Quiz 4 EE3900

1 Definitions

1. The convolution of $p_{X_1}(n)$ and $p_{X_2}(n)$ is defined as

$$p_{X_1}(n) * p_{X_2}(n) = \sum_k p_{X_1}(n-k)p_{X_2}(k) \tag{1.1}$$

2 Problems

1. Two dice, one blue and one grey, are thrown at the same time. The event defined by the sum of the two numbers appearing on the top of the dice can have 11 possible outcomes 2, 3, 4, 5, 6, 6, 8, 9, 10, 11 and 12. A student argues that each of these outcomes has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.
2. Let $X_i \in \{1,2,3,4,5,6\}, i = 1,2,$ be the random variables representing the outcome for each die. Assuming the dice to be fair, show that the probability mass function (pmf) is expressed as

$$p_{X_i}(n) = \Pr\left(X_i = n\right) = \begin{cases} \frac{1}{6} & 1 \le n \le 6 \\ 0 & otherwise \end{cases} \tag{2.1}$$

3. Let

$$X = X_1 + X_2, \tag{2.2}$$

4. Using the fact that

$$p_X(n) = \sum_k \Pr\left(X_1 = n-k|X_2 = k\right) p_{X_2}(k) \tag{2.3}$$

show that

$$p_X(n) = p_{X_1}(n) * p_{X_2}(n) \tag{2.4}$$

assuming X_1 and X_2 to be independent,

5. Show that

$$p_X(n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \leq n \leq 7 \\ \frac{13-n}{36} & 7 < n \leq 12 \\ 0 & n > 12 \end{cases} \tag{2.5}$$