Assignment 3

Vibhavasu Pasumarti - EP20BTECH11015

Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 Problem

GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

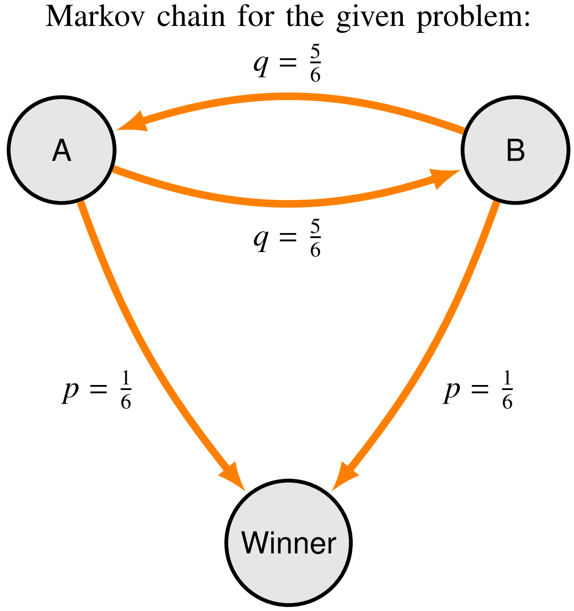
Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

2 Solution

Let the random variable X denote the win of A.
Given the die is fair.
The probability of getting 6 = $\frac{1}{6}$ = p (say)
The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

Markov chain for the given problem:



Transition matrix:

$$P = \begin{bmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{bmatrix} \implies \lim_{n\to\infty} P^n \to 0 \because p, q < 1$$

$\implies P$is reducible

$\implies \lim_{n\to\infty} Pr_{AW}(n) = lim_{n\to\infty} Pr_{BW}(n) = 0$

Constraint: A wins the game.

$$\Pr(\text{A wins}) = \sum_{n=1}^{\infty} Pr_{AW}(n) \tag{2.0.1}$$

$$= Pr_{AW}(1) + Pr_{AW}(2) + Pr_{AW}(3) + ... \tag{2.0.2}$$

$$= P_{13} + 0 + P_{13}^3 + 0 + P_{13}^5 + ... \tag{2.0.3}$$

$$= p + q^2 p + q^4 p + ... \tag{2.0.4}$$

$$= \sum_{k=1}^{\infty} q^{2k} p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1 \tag{2.0.5}$$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{p}{1-q^2} = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \tag{2.0.6}$$

$$\Pr(\text{A wins the game}) = \Pr(X) = \frac{6}{11}$$

Option D