Assignment 3

Vibhavasu Pasumarti - EP20BTECH11015

Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 Problem

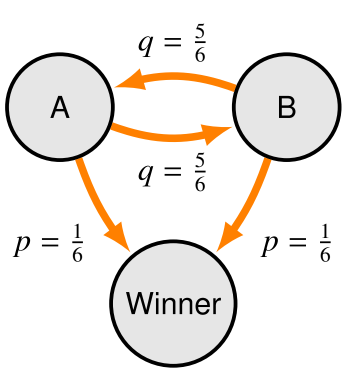
GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

2 Solution

Let the random variable X denote the win of A.
Given the die is fair.
The probability of getting 6 = $\frac{1}{6}$ = p (say)
The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)
Markov chain for the given problem:



Transition/Stochastic matrix:

$$P = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \implies \lim_{n\to\infty} P^n \to 0 \because p, q < 1$$

$\implies$ P is reducible $\implies \lim_{n\to\infty} Pr_{AW}(n) = 0$
P^n represents n rolls of dice
Given: A starts the game
Constraint: A wins the game.
The initial operator representing this constraint:
$A_1 = \begin{pmatrix} p & 0 & 0 \end{pmatrix}$
First element of each A_i represents the probability of A to win in i^{th} roll
$Pr_{AW}(1) = p$

$$A_2 = A_1 P = \begin{pmatrix} p & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} 0 & qp & 0 \end{pmatrix}$$

$\implies Pr_{AW}(2) = 0$

$$A_3 = A_2 P = \begin{pmatrix} 0 & qp & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} q^2 p & 0 & 0 \end{pmatrix}$$

$\implies Pr_{AW}(3) = q^2 p$
$A_4 = \begin{pmatrix} 0 & q^3 p & 0 \end{pmatrix} \implies Pr_{AW}(4) = 0$
$A_5 = \begin{pmatrix} q^4 p & 0 & 0 \end{pmatrix} \implies Pr_{AW}(5) = q^4 p$
By induction:
$A_{2n} = \begin{pmatrix} 0 & q^{2n-1} p & 0 \end{pmatrix} \implies Pr_{AW}(2n) = 0$
$A_{2n+1} = \begin{pmatrix} q^{2n} & 0 & 0 \end{pmatrix} \implies Pr_{AW}(2n+1) = q^{2n} p$
$\Pr(\text{A wins}) = \sum_{n=1}^{\infty} Pr_{AW}(n)$
$= Pr_{AW}(1) + Pr_{AW}(2) + Pr_{AW}(3) + ...$
$= p + 0 + q^2 p + 0 + q^4 p + ...$
$= \sum_{k=1}^{\infty} q^{2k} p = p \sum_{k=1}^{\infty} (q^2)^k$ But $|q| < 1$

$$= \text{p} \left(\frac{1}{1 - q^2} \right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D