Circuits and Transforms

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Contents

***Abstract*—This manual provides a simple introduction to Transforms**

1 Definitions

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \tag{1.1}$$

2. The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st}\,dt \tag{1.2}$$

2 Laplace Transform

1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1\,\mu C$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2\,\mu C$.

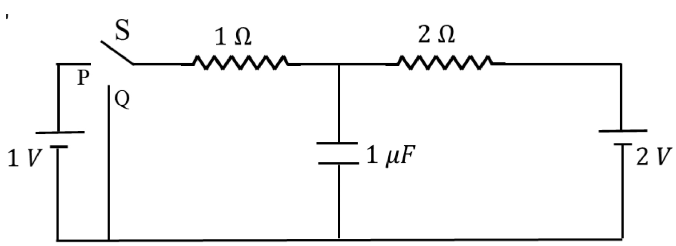

Fig. 2.1

2. Draw the circuit using latex-tikz.
3. Find q_1.
4. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC.
5. Show that

$$e^{-at}u(t) \overset{\mathcal{H}}{\longleftrightarrow} L\frac{1}{s+a}, \quad a > 0 \tag{2.1}$$

and find the ROC.

6. Now consider the following resistive circuit transformed from Fig. 2.1 where

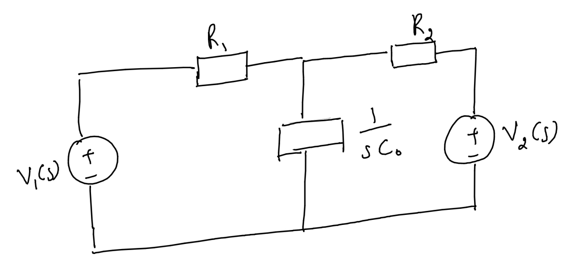

Fig. 2.2

$$u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_1(s) \tag{2.2}$$

$$2u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_2(s) \tag{2.3}$$

Find the voltage across the capacitor $V_{C_0}(s)$.

7. Find $v_{C_0}(t)$. Plot using python.
8. Verify your result using ngspice.
9. Obtain Fig. 2.2 using the equivalent differential equation.

3 Initial Conditions

1. Find q_2 in Fig. 2.1.
2. Draw the equivalent s-domain resistive circuit when S is switched to position Q. Use variables R_1, R_2, C_0 for the passive elements. Use latex-tikz.
3. $V_{C_0}(s) = ?$
4. $v_{C_0}(t) = ?$ Plot using python.
5. Verify your result using ngspice.
6. Find $v_{C_0}(0-), v_{C_0}(0+)$ and $v_{C_0}(\infty)$.

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7. Obtain the Fig. in problem 3.2 using the equivalent differential equation.