Assignment 3

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Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

2 Solution

- Given the die is fair.
- So, for any given throw by A or B:
  The probability of getting 6 = $\frac{1}{6}$ = p (say)
  The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

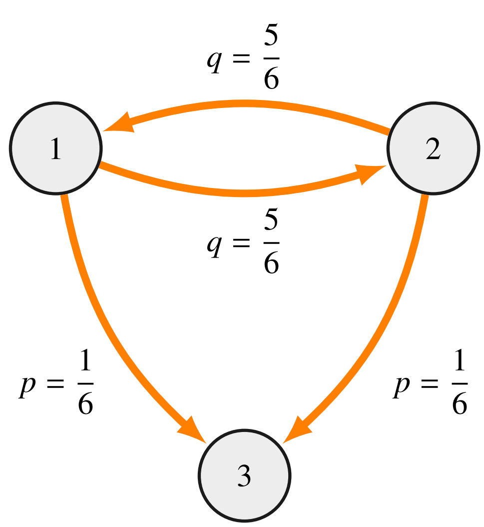

Fig. 0: Markov chain for the given problem

State	Corresponding description
1	Player who plays first. Here: PLAYER A
2	Player who plays second. Here: PLAYER B
3	WINNER

Table 1: State description table

Definition 2.1. Transition/Stochastic matrix corresponding to single throw of dice: X

$$\begin{array}{cc} & \text{(To)} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{(From)} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{bmatrix} = X \end{array}$$

X^n corresponds to n throws of dice

$$\Pr(\text{A wins in 'i' throws}) = p_{13}(i) = (X^i)_{13} \tag{2.0.1}$$

Lemma 2.1.

$$\lim_{n\to\infty} p_{13}(n) = 0 \tag{2.0.2}$$

Proof.

$$p, q < 1 \implies \text{X is reducible.} \tag{2.0.3}$$

$$\implies \lim_{n\to\infty}(X)^n \to 0 \tag{2.0.4}$$

$$\lim_{n\to\infty}(X)^n{}_{13} = 0 \therefore \lim_{n\to\infty} p_{13}(n) = 0 \tag{2.0.5}$$

□

Given: A throws first.

$$\implies p_{13}(1) = X_{13} = p \tag{2.0.6}$$

After second throw: (by B)

$$X^2 = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.7}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix} \implies p_{13}(2) = 0 \tag{2.0.8}$$

$$X^3 = \begin{pmatrix} 0 & q & p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.9}$$

$$= \begin{pmatrix} 0 & q^3 & q^2 p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \implies p_{13}(3) = q^2 p \tag{2.0.10}$$

$p_{13}(i)$	Probability
$p_{13}(1)$	$p = 1/6$
$p_{13}(2)$	0
$p_{13}(3)$	$q^2 p$
$\vdots$ $p_{13}(2n)$	$\vdots$ 0
$p_{13}(2n+1)$	$q^{2n} p$

Table 2: Summary of Probabilities

$$\Pr(\text{A wins}) = \sum_{n=1}^{\infty} p_{13}(n) \tag{2.0.11}$$

$$= p + 0 + q^2 p + 0 + q^4 p + ... \tag{2.0.12}$$

$$= \sum_{k=1}^{\infty} (q^2)^k p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1 \tag{2.0.13}$$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \tag{2.0.14}$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D is correct