EE 3900 - Assignment 2

VIBHAVASU

1 QUESTION

Opp 3.26d - Determine the Inverse *z*-Transform of:

$$X(z) = \frac{1}{1 - \frac{1}{3}z^{-3}} \quad |z| > 3^{-\frac{1}{3}}$$

2 SOLUTION

Given ROC $|z| > 3^{-\frac{1}{3}}$

$$\implies 1 - \frac{1}{3}z^{-3} < 1 \quad (2.1)$$

Taking the Taylor expansion:

$$X(z) = \left(1 - \frac{1}{3}z^{-3}\right)^{-1} \quad (2.2)$$

$$= 1 + \frac{1}{3}z^{-3} + \frac{1}{9}z^{-6} + \frac{1}{27}z^{-9} + \dots$$
 (2.3)

 ${\mathcal Z}$ - transform is defined as:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (2.4)$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + \dots$$
 (2.5)

Comparing coefficients:

$$x(0) = 1$$
 $x(1) = 0$ $x(2) = 0$ $x(3) = \frac{1}{3}$ (2.6)

$$x(4) = 1$$
 $x(5) = 0$ $x(6) = \frac{1}{9}$ $x(7) = 0$ (2.7)

$$x(n) = \begin{cases} \left(\frac{1}{3}\right)^{\frac{n}{3}} & \text{if } \frac{n}{3} = \left[\frac{n}{3}\right] \\ 0 & \text{otherwise} \end{cases}$$
 (2.8)

where [x] is the Greatest Integer function