15.11 Find the trigonometric Fourier series coefficients for the waveform in Fig. P15.11.

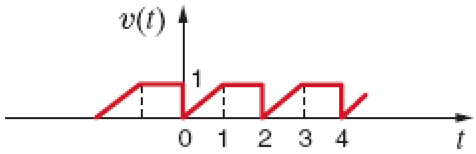

Figure P15.11

SOLUTION:

$$a_0 = \frac{1}{T_0}\int_0^{T_0} F(t)\,dt$$

$$a_0 = \frac{1}{2}\left[\frac{1}{2}(1)(1) + 1(1)\right]$$

$$a_0 = \frac{1}{2}\left[\frac{1}{2}+1\right] = \frac{1}{2}\left[\frac{1}{2}+\frac{2}{2}\right]$$

$$a_0 = \frac{3}{4}$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} F(t)\cos n\omega_0 t\,dt$$

$$a_n = \frac{2}{T_0}\int_0^{1} t\cos n\omega_0 t\,dt + \frac{2}{T_0}\int_1^{2}\cos n\omega_0 t\,dt$$

$$a_n = \frac{2}{T_0}\left[\frac{1}{(n\omega_0)^2}\cos n\omega_0 t + \frac{t}{n\omega_0}\sin n\omega_0 t\right]_0^1 + \frac{2}{T_0}\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_1^2$$

$$T_0 = 2s \qquad \omega_0 = \pi \text{ rad/s}$$

$$a_n = \left[\frac{1}{n^2\omega_0^2}\cos n\pi + \frac{1}{n\omega_0}\sin n\pi - \frac{1}{(n\omega_0)^2}\right] + \frac{2}{T_0}\left[\frac{\sin 2n\pi}{n\omega_0} - \frac{\sin n\pi}{n\omega_0}\right]$$

$$a_n = \frac{1}{n^2\omega_0^2}\cos n\pi - \frac{1}{n^2\omega_0^2}$$

$$a_n = \frac{1}{n^2\pi^2}\left[\cos n\pi - 1\right]$$

$$a_n = \frac{\cos n\pi - 1}{n^2\pi^2}$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} F(t)\sin n\omega_0 t\,dt$$

$T_0 = 2S$

$$b_n = \int_0^1 t\sin n\omega_0 t\,dt + \int_1^2 \sin n\omega_0 t\,dt$$

$$b_n = \left[\frac{1}{(n\omega_0)^2}\sin n\omega_0 t - \frac{t}{n\omega_0}\cos n\omega_0 t\right]_0^1 - \left[\frac{\cos n\omega_0 t}{n\omega_0}\right]_1^2$$

$$b_n = \left[\frac{1}{n^2\omega_0^2}\sin n\pi - \frac{1}{n\omega_0}\cos n\pi\right] - \frac{\cos 2n\pi}{n\omega_0} + \frac{\cos n\pi}{n\omega_0}$$

$$b_n = \frac{-1}{n\pi}\cos n\pi - \frac{1}{n\pi}\cos 2n\pi + \frac{1}{n\pi}\cos n\pi$$

$$b_n = \frac{-\cos 2n\pi}{n\pi}$$

$$b_n = \frac{-1}{n\pi}$$

$$a_0 = 3/4$$

$$a_n = \frac{\cos n\pi - 1}{n^2\pi^2}$$

$$b_n = \frac{-1}{n\pi}$$