Assignment 3

Vibhavasu Pasumarti - EP20BTECH11015

Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

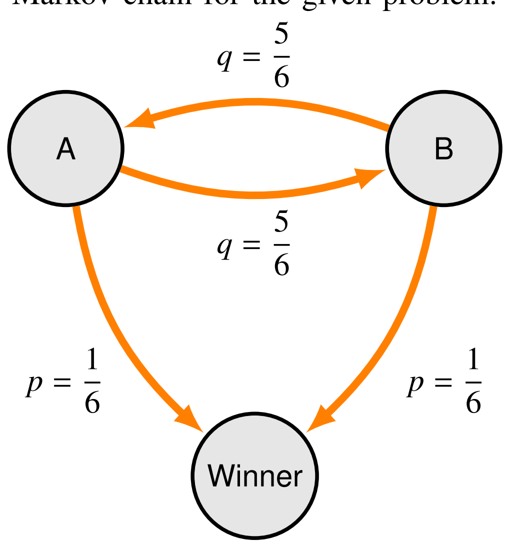
Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

2 Solution

- Given the die is fair.
- So, for any given throw by A or B:
  The probability of getting 6 = $\frac{1}{6}$ = p (say)
  The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

Markov chain for the given problem:



Definition 2.1. Transition/Stochastic matrix for the above Markov chain:

$$P = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix}$$

Lemma 2.1.

$$\lim_{n\to\infty} p_{AW}(n) = 0$$

Proof.

$$p, q < 1 \implies \lim_{n\to\infty} P^n \to 0$$
$$\implies P \text{ is reducible}$$
$$\therefore \lim_{n\to\infty} p_{AW}(n) = 0$$

□

Definition 2.2. Operator A_i corresponds the state of the game on the i_{th} roll of dice.
P^n is the corresponding Transition matrix for n rolls of dice

Lemma 2.2. The operator corresponding to the initial state i.e the first throw of dice is:

$$A_1 = \begin{pmatrix} p & 0 & 0 \end{pmatrix}$$

Proof. Given: A starts the game.
Constraint: A wins the game.
$\implies$ Pr (B winning on first throw) = $p_{BW}(1) = 0$
Pr (A winning on first throw) = $p_{AW}(1)$ = p
$\therefore$ Operator corresponding this state is:

$$A_1 = \begin{pmatrix} p & 0 & 0 \end{pmatrix}$$

□

Remark. First element of each A$_i$ represents the probability of A to win in ith roll

$$\implies p_{AW}(1) = p$$

$$A_2 = A_1 P = \begin{pmatrix} p & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} 0 & qp & 0 \end{pmatrix}$$

$$\implies p_{AW}(2) = 0$$

$$A_3 = A_2 P = \begin{pmatrix} 0 & qp & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} q^2 p & 0 & 0 \end{pmatrix}$$

$$\implies p_{AW}(3) = q^2 p$$

A	No.of Turns	Probability
$p_{AW}(1)$	1	p = 1/6
$p_{AW}(2)$	2	0
$p_{AW}(3)$	3	q^{2p}
. . $p_{AW}(2n)$	. . 2n	. . 0
$p_{AW}(2n+1)$	2n + 1	$q^{2n}p$

Table:1 Summary of Probabilities

$$\Pr(\text{A wins}) = \sum_{n=1}^{\infty} p_{AW}(n)$$

$$= p_{AW}(1) + p_{AW}(2) + p_{AW}(3) + ...$$

$$= p + 0 + q^2 p + 0 + q^4 p + ...$$

$$= \sum_{k=1}^{\infty} (q^2)^k p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1$$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{6}{11}$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D is correct