Quiz X EE3900

## 1 Definitions

1. The DFT of $x(n)$ is given by
$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.1}$$
2. Let
$$W_N = e^{-j2\pi/N} \tag{1.2}$$
Then the $N$-point *DFT matrix* is defined as
$$\mathbf{F}_N = [W_N^{mn}] \tag{1.3}$$
where $W_N^{mn}$ are the elements of $\mathbf{F}_N$.
3. Let
$$\mathbf{I}_4 = \begin{pmatrix} \mathbf{e}_4^1 & \mathbf{e}_4^2 & \mathbf{e}_4^3 & \mathbf{e}_4^4 \end{pmatrix} \tag{1.4}$$
be the $4 \times 4$ identity matrix. Then the 4 point *DFT permutation matrix* is defined as
$$\mathbf{P}_4 = \begin{pmatrix} \mathbf{e}_4^1 & \mathbf{e}_4^3 & \mathbf{e}_4^2 & \mathbf{e}_4^4 \end{pmatrix} \tag{1.5}$$
4. The 4 point *DFT diagonal matrix* is defined as
$$\mathbf{D}_4 = diag\begin{pmatrix} W_N^0 & W_N^1 & W_N^2 & W_N^3 \end{pmatrix} \tag{1.6}$$

## 2 Problems

1. Show that
$$W_N^2 = W_{N/2} \tag{2.1}$$
2. Find $\mathbf{P}_6$.
3. Find $\mathbf{D}_3$.
4. Show that
$$\mathbf{F}_6 = \begin{bmatrix} \mathbf{I}_3 & \mathbf{D}_3 \\ \mathbf{I}_3 & -\mathbf{D}_3 \end{bmatrix} \begin{bmatrix} \mathbf{F}_3 & 0 \\ 0 & \mathbf{F}_3 \end{bmatrix} \mathbf{P}_3 \tag{2.2}$$
5. Show that
$$\mathbf{F}_N = \begin{bmatrix} \mathbf{I}_{N/2} & \mathbf{D}_{N/2} \\ \mathbf{I}_{N/2} & -\mathbf{D}_{N/2} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{N/2} & 0 \\ 0 & \mathbf{F}_{N/2} \end{bmatrix} \mathbf{P}_N \tag{2.3}$$

6. Find

$$\mathbf{P}_6\mathbf{x} \tag{2.4}$$

7. Show that

$$\mathbf{X} = \mathbf{F}_N\mathbf{x} \tag{2.5}$$

where $\mathbf{x}, \mathbf{X}$ are the vector representations of $x(n), X(k)$ respectively.

8. Let

$$\begin{pmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \end{pmatrix} = F_3 \begin{pmatrix} x(0) \\ x(2) \\ x(4) \end{pmatrix} \tag{2.6}$$

$$\begin{pmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \end{pmatrix} = F_3 \begin{pmatrix} x(1) \\ x(3) \\ x(5) \end{pmatrix} \tag{2.7}$$

Show that

$$\begin{pmatrix} X(0) \\ X(1) \\ X(2) \end{pmatrix} = \begin{pmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \end{pmatrix} + \begin{pmatrix} W_6^0 & 0 & 0 \\ 0 & W_6^1 & 0 \\ 0 & 0 & W_6^2 \end{pmatrix} \begin{pmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \end{pmatrix} \tag{2.8}$$

$$\begin{pmatrix} X(3) \\ X(4) \\ X(5) \end{pmatrix} = \begin{pmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \end{pmatrix} - \begin{pmatrix} W_6^0 & 0 & 0 \\ 0 & W_6^1 & 0 \\ 0 & 0 & W_6^2 \end{pmatrix} \begin{pmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \end{pmatrix} \tag{2.9}$$

9. For

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 2 \\ 1 \end{pmatrix} \tag{2.10}$$

compte the DFT using (2.5)

10. Repeat the above exercise using (2.9)