1 Definitions

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \tag{1.1}$$

2. The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st}\,dt \tag{1.2}$$

3. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1\,\mu C$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2\,\mu C$.

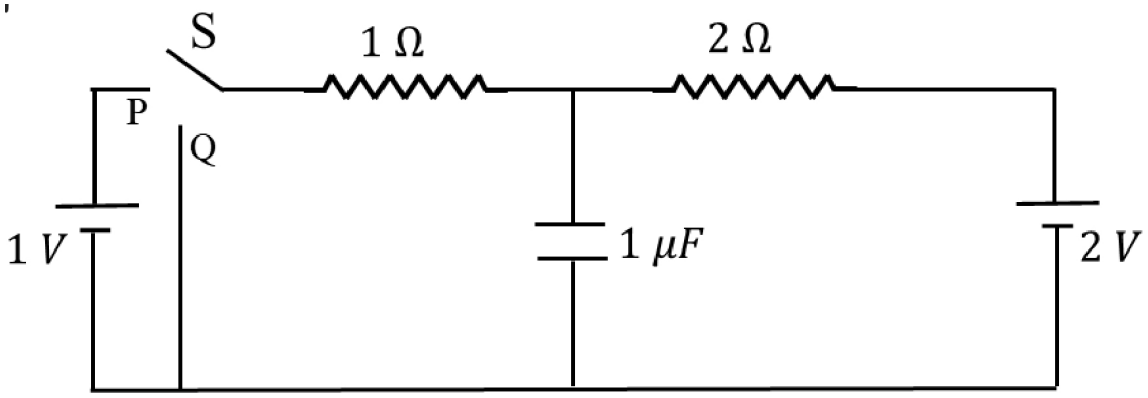

Fig. 1.1

2 Problems

1. Find q_1.
2. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC.
3. Show that

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad a > 0 \tag{2.1}$$

and find the ROC.

4. Now consider the following resistive circuit transformed from Fig. 1.1 where

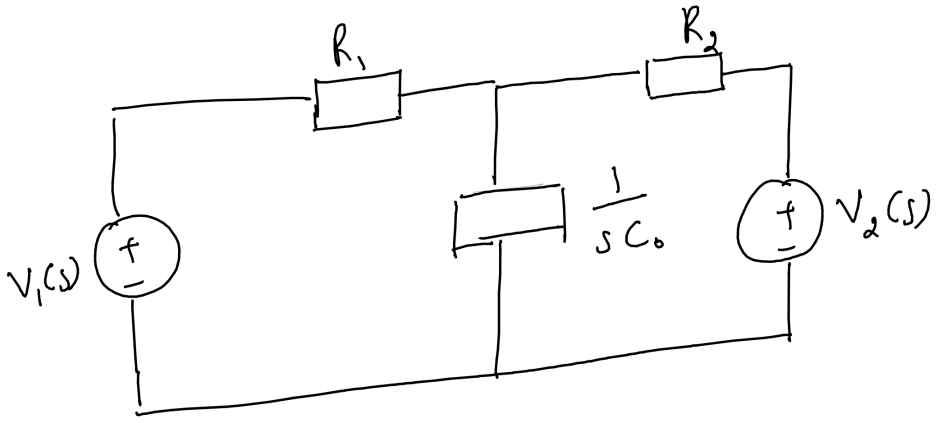

Fig. 2.1

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} V_1(s) \tag{2.2}$$

$$2u(t) \overset{\mathcal{L}}{\longleftrightarrow} V_2(s) \tag{2.3}$$

Find the voltage across the capacitor $V_{C_0}(s)$.

5. Find $v_{C_0}(t)$.