Pingala Series

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Contents

***Abstract*—This manual provides a simple introduction to Transforms**

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Let

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \ge 1 \tag{1.1}$$

$$b_n = a_{n-1} + a_{n+1}, \quad n \ge 2, \quad b_1 = 1 \tag{1.2}$$

Verify the following using a python code.

1.1

$$\sum_{k=1}^{n} a_k = a_{n+2} - 1, \quad n \ge 1 \tag{1.3}$$

1.2

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{10}{89} \tag{1.4}$$

1.3

$$b_n = \alpha^n + \beta^n, \quad n \ge 1 \tag{1.5}$$

1.4

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{8}{89} \tag{1.6}$$

2 Pingala Series

2.1 The *one sided* Z-transform of $x(n)$ is defined as

$$X^+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \quad z \in \mathbb{C} \tag{2.1}$$

2.2 The *Pingala* series is generated using the difference equation

$$x(n+2) = x(n+1) + x(n), \quad x(0) = x(1) = 1, n \ge 0 \tag{2.2}$$

Generate a stem plot for $x(n)$.

2.3 Find $X^+(z)$.

2.4 Find $x(n)$.

2.5 Sketch

$$y(n) = x(n-1) + x(n+1), \quad n \ge 0 \tag{2.3}$$

2.6 Find $Y^+(z)$.

2.7 Find $y(n)$.

3 Power of the Z transform

3.1 Show that

$$\sum_{k=1}^{n} a_k = \sum_{k=0}^{n-1} x(n) = x(n) * u(n-1) \tag{3.1}$$

3.2 Show that

$$a_{n+2} - 1, \quad n \ge 1 \tag{3.2}$$

can be expressed as

$$[x(n+1) - 1]\, u(n) \tag{3.3}$$

3.3 Show that

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{x(10)}{10^k} = \frac{1}{10} X^+ \left(\frac{1}{10}\right) \tag{3.4}$$

3.4 Show that

$$\alpha^n + \beta^n, \quad n \ge 1 \tag{3.5}$$

can be expressed as

$$w(n) = \left(\alpha^{n+1} + \beta^{n+1}\right) u(n) \tag{3.6}$$

and find $W(z)$.

3.5 Show that

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{y(k)}{10^k} = \frac{1}{10} Y^+ \left(\frac{1}{10}\right) \tag{3.7}$$

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