Assignment 3

Vibhavasu Pasumarti - EP20BTECH11015

1 GATE 2015 (EE PAPER 01 NEW 2), Q 27 (ELECTRICAL ENGG SECTION)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

(A) $\dfrac{5}{11}$

(B) $\dfrac{1}{2}$

(C) $\dfrac{7}{13}$

(D) $\dfrac{6}{11}$

2 SOLUTION

- Given the die is fair.
- So, for any given throw by A or B:

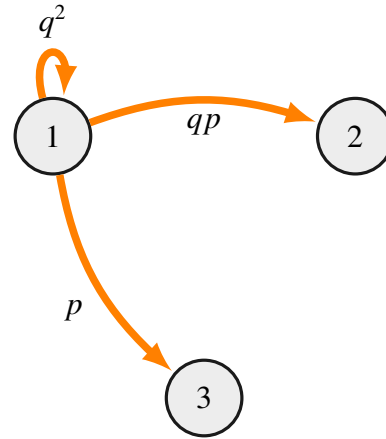 The probability of getting $6 = \dfrac{1}{6} = p$ (say)

  The probability of NOT getting $6 = \dfrac{5}{6} = q$ (say)

Constraint: A starts the game and A should win.
$\implies$ Until A wins, both A and B cannot win.

State	Corresponding description
1	A and B both lose.
2	A loses and B wins.
3	WINNER

Table 1: State description table

Corresponding Markov chain is:

Definition 2.1. 2 and 3 are ABSORBING states. Transition/Stochastic matrix is: T

$$
\begin{array}{c}
\text{(To)} \\
\text{(From)} \quad
\begin{array}{cc}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} &
\begin{bmatrix} q^2 & qp & p \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{array}
= T
\end{array}
$$

Theorem 2.1. Every transition matrix can be partitioned as $\left(\begin{array}{c|c} Q & P \\ \hline O & J \end{array} \right)$
where Q arise from transition probabilities between non-absorbing states
R arises from Transition probability from non-absorbing state to absorbing state.
O = Null matrix, J = Identity matrix
T^k approaches $\overline{T}$ as k increases. $\overline{T}$ is the limiting matrix and $\overline{T} = \begin{pmatrix} O & NP \\ O & J \end{pmatrix}$
where N is the fundamental matrix. $N = (I - Q)^{-1}$

Definition 2.2. The matrix D = NP gives the probability of ending up in the absorbing states (2 and 3) when the chain starts from a non-absorbent state 1.

$$T = \left(\begin{array}{c|cc} q^2 & qp & p \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \quad (1)$$

$$Q = \left(q^2 \right), P = \left(qp \quad p \right), O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2)$$

$$N = \left(1 - q^2 \right)^{-1} = \left(\frac{1}{1 - q^2} \right) \quad (3)$$

$$D = NP = \left(\frac{qp}{1 - q^2} \quad \frac{p}{1 - q^2} \right) \quad (4)$$

$$\text{Pr} \left(A \text{ wins} \right) = D_2 = \frac{p}{1 - q^2} = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \quad (5)$$

$$\text{Pr} \left(A \text{ wins} \right) = \frac{6}{11}$$

OPTION D is correct