Assignment 3

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Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 Problem
GATE 2015 (EE paper 01 new 2), Q 27
(Electrical Engg section)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

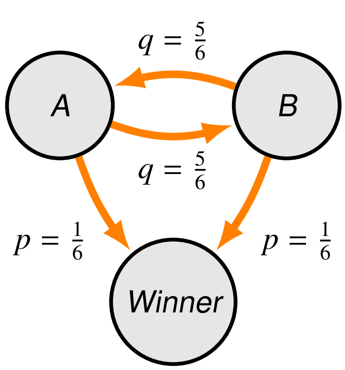
2 Solution

Definition .1. *Let the random variable X denote the win of A. Given the die is fair.*

The probability of getting 6 $= \frac{1}{6} = p$ *(say)*

The probability of NOT getting 6 $= \frac{5}{6} = q$ *(say)*

Markov chain for the given problem:



Transition/Stochastic matrix:

$$P = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \implies \lim_{n\to\infty} P^n \to 0 \because p, q < 1$$

$\implies$ *P is reducible* $\implies \lim_{n\to\infty} Pr_{AW}(n) = 0$

P^n *represents n rolls of dice*

Given: A starts the game

Constraint: A wins the game.

The initial operator representing this constraint:

$A_1 = \begin{pmatrix} p & 0 & 0 \end{pmatrix}$

First element of each A_i *represents the probability of A to win in* i^{th} *roll*

$Pr_{AW}(1) = p$

$$A_2 = A_1 P = \begin{pmatrix} p & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} 0 & qp & 0 \end{pmatrix}$$

$\implies Pr_{AW}(2) = 0$

$$A_3 = A_2 P = \begin{pmatrix} 0 & qp & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} q^2 p & 0 & 0 \end{pmatrix}$$

$\implies Pr_{AW}(3) = q^2 p$

A	No.of Turns	Probability
$\Pr_{AW}(1)$	1	p = 1/6
$\Pr_{AW}(3)$	2	0
$\Pr_{AW}(3)$	3	$q^2 p$
. . .	. . ..	. .
$\Pr_{AW}(2n)$	2n	0
$\Pr_{AW}(2n+1)$	2n + 1	$q^{2n} p$

TABLE 0: Summary of Probabilities

Definition .2. $\Pr(A\ wins) = \sum_{n=1}^{\infty} Pr_{AW}(n)$

$= Pr_{AW}(1) + Pr_{AW}(2) + Pr_{AW}(3) + ...$

$= p + 0 + q^2 p + 0 + q^4 p + ...$

$= \sum_{k=1}^{\infty} q^{2k} p = p \sum_{k=1}^{\infty} (q^2)^k$ *But* $|q| < 1$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D