Assignment 3

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Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

(A) $\frac{5}{11}$

(B) $\frac{1}{2}$

(C) $\frac{7}{13}$

(D) $\frac{6}{11}$

2 Solution

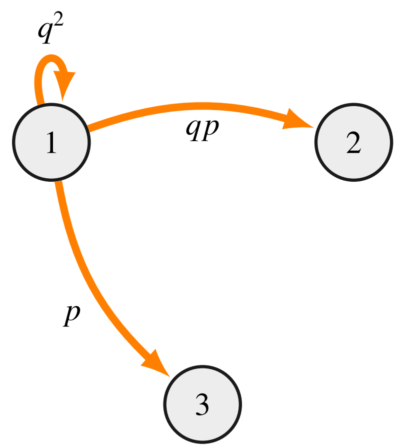
- Given the die is fair.
- So, for any given throw by A or B:
  The probability of getting 6 = $\frac{1}{6}$ = p (say)
  The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

Constraint: A starts the game and A should win.
$\implies$ Until A wins, both A and B cannot win.

State	Corresponding description
1	A and B both lose.
2	A loses and B wins.
3	A wins

Table 1: State description table

Corresponding Markov chain is:



Definition 2.1. Let X_n be a random variable corresponding to the state of the game after n^{th} throw of dice. X_0 is the initial state.

Definition 2.2. 2 and 3 are ABSORBING states. Transition/Stochastic matrix is: T

$$\begin{array}{cc} & \text{(To)} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{(From)} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} q^2 & qp & p \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{T} \end{array}$$

Theorem 2.1. Every transition matrix can be partitioned as $\left(\begin{array}{c|c} Q & P \\ \hline O & J \end{array}\right)$

T^k approaches $\overline{T}$ as k increases. $\overline{T}$ is the limiting matrix and $\overline{T} = \begin{pmatrix} O & NP \\ O & J \end{pmatrix}$

Partition	Description
Q	Transition probability between non-absorbing states
P	Transition probabilities from non- -absorbing state to absorbing states.
O	Null matrix
J	Identity matrix
N	Fundamental matrix

$$T = \left(\begin{array}{c|cc} q^2 & qp & p \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \tag{1}$$

Table 4: Matrices and corresponding values

Matrix	Description	Value
Q	$\Pr(X_{n+1} = 1 \mid X_n = 1)$	$\begin{pmatrix} q^2 \end{pmatrix}$
P	$\Pr(X_{n+1} \neq 1 \mid X_n = 1)$	$\begin{pmatrix} qp & p \end{pmatrix}$
N	$\sum_{n=0}^{\infty} Q^n = (I - Q)^{-1}$	$\mathrm{N} = \left(\frac{1}{1-q^2}\right)$

Theorem 2.2. The matrix D = NP gives the probability of ending up in the absorbing states (2 and 3) when the chain starts from a non-absorbent state 1.
By definition:

$$D = \left(\sum_{n=1}^{\infty} \Pr\left(\frac{X_n = 2}{X_0 = 1}\right) \quad \sum_{n=1}^{\infty} \Pr\left(\frac{X_n = 3}{X_0 = 1}\right) \right)$$

$$D = NP = \begin{pmatrix} \frac{qp}{1-q^2} & \frac{p}{1-q^2} \end{pmatrix} \tag{2}$$

$$\Pr(\text{A wins}) = D_2 = \frac{p}{1-q^2} = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \tag{3}$$

$$\Pr(\text{A wins}) = \frac{6}{11}$$

OPTION D is correct