Assignment 3

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Download all python codes from

```
https://github.com/VIB2020/AI1103/blob/main/
    Assignment%203/code/Assignment%203.py
```

and latex-tikz codes from

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https://github.com/VIB2020/AI1103/blob/main/
    Assignment%203/Assignment%203.pdf
```

1 Problem

GATE 2015 (EE paper 01 new 2), Q. 27

(Electrical Engg. section)

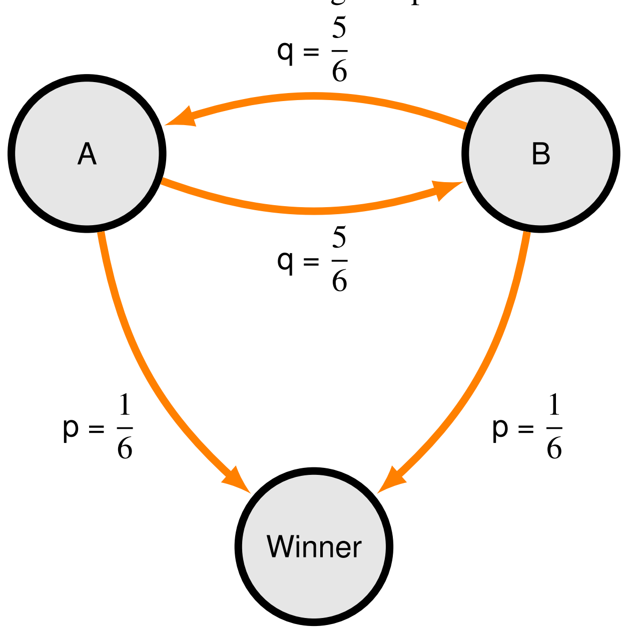
Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A: $\frac{5}{11}$ $\quad B : \frac{1}{2}$ $\quad C : \frac{7}{13}$ $\quad D : \frac{6}{11}$

2 Solution

Let the random variable X denote the win of A.
Given the die is fair.
The probability of getting 6 = $\frac{1}{6}$ = p (say)
The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

Markov chain for the given problem:



Constraint: A wins the game.
P(A wins on the first throw) = P(X_1) = p
P(A wins on the third throw) = P(X_3) = $q^2 p$
P(A wins on the $(2n+1)^{th}$ throw)= P(X_{2n+1}) = $q^{2n} p$

$$P(\text{A wins the game}) = P(X) = \sum_{n=1}^{+\infty} P(X_i) \tag{1}$$

$$P(X) = \sum_{n=1}^{+\infty} q^{2n} p \tag{2}$$

$$= p \sum_{n=1}^{+\infty} (q^2)^n \tag{3}$$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{p}{1-q^2} = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{6}{11} \tag{4}$$

$$\text{P(A wins the game) = P(X) = } \frac{6}{11}$$

Option D