorbing states (2 and 3) when the chain starts from a non-absorbent state 1.

$$T = \left(\begin{array}{c|cc} q^2 & qp & p \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right)$$

$$Q = \left(q^2\right), P = \begin{pmatrix} qp & p \end{pmatrix}, O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (1)$$

$$N = \left(1 - q^2\right)^{-1} = \left(\frac{1}{1 - q^2}\right) \quad (2)$$

$$D = NP = \begin{pmatrix} \frac{qp}{1 - q^2} & \frac{p}{1 - q^2} \end{pmatrix} \quad (3)$$

OPTION D is correct