

Assignment 3

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Download all python codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py>

and latex-tikz codes from

<https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf>

1 GATE 2015 (EE PAPER 01 NEW 2), Q 27 (ELECTRICAL ENGG SECTION)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

- (A) $\frac{5}{11}$
 (B) $\frac{1}{2}$
 (C) $\frac{7}{13}$
 (D) $\frac{6}{11}$

2 SOLUTION

- Given the die is fair.
- So, for any given throw by A or B:

The probability of getting 6 = $\frac{1}{6} = p$ (say)

The probability of NOT getting 6 = $\frac{5}{6} = q$ (say)

State	Corresponding description
1	Player who plays first. Here: PLAYER A
2	Player who plays second. Here: PLAYER B
3	WINNER

Table 1: State description table

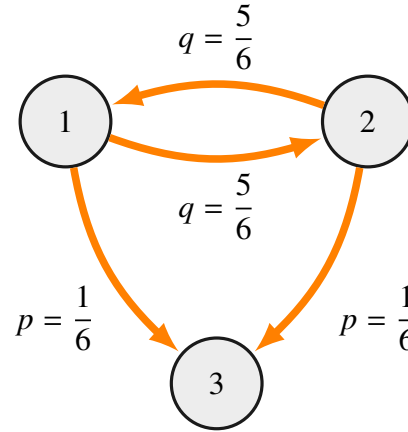


Fig. 4: Markov chain for the given problem

Definition 2.1. From Fig 4 and (??)
 Transition/Stochastic matrix corresponding to single throw of dice: X

$$\begin{matrix} & \text{(To)} \\ & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} \text{(From)} \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{bmatrix} = X \end{matrix}$$

X^n corresponds to n throws of dice

$$\Pr(\text{A wins in 'i' throws}) = p_{13}(i) = (X^i)_{13} \quad (1)$$

Lemma 2.1.

$$\lim_{n \rightarrow \infty} p_{13}(n) = 0 \quad (2)$$

Proof.

$$p, q < 1 \implies X \text{ is REDUCIBLE.} \quad (3)$$

$$\implies \lim_{n \rightarrow \infty} (X)^n \rightarrow 0 \quad (4)$$

$$\lim_{n \rightarrow \infty} (X)^n_{13} = 0 \therefore \lim_{n \rightarrow \infty} p_{13}(n) = 0 \quad (5)$$

□

Given: A throws first.

$$\implies p_{13}(1) = X_{13} = p \quad (6)$$

After second throw (by B):

$$X^2 = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \quad (7)$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix} \implies p_{13}(2) = 0 \quad (8)$$

$$X^3 = \begin{pmatrix} 0 & q & p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} 0 & q^3 & q^2p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \implies p_{13}(3) = q^2p \quad (10)$$

$p_{13}(i)$	Probability
$p_{13}(1)$	$p = 1/6$
$p_{13}(2)$	0
$p_{13}(3)$	q^2p
\vdots	\vdots
$p_{13}(2n)$	0
$p_{13}(2n+1)$	$q^{2n}p$

Table 2: Summary of Probabilities

From (4) and Table 2:

$$\Pr(\text{A wins}) = \sum_{n=1}^{\infty} p_{13}(n) \quad (11)$$

$$= p + 0 + q^2p + 0 + q^4p + \dots \quad (12)$$

$$= \sum_{k=1}^{\infty} (q^2)^k p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1 \quad (13)$$

$$= p \left(\frac{1}{1 - q^2} \right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11} \quad (14)$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D is correct