Assignment 3

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Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

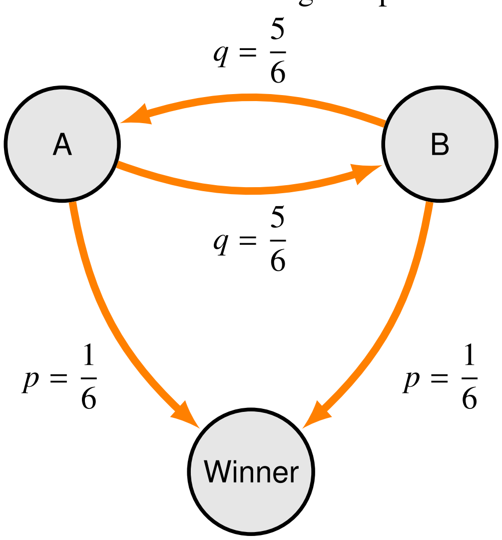
Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

2 Solution

- Given the die is fair.
- So, for any given throw by A or B:
  The probability of getting 6 = $\frac{1}{6}$ = p (say)
  The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)

Markov chain for the given problem:



Definition 2.1. Transition/Stochastic matrix corresponding to each throw of dice:

$$X = \begin{array}{c c} & \begin{array}{ccc} A & B & Winner \end{array} \\ \begin{array}{c} A \\ B \\ Winner \end{array} & \begin{bmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\implies p_{AW}(i) = (X^i)_{13}$$

Lemma 2.1.

$$\lim_{n\to\infty} p_{AW}(n) = 0$$

Proof.

$$p, q < 1 \implies \text{X is reducible.}$$
$$\implies \lim_{n\to\infty} (X)^n \to 0$$
$$\lim_{n\to\infty} (X)^n{}_{13} = 0 \therefore \lim_{n\to\infty} p_{AW}(n) = 0$$

□

Given: A throws first.

$$\implies p_{AW}(1) = X_{13} = p$$

Second throw: (by B)

$$X^2 = \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix} \implies (X^2)_{13} = 0$$
$$X^3 = \begin{pmatrix} 0 & q & p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & q^2 & qp \\ 0 & 0 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & q^3 & q^2 p \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \implies (X^3)_{13} = q^2 p$$

X	No.of Turns	Probability
$(X^1)_{13}$	1	p = 1/6
$(X^2)_{13}$	2	0
$(X^3)_{13}$	3	q^2p
$\vdots$	$\vdots$	$\vdots$
$(X^{2n})_{13}$	2n	0
$(X^{2n+1})_{13}$	2n + 1	$q^{2n}p$

Table:1 Summary of Probabilities

$$\Pr(\text{A wins}) = \sum_{n=1}^{\infty} p_{AW}(n) = \sum_{n=1}^{\infty} (X^n)13$$

$$= p + 0 + q^2 p + 0 + q^4 p + ...$$

$$= \sum_{k=1}^{\infty} (q^2)^k p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1$$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{6}{11}$$

$$\Pr(\text{A wins the game}) = \frac{6}{11}$$

Option D is correct