

Assignment 3

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Download all python codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py>

and latex-tikz codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf>

1 GATE 2015 (EE PAPER 01 NEW 2), Q 27 (ELECTRICAL ENGG SECTION)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

- (A) $\frac{5}{11}$
 (B) $\frac{1}{2}$
 (C) $\frac{7}{13}$
 (D) $\frac{6}{11}$

2 SOLUTION

- Given the die is fair.
- So, for any given throw by A or B:

The probability of getting 6 = $\frac{1}{6} = p$ (say)

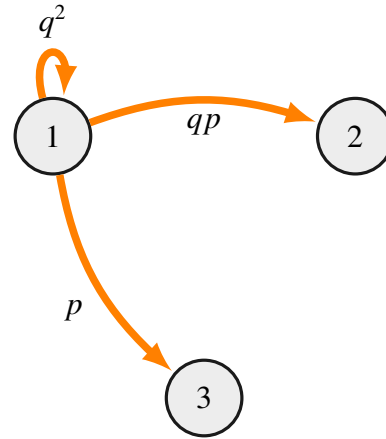
The probability of NOT getting 6 = $\frac{5}{6} = q$ (say)

Constraint: A starts the game and A should win.
 \Rightarrow Until A wins, both A and B cannot win.

State	Corresponding description
1	A and B both lose.
2	A loses and B wins.
3	WINNER

Table 1: State description table

Corresponding Markov chain is:



Definition 2.1. 2 and 3 are ABSORBING states.

Transition/Stochastic matrix is: T

$$\begin{matrix} & \text{(To)} \\ & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} \text{(From)} \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} q^2 & qp & p \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = T \end{matrix}$$

Theorem 2.1. Every transition matrix can be

partitioned as $\begin{pmatrix} Q & P \\ O & J \end{pmatrix}$

where Q arise from transition probabilities between non-absorbing states

R arises from Transition probability from non-absorbing state to absorbing state.

O = Null matrix, J = Identity matrix

T^k approaches \bar{T} as k increases. \bar{T} is the limiting

matrix and $\bar{T} = \begin{pmatrix} O & NP \\ O & J \end{pmatrix}$

where N is the fundamental matrix. $N = (I - Q)^{-1}$

Definition 2.2. The matrix D = NP gives the probability of ending up in the absorbing states (2 and 3) when the chain starts from a non-absorbent state 1.

$$T = \left(\begin{array}{c|cc} q^2 & qp & p \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right)$$

$$Q = (q^2), P = (qp \quad p), O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (1)$$

$$N = (1 - q^2)^{-1} = \left(\frac{1}{1 - q^2} \right) \quad (2)$$

$$D = NP = \left(\frac{qp}{1 - q^2} \quad \frac{p}{1 - q^2} \right) \quad (3)$$

$$\Pr(\text{A wins}) = D_2 = \frac{p}{1 - q^2} = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \quad (4)$$

$$\Pr(\text{A wins}) = \frac{6}{11}$$

OPTION D is correct