Pingala Series

VIBHAVASU - EP20BTECH11015

Contents

***Abstract*—This manual provides a simple introduction to Transforms**

1 JEE 2019

Let

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1 \tag{1.1}$$

$$b_n = a_{n-1} + a_{n+1}, \quad n \geq 2, \quad b_1 = 1 \tag{1.2}$$

Verify the following using a python code.

1.1

$$\sum_{k=1}^{n} a_k = a_{n+2} - 1, \quad n \geq 1 \tag{1.3}$$

Solution:

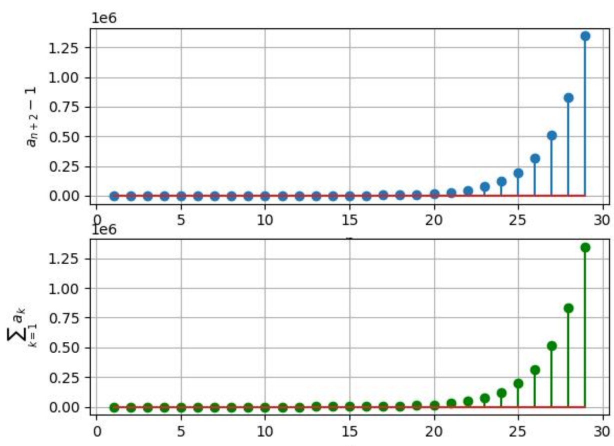

Fig. 1.1

From 1.1 above equation is **True**

1.2

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{10}{89} \tag{1.4}$$

Solution:
Proposed value of above summation = $\frac{10}{89}$ = 0.11235955056179775
Calculated value from the code = 0.11235955056179774
∴ The above equation is **True**

1.3

$$b_n = \alpha^n + \beta^n, \quad n \geq 1 \tag{1.5}$$

Solution:

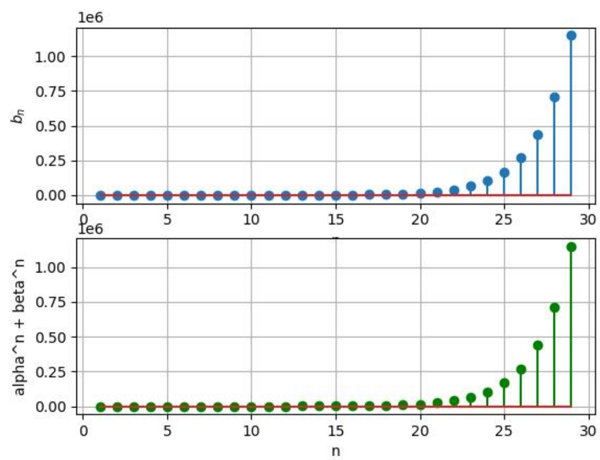

Fig. 1.3

From 2.2 above equation is **True**

1.4

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{8}{89} \tag{1.6}$$

Solution:
Proposed value of above summation = $\frac{8}{89}$ = 0.0898876404494382
Calculated value from the code = 0.1348314606741573
∴ The above relation is **False**

Solution:
The solution to all above questions can be found at:

```
wget https://github.com/DarkWake9/EE3900/
    blob/main/pingala/codes/e1.py
```

2 Pingala Series

2.1 The *one sided* Z-transform of $x(n)$ is defined as

$$X^{+}(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \quad z \in \mathbb{C} \tag{2.1}$$

2.2 The *Pingala* series is generated using the difference equation

$$x(n+2) = x(n+1) + x(n), \quad x(0) = x(1) = 1, n \geq 0 \tag{2.2}$$

Generate a stem plot for $x(n)$.
Solution:

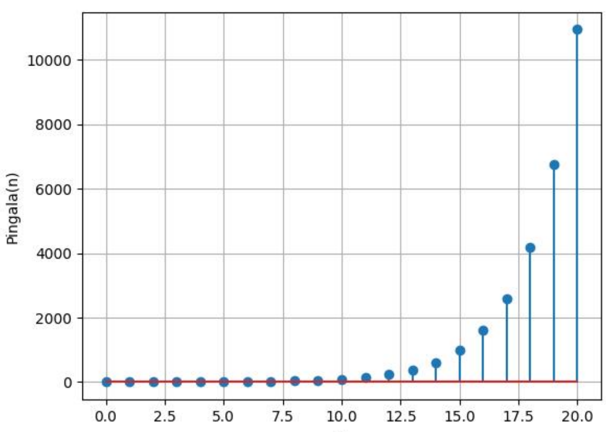

Fig. 2.2

2.3 Find $X^{+}(z)$.
Solution:

From 2.3: we see that

$$x(n) = a_{n+1} \quad n \geq 1 \tag{2.3}$$

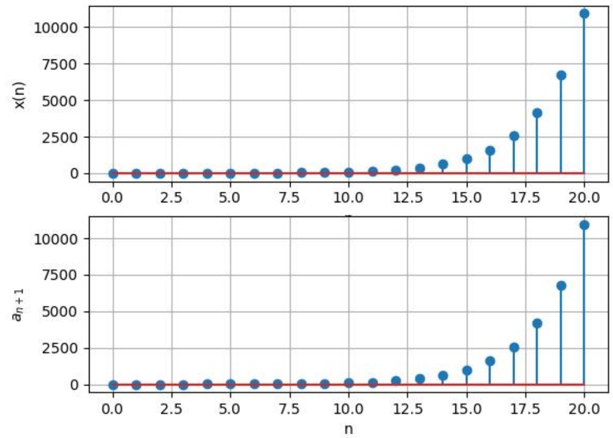

Fig. 2.3

Apply $\mathcal{Z}^{+}$ transform on both sides

$$= 1 + \sum_{n=1}^{\infty} a_{n+1} z^{-n} \tag{2.4}$$

$$= \sum_{n=0}^{\infty} \frac{\alpha^{n+1} - \beta^{n+1}}{(\alpha - \beta)} z^{-n} \tag{2.5}$$

$$= \frac{1}{\alpha - \beta} \left(\frac{\alpha}{1 - \alpha z^{-1}} - \frac{\beta}{1 - \beta z^{-1}} \right) \tag{2.6}$$

$$= \frac{1}{(1 - \alpha z^{-1})(1 - \beta z^{-1})} \tag{2.7}$$

$$X^{+}(z) = \frac{1}{1 - z^{-1} - 2^{-2}} \tag{2.8}$$

2.4 Find $x(n)$.
2.5 Sketch

$$y(n) = x(n-1) + x(n+1), \quad n \geq 0 \tag{2.9}$$

2.6 Find $Y^{+}(z)$.
2.7 Find $y(n)$.

3 Power of the Z transform

3.1 Show that

$$\sum_{k=1}^{n} a_k = \sum_{k=0}^{n-1} x(n) = x(n) * u(n-1) \tag{3.1}$$

3.2 Show that

$$a_{n+2} - 1, \quad n \geq 1 \tag{3.2}$$

can be expressed as

$$[x(n+1) - 1] u(n) \tag{3.3}$$

3.3 Show that

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{x(k)}{10^k} = \frac{1}{10} X^+(10) \quad (3.4)$$

3.4 Show that

$$\alpha^n + \beta^n, \quad n \geq 1 \quad (3.5)$$

can be expressed as

$$w(n) = \left(\alpha^{n+1} + \beta^{n+1}\right) u(n) \quad (3.6)$$

and find $W(z)$.

3.5 Show that

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{y(k)}{10^k} = \frac{1}{10} Y^+(10) \quad (3.7)$$

3.6 Solve the JEE 2019 problem.