

Assignment 3

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Download all python codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py>

and latex-tikz codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf>

1 PROBLEM

GATE 2015 (EE PAPER 01 NEW 2), Q 27
(ELECTRICAL ENGG SECTION)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

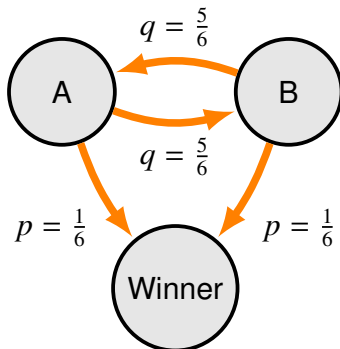
2 SOLUTION

Let the random variable X denote the win of A.
Given the die is fair.

The probability of getting 6 = $\frac{1}{6} = p$ (say)

The probability of NOT getting 6 = $\frac{5}{6} = q$ (say)

Markov chain for the given problem:



Transition/Stochastic matrix:

$$P = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \lim_{n \rightarrow \infty} P^n \rightarrow 0 \because p, q < 1$$

$$\Rightarrow P \text{ is reducible} \Rightarrow \lim_{n \rightarrow \infty} Pr_{AW}(n) = 0$$

P^n represents n rolls of dice

Given: A starts the game

Constraint: A wins the game.

The initial operator representing this constraint:

$$A_1 = \begin{pmatrix} p & 0 & 0 \end{pmatrix}$$

First element of each A_i represents the probability of A to win in i^{th} roll

$$Pr_{AW}(1) = p$$

$$A_2 = A_1 P = \begin{pmatrix} p & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} 0 & qp & 0 \end{pmatrix}$$

$$\Rightarrow Pr_{AW}(2) = 0$$

$$A_3 = A_2 P = \begin{pmatrix} 0 & qp & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} q^2 p & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Pr_{AW}(3) = q^2 p$$

$$A_4 = \begin{pmatrix} 0 & q^3 p & 0 \end{pmatrix} \Rightarrow Pr_{AW}(4) = 0$$

$$A_5 = \begin{pmatrix} q^4 p & 0 & 0 \end{pmatrix} \Rightarrow Pr_{AW}(5) = q^4 p$$

By induction:

$$A_{2n} = \begin{pmatrix} 0 & q^{2n-1} p & 0 \end{pmatrix} \Rightarrow Pr_{AW}(2n) = 0$$

$$A_{2n+1} = \begin{pmatrix} q^{2n} & 0 & 0 \end{pmatrix} \Rightarrow Pr_{AW}(2n+1) = q^{2n} p$$

$$\begin{aligned} \Pr(\text{A wins}) &= \sum_{n=1}^{\infty} Pr_{AW}(n) \\ &= Pr_{AW}(1) + Pr_{AW}(2) + Pr_{AW}(3) + \dots \\ &= p + 0 + q^2 p + 0 + q^4 p + \dots \\ &= \sum_{k=1}^{\infty} q^{2k} p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1 \\ &= p \left(\frac{1}{1 - q^2} \right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \end{aligned}$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D