

# Assignment 3

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Download all python codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py>

and latex-tikz codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf>

## 1 PROBLEM

GATE 2015 (EE PAPER 01 NEW 2), Q 27  
(ELECTRICAL ENGG SECTION)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A  $\frac{5}{11}$       B  $\frac{1}{2}$       C  $\frac{7}{13}$       D  $\frac{6}{11}$

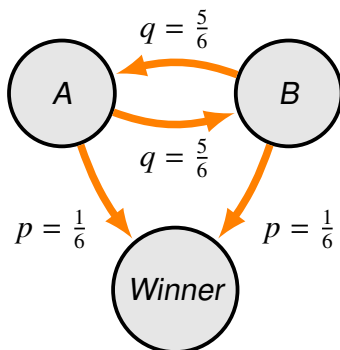
## 2 SOLUTION

1. Let the random variable  $X$  denote the win of A. Given the die is fair.

The probability of getting 6 =  $\frac{1}{6} = p$  (say)

The probability of NOT getting 6 =  $\frac{5}{6} = q$  (say)

Markov chain for the given problem:



Transition/Stochastic matrix:

$$P = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \lim_{n \rightarrow \infty} P^n \rightarrow 0 \because p, q < 1$$

$$\Rightarrow P \text{ is reducible} \Rightarrow \lim_{n \rightarrow \infty} Pr_{AW}(n) = 0$$

$P^n$  represents  $n$  rolls of dice

Given: A starts the game

Constraint: A wins the game.

The initial operator representing this constraint:

$$A_1 = \begin{pmatrix} p & 0 & 0 \end{pmatrix}$$

First element of each  $A_i$  represents the probability of A to win in  $i^{\text{th}}$  roll

$$Pr_{AW}(1) = p$$

$$A_2 = A_1 P = \begin{pmatrix} p & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} 0 & qp & 0 \end{pmatrix}$$

$$\Rightarrow Pr_{AW}(2) = 0$$

$$A_3 = A_2 P = \begin{pmatrix} 0 & qp & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} q^2 p & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Pr_{AW}(3) = q^2 p$$

A	No.of Turns	Probability
$Pr_{AW}(1)$	1	$p = 1/6$
$Pr_{AW}(2)$	2	0
$Pr_{AW}(3)$	3	$q^2 p$
$\vdots$	$\vdots$	$\vdots$
$Pr_{AW}(2n)$	$2n$	0
$Pr_{AW}(2n+1)$	$2n+1$	$q^{2n} p$

TABLE 0: Summary of Probabilities

$$\begin{aligned} 2. \Pr(A \text{ wins}) &= \sum_{n=1}^{\infty} Pr_{AW}(n) \\ &= Pr_{AW}(1) + Pr_{AW}(2) + Pr_{AW}(3) + \dots \\ &= p + 0 + q^2 p + 0 + q^4 p + \dots \end{aligned}$$

$$= \sum_{k=1}^{\infty} q^{2k} p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1$$

$$= p \left( \frac{1}{1 - q^2} \right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$$\Pr(A \text{ wins the game}) = \frac{6}{11}$$

Option D