# Assignment 3

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## 1 PROBLEM
## GATE 2015 (EE PAPER 01 NEW 2), Q 27
### (ELECTRICAL ENGG SECTION)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

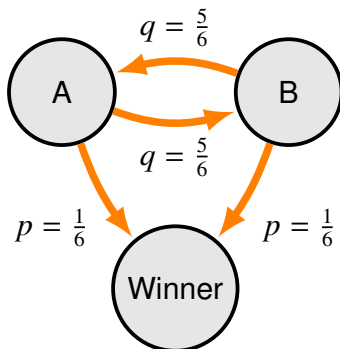A $\dfrac{5}{11}$     B $\dfrac{1}{2}$     C $\dfrac{7}{13}$     D $\dfrac{6}{11}$

## 2 SOLUTION

Let the random variable X denote the win of A. Given the die is fair.

The probability of getting 6 = $\dfrac{1}{6}$ = p (say)

The probability of NOT getting 6 = $\dfrac{5}{6}$ = q (say)

Markov chain for the given problem:



$q = \frac{5}{6}$
$q = \frac{5}{6}$
$p = \frac{1}{6}$     $p = \frac{1}{6}$

Transition/Stochastic matrix:

$P = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \implies \lim_{n\to\infty} P^n \to 0 \ \because p, q < 1$

$\implies P$ is reducible $\implies \lim_{n\to\infty} Pr_{AW}(n) = 0$

$P^n$ represents n rolls of dice
Given: A starts the game
Constraint: A wins the game.
The initial operator representing this constraint:
$A_1 = \begin{pmatrix} p & 0 & 0 \end{pmatrix}$
First element of each $A_i$ represents the probability of A to win in $i^{th}$ roll

$$Pr_{AW}(1) = p$$

$$A_2 = A_1 P = \begin{pmatrix} p & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} 0 & qp & 0 \end{pmatrix}$$

$$\implies Pr_{AW}(2) = 0$$

$$A_3 = A_2 P = \begin{pmatrix} 0 & qp & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} q^2 p & 0 & 0 \end{pmatrix}$$

$$\implies Pr_{AW}(3) = q^2 p$$

$$A_4 = \begin{pmatrix} 0 & q^3 p & 0 \end{pmatrix} \implies Pr_{AW}(4) = 0$$

$$A_5 = \begin{pmatrix} q^4 p & 0 & 0 \end{pmatrix} \implies Pr_{AW}(5) = q^4 p$$

By induction:

$$A_{2n} = \begin{pmatrix} 0 & q^{2n-1} p & 0 \end{pmatrix} \implies Pr_{AW}(2n) = 0$$

$$A_{2n+1} = \begin{pmatrix} q^{2n} & 0 & 0 \end{pmatrix} \implies Pr_{AW}(2n+1) = q^{2n} p$$

$$\Pr(A \text{ wins}) = \sum_{n=1}^{\infty} Pr_{AW}(n)$$

$$= Pr_{AW}(1) + Pr_{AW}(2) + Pr_{AW}(3) + \ldots$$

$$= p + 0 + q^2 p + 0 + q^4 p + \ldots$$

$$= \sum_{k=1}^{\infty} q^{2k} p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1$$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$$\Pr(A \text{ wins the game}) = \frac{6}{11}$$

### Option D