Assignment 3

Vibhavasu Pasumarti - EP20BTECH11015

Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

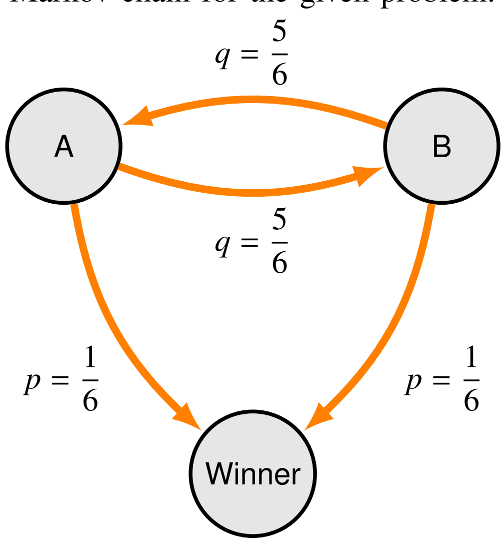
2 Solution

- Given the die is fair.
- So, for any given throw by A or B:

  The probability of getting 6 = $\frac{1}{6}$ = p (say)

  The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

Markov chain for the given problem:



Definition 2.1. Transition/Stochastic matrix corresponding to each throw of dice:

$$X = \begin{array}{c c} & \begin{array}{ccc} A & B & Winner \end{array} \\ \begin{array}{c} A \\ B \\ Winner \end{array} & \begin{bmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\implies p_{AW}(i) = p_{13}(i) = (X^i)_{13}$$

Lemma 2.1.

$$\lim_{n\to\infty} p_{13}(n) = 0$$

Proof.

$$p, q < 1 \implies \text{X is reducible.}$$
$$\implies \lim_{n\to\infty}(X)^n \to 0$$
$$\lim_{n\to\infty}(X)^n{}_{13} = 0 \therefore \lim_{n\to\infty} p_{13}(n) = 0$$

□

Given: A throws first.

$$\implies p_{13}(1) = X_{13} = p$$

Second throw: (by B)

$$X^2 = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix} \implies p_{13}(2) = 0$$
$$X^3 = \begin{pmatrix} 0 & q & p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & q^3 & q^2p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \implies p_{13}(3) = q^2p$$

$p_{13}(i)$	Probability
$p_{13}(1)$	$p = 1/6$
$p_{13}(2)$	0
$p_{13}(3)$	$q^2 p$
$\vdots$	$\vdots$
$p_{13}(2n)$	0
$p_{13}(2n+1)$	$q^{2n} p$

Table:1 Summary of Probabilities

$$\begin{aligned}
\Pr(\text{A wins}) &= \sum_{n=1}^{\infty} p_{13}(n) = p + 0 + q^2 p + 0 + q^4 p + \ldots. \\
&= \sum_{k=1}^{\infty} (q^2)^k p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1 \\
&= p\left(\frac{1}{1-q^2}\right) = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{6}{11}
\end{aligned}$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D is correct