1 Definitions

1. The *Kronecker delta* function is defined as

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{1.1}$$

2. The unit step function is

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1.2}$$

3. The *one sided* Z-transform of $x(n)$ is defined as

$$X^+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \quad z \in \mathbb{C} \tag{1.3}$$

4. The *Pingala* series is generated using the difference equation

$$x(n+2) = x(n+1) + x(n), \quad x(0) = x(1) = 1, n \geq 0 \tag{1.4}$$

5. α, β are the roots of the equation

$$z^2 - z - 1 = 0 \tag{1.5}$$

6.

$$y(n) = x(n-1) + x(n+1), \quad n \geq 0 \tag{1.6}$$

7.

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \geq 1 \tag{1.7}$$

8.

$$b_n = a_{n-1} + a_{n+1}, \quad n \geq 2, \quad b_1 = 1 \tag{1.8}$$

2 Problems

1. Show that

$$\sum_{k=1}^{n} a_k = \sum_{k=0}^{n-1} x(n) = x(n) * u(n-1) \tag{2.1}$$

2. Show that

$$a_{n+2} - 1, \quad n \geq 1 \tag{2.2}$$

can be expressed as

$$[x(n+1) - 1]\, u(n) \tag{2.3}$$

3. Show that

$$\sum_{k=1}^{\infty} \frac{a_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{x(k)}{10^k} = \frac{1}{10} X^{+}(10) \tag{2.4}$$

4. Show that

$$\alpha^n + \beta^n, \quad n \geq 1 \tag{2.5}$$

can be expressed as

$$w(n) = \left(\alpha^{n+1} + \beta^{n+1}\right) u(n) \tag{2.6}$$

and find $W(z)$.

5. Show that

$$\sum_{k=1}^{\infty} \frac{b_k}{10^k} = \frac{1}{10} \sum_{k=0}^{\infty} \frac{y(k)}{10^k} = \frac{1}{10} Y^{+}(10) \tag{2.7}$$