Assignment 3

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Download all python codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/code/Assignment%203.py

and latex-tikz codes from

https://github.com/VIB2020/AI1103/blob/main/Assignment%203/Assignment%203.pdf

1 Problem

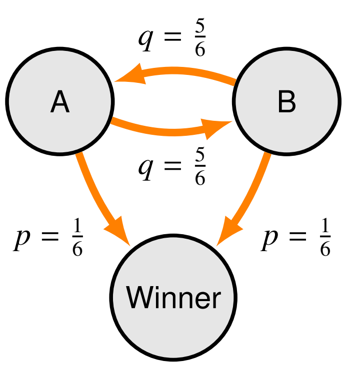
GATE 2015 (EE paper 01 new 2), Q 27 (Electrical Engg section)

Two players A, and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is:

A $\frac{5}{11}$ B $\frac{1}{2}$ C $\frac{7}{13}$ D $\frac{6}{11}$

2 Solution

Let the random variable X denote the win of A.
Given the die is fair.
The probability of getting 6 = $\frac{1}{6}$ = p (say)
The probability of NOT getting 6 = $\frac{5}{6}$ = q (say)
Markov chain for the given problem:



Transition/Stochastic matrix:

P = $\begin{pmatrix} 0 & q & p \\ q & 0 & p \\ 0 & 0 & 0 \end{pmatrix} \implies \lim_{n\to\infty}$ P$^n \to 0 \because p, q < 1$

$\implies P$ is reducible $\implies \lim_{n\to\infty} Pr_{AW}(n) = 0$

P^n represents n rolls of dice
Given: A starts the game
Constraint: A wins the game.
The initial operator representing this constraint:
A$_1$ = $\begin{pmatrix} p & 0 & 0 \end{pmatrix}$
First element of each A$_i$ represents the probability of A to win in ith roll

$$\text{Pr}_{AW}(1) = p$$

$$A_2 = A_1 P = \begin{pmatrix} p & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} 0 & qp & 0 \end{pmatrix}$$

$$\implies Pr_{AW}(2) = 0$$

$$A_3 = A_2 P = \begin{pmatrix} 0 & qp & 0 \end{pmatrix} \begin{pmatrix} 0 & q & 0 \\ q & 0 & 0 \\ p & p & 0 \end{pmatrix} = \begin{pmatrix} q^2 p & 0 & 0 \end{pmatrix}$$

$$\implies Pr_{AW}(3) = q^2 p$$

$$A_4 = \begin{pmatrix} 0 & q^3 p & 0 \end{pmatrix} \implies Pr_{AW}(4) = 0$$

$$A_5 = \begin{pmatrix} q^4 p & 0 & 0 \end{pmatrix} \implies Pr_{AW}(5) =$$

$$q^4 pBy induction : A_{2n} = \begin{pmatrix} 0 & q^{2n-1} p & 0 \end{pmatrix} \implies$$

$$Pr_{AW}(2n) = 0$$

$$A_{2n+1} = \begin{pmatrix} q^{2n} & 0 & 0 \end{pmatrix} \implies Pr_{AW}(2n+1) = q^{2n} p$$

$$\text{Pr}\left(\text{A wins}\right) = \textstyle\sum_{n=1}^{\infty} Pr_{AW}(n)$$

$$= Pr_{AW}(1) + Pr_{AW}(2) + Pr_{AW}(3) + ...$$

$$= p + 0 + q^2 p + 0 + q^4 p + ...$$

$$= \textstyle\sum_{k=1}^{\infty} q^{2k} p = p \sum_{k=1}^{\infty} (q^2)^k \quad \text{But } |q| < 1$$

$$= p\left(\frac{1}{1-q^2}\right) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$$\text{Pr}\left(\text{A wins the game}\right) = \frac{6}{11}$$

Option D