Quiz 9 EE3900

1 Definitions

1. The *Kronecker delta* function is defined as

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{1.1}$$

2. The unit step function is

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1.2}$$

3. The *one sided* Z-transform of $x(n)$ is defined as

$$X^+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \quad z \in \mathbb{C} \tag{1.3}$$

4. α, β are the roots of the equation

$$t^2 - t - 1 = 0 \tag{1.4}$$

5.

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1 \tag{1.5}$$

6.

$$b_n = a_{n-1} + a_{n+1}, \quad n \geq 2, \quad b_1 = 1 \tag{1.6}$$

2 Problems

1. Show that (1.6) can be expressed as

$$y(n) = x(n-1) + x(n+1), \quad x(0) = y(0) = 1, n \geq 0 \tag{2.1}$$

where

$$x(n) = a(n+1), \quad n \geq 0 \tag{2.2}$$
$$y(n) = b(n+1), \quad n \geq 0 \tag{2.3}$$

2. Show that the one sided Z transform of $x(n-1)$ and $x(n+1)$ are

$$x(-1) + z^{-1}X^{+}(z) \tag{2.4}$$

and

$$zX^{+}(z) - zx(0) \tag{2.5}$$

respectively.

3. Show that

$$X^{+}(z) = \frac{1}{1 - z^{-1} - z^{-2}} \tag{2.6}$$

4. Show that

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{x(k)}{10^k} \tag{2.7}$$

5. Show that

$$Y^{+}(z) = \frac{1 + 2z^{-1}}{1 - z^{-1} - z^{-2}} \tag{2.8}$$

6. Show that

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{1}{10} \sum_{k=1}^{\infty} \frac{y(k)}{10^k} \tag{2.9}$$

7. Show that

$$\sum_{k=1}^{n} a_k = x(n) * u(n-1) \tag{2.10}$$

8. Find $y(n)$.
9. Which of the following options is/are correct?

a)

$$\sum_{k=1}^{n} a_k = a_{n+2} - 1, \quad n \geq 1 \tag{2.11}$$

b)

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{10}{89} \tag{2.12}$$

c)

$$b_n = \alpha^n + \beta^n, \quad n \geq 1 \tag{2.13}$$

d)

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{8}{89} \tag{2.14}$$