15.44 The input signal for the network in Fig. P15.44 is $v_i(t) = 10e^{-5t}u(t)$ V. Determine the total 1-Ω energy content of the output $v_o(t)$.

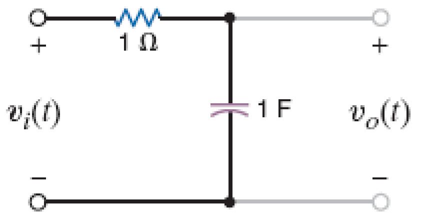

Figure P15.44

SOLUTION:

$$V_i(t) = 10e^{-5t}u(t)\ \text{V}$$

$$V_i(\omega) = \frac{10}{5+j\omega}$$

$$H(\omega) = \frac{\frac{1}{j\omega}}{1+\frac{1}{j\omega}}$$

$$H(\omega) = \frac{1}{1+j\omega}$$

$$V_o(\omega) = V_i(\omega)\,H(\omega)$$

$$V_o(\omega) = \left(\frac{10}{5+j\omega}\right)\left(\frac{1}{1+j\omega}\right)$$

$$V_o(\omega) = \frac{10}{(5+j\omega)(1+j\omega)}$$

$$|V_o(\omega)|^2 = \frac{100}{(25+\omega^2)(1+\omega^2)}$$

$$|V_o(\omega)|^2 = \frac{100}{24}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$|V_o(\omega)|^2 = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{1}{2\pi}\int_{-\infty}^{\infty} |V_o(\omega)|^2 d\omega$$

$$W = \frac{25}{12\pi}\int_{-\infty}^{\infty} \frac{d\omega}{1+\omega^2} - \frac{25}{12\pi}\int_{-\infty}^{\infty} \frac{d\omega}{25+\omega^2}$$

$$W = \frac{25}{12\pi}\left[\tan^{-1}\omega\right]_{-\infty}^{\infty} - \frac{25}{12\pi}\left[\frac{1}{5}\tan^{-1}\left(\frac{\omega}{5}\right)\right]_{-\infty}^{\infty}$$

$$W = \frac{25}{12\pi}\left[\pi - \pi/5\right]$$

$$W = \frac{25}{12\pi}\left[\frac{5\pi - \pi}{5}\right]$$

$$W = \frac{5}{3} J$$